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What Is Not Invariant under Time Reversal?

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The departure from T invariance in neutral K-meson decays is shown to be directly demonstrable through suitable measurements. The predicted asymmetries include a *CP- and T*-noninvariant effect larger than any which has been measured so far.

UNDER the assumption of TCP invariance, the observation of $K_2^0 \to 2\pi$ decays[1] demonstrates T noninvariance to the same extent that it demonstrates the failure of CP invariance. Tests of T invariance in K^0 decay, independent of the assumption of TCP invariance, have also been discussed,[2–4] and it was shown that available data cannot be easily reconciled[5,6] with the hypothesis of T invariance. In particular, if the phase of η_{00}, like the phase of η_{+-}, lies in the first quadrant,[7] the violation of T invariance can be directly demonstrated. However, as far as I am aware, none of the published analyses relates this observation to the lack of reversibility in any reaction. The purpose of this paper is to show the primitive connection between measurements of neutral K-meson decay and the breakdown of microscopic reversibility.[8]

We first demonstrate the failure of T invariance, i.e., reciprocity,[9] under the assumption of TCP invariance. Under this assumption, the short- and long-lived neutral kaon states can be written in the form[10,2]

$$|K_{1,2}^0\rangle = (1+|r|^2)^{-1/2}(|K^0\rangle \pm r|\bar{K}^0\rangle), \quad (1)$$

where the complex parameter r is related to the more commonly used[2] parameter ϵ through

$$r=(1-\epsilon)/(1+\epsilon).$$

Inversion of Eq. (1) yields

$$|K^0\rangle = \tfrac{1}{2}(1+|r|^2)^{1/2}(|K_1^0\rangle + |K_2^0\rangle), \quad (2a)$$

$$|\bar{K}^0\rangle = \tfrac{1}{2}r^{-1}(1+|r|^2)^{1/2}(|K_1^0\rangle - |K_2^0\rangle). \quad (2b)$$

After a time τ has elapsed, the amplitude of a K_1^0 state changes by a factor $\theta_1 \equiv e^{-(\frac{1}{2}\gamma_1+im_1)\tau}$, while that of a K_2^0 state is multiplied by $\theta_2 \equiv e^{-(\frac{1}{2}\gamma_2+im_2)\tau}$. Therefore, according to Eqs. (1) and (2), a state created initially as $|K^0\rangle$ will be found after a lapse of time τ to have transformed into

$$|K^0\rangle \to \tfrac{1}{2}[(\theta_1+\theta_2)|K^0\rangle + r(\theta_1-\theta_2)|\bar{K}^0\rangle], \quad (3a)$$

while an initial $\bar{K}^0$ state will change during the same time into

$$|\bar{K}^0\rangle \to \tfrac{1}{2}[r^{-1}(\theta_1-\theta_2)|K^0\rangle + (\theta_1+\theta_2)|\bar{K}^0\rangle]. \quad (3b)$$

Consequently, the probability to find, after a time τ, that a K^0 state has changed into a $\bar{K}^0$ is $|r|^4$ times the probability that an initial $\bar{K}^0$ state is found to have transformed[11] into a K^0. The parameter $|r|^2 \approx 1-4\,\mathrm{Re}\epsilon$ has already been measured, under the assumption of TCP invariance, through the leptonic charge asymmetry[12,13] in K_2^0 decays. If we accept the conclusion of Refs. 12 and 13 that $\mathrm{Re}\epsilon > 0$, we deduce that a $\bar{K}^0$ is more likely to be transformed into a K^0 than a K^0 into a $\bar{K}^0$. The probability for a neutral kaon produced with negative strangeness to reappear spontaneously with positive strangeness after a time τ should exceed that for an $S=+1$ neutral kaon to transform spontaneously into one with $S=-1$ by a fraction

$$|r|^{-4}-1 \approx 8\,\mathrm{Re}\epsilon, \quad (4)$$

which is independent of time.[14] Observation of this effect would directly demonstrate CP noninvariance and T noninvariance simultaneously. The asymmetry of Eq. (4), which according to the values of $\mathrm{Re}\epsilon$ reported in Refs. 12 and 13 has a value between 1.1 and 1.6%, is the largest CP-noninvariant effect which can be predicted with assurance at the present time.

As a concrete example of nonreversibility resulting from this effect, let us consider a *Gedanken* experiment in which a K^- beam collides head on with a proton beam. We consider the production of K^+'s in the K^-

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).
[2] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 471 (1966).
[3] M. Gourdin, Nucl. Phys. **B3**, 207 (1967).
[4] P. K. Kabir, Nature **220**, 1310 (1968).
[5] R. C. Casella, Phys. Rev. Letters **21**, 1128 (1968); **22**, 554 (1969).
[6] J. Ashkin and P. K. Kabir, Phys. Rev. D **1**, 868 (1970).
[7] J. Chollet *et al.*, Phys. Letters **31B**, 658 (1970).
[8] Some of the conclusions of this note are implicitly contained in the paper by C. P. Enz and R. R. Lewis, Helv. Phys. Acta. **38**, 860 (1965).
[9] As defined, for example by J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952), p. 529.
[10] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).
[11] If K^0 and $\bar{K}^0$ states are distinguished, as at present, by their opposite values of strangeness, these probabilities will not be measurable with arbitrary accuracy, even in principle, because strangeness is not conserved absolutely. However, since strangeness-nonconserving interactions are expected to be of order 10^{-6} times weaker than strangeness-conserving interactions (see Ref. 2, for example), this effect is neglibible for our discussion.
[12] D. Dorfan *et al.*, Phys. Rev. Letters **19**, 987 (1967).
[13] J. Steinberger, in *Proceedings of the Lund International Conference on Elementary Particles, 1969*, edited by G. von Dardel (Berlingska, Lund, 1969).
[14] The constancy of the ratio of the two probabilities does not depend on TCP invariance.

beam direction, through the two-stage process

$$K^- + p \to n + \bar{K}^0 \searrow K^0 + p \to n + K^+. \tag{5}$$

Even if we assume that the charge-exchange processes are perfectly reversible—if they were not, one would have evidence for nonreversibility already—the rate of the inverse reaction

$$K^+ + n \to p + K^0 \searrow \bar{K}^0 + n \to p + K^- \tag{6}$$

will *not* be deducible directly from that of (5) simply because the probability for a K^0 to change into a $\bar{K}^0$ is not equal to that for a $\bar{K}^0$ to undergo the inverse transformation.

A corollary of our earlier theorem is that an incoherent equal mixture of K^0 and $\bar{K}^0$ particles will always be found to have acquired a net positive strangeness at a later time. This is because *TCP* invariance requires that the fraction of K^0's which survive as K^0 be exactly equal to that of $\bar{K}^0$'s which remain $\bar{K}^0$, while we have seen that the number of $\bar{K}^0$'s transforming into K^0 is expected to exceed that of K^0's undergoing the inverse transmutation. If the $\Delta S = \Delta Q$ rule is valid, this excess of positive strangeness, which is a manifestly *CP*-noninvariant effect, will be reflected in a corresponding excess of positively charged leptons in the leptonic decays of this beam. The mean strangeness at time τ is given by

$$S(\tau) = \frac{\langle K_2^0 | K_1^0 \rangle [e^{-\gamma_1\tau} + e^{-\gamma_2\tau} - 2e^{-(\gamma_1+\gamma_2)\tau/2} \cos(m_2 - m_1)\tau]}{e^{-\gamma_1\tau} + e^{-\gamma_2\tau} - 2e^{-(\gamma_1+\gamma_2)\tau/2} \langle K_2^0 | K_1^0 \rangle^2 \cos(m_2 - m_1)\tau}. \tag{7}$$

It increases from zero at $\tau = 0$ to a maximum of $\sim 1.1 \times \langle K_2^0 | K_1^0 \rangle$ at $T \sim 5\tau_1$, and finally reaches the asymptotic value of $\langle K_2^0 | K_1^0 \rangle$.

We now show that similar effects can be predicted even without assuming *TCP* invariance. Making no symmetry assumption whatever, the short- and long-lived kaon states can be written as[15]

$$|K_1^0\rangle = \cos\alpha_1\, e^{i\beta} |K^0\rangle + \sin\alpha_1\, e^{-i\beta} |\bar{K}^0\rangle, \tag{8a}$$

$$|K_2^0\rangle = \cos\alpha_2\, e^{-i\beta} |K^0\rangle - \sin\alpha_2\, e^{i\beta} |\bar{K}^0\rangle, \tag{8b}$$

where β, $(\frac{1}{4}\pi - \alpha_1)$, and $(\frac{1}{4}\pi - \alpha_2)$ were shown to be small angles not exceeding 10^{-2} in magnitude. From equations analogous to (3a) and (3b), we find the probability for a K^0 state to transform into $\bar{K}^0$ after time τ to be[16,17]

$$P_{\bar{K}K}(\tau) = (1 - |\langle K_2^0 | K_1^0 \rangle|^2)^{-1} \times |\theta_1 - \theta_2|^2 \sin^2\alpha_1 \sin^2\alpha_2, \tag{9a}$$

while the probability for a $\bar{K}^0$ to be transmuted into K^0 in the same time is

$$P_{K\bar{K}}(\tau) = (1 - |\langle K_2^0 | K_1^0 \rangle|^2)^{-1} \times |\theta_1 - \theta_2|^2 \cos^2\alpha_1 \cos^2\alpha_2. \tag{9b}$$

Thus

$$\frac{P_{K\bar{K}}(\tau) - P_{\bar{K}K}(\tau)}{P_{K\bar{K}}(\tau) + P_{\bar{K}K}(\tau)} = \frac{2\cos(\alpha_1 + \alpha_2)\cos(\alpha_1 - \alpha_2)}{1 + \cos 2\alpha_1 \cos 2\alpha_2} \approx 2\,\mathrm{Re}\langle K_2^0 | K_1^0 \rangle \tag{10}$$

to lowest order in β, $(\frac{1}{4}\pi - \alpha_1)$, and $(\frac{1}{4}\pi - \alpha_2)$. Equation (10) is equivalent to our earlier result (4) if *TCP* invariance holds. Even if *TCP* invariance is not assumed, we can estimate the right-hand side of Eq. (10) by using the unitarity relation,[16] which yields

$$\tfrac{1}{2}(1 + 4\delta^2)^{1/2}\,\mathrm{Re}\langle K_2^0 | K_1^0 \rangle = \mathrm{Re}[e^{-i\phi_W} \sum_j \eta_j \gamma_1^j / (\gamma_1 + \gamma_2)], \tag{11}$$

where $\delta = (m_2 - m_1)/(\gamma_1 + \gamma_2)$ and $\phi_W = \tan^{-1}(2\delta) \approx 43°$. The contributions to the sum from 2π channels are expected to be much larger than those of other channels, for which we can set an upper bound by using Schwarz's inequality and known limits on the corresponding partial decay rates; a conservative estimate[4] based on available data is 6×10^{-4}. If we neglect γ_2/γ_1 in comparison with unity and set $C = \gamma_1^{+-}/\gamma_1^{00}$, the 2π contribution to (11) is given by[18]

$$[C|\eta_{+-}|\cos(\phi_{+-} - \phi_W) + |\eta_{00}|\cos(\phi_{00} - \phi_W)]/(C+1). \tag{12}$$

C is known[19,20] to be slightly greater than 2 and $|\eta_{+-}|$ is known with fair precision[21] to be

$$|\eta_{+-}| = (1.90 \pm 0.05) \times 10^{-13},$$

[15] P. Eberhard, Phys. Rev. Letters **16**, 150 (1966).

[16] J. S. Bell and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High-Energy Laboratory, Chilton, Berkshire, England, 1966).

[17] P. K. Kabir, in *Springer Tracts in Modern Physics* (Springer, Berlin, 1970), Vol. 52, p. 91.

[18] If we assume no symmetry, the composition of K_1^0 and K_2^0 states may be slightly different from that assuming *TCP* invariance. Then the same experimental results would be represented by η's which differ slightly from those deduced assuming Eq. (1). From the known limits on β, $(\frac{1}{4}\pi - \alpha_1)$, and $(\frac{1}{4}\pi - \alpha_2)$, we can be confident that this would not change (12) by more than 1%.

[19] B. Gobbi *et al.*, Phys. Rev. Letters **22**, 682 (1969).

[20] J. G. Morfin and D. Sinclair, Phys. Rev. Letters **23**, 660 (1969).

[21] J. W. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 281.

and ϕ_{+-} coincides with ϕ_W within the experimental error of a few degrees.[22] The $\pi^+\pi^-$ contribution to (12) is therefore 1.3×10^{-3}. There is far greater uncertainty about η_{00}, but it is consistent with all reported measurements to say that it has a magnitude comparable to $|\eta_{+-}|$. The only reported[7] measurement of ϕ_{00} yielded a value in the first quadrant, which assures a positive contribution to (12). We can therefore predict a finite nonvanishing T asymmetry [Eq. (10)] of several parts in a thousand, independent of any symmetry assumptions. The expected asymmetry could vanish only in the unlikely circumstance that $|\eta_{00}|$ were significantly larger than $|\eta_{+-}|$ and had a phase ϕ_{00} differing from ϕ_W by considerably more than 90°.

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[22] D. A. Jensen *et al.*, Phys. Rev. Letters **23**, 615 (1969).

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Current Algebra, $\bar{K}_{l3}{}^0$ Form Factors, and Radiative $\bar{K}_{l3}{}^0$ Decay*

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The complete gauge-invariant matrix element for the decays $\bar{K}^0\to\pi^+l^-\bar{\nu}\gamma$ ($l=e$ or μ) is derived using the soft-photon theorems of Low and of Adler and Dothan. These theorems, along with several corollaries of them, are reviewed in detail and their application demonstrated by reference to the radiative $\bar{K}_{l3}{}^0$ decay mode. The square of the matrix element is calculated using the theorem of Burnett and Kroll, and is compared with the result of a direct computer evaluation of the appropriate traces. Structure-dependent terms are discussed, and the dominant terms among those linear in the photon energy are estimated in the soft-pion and kaon limits. Results for the radiative photon spectra are given, together with the decay rates for a specific value of the minimum photon energy E_0.

I. INTRODUCTION

THE present paper, which is the sequel to a previous paper[1] on radiative $K_{l3}{}^+$ decays, has two purposes. One is to discuss in detail the matrix element for the radiative decay $\bar{K}^0\to\pi^+l^-\bar{\nu}\gamma$ ($l=e$ or μ), and then calculate results for decay branching ratios and photon spectra. The other is to use this calculation as a vehicle for reviewing a number of soft-photon theorems and corollaries which are useful for discussing radiative processes in general. Of particular interest are the theorems of Low[2] and Adler and Dothan[3] for the radiative matrix element, and Burnett and Kroll[4] and Bell and Van Royen[5] for the square of the radiative matrix element. Additional references to radiative decays are given in I.

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[1] E. Fischbach and J. Smith, Phys. Rev. **184**, 1645 (1969); hereafter called I. A compilation of rates and spectra for both charged and neutral K decays (with different values of E_0) has also been published. See H. W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. Letters **24**, 189 (1970). With regard to the comment made in this Letter on the first number in the branching ratio $\Gamma(K\to\pi\mu\nu)/\Gamma(K\to\pi e\nu)$ published by N. Cabibbo in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 34, we would like to thank Dr. Cabibbo for confirming the fact that this number was misprinted and should read 0.6457 (for charged K decays) and not 0.6487. Details of Dr. Cabibbo's calculation have been given by C. T. Murphy in University of Michigan Bubble Chamber Group Research Note No. 58/66 (unpublished). The correct branching ratios for charged and neutral K decays can be obtained from Eqs. (2)–(5) of the Letter referred to above, and are given in Appendix B below.

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[3] S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966).

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