Numerical Study of Regge Cuts due to Absorption in Neutral-Pion Photoproduction*

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The relationship between Regge cuts and absorption is reviewed. A detailed numerical study is carried through for the Gottfried-Jackson absorption model as applied to the reaction $\gamma p \rightarrow \pi^0 p$ with Reggeized ω exchange. The absorption parameters are set from πp elastic scattering data. Calculations of s- and t-channel amplitudes and cut discontinuities are presented and discussed as regards their advantages and shortcomings for fitting the neutral-pion photoproduction data. Several published fits are compared. We conclude that the cut contribution must be roughly doubled to obtain a successful fit.

I. INTRODUCTION

It has been known for some time that double Reggeon exchange or an absorptive correction to single Reggeon exchange leads to Regge cuts. References 1–6 list some of the papers which discuss the relationship between cuts and absorption. On the other hand, it has not always been clear that the cuts given by the Reggeized absorption model are the same as the cuts necessary to fit the data, since sometimes the fit to the data has involved varying the parameters in a somewhat arbitrarily parametrized cut.[7–9] In the following, we carry through a detailed numerical study of this question for a particular reaction $\gamma p \rightarrow \pi^0 p$ with Reggeized ω exchange.

In Sec. II we derive the cut amplitude and two forms of the absorptive amplitude: the Gottfried-Jackson form and the intermediate angle form used by Henyey, Kane, Pumplin, and Ross.[10] In Sec. III we report numerical calculations of the discontinuities and the scattering amplitudes in the s and t channels. We close in Sec. IV by comparing several absorptive fits with each other and with the calculation presented here and conclude that the absorptive-cut contribution must be roughly doubled to obtain a successful fit.

II. MULTIPLE SCATTERING, ABSORPTION, AND REGGE CUTS

The results of this section are well known to experts. For pedagogical reasons and for the sake of a consistent notation we collect all the relevant formulas and indicate their derivation.

Consider the two-stage process (S'') involving a reaction (S) with final-state absorption (S') illustrated in Fig. 1. We will retain generality for as long as it is convenient, but for definiteness think of S as $\gamma p \rightarrow \pi^0 p$ by ω exchange, S' as $\pi^0 p$ elastic scattering, and S'' as the combined process. The Greek letters α, β, and γ label two-particle states; the letters a–f label particles and their helicities; θ_1, θ_2, and Θ label s-channel c.m. scattering angles corresponding to squared four-momentum transfers t_1, t_2, and t, respectively.

We start by assuming

$$S_{\gamma\alpha}'' = \sum_{\beta} S_{\gamma\beta}' S_{\beta\alpha}, \tag{1}$$

where $S_{\beta\alpha}$, etc., are elements of the appropriate S matrices. This is the basic assumption of absorption models of the type discussed by Gottfried and Jackson.[11] It cannot be derived in any simple way. In a complete dynamical model, the sum over intermediate states in (1) would be more complicated and would involve off-

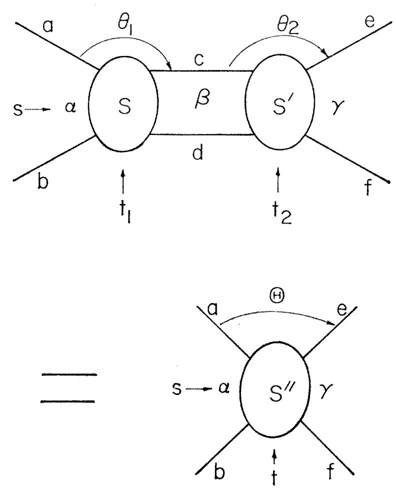

FIG. 1. Illustration of nomenclature.

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† NSF Predoctoral Fellow.

[1] R. C. Arnold, Phys. Rev. **140**, B1022 (1965); **153**, 1523 (1967).

[2] L. Van Hove, Phys. Letters **24B**, 183 (1967).

[3] E. J. Squires, Phys. Letters **26B**, 461 (1968); **26B**, 736(E) (1968).

[4] J. Finkelstein and M. Jacob, Nuovo Cimento **56A**, 681 (1968).

[5] C. B. Chiu and J. Finkelstein, Nuovo Cimento **48A**, 820 (1967).

[6] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Nuovo Cimento **48A**, 1075 (1967).

[7] A. Capella and J. Tran Thanh Van, Nuovo Cimento Letters **1**, 321 (1969).

[8] J. Frøyland, Nucl. Phys. **B11**, 204 (1969).

[9] A. Borgese and M. Colocci, CERN Report No. Th. 1024, 1969 (unpublished).

[10] F. Henyey, G. L. Kane, Jon Pumplin, and M. H. Ross, Phys. Rev. **182**, 1579 (1969).

[11] J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965); or K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).

shell contributions and multiparticle states. In the following, we keep only the on-shell contributions from elastic intermediate states.

Introducing invariant matrix elements M defined by

$$S_{\beta\alpha}=\delta_{\beta\alpha}+i(2\pi)^4\delta^4(P_\beta-P_\alpha)\frac{M_{\beta\alpha}}{(2E_a2E_b2E_c2E_d)^{1/2}}, \quad (2)$$

the δ functions in (2) cause the sum over intermediate states in (1) to reduce to an integral over the scattering angle and a sum over intermediate helicities:

$$M_{\gamma\alpha}''=(M_{\gamma\alpha}'+M_{\gamma\alpha})+\frac{ik_c}{16\pi^2\sqrt{s}}\sum_\beta\int d\Omega_c\, M_{\gamma\beta}'M_{\beta\alpha}. \quad (3)$$

From here on, we will assume that s is much larger than any of the relevant masses. We can write a Jacob-Wick[12] partial-wave expansion of the first scattering matrix:

$$M_{\beta\alpha}=\sum_{jm}(j+\tfrac{1}{2})M_{cdab}{}^jD_{m,c-d}{}^{j*}(\varphi_c,\theta_c,-\varphi_c)$$
$$\times D_{m,a-b}{}^j(\varphi_a,\theta_a,-\varphi_a) \quad (4a)$$
$$=4(\sqrt{\pi})se^{i\varphi_1(a-b-c+d)}f_{cdab}(\theta_1), \quad (4b)$$
$$4(\sqrt{\pi})sf_{cdab}(\theta_1)=\sum_j(j+\tfrac{1}{2})M_{cdab}{}^jd_{a-b,c-d}{}^j(\theta_1). \quad (4c)$$

We have taken $\theta_a=0$, $\varphi_a=0$, $\theta_c=\theta_1$, and $\varphi_c=\varphi_1$ and have used $D_{m\lambda}{}^j(0,0,0)=\delta_{m\lambda}$. The amplitudes which we will use are normalized so that $d\sigma/dt=\sum_{abcd}|f_{cdab}|^2$. Similarly,

$$M_{\gamma\beta}'=\sum_{jm}(j+\tfrac{1}{2})M_{efcd}'{}^jD_{m,e-f}{}^{j*}(\varphi_e,\theta_e,-\varphi_e)$$
$$\times D_{m,c-d}{}^j(\varphi_c,\theta_c,-\varphi_c) \quad (5a)$$
$$=4(\sqrt{\pi})se^{i\varphi_2(c-d)}e^{-i\varphi_l(e-f)}g_{efcd}(\theta_2), \quad (5b)$$
$$4(\sqrt{\pi})sg_{efcd}(\theta_2)=\sum_j(j+\tfrac{1}{2})M_{efcd}'{}^jd_{c-d,e-f}{}^j(\theta_2). \quad (5c)$$

To obtain this expression, we took $\theta_c=\theta_1$, $\varphi_c=\varphi_1$, $\theta_e=\Theta$, $\varphi_e=0$, and used the fact that the D's are representations of the rotation group to evaluate the sum over m:

$$\sum_m D_{m,e-f}{}^{j*}(0,\Theta,0)D_{m,c-d}{}^j(\varphi_1,\theta_1,-\varphi_1)$$
$$=D_{c-d,e-f}{}^{j*}(\varphi_2,\theta_2,-\varphi_l). \quad (6)$$

Equation (6) defines φ_2, θ_2, and φ_l, and any explicit representation (e.g., $j=1$) can be used to get the relations for these angles. Thus,

$$\cos\theta_2=\cos\theta_1\cos\Theta+\sin\theta_1\sin\Theta\cos\varphi_1,$$
$$\sin(\varphi_2-\varphi_1)=-\sin\Theta\sin\varphi_1/\sin\theta_2,$$
$$\cos(\varphi_2-\varphi_1)=-(\sin\theta_1\cos\Theta-\cos\theta_1\sin\Theta\cos\varphi_1)/\sin\theta_2, \quad (7)$$
$$\sin\varphi_l=-\sin\theta_1\sin\varphi_1/\sin\theta_2,$$
$$\cos\varphi_l=(\cos\theta_1\sin\Theta-\sin\theta_1\cos\Theta\cos\varphi_1)/\sin\theta_2.$$

[12] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959), Eqs. (30) and (31).

All the angles mentioned above can be visualized and (7) can be checked from spherical trigonometry, but it requires some patience.

Substituting (4b) and (5b) into (3) and letting $h_{efab}(\Theta)=M_{\gamma\alpha}''/4(\sqrt{\pi})s$ be the total scattering amplitude, having the same normalization as f and g, we obtain our basic formula

$$h_{efab}(\Theta)=f_{efab}(\Theta)+\frac{is}{8\pi\sqrt{\pi}}\sum_{cd}\int d\Omega_1\,f_{cdab}(\theta_1)g_{efcd}(\theta_2)$$
$$\times e^{i\varphi_1(a-b)}e^{i(\varphi_2-\varphi_1)(c-d)}e^{-i\varphi_l(e-f)}. \quad (8)$$

We have used the fact that $M_{\gamma\alpha}'=0$ because M' is π^0p elastic scattering and cannot account for $\gamma p\to\pi^0p$. This formula is used by Henyey, Kane, Pumplin, and Ross,[10] although their amplitudes are defined differently.

We can get the Gottfried-Jackson partial-wave expression if we do the angular integration first after substituting (4a) and (5a) in (3):

$$h_{efab}(\Theta)=f_{efab}(\Theta)+\frac{is}{4\sqrt{\pi}}\sum_{cd}\sum_j(j+\tfrac{1}{2})$$
$$\times g_{efcd}{}^jf_{cdab}{}^jd_{a-b,e-f}{}^j(\Theta), \quad (9)$$

where $f_{cdab}{}^j=M_{cdab}{}^j/4(\sqrt{\pi})s$, and similarly for g^j.

To make this look more familiar we note that πp elastic scattering is given experimentally[13–15] to be

$$g_{0\frac{1}{2}0\frac{1}{2}}(t)=g_{0-\frac{1}{2}0-\frac{1}{2}}(t)=i\sigma_{\text{tot}}/(4\sqrt{\pi})e^{at/2}, \quad (10)$$

where σ_{tot} is the total π^0p cross section and $a\approx8$ GeV^{-2}. We have used the optical theorem $\text{Im}g(0)=\sigma_{\text{tot}}/4\sqrt{\pi}$. We also assume the spin-flip amplitudes are zero, which is only approximate. Making use of the impact parameter approximation $d_{\lambda\mu}{}^j(\theta)\approx J_{\mu-\lambda}[b(-t)^{1/2}]$ for small θ, where $j=qb$, and J is a Bessel function, we can perform the partial-wave expansion to find

$$g^j\approx[i\sigma_{\text{tot}}/(\sqrt{\pi})as]e^{-2j^2/as}.$$

Substituting this into (9) gives

$$h_{efab}(\Theta)=\sum_j(j+\tfrac{1}{2})\left(1-\frac{\sigma_{\text{tot}}}{4\pi a}e^{-2j^2/as}\right)$$
$$\times f_{efab}{}^jd_{a-b,e-f}{}^j(\Theta). \quad (11)$$

To obtain the Regge-cut expression for the double scattering process we have been considering, assume that $f(t_1)$ and $g(t_2)$ have the form

$$f_{cdab}(t_1)=F_{cdab}(t_1)e^{a_1t_1}s^{\alpha_1(t_1)-1},$$
$$g_{efcd}(t_2)=G_{efcd}(t_2)e^{a_2t_2}s^{\alpha_2(t_2)-1}, \quad (12)$$

[13] Stephen Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966), pp. 473 and 481.

[14] L. Van Hove, Rapporteur talk in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, Calif., 1967* (California U.P., Berkeley, 1967), p. 253.

[15] M. Borghini, C. Coignet, L. Dick, L. di Lella, A. Michalowicz, P. D. Macq, and J. C. Olivier, Phys. Letters **21**, 114 (1966).

where

$t_1=\frac{1}{2}(\cos\theta_1-1)s, \quad t_2=\frac{1}{2}(\cos\theta_2-1)s, \quad t=\frac{1}{2}(\cos\Theta-1)s.$

Substituting these forms in (8) leads to

$$h_{efab}(\Theta)=f_{efab}(\Theta)+\int_{-\infty}^{\alpha_c} d\beta\, s^{\beta-1}K_{efab}(\beta,\Theta), \quad (13)$$

where $K(\beta,\Theta)$ is essentially the discontinuity across the cut. It is given by

$$K_{efab}(\beta,\Theta)=\frac{is}{8\pi\sqrt{\pi}}\sum_{cd}\int d\cos\theta_1 d\varphi_1\, F_{cdab}(t_1)G_{efcd}(t_2) \times e^{a_1t_1+a_2t_2}e^{i\varphi_1(a-b)}e^{i(\varphi_2-\varphi_1)(c-d)}e^{-i\varphi l(e-f)} \times\delta(\beta-\alpha_1(t_1)-\alpha_2(t_2)+1). \quad (14)$$

Amati, Fubini, and Stanghellini[16] were the first to obtain this type of expression for a Regge cut.[17] The branch point α_c is the maximum value of $\alpha_1(t_1)+\alpha_2(t_2)-1$ subject to the restriction that the intermediate scattering angles add to give the total scattering angle. For linear trajectories,

$$\alpha_c(t)=\alpha_1(0)+\alpha_2(0)-1+[\alpha_1'\alpha_2'/(\alpha_1'+\alpha_2')]t.$$

Notice that if K is independent of β, (13) takes the traditional form for the cut amplitude:

$$h_{efab}(t)=f_{efab}(t)+K_{efab}(t)s^{\alpha_c-1}/\ln s. \quad (15)$$

In general, however, K can be a rapidly varying function of β. We can do the integral for K analytically anyway if we restrict ourselves to the extreme forward direction, $\Theta=0$, where $\theta_1=\theta_2$. For linear trajectories,

$$K_{efab}(\beta,0)=\frac{i}{(2\sqrt{\pi})(\alpha_1'+\alpha_2')}\sum_{cd}F_{cdab}(t_\beta)G_{efcd}(t_\beta) \times e^{(a_1+a_2)t_\beta}(-1)^{c-d-e+f}\delta_{a-b,e-f}, \quad (16)$$

where $t_\beta=(\beta-\alpha_c)/(\alpha_1'+\alpha_2')$. If, for example, we take $F_{0\frac{1}{2}-1-\frac{1}{2}}(t_\beta)=t_\beta F'$, $G_{0\frac{1}{2}0\frac{1}{2}}(t)=i\sigma_{\rm tot}/4\sqrt{\pi}$, and $G_{0\frac{1}{2}0-\frac{1}{2}}=0$, we can put this expression in (13), giving

$$h_{0\frac{1}{2}-1-\frac{1}{2}}(0)=\frac{\sigma_{\rm tot}F'}{8\pi}\frac{s^{\alpha_c-1}}{[(\alpha_1'+\alpha_2')\ln s+a_1+a_2]^2}. \quad (17)$$

This is exact only if F' is a constant. It should, however, be a reasonable approximation even if F' has a slow t dependence, such as the signature factor, because the $\exp[(a_1+a_2)t_\beta]$ term will kill off the integrand a short distance from $t_\beta=0$.

Away from the forward direction, the following expression is useful for calculations and for understanding qualitative features of the discontinuity:

$$K_{efab}(\beta,\Theta)=\frac{i}{4\pi(\sqrt{\pi})\alpha_i'}\sum_{cd}F_{cdab}(t_\beta)e^{a_1t_\beta} \times\int_0^{2\pi}d\varphi_1\, G_{efcd}(t_2)e^{a_2t_2}e^{i\varphi_1(a-b)} \times e^{i(\varphi_2-\varphi_1)(c-d)}e^{-i\varphi l(e-f)}. \quad (18)$$

This expression was obtained from (14) by taking $\alpha_2'=0$ (flat Pomeranchukon) and using the δ function to do the $\cos\theta_1$ integration.

III. NUMERICAL CALCULATIONS

We will use the formalism just presented to make detailed numerical calculations of the scattering amplitudes and Regge-cut discontinuities for the reaction $\gamma p\to\pi^0 p$. First we must choose the inputs. We will use the s- and t-channel scattering amplitudes in terms of invariant amplitudes[18]:

$$\begin{aligned} &f_{0\frac{1}{2}1-\frac{1}{2}}{}^s=-tA_2, \quad f_{0\frac{1}{2}-1-\frac{1}{2}}{}^s=2A_1+tA_2,\\ &f_{0\frac{1}{2}1\frac{1}{2}}{}^s=(-t)^{1/2}(A_3+A_4),\\ &f_{0\frac{1}{2}-1\frac{1}{2}}{}^s=(-t)^{1/2}(-A_3+A_4), \end{aligned} \quad (19)$$

and

$$\begin{aligned} &\bar{f}_{01}{}^{t+}=A_1-2mA_4=-4(t-\mu^2)^{-1}\bar{f}_{01}{}^{t+},\\ &\bar{f}_{01}{}^{t-}=A_1+tA_2=4t^{1/2}(t-\mu^2)^{-1}(t-4m^2)^{-1/2}\bar{f}_{01}{}^{t-},\\ &\bar{f}_{11}{}^{t+}=2mA_1-tA_4=4t^{1/2}(t-\mu^2)^{-1}\bar{f}_{11}{}^{t+},\\ &\bar{f}_{11}{}^{t-}=A_3=4(t-\mu^2)^{-1}(t-4m^2)^{-1/2}\bar{f}_{11}{}^{t-}. \end{aligned} \quad (20)$$

The context of these amplitudes is given by

$$\begin{aligned} \frac{d\sigma}{dt}&=\sum_{abcd}|f_{abcd}{}^s|^2\\ &=(8/s^2)[(|\bar{f}_{01}{}^{t+}+\bar{f}_{01}{}^{t-}|^2+|\bar{f}_{01}{}^{t+}-f_{01}{}^{t-}|^2)|\sin\theta_t|^2\\ &\quad+(1+\cos\theta_t)^2|\bar{f}_{11}{}^{t+}+\bar{f}_{11}{}^{t-}|^2\\ &\quad+(1-\cos\theta_t)^2|\bar{f}_{11}{}^{t+}-\bar{f}_{11}{}^{t-}|^2]. \quad (21) \end{aligned}$$

If we assume only natural-parity ω exchange in the t channel, then $A_3=0$, $A_1=-tA_2$, and

$$f_{0\frac{1}{2}1-\frac{1}{2}}{}^s=f_{0\frac{1}{2}-1-\frac{1}{2}}{}^s, \quad f_{0\frac{1}{2}1\frac{1}{2}}{}^s=f_{0\frac{1}{2}-1\frac{1}{2}}{}^s.$$

We give these two amplitudes a standard Regge parametrization[19]:

$$\begin{aligned} &f_{0\frac{1}{2}1-\frac{1}{2}}{}^s(s,t)=-\gamma_2 t\xi^\omega, \quad f_{0\frac{1}{2}1\frac{1}{2}}{}^s(s,t)=\gamma_4(-t)^{1/2}\xi^\omega,\\ &\xi^\omega=\Gamma(1-\alpha_\omega(t))[e^{-i\pi\alpha_\omega(t)}-1]s^{\alpha_\omega(t)-1}. \end{aligned} \quad (22)$$

[16] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento **26**, 896 (1962).

[17] The Amati-Fubini-Stanghellini expression was in terms of t_1 and t_2 according to

$$\int d\Omega_1\to\int dt_1dt_2\, 2\theta(H)/(H^{1/2}s),$$

where $H=-\frac{1}{4}(t_1^2+t_2^2+t^2-2t_1t_2-2t_1t-2t_2t)$. We find the angular variables more convenient.

[18] J. S. Ball, Phys. Rev. **124**, 2014 (1961); G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, *ibid.* **106**, 1345 (1957).

[19] The input Regge exchange has linear zeros at nonsense wrong-signature points in all s- and t-channel amplitudes. In terms of the standard nomenclature, we are using the nonsense-choosing or Gell-Mann mechanism. See C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. **161**, 1563 (1967), for a catalog of ways of assigning α factors.

These are s-channel amplitudes, but the crossing relations (23) given below will show that the resultant t-channel parametrization is reasonable. They include a nonsense wrong-signature zero (NWSZ). We take the trajectory to be $\alpha_\omega(t)=0.4+t$. Our best fit to the data,

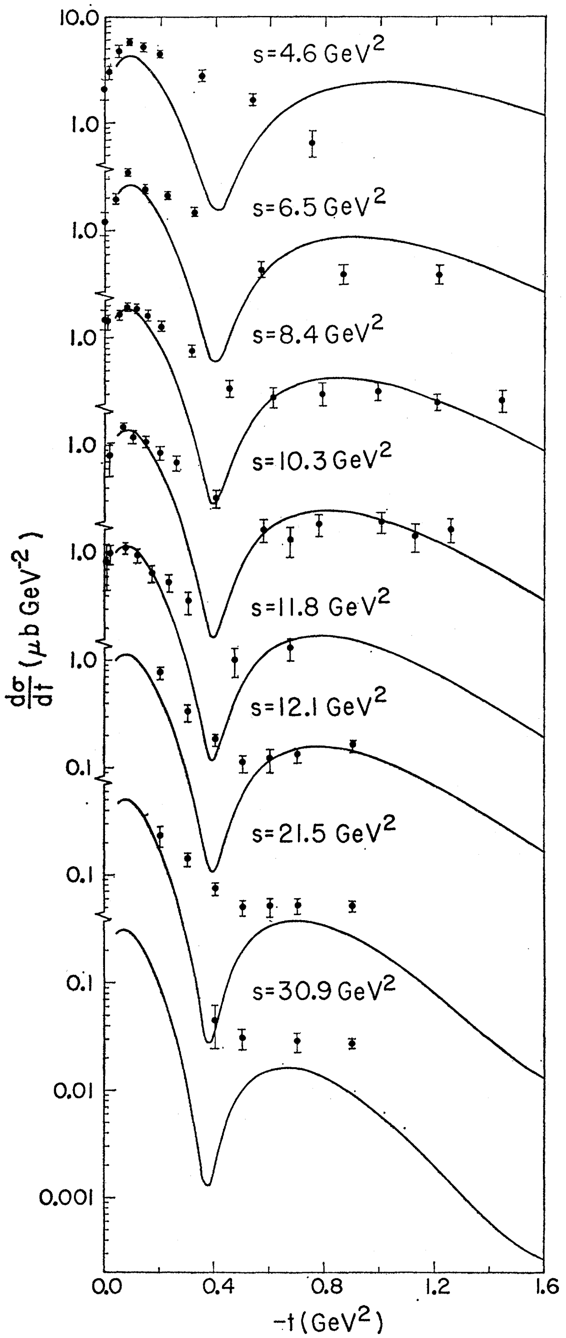

FIG. 2. Comparison of the absorbed Regge model with the data for $\gamma p\to\pi^0 p$ from M. Braunschweig, W. Braunschweig, D. Husmann, K. Lübelsmeyer, and D. Schmitz, Phys. Letters **26B**, 405 (1968) and R. Anderson, D. Gustavson, J. Johnson, D. Ritson, W. G. Jones, D. Kreinick, F. Murphy, and R. Weinstein, Phys. Rev. Letters **21**, 384 (1968). The only free parameters are the two coefficients of the Regge amplitudes, $\gamma_2\approx 0$ and $\gamma_4\approx 10$.

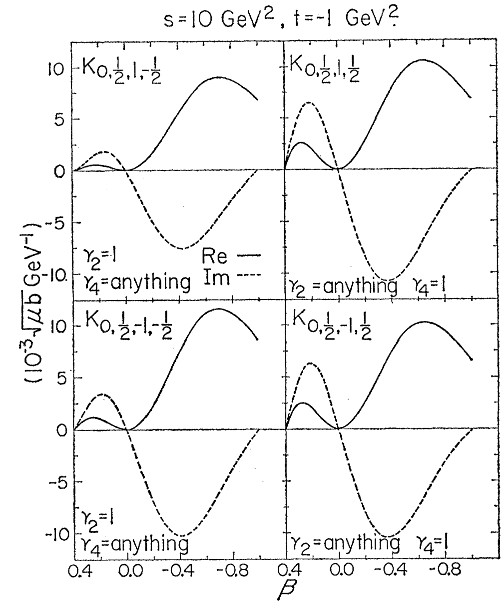

FIG. 3. Discontinuities of the Regge cut generated by absorbed Regge-pole exchange. A value of "anything" for a Regge coefficient γ_i signifies that it does not affect the associated discontinuity.

which is not very good and is shown in Fig. 2, had $\gamma_2=0$, $\gamma_4\approx 10$. Since these numbers are probably not very fundamental, we will present the calculations so that the effect of any parameter choice can be inferred. We do this by calculating everything for both $\gamma_2=0$, $\gamma_4=1$ and $\gamma_2=1$, $\gamma_4=0$. An amplitude for general γ_2 and γ_4 is found by taking a linear combination of the two cases. On the graphs, "γ_4= anything" signifies that the quantity plotted is proportional to γ_2 only, and similarly for "γ_2= anything."

We use for the elastic amplitude the parameter values[13–15]

$$G_{0\frac{1}{2}0\frac{1}{2}}=i\sigma_{\rm tot}/4\sqrt{\pi}\,,\quad G_{0\frac{1}{2}0-\frac{1}{2}}=0\,,$$

$$\sigma_{\rm tot}=24\text{ mb}=62\text{ GeV}^{-2}\,,\quad a_2=4\text{ GeV}^{-2}\,,\quad \alpha_2(t)=1\,,$$

where the notation is defined in (12). From πp polarization data,[15] we know that the spin-flip amplitude is not zero; we estimate it to be about one-tenth the nonflip amplitude, so we ignore it.

The integration for $K_{efab}(\beta,t)$ was done numerically using the inputs discussed. A sample of the results is shown in Fig. 3. Notice that the discontinuity always vanishes at the branch point $\beta=0.4$ and at $\beta=0$. This can be seen from (18) with the help of (12) and (22). At $\beta=0.4$, $t_\beta=0$ and the evasive factors in (22) make $F_{cdab}(t_\beta)$ vanish. At $\beta=0$, $\alpha_\omega(t_\beta)=0$ and $F_{cdab}(t_\beta)$ has a NWSZ. Bronzan and Jones[20] predict that the discontinuity will always vanish at the branch point due to elastic unitarity. The discontinuities $K_{0\frac{1}{2}1\frac{1}{2}}$ and $K_{0\frac{1}{2}-1\frac{1}{2}}$ are equal to within a few percent. This is a numerical feature and apparently not of qualitative importance. Specialists in analytic properties will notice that the

[20] J. B. Bronzan and C. E. Jones, Phys. Rev. **160**, 1494 (1967).

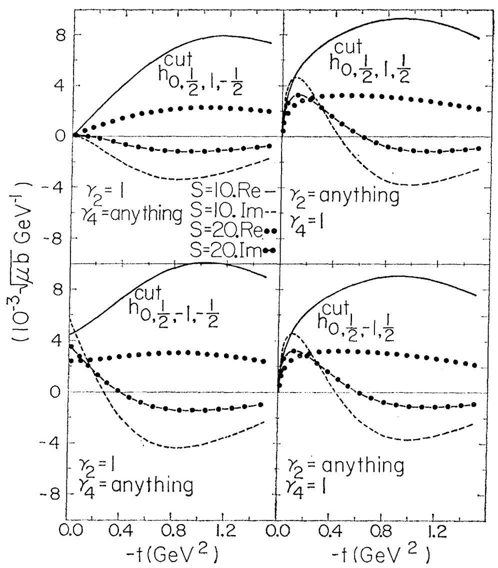

FIG. 4. Contributions of the cut to the s-channel scattering amplitudes. The s dependence is not a simple scaling. The cut contributions tend to cancel the Regge contributions in the forward direction. A value of "anything" for a Regge coefficient γ_i signifies that it does not affect the associated amplitude.

definition of K differs somewhat from the complex-variable definition of a cut discontinuity. The details can be seen by doing a Sommerfeld-Watson transformation with a cut included.

Once the discontinuity K is known, we could find the cut contribution to the scattering amplitude by evaluating the integral in (13). It is more direct and more convenient, however, to do the equivalent integration in (8). We will look only at the cut contribution, $h^{\text{cut}}=h-f$. The results are shown in Fig. 4. Remember that these are s-channel amplitudes. Curves are shown for both $s=10$ GeV^2 and $s=20$ GeV^2 to show that the s dependence is not a simple scaling. For comparison, the input Reggeon exchange amplitudes are shown in Fig. 5. Notice that the Regge amplitudes are drawn for $s=10$ GeV^2 and so should be compared with the corresponding cut contributions. The cut contributions tend

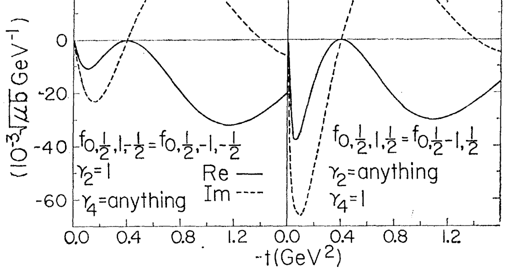

FIG. 5. Input s-channel Regge amplitudes.

to be smaller, but are not everywhere negligible, and of course they dominate around the zeros of the Regge amplitude. Except for the imaginary part of $h_{0\frac{1}{2}1-\frac{1}{2}}^{\text{cut}}$, the cut terms interfere destructively with the pole terms in the forward direction. Henyey, Kane, Pumplin, and Ross[10] agree with this relative sign. Finkelstein and Jacob[4] have observed that this sign depends on interpreting the process as absorption rather than rescattering.

The s dependence of corrections to ω exchange is an important feature of neutral-pion photoproduction. Since the dip fills in with increasing energy, the correction must have a slower s dependence than the Regge pole part. A common way of describing s dependence is in terms of an effective Regge trajectory α_{eff} defined by

$$d\sigma/dt\sim s^{2(\alpha_{\text{eff}}-1)}.$$

The dominant parts of the cut contributions presented here have an s dependence in the dip region which gives $\alpha_{\text{eff}}(-0.4)\approx-0.4$ compared with the phenomenological cut contribution of Capella and Tran Thanh Van,[7] which gives $\alpha_{\text{eff}}(-0.4)\approx 0$. [Notice that α_{eff} is not the same as α_c because of the $(\ln s)^{-1}$ factor.] The data imply an $\alpha_{\text{eff}}(-0.4)$ of as much as $+0.4$.

Because most fits are done in terms of t-channel amplitudes, we have plotted the t-channel cut contributions in Fig. 6. The crossing relations (23) were obtained by solving for A_1 through A_4 in (19) and substituting the result in (20):

$$\begin{aligned}
\tilde{f}_{01}^{t+}&=\tfrac{1}{2}(f_{0\frac{1}{2}-1-\frac{1}{2}}^{s}+f_{0\frac{1}{2}1-\frac{1}{2}}^{s})\\
&\qquad-m(-t)^{-1/2}(f_{0\frac{1}{2}1\frac{1}{2}}^{s}+f_{0\frac{1}{2}-1\frac{1}{2}}^{s}),\\
\tilde{f}_{01}^{t-}&=\tfrac{1}{2}(f_{0\frac{1}{2}-1-\frac{1}{2}}^{s}-f_{0\frac{1}{2}1-\frac{1}{2}}^{s}),\\
\tilde{f}_{11}^{t+}&=m(f_{0\frac{1}{2}-1-\frac{1}{2}}^{s}+f_{0\frac{1}{2}1-\frac{1}{2}}^{s})\\
&\qquad+\tfrac{1}{2}(-t)^{1/2}(f_{0\frac{1}{2}1\frac{1}{2}}^{s}+f_{0\frac{1}{2}-1\frac{1}{2}}^{s}),\\
\tilde{f}_{11}^{t-}&=[2(-t)^{1/2}]^{-1}(f_{0\frac{1}{2}1\frac{1}{2}}^{s}-f_{0\frac{1}{2}-1\frac{1}{2}}^{s}).
\end{aligned}\tag{23}$$

The numbers in Fig. 6 were obtained by substituting h_{efab}^{cut} for f_{efab}^{s} in (23). The unnatural-parity amplitudes were always zero or much smaller than the natural-parity amplitudes. As a result, the polarized photon production ratio was always $+1$, in contrast with the dip to 0.5 of the data.[21]

In Fig. 7 we have plotted the cut amplitudes used by Capella and Tran Thanh Van[7] in their fit of neutral-pion photoproduction. We have converted their amplitudes to our conventions. That their amplitudes are substantially different from the cut amplitudes generated by absorption could have been foreseen since we could not fit the reaction well using absorption, while they fit it very well with their phenomenological cut.

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[21] D. Bellenger, R. Bordelon, K. Cohen, S. B. Deutsch, W. Lobar, D. Luckey, L. S. Osborne, E. Pothier, and R. Schwitters, Phys. Rev. Letters 23, 540 (1969).

IV. COMPARISON WITH SOME PUBLISHED FITS—FACTOR OF 2

We have examined the workings of a simple absorptive model of neutral-pion photoproduction in some detail. By comparing this simple model with several published fits, we can pick out the most useful modifications. We will confine our attention to fits which produce cut effects from absorption, unlike Ref. 7, for instance, so that there will be a reasonable amount of common ground for comparison. We will make a case that one modification, the Michigan[10] λ or an equivalent enhancement of the cut contribution, is necessary in the types of fits that have been tried up until now.

Most successful fits which calculate the cut contribution from multiple scattering do, in fact, make use of some mechanism to enhance the absorptive cut effect. Contogouris, Lebrun, and Von Bochman,[22,23] and Colocci,[24] using an absorption prescription which differs from ours, need a "Pomeranchukon coupling constant" of about 8 (in their units) compared with a value of 4 implied by π-nucleon elastic scattering. Henyey, Kane, Pumplin, and Ross[10] have their λ factor equal to about 2 to account for the effect of inelastic intermediate states.

Blackmon, Kramer, and Schilling[25] increase the effect of the absorption in two ways. They increase their absorption parameter from $\sigma_{tot}/4\pi a \approx 0.6$ as in (11) to 0.9 to account for inelastic intermediate states. They also find they need a B contribution to get a quantitative fit. Note that their B with a small slope (0.4) and a large intercept (0.4) could simulate to some extent the s dependence of a cut.

Benfatto, Nicolo, and Rossi[26] have produced a fit using the multiple Pomeranchukon exchange model of Frautschi and Margolis.[27] They have an arbitrary cut multiplier for each of the three isospin combinations (+, −, 0). They have a cut multiplier of 1.54 for the dominant ω amplitude.

In Fig. 8 we have plotted the fit we obtain if we allow an arbitrary parameter multiplying the cut contribution. The cut multiplier comes out to be 2.4. In addition, the parameter a_1 in (12) was varied, resulting in a best value of 0.5 GeV^{-2}. χ^2 is 580 for 74 data, compared with 850 for the simple model, as in Fig. 2. (The effect of a_1 is not overwhelming. If it is made 0.0, χ^2 increases to 610.)

There is a further important feature of these fits which we have not yet mentioned. That is the presence or absence of a NWSZ. Our model has one as can be be checked from (22). Henyey *et al.*[10] and Benfatto *et al.*[26] leave out the NWSZ. If we remove our NWSZ

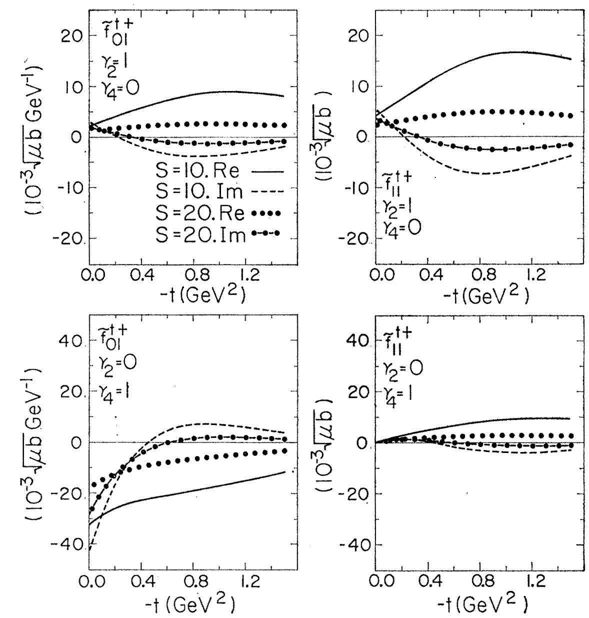

FIG. 6. Cut contributions to the t-channel amplitudes calculated from the s-channel amplitudes in Fig. 4. The amplitude for general γ_2 and γ_4 is a linear combination of the amplitude for $\gamma_2=1$, $\gamma_4=0$ and $\gamma_2=0$, $\gamma_4=1$. Unnatural parity amplitudes are small or zero resulting in a constant value of +1 for the polarization ratio of Ref. 21 in contrast to their data. Compare with Fig. 7.

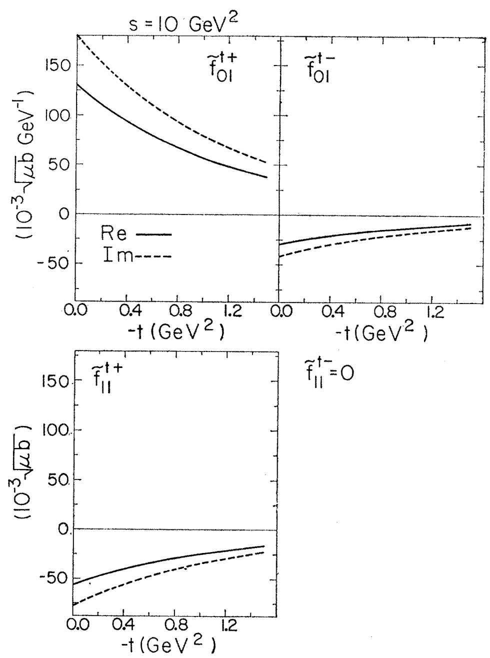

FIG. 7. Cut contributions to the t-channel amplitudes from the fit of Ref. 7. The normalizations of these amplitudes have been changed to our conventions.

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[22] A. P. Contogouris and J. P. Lebrun, Nuovo Cimento **64A**, 627 (1969).

[23] A. P. Contogouris, J. P. Lebrun, and G. Von Bochman, Nucl. Phys. **B13**, 246 (1969).

[24] M. Colocci, CERN Report No. TH. 1150, 1970 (unpublished).

[25] M. Blackmon, G. Kramer, and K. Schilling, Phys. Rev. **183**, 1452 (1969).

[26] G. Benfatto, F. Nicolo, and G. C. Rossi, Nuovo Cimento **64A**, 1033 (1969).

[27] S. Frautschi and B. Margolis, Nuovo Cimento **56A**, 1155 (1968); **57A**, 427 (1968).

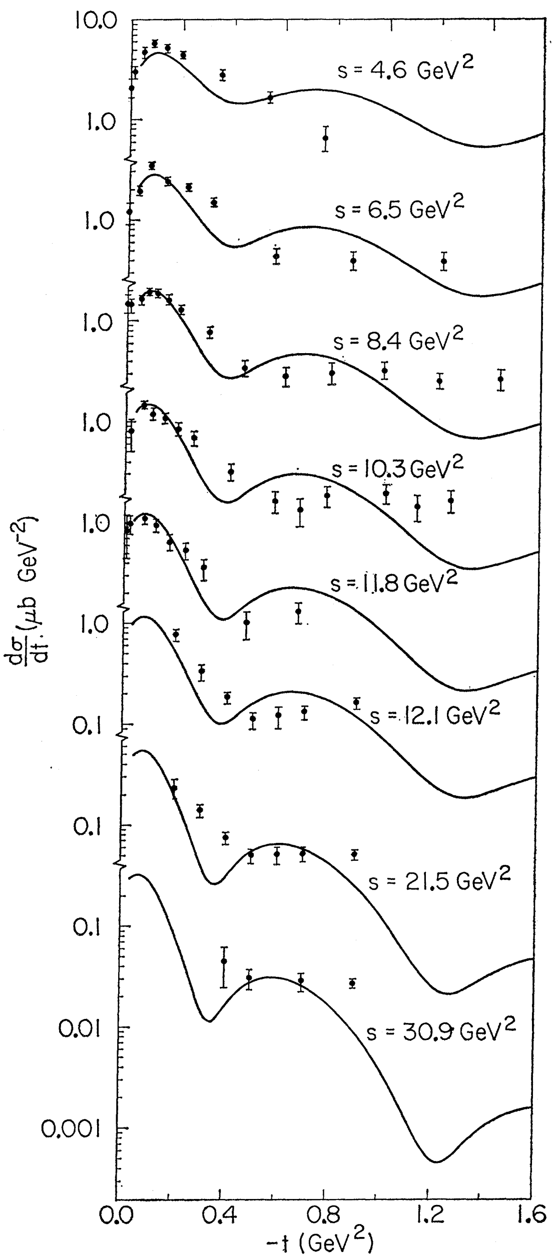

FIG. 8. A fit to the data where an arbitrary multiplier of the cut amplitude has been included as a free parameter. χ^2 is 580 for 74 points, the cut multiplier is 2.4, and the other parameters are $\gamma_2=16.5$, $\gamma_4=9.75$, $a_1=0.5$ GeV^{-2}.

by dividing by α in (22), we get the fit of Fig. 9, which has a cut multiplier of 2.5 and $a_1=0.0$ GeV^{-2}. χ^2 for Fig. 9 is 160, which is dramatically better than that for Fig. 2 or Fig. 8. Leaving out the NWSZ is attractive phenomenologically. However, as beneficial as the absence of NWSZ may be, the multiplication of the cut amplitude is still necessary. If we return the cut multiplier to a value of 1, our best no-NWSZ fit, shown in Fig. 10, is no better than the fit with NWSZ, χ^2 being about 800 in both cases.

We tried adding the complications of a fixed pole to the fits to see if we could remove the necessity of a cut multiplier. The effect of a fixed pole at nonsense wrong-signature values was discussed by Mandelstam and Wang.[28] The effect on the parametrization is to replace

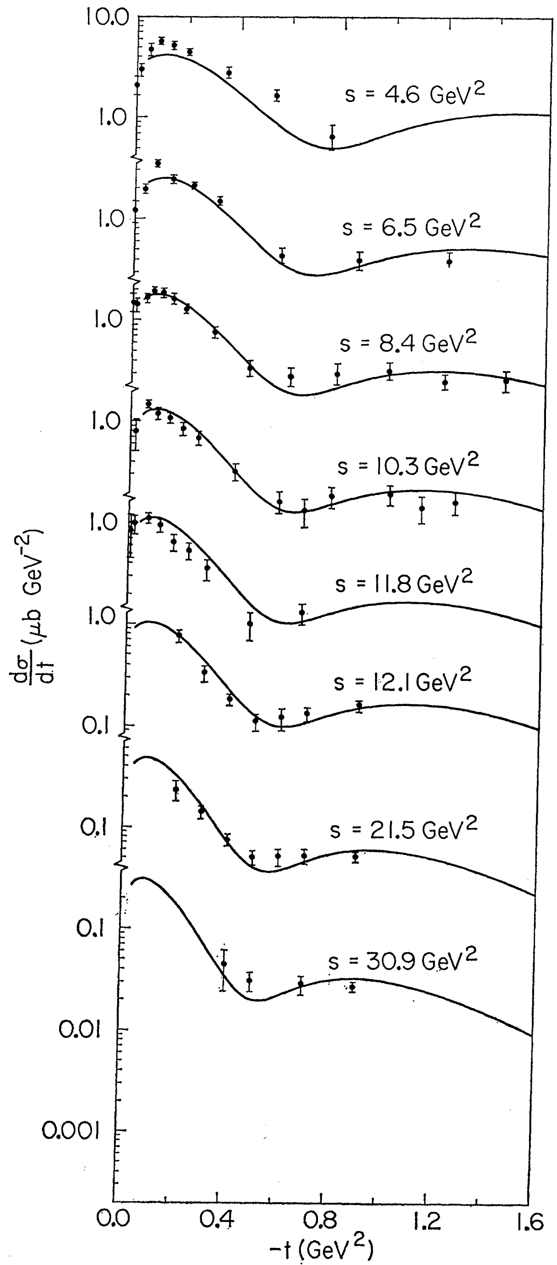

FIG. 9. A fit to the data with a variable-cut multiplier and no nonsense wrong-signature zero in the ω amplitude. χ^2 is 160 for 74 points, the cut multiplier is 2.5, and the remaining parameters are $\gamma_2=4.76$, $\gamma_4=3.89$, $a_1=0.0$ GeV^{-2}.

[28] S. Mandelstam and L. L. Wang, Phys. Rev. **160**, 1490 (1967).

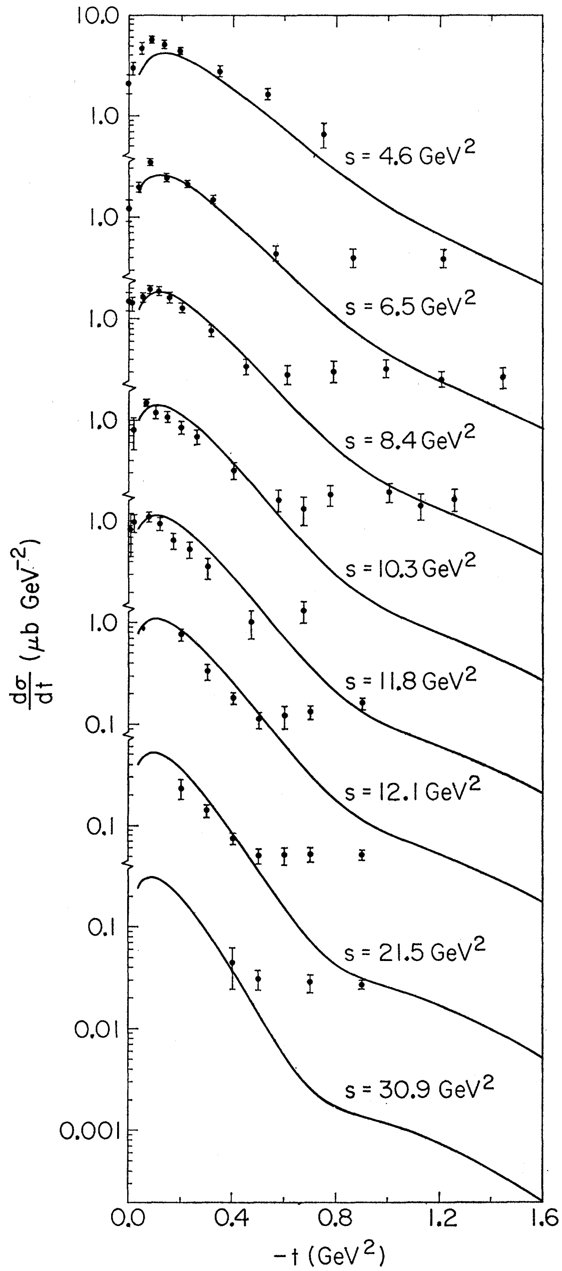

FIG. 10. A fit without a nonsense wrong-signature zero and with no arbitrary enhancement of the cut contribution, i.e., the cut multiplier is 1. χ^2 is 820 for 74 points and the remaining parameters are $\gamma_2=2.23$, $\gamma_4=3.16$, $a_1=1.0$ GeV^{-2}.

α by $\alpha+\epsilon$,[29] where ϵ need not be small. If we reparametrize this as $\alpha \sin\eta+\cos\eta$, we see that this modification can be thought of as mixing the NWSZ and no-NWSZ amplitudes. Since the flaw in NWSZ fits, like Fig. 2, is too much dip, and that of the no-NWSZ fits, like Fig. 10, is too little dip, there was some hope that a mixture would give the proper amount of dip without invoking a cut multiplier. This hope was not vindicated. We found no fit better than those shown in Fig. 2 or Fig. 10. If the cut multiplier was included as an additional free parameter, we obtained essentially the fit of Fig. 9 and nothing better.

It appears that a good fit requires roughly doubling the cut contribution over that calculated from the simple form of absorption presented here. In the absence of a NWSZ, this expedient gives reasonable fits. In the presence of NWSZ it helps, but further improvements are needed: Blackmon *et al.*[25] have a B contribution and Contogouris *et al.*[22,23] and Colocci[24] use a different absorption prescription and a different behavior for the Pomeranchukon contribution.

Note that the absorptive-cut correction in Fig. 4 is not small compared with the pole contribution in Fig. 5, and when the cut contribution is doubled it is about the same magnitude as the pole contribution. In this reaction at least, absorptive Regge fits are definitely not cases of corrections to a pure Regge model. The cut dominates over large regions in t and in the remaining regions it interferes strongly with the pole.

A frequently cited mechanism for the cut enhancement is the effect of inelastic and off-mass-shell intermediate states. Rivers[30] and Ebel and Moore[31] have found that these states can produce important corrections, but Ravenhall and Wyld,[32] using a simplified model, estimate that the effect is not large enough. Harrington[33] has done a quark-model calculation giving a t-dependent multiplier which varies between 1.0 to 1.4. A firm calculation of the cut multiplier would definitely be useful.

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[33] D. Harrington, Phys. Rev. D **1**, 2615 (1970).