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Some Applications of a Simple Approach to Unitarization in Hard-Meson Calculations*

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Several applications of a simple approach to unitarization in the case of the three-point functions of hard-meson theory are made. For the first of these, effective-range formulas are obtained for the pion form factor in the timelike region for both hard-pion current-algebra ($\delta=-\frac{1}{2}$) and Veneziano models. Next, the extension of this procedure to the case of several coupled channels is used to obtain an effective-range formula for the pion form factor which includes the small effect of the closed $K\bar{K}$ channel. Finally, an expression for the pion form factor is derived which allows for the inclusion of some nonresonant p-wave ($I=1$) $\pi\pi$ scattering. Since this component is parametrized via the introduction of an additional (nonresonant) p-wave scattering length, the resulting expression for the pion form factor generates a one-parameter family of fits to the experimental data. The data appear to be consistent with a small positive nonresonant scattering length and favor a value of Γ_ρ slightly less than 124 MeV.

I. INTRODUCTION

In a previous communication[1] dealing with an approach to unitarization in hard-meson calculations in the case of three-point functions, it was shown how effective-range formulas may be derived by the application of the principles of unitarity and meson dominance, assuming real tree form-factor inputs. In the case of the pion form factor, we[1] are led to the selfsame hard-pion effective-range formula derived earlier by Brehm, Golowich, and Prasad[2] from the specific hard-pion current-algebra method of Ward identities.[3] Important features of our derivation[1] are its independence of any particular current-algebra procedure which produces hard-meson results and its easy extension to other three-point problems, e.g., the π-A_1-ρ system. Such an extension, along with the appropriate application to a hitherto intractable problem, the calculation of the soft π^+-π^0 mass difference for subtracted tree form factors, was undertaken in our previous note.[1] In this paper we are primarily concerned with increasing the range of application of this procedure; as before, we find the pion form factor well suited for purposes of illustration. After some recapitulation below, we turn to a consideration of the perturbation produced by an additional coupled two-particle channel in our approach, specifically the effect of the $K\bar{K}$ channel in the case of the pion form factor, in Sec. II. The resulting expression for the pion form factor is still a "zero-parameter"[4] prediction in the sense of Ref. 1. On the other hand, in Sec. III, we show how the unitarity-preserving addition to the resonant p-wave (reducible) amplitude of an admissible small nonresonant scattering (irreducible) amplitude can generate a "one-parameter" family of pion form factors which allows for some additional freedom in fitting to experiment.

To make things simpler, we present first a capsule version of the derivation of our approach[1] [in the case of the pion form factor $F(t)$] to which we may later refer when discussing possible modifications. Thus for

$$\langle 0|V_\mu^{(3)}(0)|\pi^+(p)\pi^-(q)\ \text{in}\rangle=F(t)(-p+q)_\mu\,, \tag{1}$$

we have the usual two-particle unitarity relation in the $\pi\pi$ region,

$$\text{Abs}F(t)=F(t)(e^{i\delta_{11}}\sin\delta_{11})^*,\quad t\geq 4m_\pi^2 \tag{2}$$

with the partial-wave projection given by[5]

$$\frac{\sqrt{t}}{Q}e^{-i\delta_{11}}\sin\delta_{11}=-\int\frac{d\Omega}{32\pi^2}P_1\left(\frac{(\mathbf{p}'-\mathbf{q}')\cdot(\mathbf{p}-\mathbf{q})}{4Q^2}\right)$$
$$\times\langle\pi^+(p')\pi^-(q')\ \text{in}|J^{(1+i2)/\sqrt{2}}(0)|\pi^+(p)\rangle|_{(p'+q'=p+q)}. \tag{3}$$

The mutilation of the $\pi\pi$ amplitude,[6]

$$\langle\pi^+(p')\pi^-(q')\ \text{in}|J^{(1-i2)/\sqrt{2}}(0)|\pi^-(q)\rangle$$
$$\simeq -F^*(t)(-p'+q')_\nu g_\rho^{-2}[(-\Box+m_\rho^2)\times\langle 0|V_\nu^{(3)}(0)|\pi^+(p)\pi^-(q)\ \text{out}\rangle]_{\text{tree}}, \tag{4}$$

which is suggested by considerations[1] of reality, reducibility, and vector dominance, yields the "effective-bubble approximation" of Ref. 1,

$$\text{Abs}F(t)=|F(t)|^2\frac{Q^3}{\sqrt{t}}\frac{f(t)}{12\pi F_\pi^2 m_\rho^2}, \tag{5}$$

where

$$f(t)(-p+q)_\nu=[(m_\rho^2-t)\langle 0|V_\nu^{(3)}(0)|\pi^+(p)\pi^-(q)\ \text{out}\rangle]_{\text{tree}} \tag{6}$$

and

$$g_\rho^2=2F_\pi^2 m_\rho^2. \tag{7}$$

* Research supported in part by the National Science Foundation, under Grant No. GP-7082.

[1] R. Rockmore, Phys. Rev. Letters 24, 541 (1970).

[2] J. J. Brehm, E. Golowich, and S. C. Prasad, Phys. Rev. Letters 23, 666 (1969).

[3] H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

[4] There are no free parameters in the one-channel approximation if the parameter δ_π (which essentially characterizes subtractedness) has the value $\delta_\pi\simeq-\frac{1}{2}$ which fits both A_1 and ρ decay. (δ_π is the parameter δ of Refs. 1–3.) In the two-channel approximation of Sec. II, for want of a better *modus operandi* we again take $\delta_K\simeq-\frac{1}{2}$.

[5] As in Ref. 1, $Q(t)=(\frac{1}{4}t-m_\pi^2)^{1/2}$ with $t=-(p+q)^2$.

[6] We discuss the form-factor approximation in an off-shell context in the Appendix.

Hard-pion current algebra gives

$$f(t)=m_\rho^2\left(1-\frac{1+\delta_\pi}{4}\frac{t}{m_\rho^2}\right), \tag{8a}$$

while from considerations of the Veneziano model,[7]

$$f(t)=\frac{2\Gamma(\frac{3}{4})}{\sqrt{2}\pi b}\frac{\Gamma(\frac{3}{2}-\frac{1}{2}\alpha_t)}{\Gamma(5/4-\frac{1}{2}\alpha_t)}, \tag{8b}$$

with $b\simeq 1/(2m_\rho^2)$, $\alpha_t\simeq bt+\frac{1}{2}$.

In Ref. 1 the "solution" of the unitarity equation, Eq. (5), was argued from the standpoint of dispersion relations, but note that that solution is also obtained if one utilizes the device of dispersing only the phase-space factor $Q/\sqrt{t}$. Thus one introduces

$$h(t)-h(0)=-t\int_{4m_\pi^2}^{\infty}\frac{dt'}{\sqrt{t'}}\frac{Q(t')}{t'(t'-t)}, \tag{9}$$

with

$$\begin{aligned} h(t)&=\frac{2Q(t)}{\sqrt{t}}\ln\left(\frac{\sqrt{t}+2Q}{2m_\pi}\right)-\frac{i\pi Q(t)}{\sqrt{t}}, \quad t\geq 4m_\pi^2\\ &=\frac{2|Q(t)|}{\sqrt{|t|}}\ln\left(\frac{\sqrt{|t|}+2|Q|}{2m_\pi}\right), \quad t<0\\ &=\frac{2|Q(t)|}{\sqrt{t}}\tan^{-1}\left(\frac{\sqrt{t}}{2|Q|}\right), \quad 0\leq t\leq 4m_\pi^2 \end{aligned} \tag{10}$$

and

$$h(0)=1;$$

then the "class of solutions" of (5), of the form

$$[F(t)]^{-1}=1+t(\text{polynomial in } t)+\frac{f(t)Q^2[h(t)-h(0)]}{12\pi^2F_\pi^2m_\rho^2},$$

reduces on making the one-parameter linear (in t) approximation

$$t(\text{polynomial in } t)-\frac{f(t)Q^2h(0)}{12\pi^2F_\pi^2m_\rho^2} \simeq \frac{f(0)m_\pi^2h(0)}{12\pi^2F_\pi^2m_\rho^2}+\beta t \tag{11}$$

to the one-parameter solution of Ref. 1,

$$[F(t)]^{-1}=1+\beta t+\frac{f(t)Q^2h(t)}{12\pi^2F_\pi^2m_\rho^2}+\frac{f(0)m_\pi^2h(0)}{12\pi^2F_\pi^2m_\rho^2}. \tag{12}$$

In Fig. 1 we have plotted $|F|^2$ in the timelike region for both the hard-pion current-algebra model [Eq. (8a)] (solid line) and the Veneziano model [Eq. (8b)] (dashed line). Note that the peak in the latter model is only 5% lower than that predicted by hard-pion current algebra for the favored value[2] of $\delta_\pi=-\frac{1}{2}$, so that both models would appear to fit the data equally well. Finally, we remark that $\Gamma_\rho\simeq 116$ MeV in the Veneziano model as compared with the currently favored value $\Gamma_\rho\simeq 124$ MeV (for $\delta_\pi=-\frac{1}{2}$) in the former case.[8]

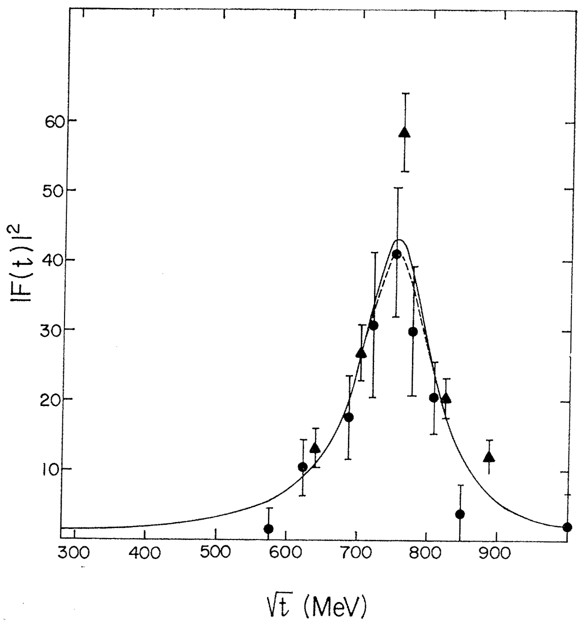

FIG. 1. Results of the form-factor calculation in the timelike region. The solid line is the prediction of hard-pion current algebra ($\delta_\pi=-\frac{1}{2}$), while the dashed line is that of the Veneziano model of Ref. 7. The closed circle refers to the experimental results of V. L. Auslander *et al.* (Ref. 8), the closed triangle to that of J. E. Augustin *et al.* (Ref. 8).

II. PION FORM FACTOR AND EFFECT OF (CLOSED) $K\bar{K}$ CHANNEL

Our procedure is easily extended to several coupled two-pseudoscalar-meson channels; in the case of the pion form factor, we are led to a matrix version of the unitarity equation, Eq. (5),

$$\text{Abs}\mathbf{F}(t)=(1/12\pi F_\pi^2m_\rho^2)\,\text{Tr}[\mathbf{F}^\dagger(t)\boldsymbol{\varrho}(t)\mathbf{F}(t)]\mathbf{f}(t). \tag{13}$$

One can then determine the perturbation on the pion form factor which results from the coupling to the (closed) $K\bar{K}$ channel. From considerations of hard-meson current algebra,[9] one has

[7] H. Suura, Phys. Rev. Letters **23**, 551 (1969); **23**, 1007(E) (1969).

[8] Our plot of $|F(t)|^2$ as predicted by hard-pion current algebra ($\delta_\pi=-\frac{1}{2}$) is identical with Fig. 1(a) of Ref. 2. We have also plotted the experimental points plotted there, i.e., the experimental results of V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967); in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968) (closed circles), and those of J. E. Augustin *et al.*, Phys. Letters **28B**, 508 (1968) (closed triangles).

[9] Y. Ueda, Phys. Rev. **174**, 2082 (1968).

$$f_1(t)\equiv f_\pi(t)=m_\rho^2\left(1-\frac{1+\delta_\pi}{4}\frac{t}{m_\rho^2}\right), \tag{14}$$

$$f_2(t)\equiv f_K(t)=m_\rho^2\left[1-\frac{t}{2m_\rho^2}\left(1-\frac{g_{K_A}{}^2}{F_K{}^2m_{K_A}{}^4}\times(m_{K_A}{}^2-m_\rho^2)+\frac{\delta_K g_{K_A}{}^2 m_\rho^2}{F_K{}^2 m_{K_A}{}^4}\right)\right]. \tag{15}$$

Our solution to the unitarity equation for $F_1(t)$ [or $F(t)$] follows from the ansatz $F_i=\phi(t)f_i(t)$, with[10]

$$\mathrm{Abs}F(t)=|F(t)|^2\left[\rho_\pi(t)f_\pi(t)+\rho_K(t)\frac{[f_K(t)]^2}{2f_\pi(t)}\right]; \tag{16}$$

thus,[11]

$$F(t)=12\pi F_\pi{}^2\left[12\pi F_\pi{}^2+\beta t+\frac{Q_\pi{}^2(t)f_\pi(t)}{\pi m_\rho{}^2}h_\pi(t)+\frac{m_\pi{}^2}{\pi}+\frac{1}{2}\frac{Q_K{}^2(t)[f_K(t)]^2}{\pi m_\rho{}^2 f_\pi(t)}h_K(t)+\frac{m_K{}^2}{2\pi}\right]^{-1}, \tag{17}$$

where, below the KK threshold ($t\leq 4m_K{}^2$),

$$h_K(t)=\frac{2}{\pi}\left(\frac{m_K{}^2-\frac{1}{4}t}{t}\right)^{1/2}\tan^{-1}\left[\frac{1}{2}\left(\frac{t}{m_K{}^2-\frac{1}{4}t}\right)^{1/2}\right]. \tag{18}$$

Unfortunately, in the numerical evaluation of the expression for $|F(t)|^2$ which results, we are confronted with the rather wide yet permissible variation in several of the parameters characterizing the $K\bar{K}$ channel. If we make the convenient,[9] though not at all necessary, simplification $\delta_\pi=\delta_K=-\frac{1}{2}$, then there still remains[12] $1\leq F_K/F_\pi \lesssim 1.3$, with either $m_{K_A}\simeq 1320$ or 1260 MeV. Furthermore, the possible existence of the κ meson[9,12] will have some effect on our result. In spite of these uncertainties, we do find, for example, that the conglomerate parameter $(g_{K_A}{}^2/F_K{}^2m_{K_A}{}^4)(m_{K_A}{}^2-m_\rho{}^2)$ does not show much variation. For $\delta_\pi=\delta_K=-\frac{1}{2}$, we find that a reasonable value for

$$\eta\equiv 1-(g_{K_A}{}^2/F_K{}^2m_{K_A}{}^4)(m_{K_A}{}^2-\tfrac{1}{2}m_\rho{}^2)=1-\left(\frac{F_K}{F_\pi}\right)^{-2}\left(2-\frac{F_K{}^2}{F_\pi{}^2}-\frac{F_\kappa{}^2}{F_\pi{}^2}\right)\left(1-\frac{m_\rho{}^2}{2m_{K_A}{}^2}\right) \tag{19}$$

is $\eta\simeq 0.7$.[13] If, as in Ref. 2, we compare

$$\Gamma_\rho=-\left(\frac{d}{dt}\mathrm{Re}\frac{12\pi F_\pi{}^2}{F(t)}\right)^{-1}_{m_\rho{}^2}\frac{3-\delta_\pi}{4}\frac{[Q_\pi(m_\rho{}^2)]^3}{m_\rho{}^2}, \tag{20}$$

with

$$\Gamma_\rho=\frac{2}{3}\frac{f_{\rho\pi\pi}{}^2}{4\pi}\frac{[Q_\pi(m_\rho{}^2)]^3}{m_\rho{}^2}, \tag{21}$$

then we obtain the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[14] modified by the factor $\frac{1}{4}(3-\delta_\pi)$ within 0.1%. (In the same comparison in Ref. 2, this result was obtained within 5%.) Moreover, the change in the p-wave scattering length obtained from

$$\left(\frac{Q_\pi{}^3}{\sqrt{t}}\cot\delta_{11}\right)_{t=4m_\pi{}^2}-14.9m_\pi{}^2=m_\pi{}^2\left\{\frac{1}{2\pi}\left(\frac{m_K}{m_\pi}\right)^2+\frac{(2-\eta)^2}{(3-\delta_\pi)}\frac{|Q_K(m_\rho{}^2)|^3}{\pi m_\rho m_\pi{}^2}\tan^{-1}\left(\frac{m_\rho}{2|Q_K(m_\rho{}^2)|}\right)\left[\frac{m_\rho{}^2}{4[Q_\pi(m_\rho{}^2)]^2}-1\right]\right.$$
$$\left.-\frac{\{1+\frac{1}{2}\eta(4[Q_\pi(m_\rho{}^2)]^2/m_\rho{}^2-1)\}^2}{\{4[Q_\pi(m_\rho{}^2)]^2/m_\rho{}^2\}^2\{1+\frac{1}{4}(3-\delta_\pi)m_\pi{}^2/[Q_\pi(m_\rho{}^2)]^2\}}\frac{(m_K{}^2-m_\pi{}^2)^{3/2}}{2\pi m_\pi{}^3}\times\tan^{-1}\left(\frac{m_\pi}{(m_K{}^2-m_\pi{}^2)^{1/2}}\right)\right\}\left[1+\frac{3-\delta_\pi}{4}\frac{m_\pi{}^2}{[Q_\pi(m_\rho{}^2)]^2}\right] \tag{22}$$

is just $0.1m_\pi{}^2$. The shape of $|F(t)|^2$ is only very mildly depressed on the low side of the resonance by the correction and very mildly enhanced on its high side; there is no significant alteration in the peak height. Altogether, the $K\bar{K}$ channel appears to constitute a negligible perturbation in this model.

III. PION FORM FACTOR AND EFFECT OF NONRESONANT P-WAVE ($I=1$) $\pi\pi$ SCATTERING

So far in our discussion of the pion form factor $F(t)$ we have assumed the validity of the (unitary) form-factor approximation to T_{11}, namely,[15]

$$T_{11}=\frac{\sqrt{t}}{Q}e^{i\delta_{11}}\sin\delta_{11}\simeq\frac{Q^2F(t)f(t)}{a_{11}m_\rho{}^2}. \tag{23}$$

However, it is realistic to expect an additional small nonresonant contribution to T_{11} (what is effectively the

[10] $\rho_i(t)=Q_i{}^3(t)/\sqrt{t}$ ($i=\pi, K$), with $Q_i=(\frac{1}{4}t-m_i{}^2)^{1/2}$.

[11] Note that there is an additional factor of $\frac{1}{2}$ arising from unitary spin considerations in the case of the subchannels (K^+K^-) and ($K^0\bar{K}^0$).

[12] I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **175**, 1876 (1968).

[13] This value is close to that obtained using the values $F_K/F_\pi=1.2$, $F_\kappa/F_\pi=0.2$ [S. Okubo and V. S. Mathur, Phys. Rev. Letters **23**, 1412 (1969)], or with $F_K/F_\pi=1.1$, $F_\kappa/F_\pi=0.6$.

[14] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[15] We drop the subscript π on the variable Q henceforth; $a_{11}=12\pi F_\pi{}^2$ as in Ref. 2.

previously[1] neglected irreducible amplitude) arising from ρ and ϵ exchange in the crossed-channel plus seagull terms.[16] Such a (unitary) nonresonant contribution can be easily accommodated in the construction of T_{11} with but slight modification. We define

$$T_{11}=Q^2G(t)f(t)/a_{11}m_\rho^2+t_{11}, \tag{24}$$

where t_{11} is the nonresonant amplitude which satisfies (elastic) unitarity by itself,

$$\mathrm{Im}t_{11}=(Q/\sqrt{t})|t_{11}|^2, \quad t\geq 4m_\pi^2. \tag{25}$$

One might conveniently write such an amplitude t_{11} as

$$t_{11}=Q^2\mathfrak{N}_{11}/\mathfrak{D}_{11}(t), \tag{26}$$

with

$$\mathfrak{D}_{11}(t)=1+\mathfrak{N}_{11}Q^2h(t)/\pi, \tag{27}$$

thereby enabling the introduction of the nonresonant p-wave ($I=1$) scattering length $a_{11}'=1/\mathfrak{N}_{11}$ as a free parameter. In our earlier dispersion solution for $F(t)$, we had to deal with the subtracted integral

$$I(t)=-t\int_{4m_\pi^2}^{\infty}\frac{dt'Q^3(t')f(t')}{(\sqrt{t'})t'(t'-t)}; \tag{28}$$

this was replaced with

$$I_{\text{approx}}(t)=Q^2(t)f(t)h(t)-Q^2(0)f(0), \tag{29}$$

which satisfies the weaker requirements of local unitarity.[2] Now, in the presence of left-hand singularities, etc., the subtracted integral (28) is changed to

$$I'(t)=-t\int_{4m_\pi^2}^{\infty}\frac{dt'Q^3(t')f(t')}{(\sqrt{t'})t'|\mathfrak{D}_{11}(t')|^2(t'-t)}, \tag{28'}$$

so that

$$G(t)=\left\{[\mathfrak{D}_{11}(t)]^2\left[1+\beta t-\frac{t}{a_{11}\pi m_\rho^2}\times\int_{4m_\pi^2}^{\infty}\frac{dt'Q^3(t')f(t')}{(\sqrt{t'})t'|\mathfrak{D}_{11}(t')|^2(t'-t)}\right]\right\}^{-1}. \tag{30}$$

The integral (28′) can be evaluated with the aid of an old trick.[17] Since

$$\mathrm{Im}[\mathfrak{D}_{11}(t)]^{-1}=\frac{Q^3(t)\mathfrak{N}_{11}}{(\sqrt{t})|\mathfrak{D}_{11}(t)|^2}, \quad t\geq 4m_\pi^2 \tag{31}$$

$I'(t)$ can be transformed into a contour integral around the elastic cut

$$I'(t)=-\frac{t}{2i}\int_{\circlearrowleft}\frac{dt'f(t')}{\mathfrak{N}_{11}t'\mathfrak{D}_{11}(t')(t'-t)}, \tag{32}$$

which in our simple model reduces to

$$I'(t)=-\frac{\pi f(t)}{\mathfrak{N}_{11}\mathfrak{D}_{11}(t)}+\frac{\pi f(0)}{\mathfrak{N}_{11}\mathfrak{D}_{11}(0)}. \tag{33}$$

It is a tedious matter to show that T_{11} given by Eq. (24), with $G(t)$ given by Eq. (29), does indeed satisfy elastic unitarity,

$$\mathrm{Im}T_{11}=(Q/\sqrt{t})|T_{11}|^2, \quad t\geq 4m_\pi^2. \tag{34}$$

The resulting "phase-modified" pion form factor,

$$F(t)=\left(\frac{\mathfrak{D}_{11}(0)}{\mathfrak{D}_{11}(t)}\right)\left\{1+\beta t-\frac{1}{m_\rho^2}\left(\frac{a_{11}'}{a_{11}}\right)\times\left(\frac{f(t)}{\mathfrak{D}_{11}(t)}-\frac{f(0)}{\mathfrak{D}_{11}(0)}\right)\right\}^{-1}, \tag{35}$$

satisfies both elastic unitarity,

$$\mathrm{Im}F(t)=F^*(t)(Q/\sqrt{t})T_{11}(t), \quad t\geq 4m_\pi^2 \tag{36}$$

as well as the normalization condition $F(0)=1$. As in our earlier discussion, β is determined by requiring that $\mathrm{Re}[F(m_\rho^2)]^{-1}=0$. Thus,

$$\beta=[m_\rho^2\,\mathrm{Re}\mathfrak{D}_{11}(m_\rho^2)]^{-1}\times\left[\frac{f(m_\rho^2)}{m_\rho^2\xi}-\mathrm{Re}\mathfrak{D}_{11}(m_\rho^2)\left(1+\frac{f(0)}{m_\rho^2\xi\mathfrak{D}_{11}(0)}\right)\right], \tag{37}$$

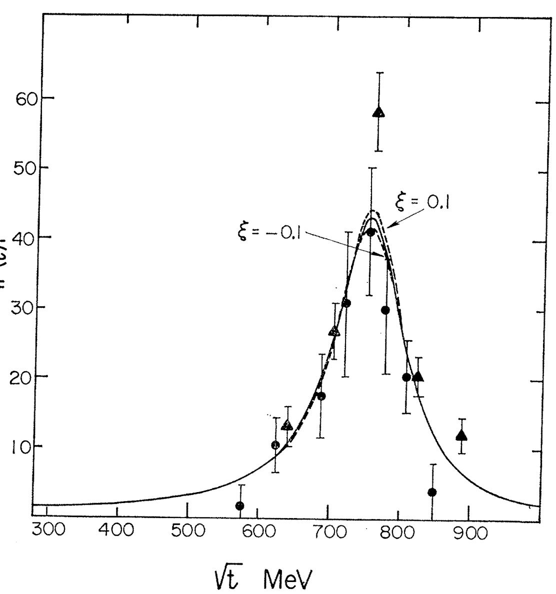

FIG. 2. Comparison of calculations of $|F(t)|^2$ for the three values of the parameter $\xi=(a_{11}/a_{11}')$, $\xi=-0.1, 0, 0.1$. $a_{11}=12\pi F_\pi^2$ and a_{11}' is the p-wave scattering length associated with permissible nonresonant p-wave ($I=1$) $\pi\pi$ scattering. The solid line refers to the form-factor approximation $\xi=0$. The experimental results of Ref. 8 are plotted as in Fig. 1.

[16] R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. Letters **20**, 475 (1968).
[17] R. Rockmore, Phys. Rev. **151**, 1228 (1966).

with

$$\xi = a_{11}/a_{11}'. \quad (38)$$

Of this one-parameter family of pion form factors $F(t)$, those corresponding to the values of $\xi = -0.1$, 0, and 0.1 are plotted in Fig. 2. The experimental data appear to be consistent with small positive ξ, i.e., small positive $\mathfrak{N}_{11}$. Since the width is given approximately by

$$\Gamma_\rho \simeq (124 - 54.7\xi) \text{ MeV}, \quad (39)$$

for small ξ, a slightly smaller ρ width seems indicated.

ACKNOWLEDGMENTS

We thank Professor D. R. Harrington for his interest in this work. We are grateful to the Rutgers Bubble Chamber Group for making the PDP-6 computer facility available to us.

APPENDIX: FORM-FACTOR APPROXIMATION IN OFF-SHELL CONTEXT

We indicate briefly how the form-factor approximation introduced in Eq. (4) is capable of extension to off-shell situations so that one might, for example, undertake to correct pole-dominance calculations[18] for the finite ρ width. Consider, for example, the calculation of the matrix element $T_{\mu\nu}$:

$$T_{\mu\nu} = i \int d^4x\, e^{iq\cdot x} \langle \pi^+(p) | T[A_\mu^{(1-i2)/\sqrt{2}}(x), V_\nu^{(3)}(0)] | 0 \rangle; \quad \text{(A1)}$$

this requires that one study the absorptive part[18]

$$\text{Abs}T_{\mu\nu} = \tfrac{1}{2} \sum_n (2\pi)^4 \delta^4(n - (p-q)) \langle \pi^+(p) | A_\mu^{(1-i2)/\sqrt{2}}(0) | n \rangle \langle n | V_\nu^{(3)}(0) | 0 \rangle - \tfrac{1}{2} \sum_{n'} (2\pi)^4 \delta^4(n' - q) \langle \pi^+(p) | V_\nu^{(3)}(0) | n' \rangle \langle n' | A_\mu^{(1-i2)/\sqrt{2}}(0) | 0 \rangle \equiv \text{Abs}T_{\mu\nu}^{(1)} - \text{Abs}T_{\mu\nu}^{(2)}. \quad \text{(A2)}$$

In the case of a finite ρ width, one need concern oneself only with $\text{Abs}T_{\mu\nu}^{(1)}$; in pole-dominance calculations, this is given by

$$\text{Abs}T_{\mu\nu}^{(1)} = \sum_\lambda \pi\delta((p-q)^2 + m_\rho^2) \langle \pi^+(p) | A_\mu^{(1-i2)/\sqrt{2}}(0) | \rho^0(p-q); \lambda \rangle \langle \rho^0(p-q); \lambda | V_\nu^{(3)}(0) | 0 \rangle. \quad \text{(A3)}$$

In our procedure, we replace the intermediate ρ^0-meson state in (A3) by the appropriate two-pion state, so that

$$\text{Abs}T_{\mu\nu}^{(1)} = \frac{1}{2} \int \frac{d\mathbf{p}' d\mathbf{q}'}{(2\pi)^6} \frac{1}{4\omega(p')\omega(q')} (2\pi)^4 \delta^4(n - (p-q)) \times \langle \pi^+(p) | A_\mu^{(1-i2)/\sqrt{2}}(0) | \pi^+(p')\pi^-(q') \text{ in} \rangle \langle \pi^+(p')\pi^-(q') \text{ in} | V_\nu^{(3)}(0) | 0 \rangle. \quad \text{(A3')}$$

Then the proper mutilation of the off-shell amplitude $\langle \pi^+(p) | A_\mu^{(1-i2)/\sqrt{2}}(0) | \pi^+(p')\pi^-(q') \text{ in} \rangle$ is

$$\langle \pi^+(p) | A_\mu^{(1-i2)/\sqrt{2}}(0) | \pi^+(p')\pi^-(q') \text{ in} \rangle \simeq -\left\{ -i \int d^4x\, e^{-iq\cdot x} \langle \pi^+(p) | T[A_\mu^{(1-i2)/\sqrt{2}}(x), V_\lambda^{(3)}(0)] | 0 \rangle [(p+q)^2 + m_\rho^2] \right\} \times g_\rho^{-2} \langle 0 | V_\lambda^{(3)}(0) | \pi^+(p')\pi^-(q') \text{ in} \rangle, \quad \text{(A4)}$$

with the "tree" amplitude to be given by a pole-dominance calculation; thus we have to deal with

$$\text{Abs}T_{\mu\nu}^{(1)} = \int d\Omega \frac{Q(t)}{8(2\pi)^2\sqrt{t}} \left\{ i \int d^4x\, e^{-iq\cdot x} \langle \pi^+(p) | T[A_\mu^{(1-i2)/\sqrt{2}}(x), V_\lambda^{(3)}(0)] | 0 \rangle [(p+q)^2 + m_\rho^2] \right\}_{\text{tree}} g_\rho^{-2} |F(t)|^2 (p'-q')_\lambda (p'-q')_\nu. \quad \text{(A5)}$$

[18] S. G. Brown and G. B. West, Phys. Rev. 168, 1605 (1968).