a possibility would lead to reexamination of soft-pion theorems which work rather well in other applications.

In conclusion, we observe that if ξ as determined from $K_{\mu 3}$ decay turns out to be large ($\xi = -1$) as claimed,[11] the theory will be hard put to explain such a result. A κ meson is certainly necessary, and its properties are such that Gell-Mann–Oakes–Renner type of symmetry breaking is ruled out. Further, if such a meson is broad, then in the pole-dominance approximation, $(3^*,3)\oplus(3,3^*)$-type breaking, as well as soft-pion theorems, are ruled out. As pointed out by Dashen and Weinstein, the alternative is to give up μ-e universality and/or current algebra.

[11] For a recent report on K_{l3} decays, see J. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968).

ACKNOWLEDGMENTS

The author has benefited from discussions with Professor R. Oakes, which led to the proof of the low-energy theorems; and he is indebted to Professor Arnowitt for a discussion of their model of K_{l3} decays.

PHYSICAL REVIEW D VOLUME 2, NUMBER 3 1 AUGUST 1970

Very-High-Energy Limit of Proton-Proton Scattering*

A. O. Barut† and W. Plywaski†

International Atomic Energy Agency, International Centre for Theoretical Physics, Trieste, Italy

(Received 8 September 1969)

We have evaluated a contact interaction between the protons using the vector currents and wave functions provided by the relativistic $O(4,2)$ model. The resulting enveloping differential cross section at $s=\infty$ is discussed and compared with experiment over the entire range of accessible momentum-transfer values.

I. INTRODUCTION

THE purpose of this paper is to suggest a formula, Eq. (6), for the infinite-energy limit of the N-N differential cross section $d\sigma/dt$. The result is based on the use of a covariant wave function for the nucleon which takes into account its composite structure (e.g., excited states N^*) and also based on a postulated contact interaction of the $J_\mu J^\mu$ type. We shall see that at high energies ($s\rightarrow\infty$) this interaction gives a finite contribution to $d\sigma/dt$, when the contribution of all other interactions has died out.

II. VECTOR CONTACT INTERACTION

The relation between the p-p scattering and the electromagnetic charge form factor of the proton was first suggested by Wu and Yang.[1] More recently, Abarbanel, Drell, and Gilman[2] have considered a vector contact interaction between the protons with phenomenological form factors [for $|t|>4(\text{GeV})^2$]. Since it is possible to describe the composite structure of the proton covariantly by an infinite-component wave function $\psi(p)$, one can consider, among others, contact interactions of the form

$$g_i\psi^\dagger(p_3)O_i\psi(p_1)\psi^\dagger(p_4)O^i\psi(p_2) \qquad (1)$$

to describe the simple momentum transfer between two composite particles without any exchange of constituents. This was done in a previous paper[3] for the scalar interaction, i.e., $O_i=I$. In this paper we consider the more interesting case of vector interaction. More precisely, we write the amplitude in the form

$$A=g_V(\langle\bar{n}_3p_3|J_\mu|\bar{n}_1p_1\rangle\langle\bar{n}_4p_4|J^\mu|\bar{n}_2p_2\rangle-\text{exchange}), \qquad (2)$$

where g_V is a constant, $|\bar{n}p\rangle$ is the state of the nucleon of momentum p and other quantum numbers $\bar{n}$, and J_μ is a vector operator to be specified. For the wave function, we use the relativistic $O(4,2)$ states used in the derivation of the form factors and mass spectrum of baryons,[4] and in I. The current operator J_μ in (2) is a function of the $O(4,2)$ generators and of the momenta. It will be taken to be more general than the one used in obtaining the electromagnetic form factors because here it need not be a conserved current. From Eq. (2), we can calculate the contact amplitude for any reaction of the form

$$N^{\text{I}}+N^{\text{II}}\rightarrow N^{\text{III}}+N^{\text{IV}},$$

where N^{I}, N^{II}, ... are arbitrary nucleon resonances N^* in the $O(4,2)$ tower (the nucleon being the ground state)

* Supported in part by the U. S. Air Force Office of Scientific Research under Grant No. AF-AFOSR-30-67.

† Present address: University of Colorado, Boulder, Colo.

[1] T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965); see also N. Byers and C. N. Yang, *ibid.* **142**, 976 (1966); T. T. Chou and C. N. Yang, in *Proceedings of the Conference on High-Energy Physics and Nuclear Structure*, edited by C. Alexander (North-Holland, Amsterdam, 1967), p. 348; L. Durand III and R. Lipes, Phys. Rev. Letters **20**, 637 (1968).

[2] H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. Letters **20**, 280 (1968); Phys. Rev. **177**, 2458 (1969).

[3] A. O. Barut and D. Corrigan, Phys. Rev. **172**, 1593 (1968), referred to as I.

[4] A. O. Barut, D. Corrigan, and H. Kleinert, Phys. Rev. Letters **20**, 167 (1968).

by appropriately inserting the quantum numbers n_i. For the $j=\frac{1}{2}$ states one can simplify the calculation, for then from general invariance considerations we can write

$$\langle \bar{n}_3 p_3 | J_\mu | \bar{n}_1 p_1 \rangle = f_1(t)\bar{u}_3\gamma_\mu u_1 + f_2(t)(i\kappa/2m)\bar{u}_3\sigma_{\mu\nu}q^\nu u_1 + f_3(t)(1/2m)\bar{u}_3 q^\mu u_1 , \quad (3)$$

where now the u's are the Dirac spinors and the form factors $f_i(t)$ are determined from the matrix elements on the left.

The calculation of the amplitude for this case is discussed in the Appendix. We give the final result in terms of the five helicity amplitudes:

$$\begin{aligned}
M_1\equiv M_{++;++} &= f_1^2(t)[(\gamma\delta-2t)/4m^2]+f_2^2(t)[-tx]+f_3^2(t)[-t\delta]+f_1(t)f_2(t)[-2t/m] \\
&\quad +f_1(t)f_3(t)[0]+f_2(t)f_3(t)[0]+f_1^2(u)[(2\alpha\delta-\gamma t/\alpha)(4m^2)^{-1}]+f_2^2(u)[\alpha\delta(1+\delta)]+f_3^2(u)[-t\delta] \\
&\quad +f_1(u)f_2(u)[2\alpha\delta/m]+f_1(u)f_3(u)[0]+f_2(u)f_3(u)[0], \\
M_3\equiv M_{++;+-} &= f_1^2(t)[-y]+f_2^2(t)[-ty]+f_3^2(t)[ty]+f_1(t)f_2(t)[\alpha y/m] \\
&\quad +f_1(t)f_3(t)[0]+f_2(t)f_3(t)[0]+f_1^2(u)[y]+f_2^2(u)[-\alpha\delta y]+f_3^2(u)[\alpha\delta y] \\
&\quad +f_1(u)f_2(u)[-\alpha y/m]+f_1(u)f_3(u)[0]+f_2(u)f_3(u)[0], \\
M_4\equiv M_{++;--} &= f_1^2(t)[t/\alpha]+f_2^2(t)[(\alpha+s\delta)t/4m^2]+f_3^2(t)[-st^2/4m^2\alpha]+f_1(t)f_2(t)[-t/m] \\
&\quad +f_1(t)f_3(t)[0]+f_2(t)f_3(t)[0]+f_1^2(u)[-\delta]+f_2^2(u)[(st-\alpha^2)\delta/4m^2]+f_3^2(u)[-\alpha s\delta/4m^2] \qquad (4)\\
&\quad +f_1(u)f_2(u)[\alpha\delta/m]+f_1(u)f_3(u)[0]+f_2(u)f_3(u)[0], \\
M_5\equiv M_{+-;+-} &= f_1^2(t)[\gamma\delta/4m^2]+f_2^2(t)[t\delta]+f_3^2(t)[-t\delta]+f_1(t)f_2(t)[0] \\
&\quad +f_1(t)f_3(t)[0]+f_2(t)f_3(t)[0]+f_1^2(u)[-\delta]+f_2^2(u)[(st-\alpha^2)\delta/4m^2]+f_3^2(u)[\alpha s\delta^2/4m^2] \\
&\quad +f_1(u)f_2(u)[-\alpha\delta/m]+f_1(u)f_3(u)[0]+f_2(u)f_3(u)[0], \\
M_8\equiv M_{+-;-+} &= f_1^2(t)[-t/\alpha]+f_2^2(t)[-(\alpha+s\delta)t/4m^2]+f_3^2(t)[st^2/4m^2\alpha]+f_1(t)f_2(t)[t/m] \\
&\quad +f_1(t)f_3(t)[0]+f_2(t)f_3(t)[0]+f_1^2(u)[\gamma t/4m^2\alpha]+f_2^2(u)[-t\delta]+f_3^2(u)[t\delta] \\
&\quad +f_1(u)f_2(u)[0]+f_1(u)f_3(u)[0]+f_2(u)f_3(u)[0],
\end{aligned}$$

in which

$$\alpha=s-4m^2,\quad \delta=1+t/(s-4m^2),$$
$$x=1-t/(s-4m^2),\quad \gamma=2s-4m^2,$$
$$y=(2m)^{-1}[-st\delta(s-4m^2)^{-1}]^{1/2},\quad m\equiv m_p,$$

and the remaining three amplitudes are just the negatives of M_3. Within the context of the $O(4,2)$ theory, the form factors are determined as we have mentioned. Also, if we use the particular model developed for the electromagnetic form factors,[4] we obtain

$$\begin{aligned}
f_1(t)&=(1-t/4M^2)^{-1}(1-at)^{-2}[1-t(\mu/4m^2-b(t))],\\
f_2(t)&=(1-t/4m^2)^{-1}(1-at)^{-2}\{\mu-[1+tb(t)]\}/2m, \qquad (5)\\
f_3(t)&=(1/2m)(1-at)^{-2},
\end{aligned}$$

where

$$b(t)\cong-(1-at)^{-1}(0.168+0.524t)$$

and

$$a=1.41 \quad (\mathrm{GeV}/c)^2,$$
$$\mu=2.79, \quad \text{total magnetic moment.}$$

The parameters in the current, in particular a, are determined from other considerations.[4] Thus we have only one new over-all normalization constant g_V. We determine this over-all constant as follows. The amplitude (2) is a real amplitude. The complete amplitude will have a very large imaginary part at large but finite energy due to the exchange of mesons which, for example, can be parametrized by Regge terms. The reason for the amplitude (2) is that for $s\rightarrow\infty$ the Regge terms will vanish and we will get a real residual term expressing the composite structure. Therefore, we can obtain g_V from the ratio $R=\mathrm{Re}A(s,t)/\mathrm{Im}A(s,t)$ if the only real part of the amplitude at high energy is the term (2). At $t=0$, the ratio R is measured[5] and, because a Pomeranchuk term is pure imaginary, we can determine g_V from its experimental value. In practice this determination would be ambiguous, because secondary Regge poles and cuts give also a real part. (The exact value of R is not very critical at the moment, because of the log scale in the figure.)

The result for $d\sigma/dt$ is shown in Fig. 1 together with the experimental points at various energies. The infinite-energy limit is given by

$$\left.\frac{d\sigma}{dt}\right|_{s\rightarrow\infty}=g_V^2 8\pi(2m)^{-4}\left(1-\frac{t}{4m^2}\right)^{-4}(1-at)^{-8} \times\left[2\left(1-\frac{\mu t}{4m^2}\right)^4+2\kappa^2(2m)^{-4}t^2 +4\kappa^2(2m)^{-2}t\left(1-\frac{\mu t}{4m^2}\right)^2\right] \quad (6)$$

[5] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters 19, 857 (1967).

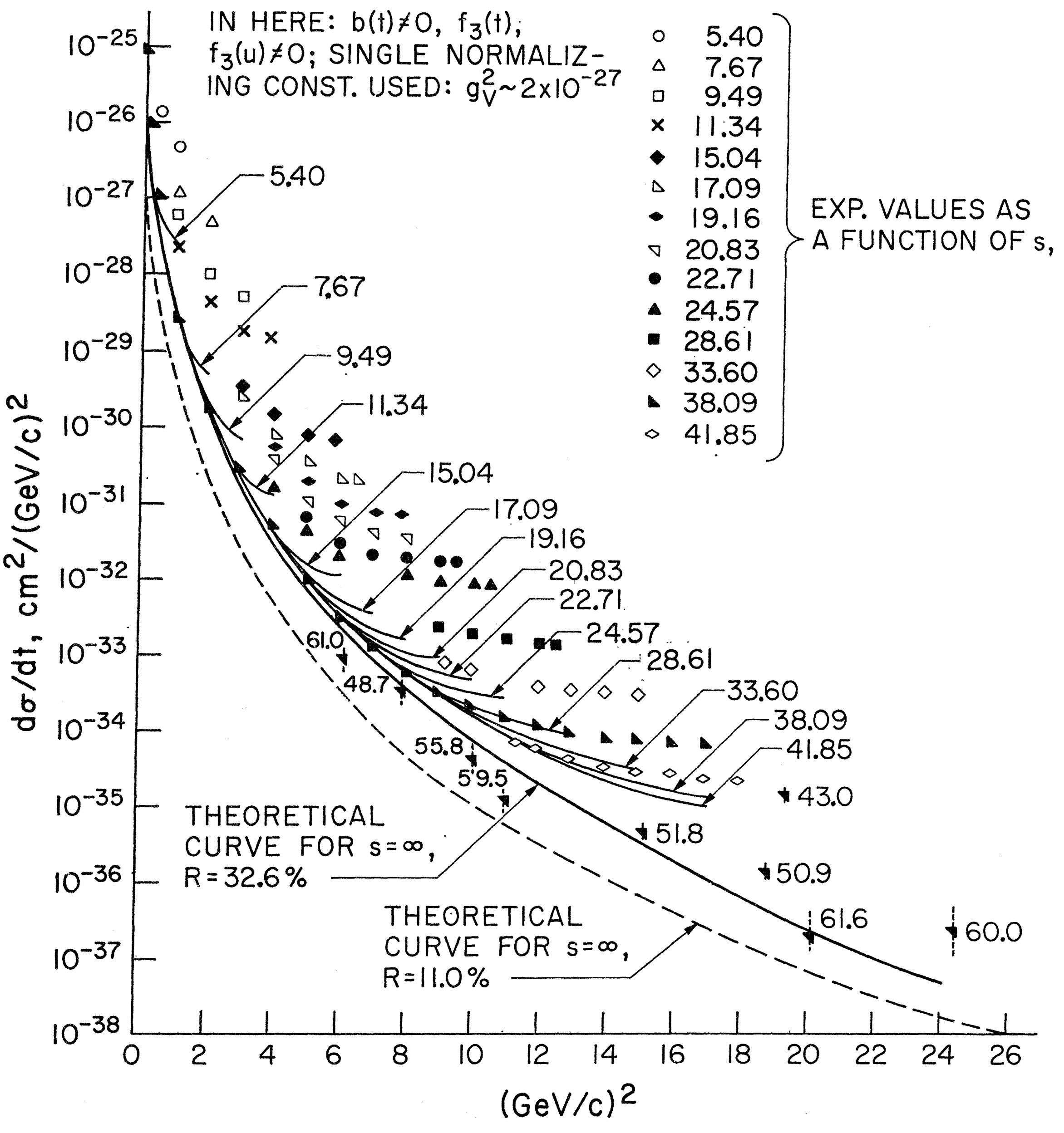

FIG. 1. Differential scattering cross section for elastic p-p scattering based on contact current-current interaction versus $-t$ with s as a parameter. [$b(t)\neq 0$, $f_3(t)$, $f_3(u)\neq 0$. Single normalizing constant used: $g_V^2 \sim = 2\times 10^{-27}$.] Comparison with experimental points of J. V. Allaby *et al.* [Phys. Letters **28B**, 67 (1968)] and G. Cocconi *et al.* [Phys. Rev. **138**, B165 (1965)]. Cocconi *et al.* represents early exploratory measurements and is signified by the symbol ◥. The other symbols are taken from Allaby *et al.*

in units of $\mathrm{cm}^2/(\mathrm{GeV}/c)^2$, $\mu=2.79$, $\kappa=1.79$, while

$$\left.\frac{d\sigma}{dt}\right|_{s\to\infty} \xrightarrow[t\to\infty]{} t^{-8}. \tag{6'}$$

In contrast to this, a scalar interaction[3] between the protons does not give an s-independent lower limit, but vanishes like

$$\left.\frac{d\sigma}{dt}\right|_{s\to\infty} \xrightarrow[t\to\infty]{} s^{-2}t^{-6}. \tag{7}$$

We have further from (6)

$$\left.\frac{d\sigma}{dt}\right|_{s\to\infty} \xrightarrow[t\to 0]{} g_V^2 16\pi(2m)^{-4}, \tag{8}$$

so that

$$R^2 = [\mathrm{Re}A(s,0)/\mathrm{Im}A(s,0)]^2 \cong 9\times 10^{-2}. \tag{9}$$

We have also evaluated one of the polarization parameters, specifically,

$$A(s,t)\equiv\frac{|M_{++;-+}|^2+|M_{+-;-+}|^2-|M_{++;--}|^2-|M_{+-;--}|^2}{|M_{++;-+}|^2+|M_{+-;-+}|^2+|M_{++;--}|^2+|M_{+-;--}|^2}. \tag{10}$$

In Fig. 2, A is plotted against t for some values of s.

III. FURTHER CONSEQUENCES

If we have the postulated contact interaction, the following important consequences have to be emphasized.

(a) The Regge asymptotic behavior of the amplitude at fixed t ($t<0$) as $s\rightarrow\infty$ of the form $s^{\alpha(t)}$, $\alpha(t)<1$ for $t<0$, cannot be an exact mathematical input, although it may be correct at finite but large s. Instead, the $s\rightarrow\infty$ limit is given in (6).[6]

(b) It had been conjectured that $d\sigma^{\text{em}}/dt$ will eventually dominate $d\sigma^{\text{strong}}/dt$ (everywhere except in the forward direction). With the residual amplitude (6) this is no longer true except in the very forward direction. Instead, we have

$$\frac{d\sigma^{\text{strong}}/dt}{d\sigma^{\text{em}}/dt}\underset{s\to\infty}{\longrightarrow}\frac{g_V{}^2}{e^4}\left(\frac{t^2}{(4m^2)^2}\right)=\frac{(g_V/4m^2)^2}{e^4}t^2$$

$$\cong 3.4\times10^3t^2. \tag{11}$$

(c) The slope of the diffraction peak at $s\rightarrow\infty$ can be evaluated from (6). For small t, we can write $d\sigma/dt=Ae^{Bt}=A(1+Bt+\cdots)$ and compare this expansion with (6). We find

$$B=8a-\kappa(2-\kappa)(2m^2)^{-1}-3\kappa/s-2m^2/s^2, \tag{12}$$

which for $s\rightarrow\infty$ approaches the value 10.96 $(\text{GeV}/c)^{-2}$. The comparison of the slope with experiment at finite s can be seen from Fig. 1.

(d) It is interesting to compare the current-current amplitude with the lower limit derived from analyticity assumptions by Cerulus and Martin.[7] This lower bound is at a fixed angle and is given by

$$|A(s,\theta)|>e^{-(\sqrt{2})\,\ln s\,C(\theta)},\quad C(\theta)\sim\sqrt{\theta}$$

for small angle. In contrast, the amplitude (2) gives at fixed angle, i.e., s and t large, but s/t fixed,

$$A(s,\theta)|_{\theta\neq0,\text{fixed}}\underset{s,t\to\infty}{\longrightarrow}s^{-5/2}\frac{16m^2}{a^2}\frac{\cos^2\theta}{1-\cos^2\theta}, \tag{13}$$

i.e., a much slower decrease than (7). Consequently, there is considerable dynamical information in Eq. (2) beyond the general principles, and so the hypothesis of minimal interaction[8] which equates, at high energies, the amplitude to the lower limit (7) cannot be correct if there is vector contact interaction.

Finally, we comment on the calculation of the ampli tude at finite s. One might postulate a Regge term in addition to the residual contact interaction, as was done in Ref. 2. However, a single or a few Regge terms do not account for the observed cross section over the whole known t range up to 20 $(\text{GeV}/c)^2$. In the formalism using infinite multiplets, Regge-type terms arise from the exchange of such infinite multiplets in the crossed channel.[9] But there is not yet a definite procedure to calculate such interaction between the meson clouds of the two protons in a nonphenomenological way.

Note added in manuscript. Two recent works, in the meantime, also argue for a real part of the amplitude and a non-Regge behavior as $s\rightarrow\infty$: D. Horn, Phys. Letters **31B**, 30 (1970); and G. Höhler, F. Steiner, and R. Strauss, Karlsruhe report, 1969 (unpublished).

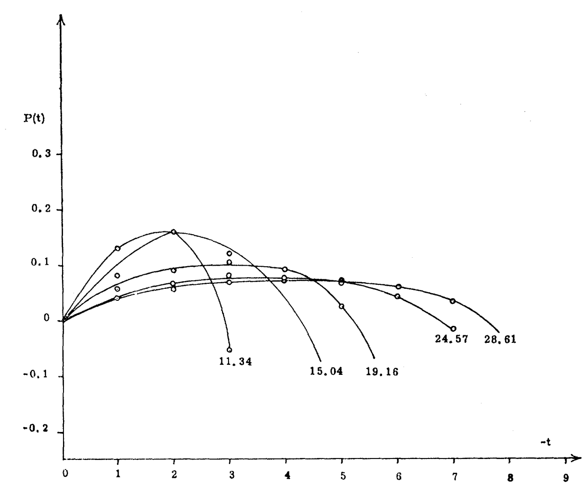

FIG. 2. Polarization as a function of $-t$ (in GeV^2) with s as a parameter corresponding to double p-p scattering. The model used is one equivalent to double scattering, i.e., a completely polarized incident beam and an unpolarized target.

[6] The point $t=0$ needs special consideration: If $\alpha(0)=1$, then we have a nonvanishing imaginary part of the amplitude as $s\rightarrow\infty$, in agreement with the optical theorem, so that the experimental value of $d\sigma/dt$ will always remain above the value given by contact interaction. For this reason, an exact determination of the over-all constant g_V and an exact comparison of the slope B [Eq. (12)] with experiment cannot be made.

[7] E. Cerulus and A. Martin, Phys. Letters **8**, 80 (1964); see also A. Martin, Nuovo Cimento **37**, 671 (1965).

[8] T. Kinoshita, Phys. Rev. Letters **12**, 257 (1964); C. B. Chiu, J. Harte, and C.-I. Tan, Nuovo Cimento **53**, 174 (1968).

[9] L. Van Hove, Phys. Letters **24B**, 183 (1967); see also R. Blankenbecler, in *Proceedings of the 1967 International Conference on Particles and Fields* (Interscience, New York, 1967).

ACKNOWLEDGMENTS

The authors would like to thank Professor Abdus Salam and Professor P. Budini as well as the International Atomic Energy Agency for hospitality at the International Centre for Theoretical Physics, Trieste. The authors are grateful to the Centro di Calcolo dell'Universitá di Trieste for the use of their IBM 7044 computer, and to Dennis Corrigan for a number of discussions.

APPENDIX: CURRENT-CURRENT COUPLING WITH THREE FORM FACTORS

In the c.m. frame the momenta are given by

$$p_1=m(\cosh\xi,0,0,\sinh\xi)\,,$$
$$p_2=m(\cosh\xi,0,0,-\sinh\xi)\,,$$
$$p_3=m(\cosh\xi,\sin\theta\sinh\xi,0,\cos\theta\sinh\xi)\,,$$
$$p_4=m(\cosh\xi,-\sin\theta\sinh\xi,0,-\cos\theta\sinh\xi)\,.$$

Hence

$$s=4m^2\cosh^2\xi\,,\quad t=2m^2\sinh^2\xi(1-\cos\theta)\,.$$

For the ground state we can use the identification expressed in Eq. (3) and, consequently, the Dirac spinor formalism. In the case of two form factors f_1 and f_2, the $j_\mu j^\mu$ interaction has been recently evaluated by Shieh.[10] We give the general case with three form factors f_1, f_2, and f_3, which does not seem to be in the literature. In the c.m. frame of the s channel we evaluate, in the helicity basis, matrix elements of the form

$$\langle p_is_i|j_\mu|p_js_j\rangle=[e^{-i(\theta/2)\sigma_2}e^{r_i(\xi/2)\alpha_3}u(0,s_i)]^\dagger\gamma_0 j_\mu \times[e^{r_j(\xi/2)\alpha_3}u(0,s_j)]\,,$$

where

$$r_i=+1\,,\quad i=1,3$$
$$\quad=-1\,,\quad i=2,4$$

with j_μ given by Eq. (3). Denoting the helicity states by $|p_1+\rangle\equiv|p_1\uparrow\rangle$, etc., we obtain the following matrix elements (in these $\chi=\frac{1}{2}\theta$, where θ is the scattering angle in the barycentric system, 3 refers to $\mathbf{p}_3$, $m\equiv m_p$, etc.):

$$\langle 3+|j^\mu|1+\rangle=\{(\epsilon/m)\cos\chi\, f_1(t)\,,\ (p/m)\sin\chi[f_1(t)+2m\sin^2\chi\, f_2(t)+2m\cos^2\chi\, f_3(t)]\,,\ i(p/m)\sin\chi[f_1(t)+2mf_2(t)]\,,\ +(p/m)\cos\chi[f_1(t)+2m\sin^2\chi\, f_2(t)-2m\sin^2\chi\, f_3(t)]\}\,,$$

$$\langle 3+|j^\mu|1-\rangle=\{\sin\chi[f_1(t)-2m(p/m)^2f_2(t)]\,,\ -2m(p\epsilon/m^2)\cos\chi\sin^2\chi[f_2(t)-f_3(t)]\,,\ 0\,,\ -2m(p\epsilon/m^2)\sin\chi[\cos^2\chi\, f_2(t)+\sin^2\chi\, f_3(t)]\}\,,$$

$$\langle 3+|j^\mu|2+\rangle=\{(\epsilon/m)\sin\chi\, f_1(u)\,,\ (p/m)\cos\chi[f_1(u)+2m\cos^2\chi\, f_2(u)+2m\sin^2\chi\, f_3(u)]\,,\ -i(p/m)\cos\chi[f_1(u)+2mf_2(u)]\,,\ -(p/m)\sin\chi[f_1(u)+2m\cos^2\chi\, f_2(u)-2m\cos^2\chi\, f_3(u)]\}\,,$$

$$\langle 3+|j^\mu|2-\rangle=\{\cos\chi[f_1(u)-2m(p/m)^2f_2(u)]\,,\ -2m(p\epsilon/m^2)\sin\chi\cos^2\chi[f_2(u)-f_3(u)]\,,\ 0\,,\ 2m(p\epsilon/m^2)\cos\chi[\sin^2\chi\, f_2(u)+\cos^2\chi\, f_3(u)]\}\,,$$

$$\langle 4+|j_\mu|2+\rangle=\{(\epsilon/m)\cos\chi\, f_1(t)\,,\ (p/m)\sin\chi[f_1(t)+2m\sin^2\chi\, f_2(t)+2m\cos^2\chi\, f_3(t)]\,,\ -i(p/m)\sin\chi[f_1(t)+2mf_2(t)]\,,\ (p/m)\cos\chi[f_1(t)+2m\sin^2\chi\, f_2(t)-2m\sin^2\chi\, f_3(t)]\}\,,$$

$$\langle 4+|j_\mu|2-\rangle=\{-\sin\chi[f_1(t)-2m(p/m)^2f_2(t)]\,,\ 2m(p\epsilon/m^2)\cos\chi\sin^2\chi[f_2(t)-f_3(t)]\,,\ 0\,,\ 2m(p\epsilon/m^2)\sin\chi[\cos^2\chi\, f_2(t)+\sin^2\chi\, f_3(t)]\}\,,$$

$$\langle 4-|j_\mu|2-\rangle=\{(\epsilon/m)\cos\chi\, f_1(t)\,,\ (p/m)\sin\chi[f_1(t)+2m\sin^2\chi\, f_2(t)+2m\cos^2\chi\, f_3(t)]\,,\ i(p/m)\sin\chi[f_1(t)+2mf_2(t)]\,,\ (p/m)\cos\chi[f_1(t)+2m\sin^2\chi\, f_2(t)-2m\sin^2\chi\, f_3(t)]\}\,,$$

$$\langle 4-|j_\mu|2+\rangle=\{\sin\chi[f_1(t)-2m(p/m)^2f_2(t)]\,,\ -2m(p\epsilon/m^2)\cos\chi\sin^2\chi[f_2(t)-f_3(t)]\,,\ 0\,,\ -2m(p\epsilon/m^2)\sin\chi[\cos^2\chi\, f_2(t)+\sin^2\chi\, f_3(t)]\}\,,$$

$$\langle 4+|j_\mu|1+\rangle=\{-(\epsilon/m)\sin\chi\, f_1(u)\,,\ -(p/m)\cos\chi[f_1(u)+2m\cos^2\chi\, f_2(u)+2m\sin^2\chi\, f_3(u)]\,,\ -i(p/m)\cos\chi[f_1(u)+2mf_2(u)]\,,\ (p/m)\sin\chi[f_1(u)+2m\cos^2\chi\, f_2(u)-2m\cos^2\chi\, f_3(u)]\}\,,$$

$$\langle 4+|j_\mu|1-\rangle=\{\cos\chi[f_1(u)-2m(p/m)^2f_2(u)]\,,\ -2m(p\epsilon/m^2)\sin\chi\cos^2\chi[f_2(u)-f_3(u)]\,,\ 0\,,\ 2m(p\epsilon/m^2)\cos\chi[\sin^2\chi\, f_2(u)+\cos^2\chi\, f_3(u)]\}\,,$$

$$\langle 4-|j_\mu|1-\rangle=\{(\epsilon/m)\sin\chi\, f_1(u)\,,\ (p/m)\cos\chi[f_1(u)+2m\cos^2\chi\, f_2(u)+2m\sin^2\chi\, f_3(u)]\,,\ -i(p/m)\cos\chi[f_1(u)+2mf_2(u)]\,,\ -(p/m)\sin\chi[f_1(u)+2m\cos^2\chi\, f_2(u)-2m\cos^2\chi\, f_3(u)]\}\,,$$

$$\langle 4-|j_\mu|1+\rangle=\{\cos\chi[f_1(u)-2m(p/m)^2f_2(u)]\,,\ -2m(p\epsilon/m^2)\sin\chi\cos^2\chi[f_2(u)-f_3(u)]\,,\ 0\,,\ 2m(p\epsilon/m^2)\cos\chi[\sin^2\chi\, f_2(u)+\cos^2\chi\, f_3(u)]\}\,.$$

Next, we take products of matrix elements and sum over μ. With the notation

$$M_{++;++}=\langle 3+|j^\mu|1+\rangle\langle 4+|j_\mu|2+\rangle-\langle 3+|j^\mu|2+\rangle\langle 4+|j_\mu|1+\rangle\,,\quad \text{etc.,}$$

[10] S. Y. Shieh, Nuovo Cimento **53A**, 790 (1968).

this gives the following amplitudes:

$$M_1 \equiv \langle 3+|j^\mu|1+\rangle\langle 4+|j_\mu|2+\rangle - \langle 3+|j^\mu|2+\rangle\langle 4+|j_\mu|1+\rangle \equiv M_{++;++}$$
$$= f_1^2(t)[2(p/m)^2\sin^2\chi + (p^2+\epsilon^2)m^{-2}\cos^2\chi] + f_2^2(t)[4(p/m)^2m^2\sin^2\chi(1+\sin^2\chi)] + f_3^2(t)[4(p/m)^2m^2\cos^2\chi\sin^2\chi]$$
$$+ f_1(t)f_2(t)[8(p/m)^2m\sin^2\chi] + f_1(t)f_3(t)[0] + f_2(t)f_3(t)[0]$$
$$+ f_1^2(u)[2(p/m)^2\cos^2\chi + (p^2+\epsilon^2)m^{-2}\sin^2\chi] + f_2^2(u)[4(p/m)^2m^2\cos^2\chi(1+\cos^2\chi)]$$
$$+ f_3^2(u)[4(p/m)^2m^2\cos^2\chi\sin^2\chi] + f_1(u)f_2(u)[8(p/m)^2m\cos^2\chi] + f_1(u)f_3(u)[0] + f_2(u)f_3(u)[0],$$

$$M_3 \equiv \langle 3+|j^\mu|1+\rangle\langle 4+|j_\mu|2-\rangle - \langle 3+|j^\mu|2-\rangle\langle 4+|j_\mu|1+\rangle \equiv M_{++;+-}$$
$$= f_1^2(t)[-(\epsilon/m)\cos\chi\sin\chi] + f_2^2[4m^2(p/m)(p\epsilon/m^2)\cos\chi\sin^3\chi] + f_3^2(t)[-4m^2(p/m)(p\epsilon/m^2)\cos\chi\sin^3\chi]$$
$$+ f_1(t)f_2(t)[4m(p/m)(p\epsilon/m^2)\cos\chi\sin\chi] + f_1(t)f_3(t)[0] + f_2(t)f_3(t)[0]$$
$$+ f_1^2(u)[(\epsilon/m)\cos\chi\sin\chi] + f_2^2(u)[-4m^2(p/m)(p\epsilon/m^2)\sin\chi\cos^3\chi] + f_3^2(u)[4m^2(p/m)(p\epsilon/m^2)\sin\chi\cos^3\chi]$$
$$+ f_1(u)f_2(u)[-4m(p/m)(p\epsilon/m^2)\cos\chi\sin\chi] + f_1(u)f_3(u)[0] + f_2(u)f_3(u)[0],$$

$$M_4 \equiv \langle 3+|j^\mu|1-\rangle\langle 4+|j_\mu|2-\rangle - \langle 3+|j^\mu|2-\rangle\langle 4+|j_\mu|1-\rangle \equiv M_{++;--}$$
$$= f_1^2(t)[-\sin^2\chi] + f_2^2(t)[-4m^2\sin^2\chi((p/m)^4 + (p\epsilon/m^2)^2\cos^2\chi)] + f_3^2(t)[-4m^2(p\epsilon/m^2)^2\sin^4\chi]$$
$$+ f_1(t)f_2(t)[4m(p/m)^2\sin^2\chi] + f_1(t)f_3(t)[0] + f_2(t)f_3(t)[0]$$
$$+ f_1^2(u)[-\cos^2\chi] + f_2^2(u)[-4m^2\cos^2\chi((p/m)^4 + (p\epsilon/m^2)^2\sin^2\chi)] + f_3^2(u)[-4m^2(p\epsilon/m^2)^2\cos^4\chi]$$
$$+ f_1(u)f_2(u)[4m(p/m)^2\cos^2\chi] + f_1(u)f_3(u)[0] + f_2(u)f_3(u)[0],$$

$$M_5 \equiv \langle 3+|j^\mu|1+\rangle\langle 4-|j_\mu|2-\rangle - \langle 3+|j^\mu|2-\rangle\langle 4-|j_\mu|1+\rangle \equiv M_{+-;+-}$$
$$= f_1^2(t)[(p^2+\epsilon^2)m^{-2}\cos^2\chi] + f_2^2(t)[-4m^2(p/m)^2\sin^2\chi\cos^2\chi] + f_3^2(t)[4m^2(p/m)^2\sin^2\chi\cos^2\chi]$$
$$+ f_1(t)f_2(t)[0] + f_1(t)f_3(t)[0] + f_2(t)f_3(t)[0]$$
$$+ f_1^2(u)[-\cos^2\chi] + f_2^2(u)[-4m^2\cos^2\chi((p/m)^4 + (p\epsilon/m^2)^2\sin^2\chi)] + f_3^2(u)[4m^2(p\epsilon/m^2)^2\cos^4\chi]$$
$$+ f_1(u)f_2(u)[-4m(p/m)^2\cos^2\chi] + f_1(u)f_3(u)[0] + f_2(u)f_3(u)[0],$$

$$M_8 \equiv \langle 3+|j^\mu|1-\rangle\langle 4-|j_\mu|2+\rangle - \langle 3+|j^\mu|2+\rangle\langle 4-|j_\mu|1-\rangle \equiv M_{+-;-+}$$
$$= f_1^2(t)[\sin^2\chi] + f_2^2(t)[4m^2\sin^2\chi((p/m)^4 + (p\epsilon/m^2)^2\cos^2\chi)] + f_3^2(t)[4m^2(p\epsilon/m^2)^2\sin^4\chi]$$
$$+ f_1(t)f_2(t)[-4m(p/m)^2\sin^2\chi] + f_1(t)f_3(t)[0] + f_2(t)f_3(t)[0]$$
$$+ f_1^2(u)[-(p^2+\epsilon^2)m^{-2}\sin^2\chi] + f_2^2(u)[4m^2(p/m)^2\cos^2\chi\sin^2\chi] + f_3^2(u)[-4m^2(p/m)^2\cos^2\chi\sin^2\chi]$$
$$+ f_1(u)f_2(u)[0] + f_1(u)f_3(u)[0] + f_2(u)f_3(u)[0],$$

and

$$M_2 \equiv M_{++;-+}, \quad M_7 \equiv M_{+-;++}, \quad M_6 \equiv M_{+-;--}$$

are each equal to $-\{M_3 \equiv M_{++;+-}\}$ and these labels are in terms of $\lambda_3\lambda_4;\lambda_1\lambda_2$.

The differential cross section is given by

$$s\frac{d\sigma}{d\Omega} = g_V^2\tfrac{1}{4}\sum_{i=1}^{16}|M_i|^2 = g_V^2\tfrac{1}{2}\sum_{i=1}^{8}|M_i|^2$$

or

$$\frac{d\sigma}{dt} = \frac{4\pi}{s(s-4m^2)}\frac{g_V^2}{2}\sum_{i=1}^{8}|M_i|^2.$$