symmetry group is found that contains both $\bar{p}$ and p in the same multiplet.

Further measurements of the phase of forward amplitudes would help reduce the theoretical uncertainties that have been mentioned, and they would assist in resolving the most interesting dilemma raised by the Serpukhov data on total cross sections.

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Dispersive Sum-Rule Approach to π-K Scattering*†

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The dispersive sum-rule method, originally developed by Fubini and Furlan, is applied to π-K elastic scattering. Sum rules are derived for the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ scattering amplitudes, and the isospin-antisymmetric combination of the s-wave scattering lengths is calculated. These expressions contain terms involving the off-mass-shell kappa-kaon-pion coupling constant. By following a procedure introduced by Dashen and Weinstein, and assuming that the $SU(3)\times SU(3)$ symmetry-breaking part of the strong-interaction Hamiltonian transforms according to the $(\mathbf{3},\mathbf{3}^*)+(\mathbf{3}^*,\mathbf{3})$ representation of $SU(3)\times SU(3)$, we evaluate the off-shell corrections. In the evaluation of the off-shell corrections, we obtain expressions for the postulated κ-meson mass and decay width, consistent with a recent experimental indication. The s-wave scattering lengths are consistent with other current-algebra and phenomenological-Lagrangian calculations, but smaller than those recently reported from calculations based on the leading term Veneziano model.

I. INTRODUCTION

SINCE its initial proposal,[1] current algebra has had a great deal of success in dealing with low-energy processes involving the weak and electromagnetic interactions.[2] However, in most of its applications to processes involving mesons, for example, one is forced, through ignorance of certain terms, to take a soft-meson limit. One must then make certain smoothness arguments in order to relate the final result to the real world. Through a great many current-algebra calculations, the idea has generally evolved that the soft-meson limit gives reasonable results when the mass of the meson is small in comparison to other masses in the process. In the case of π-N scattering, for example, the good agreement with the calculated and experimental scattering lengths is presumably due to the fact that the neglected terms are of order $(m_\pi/M)^2$ and, therefore, small. However, for processes in which this is not true, such as π-π scattering where, the soft-meson limit is not valid, a recourse to other methods is necessary.[3] In this connection it has been recently pointed out[4] that there are also processes, such as $A_1\rightarrow\rho+\pi$ decay, in which the pion is "hard" rather than "soft," and in which the use of the soft-meson limit leads to results which are in severe disagreement with experiment. From this initial observation there has grown a vast literature on "hard-meson" processes[4,5] leading to good agreement with experiment. Recently, Fubini and Furlan[6] have developed a dispersive sum-rule formulation within which the results of current algebra stated for zero-mass pions can be extrapolated to those for real pions, in addition to giving conditions under which the uncorrected soft-pion results are valid. In the author's opinion this approach represents an alternative, yet simpler, method than the previously mentioned hard-meson methods.

In this paper we shall apply the method of Fubini and Furlan to elastic π-K scattering. In Sec. II we derive the sum rules for π-K scattering and briefly illustrate the method of Fubini and Furlan. In Sec. III we evaluate the sum rules retaining the connected and semidisconnected contributions, where the continuum contributions are approximated by retaining

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[2] See, e.g., S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).

[3] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[4] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[5] Here we quote only some particular references, in addition to Ref. 4. A more complete list can be found by consulting these papers. I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **175**, 1876 (1968); R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, *ibid.* **175**, 1820 (1968).

[6] S. Fubini and G. Furlan, Ann. Phys. (N. Y.) **48**, 322 (1968).

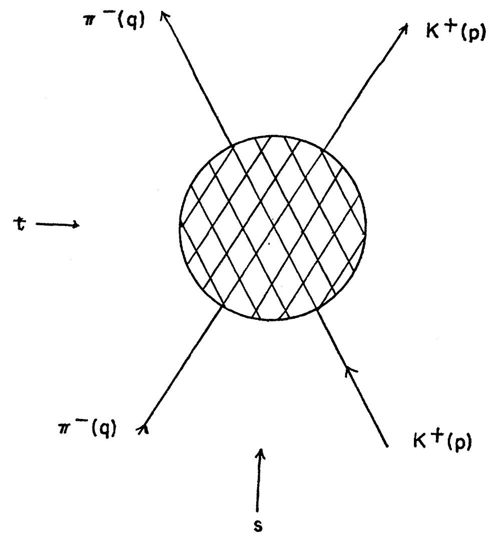

FIG. 1. Channel conventions for $\pi^-(q)-K^+(p)$ elastic scattering.

only the contribution of an $I=\frac{1}{2}$, $J^P=0^+$, strange ($S=1$) κ meson. The relations obtained involve the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ scattering amplitudes evaluated at threshold plus corrections involving the off-shell kappa-kaon-pion coupling constant, and a third relation connecting the off-shell corrections to masses and leptonic decay amplitudes. In Sec. IV, in order to evaluate the off-shell corrections to the amplitudes, we approximate the kappa-kaon-pion coupling by an expansion in the square of the momentum transfer. Under the assumptions of current algebra, pole dominance, partial conservation of axial-vector current (PCAC) applied to both the pion and kaon, partially conserved vector current (PCVC) with which we associate a strangeness-carrying scalar meson, κ,[7] and under the assumption that the part of the strong-interaction Hamiltonian which breaks the chiral $SU(3)\times SU(3)$ symmetry transforms according to the $(\mathbf{3},\mathbf{3}^*)+(\mathbf{3}^*,\mathbf{3})$ representation of the group,[8] we calculate the first two coefficients of the expansion using a method introduced by Dashen and Weinstein (DW)[9] in their derivation of a theorem on K_{l3} form factors. We find that the second coefficient vanishes, and if we keep only terms up to $O(q^4)$, where q^2 is the square of the momentum transfer, we can use one of the results obtained in Sec. III to calculate the third coefficient in the expansion. Therefore, we can explicitly evaluate the off-shell corrections in the relations for the scattering amplitudes. In Sec. V we present and discuss our results. We obtain, in addition to expressions for the mass and width of the postulated κ meson consistent with a recent experimental indication, s-wave π-K scattering lengths consistent with other calculations based on phenomenological-Lagrangian methods and current algebra, but smaller than those recently reported based on the leading term Veneziano model.

II. DERIVATION OF SUM RULES

In this section we illustrate the method of Fubini and Furlan[6] for deriving "on-mass-shell" sum rules relevant to π^--K^+ elastic scattering. We confine ourselves to forward scattering with the s and t channels defined as in Fig. 1 and closely follow the notation of Carbone *et al.*[10] in their application of the method to π-π scattering.

We begin by taking the kaon as the target particle and define the amplitude

$$T_{\mu\nu}=-i\int d^4x\, e^{iq\cdot x} \times\langle K^+(p)|T(A_\mu^+(x),A_\nu^-(0))|K^+(p)\rangle\,, \quad (2.1)$$

where in the above the axial-vector currents are defined as $A_\mu^\pm(x)\equiv A_\mu^1(x)\pm iA_\mu^2(x)$. Forming the scalar product $q^\mu T_{\mu\nu}$ and using the equal-time commutation relations of Gell-Mann[1,11]

$$\delta(x_0)[A_0^+(x),A_\nu^-(0)]=2V_\nu^3(x)\delta^4(x)\,, \quad (2.2)$$

we find

$$q^\mu T_{\mu\nu}=\int d^4x\, e^{iq\cdot x}\langle K^+(p)|T(D_A^+(x),A_\nu^-(0))|K^+(p)\rangle +(1/2\pi)^3(1/2p^0)2p_\nu\,, \quad (2.3)$$

where we have defined $D_A^\pm(x)\equiv\partial^\mu[A_\mu^1(x)\pm iA_\mu^2(x)]$. Using the translational invariance of the matrix element in (2.3), we write the generalized Ward identity for $T_{\mu\nu}$ as

$$q^\mu q^\nu T_{\mu\nu}=W-2S(1/2\pi)^3(1/2p^0) +2p\cdot q(1/2\pi)^3(1/2p^0)\,, \quad (2.4)$$

where we have defined

$$W=-i\int d^4x\, e^{iq\cdot x} \times\langle K^+(p)|T(D_A^+(x),D_A^-(0))|K^+(p)\rangle\,, \quad (2.5)$$

have assumed the equal-time commutation relation

$$\delta(x_0)[D_A^+(0),A_0^-(x)]=2iS(x)\delta^4(x)\,, \quad (2.6)$$

[7] Such an approach was made previously by Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **18**, 507 (1963), and subsequently by several other authors.

[8] See, e.g., Ref. 1; S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968); and M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[9] R. Dashen and M. Weinstein, Phys. Rev. Letters **22**, 1337 (1969).

[10] G. Carbone, E. Donini, and S. Sciuto, Nuovo Cimento **58A**, 688 (1968).

[11] We use the metric $g^{00}=1$, $g^{11}=g^{22}=g^{33}=-1$, so that $p^2=(p^0)^2-\mathbf{p}^2$. Our currents are normalized to satisfy $\delta(x_0)[V_0^\alpha(x),V_0^\beta(0)]=if^{\alpha\beta\gamma}V_0^\gamma(x)\delta^4(x)$, where in a quark model $V_\mu^\alpha(x)=\frac{1}{2}\bar\psi(x)\gamma_\mu\lambda^\alpha\psi(x)$. In addition, we assume that the Schwinger terms occurring in commutators of space-time components are c-number constants as suggested by the algebra of fields. See T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 129 (1967).

and have defined

$$\langle K^+(p)|S(0)|K^+(p)\rangle=(1/2\pi)^3(1/2p^0)S. \quad (2.7)$$

We shall not make any assumptions on the quantity $S(x)$ or the constant S.

We now follow Fubini and Furlan,[6] specialize to the collinear frame, and work in the kaon rest frame where we set

$$q=p\cdot x. \quad (2.8)$$

Equation (2.4) then becomes

$$x^2p^\mu p^\nu T_{\mu\nu}=W(x)-2S(1/2\pi)^3(1/2p^0)+2m_K{}^2x(1/2\pi)^3(1/2p^0). \quad (2.9)$$

Since there is only one independent four-momentum in this frame, we set

$$p^\mu p^\nu T_{\mu\nu}=p^\mu(T_\mu)=p^\mu p_\mu T=m_K{}^2T(x) \quad (2.10)$$

so that Eq. (2.9) becomes

$$m_K{}^2x^2T(x)=W(x)-2S(1/2\pi)^2(1/2p^0)+(1/2\pi)^3(1/2p^0)2m_K{}^2x. \quad (2.11)$$

Following the example of Carbone *et al.*,[10] we can obtain an immediate simplification by going to the t channel. In the notation of Carruthers,[12] the isospin amplitudes for the s and t channels are related as

$$M_s(\tfrac{1}{2})=(1/\sqrt{6})M_t(0)+M_t(1), \quad (2.12)$$

$$M_s(\tfrac{3}{2})=(1/\sqrt{6})M_t(0)-\tfrac{1}{2}M_t(1), \quad (2.13)$$

where in the above $M_v(v')$ is the amplitude for scattering in channel v in the isospin state v'. Therefore, in the t channel we are led to the relations

$$x^2m_K{}^2T_t{}^{(0)}(x)=W_t{}^{(0)}(x)-2(1/2\pi)^3(1/2p^0)S_t{}^{(0)} \quad (2.14)$$

and

$$x^2m_K{}^2T_t{}^{(1)}(x)=W_t{}^{(1)}(x)+2xm_K{}^2(1/2\pi)^3(1/2p^0), \quad (2.15)$$

where $T_t{}^{(0)}$ is an $I=0$, t-channel amplitude, etc. Because of the fact that $T_t{}^{(0)}$ and $T_t{}^{(1)}$ do not have poles at $x=0$, we are led to the identifications

$$W_t{}^{(0)}(x=0)=W_t{}^{(0)}(0)=2S_t{}^{(0)}(1/2\pi)^3(1/2p^0) \quad (2.16)$$

and

$$W_t{}^{(1)}(x=0)=W_t{}^{(1)}(0)=-2m_K{}^2(1/2\pi)^3(1/2p^0), \quad (2.17)$$

where we have defined $W_t{}^{(1)}(x)\equiv x\bar{W}_t{}^{(1)}(x)$ and $T_t{}^{(1)}(x)\equiv x\bar{T}_t{}^{(1)}(x)$. Under the assumption that $W_t{}^{(0)}(x)$ and $\bar{W}_t{}^{(1)}(x)$ satisfy unsubtracted dispersion relations in the variable x, we obtain from Eqs. (2.16) and (2.17) the sum rules

$$\frac{1}{\pi}\int\frac{dx\,\mathrm{Im}W_t{}^{(0)}(x)}{x}=2S_t{}^{(0)}\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0} \quad (2.18)$$

and

$$\frac{1}{\pi}\int\frac{dx\,\mathrm{Im}W_t{}^{(1)}(x)}{x^2}=-2m_K{}^2\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}. \quad (2.19)$$

The advantages of going to the kaon rest frame and choosing x as a dispersion variable have been thoroughly discussed by Fubini and Furlan.[6] For completeness we cite here two of the advantages relevant to the present situation. (1) The analytic properties of the amplitude in the x plane are very simple. In particular, the amplitudes are free from anomalous singularities. (2) Most importantly, the assumption of unsubtracted dispersion relations for W in the collinear frame can be justified by appealing to Bjorken's limit.[13]

As was pointed out by Carbone *et al.*,[10] the sum rules in Eqs. (2.18) and (2.19) are unsuitable if we wish to use single-pole methods. The difficulty follows from the observation that $W_t{}^{(0)}(x)$ and $W_t{}^{(1)}(x)$ receive double-pole contributions from the pion. This results in the introduction of not only threshold amplitudes but in addition their derivatives with respect to x. To alleviate this difficulty, an appeal is made to Bjorken's limit,[13] which, when applied to $W(x)$ in the collinear configuration, gives

$$W(x)\underset{x\to\infty}{\sim}\frac{1}{xm_\pi}\int d^3x\,e^{-i\mathbf{q}\cdot\mathbf{x}}\times\langle K^+(p)|[D_A{}^+(0,\mathbf{x}),D_A{}^-(0)]|K^+(p)\rangle+\frac{i}{x^2m_\pi{}^2}\times\int d^3x\,e^{-i\mathbf{q}\cdot\mathbf{x}}\langle K^+(p)|[\dot{D}^+(0,\mathbf{x}),D_A{}^-(0)]|K^+(p)\rangle+\frac{1}{x^3m_\pi{}^3}\int d^3x\,e^{-i\mathbf{q}\cdot\mathbf{x}}\times\langle K^+(p)|[\dot{D}_A{}^+(0,\mathbf{x}),\dot{D}_A{}^-(0)]|K^+(p)\rangle+\cdots. \quad (2.20)$$

In terms of the t-channel amplitudes, (2.20) carries the implications

$$W_t{}^{(1)}(x)\underset{x\to\infty}{\sim}C^{(1)}/x, \quad (2.21)$$

$$W_t{}^{(0)}(x)\underset{x\to\infty}{\sim}C^{(0)}/x^2, \quad (2.22)$$

$$W_t{}^{(1)}(x)\underset{x\to\infty}{\sim}\tilde{C}^{(1)}/x^3, \quad (2.23)$$

where in the above we have defined the constants through the commutation relations

$$\delta(x_0)[D_A{}^+(x),D_A{}^-(0)]=\delta^4(x)C^{(1)}, \quad (2.24)$$

$$\delta(x_0)[\dot{D}_A{}^+(x),D_A{}^-(0)]=-i\delta^4(x)C^{(0)}, \quad (2.25)$$

and

$$\delta(x_0)[\dot{D}_A{}^+(x),\dot{D}_A{}^-(0)]=\delta^4(x)\tilde{C}^{(1)}. \quad (2.26)$$

[12] P. Carruthers, *Introduction to Unitary Symmetry* (Wiley, New York, 1966); P. Carruthers and J. P. Krisch, Ann. Phys. (N. Y.) **33**, 1 (1965).

[13] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

Under the assumptions of unsubtracted dispersion relations for $W_t^{(1)}(x)$ and $W_t^{(0)}(x)$, Eqs. (2.21) and (2.22) lead to the sum rules

$$\frac{1}{\pi}\int dx\,\mathrm{Im}W_t^{(1)}(x)=-C^{(1)}, \quad (2.27)$$

$$\frac{1}{\pi}\int dx\,x\mathrm{Im}W_t^{(0)}(x)=-C^{(0)}. \quad (2.28)$$

In any model in which the divergence of the axial-vector current is rigorously proportional to the pion field,

$$C^{(1)}=\tilde{C}^{(1)}=0. \quad (2.29)$$

Because $C^{(0)}$ will only contribute to the disconnected graphs, which are always understood to be subtracted off, Eqs. (2.27) and (2.28) lead immediately to

$$\frac{1}{\pi}\int dx\,x\mathrm{Im}W_t^{(0)}(x)=0 \quad (2.30)$$

and

$$\frac{1}{\pi}\int dx\,\mathrm{Im}W_t^{(1)}(x)=0. \quad (2.31)$$

In addition, from (2.23) and (2.29) we find

$$\frac{1}{\pi}\int dx\,x^2\,\mathrm{Im}W_t^{(1)}(x)=0. \quad (2.32)$$

Then by combining Eqs. (2.18) and (2.30), (2.19) and (2.31), and (2.19), (2.31), and (2.32), we are led, respectively, to the sum rules

$$\frac{1}{\pi}\int\frac{dx}{x}\left(1-\frac{m_K^2}{m_\pi^2}x^2\right)\mathrm{Im}W_t^{(0)}(x)=\frac{1}{\pi}\int\frac{dx\,\mathrm{Im}F_t^{(0)}(x)}{x}=2S_t^{(0)}\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}, \quad (2.33)$$

$$\frac{1}{\pi}\int\frac{dx}{x^2}\left(1-\frac{m_K^2}{m_\pi^2}x^2\right)\mathrm{Im}W_t^{(1)}(x)=\frac{1}{\pi}\int\frac{dx\,\mathrm{Im}F_t^{(1)}(x)}{x^2}=-2m_K^2\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}, \quad (2.34)$$

and

$$\frac{1}{\pi}\int\frac{dx}{x^2}\left(1-\frac{m_K^2}{m_\pi^2}x^2\right)^2\mathrm{Im}W_t^{(1)}(x)=\frac{1}{\pi}\int\frac{dx\,\mathrm{Im}G_t^{(1)}(x)}{x^2}=-2m_K^2\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}, \quad (2.35)$$

where in the above relations we have defined the amplitudes

$$F=-i\sqrt{2}F_\pi\int d^4x\,e^{iq\cdot x}\times\langle K^+(p)|T(j_{\pi^-}(x),D_A^-(0))|K^+(p)\rangle \quad (2.36)$$

and

$$G=-2iF_\pi^2\int d^4x\,e^{iq\cdot x}\times\langle K^+(p)|T(j_{\pi^-}(x),\,j_{\pi^+}(0))|K^+(p)\rangle, \quad (2.37)$$

with $j_{\pi^\pm}(x)$ defined as

$$(\Box+m_\pi^2)\varphi_{\pi^\pm}(x)=j_{\pi^\pm}(x). \quad (2.38)$$

In the following we shall work only with the sum rules (2.33)–(2.35). Equations (2.33) and (2.34) now contain only a single pion-pole contribution and introduce the pion-kaon scattering amplitudes at threshold.

III. EVALUATION OF SUM RULES

In this section we turn to an evaluation of the three sum rules (2.33)–(2.35) developed in the preceding section. From the definition of F in (2.36), we insert a complete set of intermediate states and obtain the absorptive part as

$$\mathrm{Im}F=-\pi(2\pi)^3\sqrt{2}\sum_n\delta^4(p+q-n)\langle K^+(p)|j_{\pi^-}(0)|n\rangle\times\langle n|D_A^-(0)|K^+(p)\rangle+\text{c.t.}, \quad (3.1)$$

where c.t. refers to the crossed term. In evaluating (3.1), we shall retain only the π-K and κ intermediate-state contributions. In the collinear configuration the π-K contributions are[14]

$$\mathrm{Im}F_t^{(0)}|_{\pi K}=-\pi\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}\frac{F_\pi^2m_\pi}{m_K}\times\delta\left(x-\frac{m_\pi}{m_K}\right)M_t^{(0)}(x) \quad (3.2)$$

and

$$\mathrm{Im}F_t^{(1)}|_{\pi K}=-\pi\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}\frac{F_\pi^2m_\pi}{m_K}\times\delta\left(1-\frac{m_\pi}{m_K}\right)M_t^{(1)}(x). \quad (3.3)$$

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[14] We define the T matrix by the convention $S_{fi}=\delta_{fi}+i(2\pi)^4\delta(p_f-p_i)T_{fi}=\delta_{fi}+i(2\pi)^4\delta(p_f-p_i)\Pi_i\Pi_fM_{fi}$.

The contributions of the κ are

$$\mathrm{Im}F_t{}^{(0)}|_\kappa=\pi\left(\frac{1}{2\pi}\right)^3\left(\frac{1}{2p^0}\right)\frac{(\sqrt{6})F_\pi{}^2m_\pi{}^2g^2(m_K{}^2x^2)\delta((1+x)^2-(m_\kappa{}^2/m_K{}^2))\epsilon(1+x)}{2m_K{}^2(m_K{}^2x^2-m_\pi{}^2)} \quad (3.4)$$

and

$$\mathrm{Im}F_t{}^{(1)}|_\kappa=\pi\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}\frac{F_\pi{}^2m_\pi{}^2g^2(m_K{}^2x^2)\delta((1+x)^2-(m_\kappa{}^2/m_K{}^2))\epsilon(1+x)}{m_K{}^2(m_K{}^2x^2-m_\pi{}^2)}. \quad (3.5)$$

In obtaining the above we have used PCAC as

$$D_A{}^\alpha(x)=\partial^\mu A_\mu{}^\alpha(x)=F_\pi m_\pi{}^2\varphi_\pi{}^\alpha(x),\quad \alpha=1,2,3 \quad (3.6)$$

where F_π is defined as

$$\langle 0|A_\mu{}^\alpha(0)|\pi^\beta(p)\rangle=iF_\pi p_\mu\delta^{\alpha\beta}(1/2\pi)^{\frac{3}{2}}(1/2p^0)^{\frac{1}{2}}, \quad (3.7)$$

and have defined

$$\langle K^+(k)|j_{\pi^-}(0)|\kappa^0(p)\rangle=-i\sqrt{2}g(q^2)(1/2\pi)^3(1/2p^0). \quad (3.8)$$

Substituting Eqs. (3.2) and (3.4), (3.3), and (3.5) into (2.33) and (2.34), we obtain, respectively,

$$M_t{}^{(0)}=\frac{-2S_t{}^{(0)}}{F_\pi{}^2}+\frac{(\sqrt{6})m_\pi{}^2}{4m_\kappa}\left\{\frac{g^2((m_\kappa-m_K)^2)}{(m_\kappa-m_K)[(m_\kappa-m_K)^2-m_\pi{}^2]}+\frac{g^2((m_\kappa+m_K)^2)}{(m_\kappa+m_K)[(m_\kappa+m_K)^2-m_\pi{}^2]}\right\} \quad (3.9)$$

and

$$M_t{}^{(1)}=\frac{2m_\kappa m_\pi}{F_\pi{}^2}+\frac{m_\pi{}^3}{2m_\kappa}\left\{\frac{g^2((m_\kappa-m_K)^2)}{(m_\kappa-m_K)^2[(m_\kappa-m_K)^2-m_\pi{}^2]}-\frac{g^2((m_\kappa+m_K)^2)}{(m_\kappa+m_K)^2[(m_\kappa+m_K)^2-m_\pi{}^2]}\right\}. \quad (3.10)$$

In a similar fashion we return to the definition of G in Eq. (2.37), insert a complete set of intermediate states, and obtain the absorptive part as

$$\mathrm{Im}G=-\pi(2\pi)^3F_\pi{}^2 2\sum_n\delta^4(p+q-n)\langle K^+(p)|j_{\pi^-}(0)|n\rangle\times\langle n|j_{\pi^+}(0)|K^+(p)\rangle+\text{c.t.} \quad (3.11)$$

By retaining only the κ contribution and going to the collinear configuration, we have

$$\mathrm{Im}G^{(1)}=-\pi\left(\frac{1}{2\pi}\right)^3\frac{1}{2p^0}\times\frac{g^2(m_K{}^2x^2)F_\pi{}^2\delta((1+x)^2-(m_\kappa{}^2/m_K{}^2))\epsilon(1+x)}{m_K{}^2}. \quad (3.12)$$

Substitution of (3.12) into (2.35) gives

$$\frac{4m_\kappa m_K}{F_\pi{}^2}=\frac{g^2((m_\kappa-m_K)^2)}{(m_\kappa-m_K)^2}-\frac{g^2((m_\kappa+m_K)^2)}{(m_\kappa+m_K)^2}. \quad (3.13)$$

As is apparent from inspection of the above expressions, the form factor $g(q^2)$ is evaluated at two different off-shell points: $q^2=(m_\kappa-m_K)^2$ and $q^2=(m_\kappa+m_K)^2$. The reason for this circumstance has been thoroughly discussed by Fubini and Furlan and arises from the contribution of the connected and Z graphs shown in Fig. 2.

By following the same procedures outlined above, we can treat the case in which the pion is taken as the target particle with the kaon incoming, and similar expressions to (3.9), (3.10), and (3.13) are obtained with

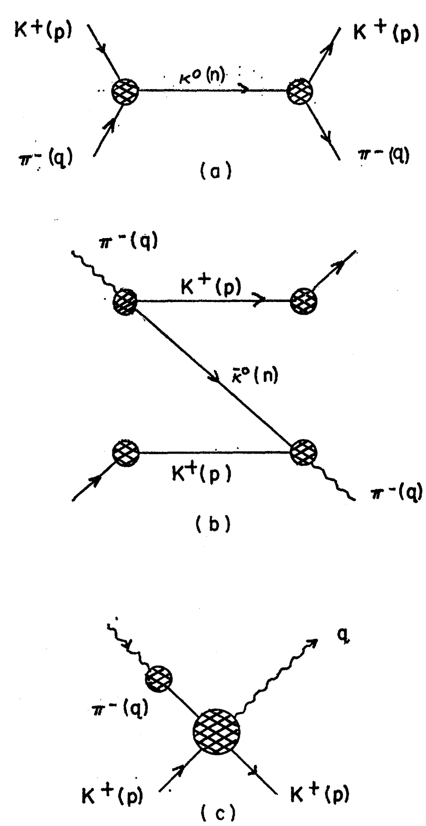

FIG. 2. Graphs which contribute to the sum rules. Graphs (a) and (b) give rise to $g(q^2)$ evaluated at two off-shell points. Graph (a) is the connected contribution, (b) is the so-called Z graph, and (c) introduces the threshold amplitudes.

the π and K labels interchanged. Here we state only the obvious analog to (3.13), namely,

$$\frac{4m_\kappa m_\pi}{F_K^2}=\frac{\tilde{g}^2((m_\kappa-m_\pi)^2)}{(m_\kappa-m_\pi)^2}-\frac{\tilde{g}^2((m_\kappa+m_\pi)^2)}{(m_\kappa+m_\pi)^2}, \quad (3.14)$$

where in the above we have defined

$$\langle\pi^+(k)|j_{K^-}^{(0)}|\kappa^0(p)\rangle\equiv i\sqrt{2}(1/2\pi)^3(1/2p^0)\tilde{g}(q^2). \quad (3.15)$$

As was mentioned above, the occurrence of form factors evaluated at two separate points is inherent in the method of Fubini and Furlan. A simple evaluation of the above results would be to completely neglect the off-shell dependence and approximate

$$g((m_\kappa-m_K)^2)=g((m_\kappa+m_K)^2)=\tilde{g}((m_\kappa-m_\pi)^2)$$
$$=\tilde{g}((m_\kappa+m_\pi)^2)=g=\text{const.} \quad (3.16)$$

If this is done, the g's can be eliminated from Eqs. (3.13) and (3.14) with the κ mass determined as

$$m_\kappa^2=\frac{(F_K/F_\pi)m_K^2-m_\pi^2}{(F_K/F_\pi)-1}, \quad (3.17)$$

which expresses the κ mass entirely in terms of known quantities. The relation (3.17) and the approximation of (3.16) are interesting in that for the choice of $F_K/F_\pi=1.28$ they lead to the predictions $m_\kappa\simeq 1020$ MeV, and $\Gamma(\kappa\to K+\pi)\approx 740$ MeV, consistent with a recent experimental indication.[15] These results have been obtained by several authors.[16,17] Although one would like to retain (3.17), it leaves one rather puzzled in that, if the κ mass is indeed $\simeq 1$ BeV, the approximation of (3.16) requires $g(q^2)$ to be roughly constant over a range of q extending from the physical decay point of $q\cong 140$ MeV to $q\cong 1500$ MeV. In an attempt to understand the above circumstance, we shall approximate $g(q^2)$ by a polynomial in q^2 as

$$g(q^2)=g_0+g_1q^2+g_2q^4. \quad (3.18)$$

The above is reminiscent of the proper-vertex expansion used in the hard-meson calculations first introduced by Schnitzer and Weinberg.[4] In the following section we shall apply the method of DW, used in deriving their theorem on the K_{l3} form factors, to calculate the coefficients g_0 and g_1 appearing in (3.18). We find that the κ mass is determined by (3.17), and, under some additional assumptions (see below), that $g_1=0$. Since g_0 is known, the demand that (3.18) satisfy (3.13) then determines the coefficient g_2. Therefore, we can evaluate the off-mass-shell corrections to the sum rules (3.9) and (3.10). Before discussing our results, however, we first turn to the calculation of the coefficients g_0 and g_1.

IV. EVALUATION OF $q(q^2)$

In this section we apply the method of DW to calculate the first two coefficients of $g(q^2)$. For notational convenience we work with Cartesian variables rather than in the charged notation. We define the form factor $g(q^2)$ as[18]

$$\langle K^\beta(k)|j_\pi{}^\alpha(0)|\kappa^\gamma(p)\rangle$$
$$=-(1/2\pi)^3(1/4p^0k^0)^{1/2}f^{\alpha\beta\gamma}g(q^2)$$
$$=-(1/2\pi)^3(1/4p^0k^0)^{1/2}f^{\alpha\beta\gamma}(g_0+g_1q^2+g_2q^4), \quad (4.1)$$

where $(\Box+m_\pi^2)\varphi_\pi{}^\alpha(x)=j_\pi{}^\alpha(x)$, α, β, and γ are $SU(3)$ indices, and $q=p-k$. In order to avoid any assumptions on the commutators involving the pion source operator $j_\pi{}^\alpha(x)$, and to apply the method of DW, we introduce PCAC and use the definition of $j_\pi{}^\alpha(x)$ in terms of the pion field to write

$$\langle K^\beta(k)|j_\pi{}^\alpha(0)|\kappa^\gamma(p)\rangle$$
$$=-\frac{(q^2-m_\pi^2)}{F_\pi m_\pi^2}\langle K^\beta(k)|D_A{}^\alpha(0)|\kappa^\gamma(p)\rangle, \quad (4.2)$$

where $D_A{}^\alpha(x)=\partial^\mu A_\mu{}^\alpha(x)=F_\pi m_\pi^2\varphi_\pi{}^\alpha(x)$. Following DW, we expand the matrix element involving the divergence as

$$\langle K^\beta(k)|D_A{}^\alpha(0)|\kappa^\gamma(p)\rangle=(1/2\pi)^3(1/4p^0k^0)^{1/2}$$
$$\times(a_0+a_1q^2+a_2q^4+\cdots). \quad (4.3)$$

By combining (4.1)–(4.3) and equating coefficients, we are led to the identifications

$$f^{\alpha\beta\gamma}g_0=-a_0/F_\pi \quad (4.4)$$

and

$$f^{\alpha\beta\gamma}g_1=(1/F_\pi)(a_0/m_\pi^2-a_1). \quad (4.5)$$

Following DW and appealing to the LSZ reduction technique, we define

$$M\equiv\int d^4x\,d^4y\,e^{ik\cdot x}e^{-ip\cdot y}$$
$$\times\langle 0|T(D_A{}^\beta(x),D_V{}^\gamma(y),D_A{}^\alpha(0))|0\rangle$$
$$=\frac{-F_\kappa F_K m_\kappa^2 m_K^2(a_0+a_1q^2+\cdots)}{(p^2-m_\kappa^2)(k^2-m_K^2)}, \quad (4.6)$$

[15] T. G. Trippe, C. Y. Chien, E. Malamud, J. Mellema, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Letters **28B**, 203 (1968). These authors report experimental evidence for a K-π enhancement of mass ≈ 1100 MeV and width ≈ 450 MeV. The κ width of ≈ 750 MeV calculated from (3.16) and (3.17) is consistent with this experiment (private communication from Professor Malamud to Professor Wada). More recently, evidence of a K-π enhancement of mass$=1160\pm 10$ MeV and width $=90\pm 30$ MeV has been reported. See D. J. Crennell, U. Karshon, K. W. Lai, J. S. O'Neall, and J. M. Scarr, Phys. Rev. Letters **22**, 487 (1969). We are not in agreement with the quoted decay width from this experiment.

[16] D. W. McKay, J. M. McKisic, and W. W. Wada, Phys. Rev. **184**, 1609 (1969). The quoted κ widths in this paper are for the neutral κ decay mode [Phys. Rev. D 1, 957(E) (1970)].

[17] See, e.g., S. P. DeAlwis and D. A. Nutbrown, Nuovo Cimento **58**, 876 (1968); D. H. Dahmen, K. D. Rothe, and L. Schülke, Nucl. Phys. **B7**, 472 (1968).

[18] Here, as in Sec. III, we are assuming a kappa-kaon-pion interaction density of the form $\mathcal{H}(x)=g_{\kappa K\pi}f^{\alpha\beta\gamma}\pi^\alpha(x)K^\beta(x)\kappa^\gamma(x)$. This implies, for example, that under charge conjugation, the $|\kappa^+\rangle$ state goes into $-|\kappa^-\rangle$.

where in the above we use kaon PCAC and PCVC in the forms

$$\partial^\mu A_\mu{}^\beta(x)=F_K m_K{}^2\varphi_K{}^\beta(x),\quad \beta=4,5,6,7 \tag{4.7}$$

$$\partial^\mu V_\mu{}^\gamma(x)=F_\kappa m_\kappa{}^2\varphi_\kappa{}^\gamma(x),\quad \gamma=4,5,6,7. \tag{4.8}$$

Integrating the left-hand side of (4.6) by parts three times yields

$$\begin{aligned}M=&-ik^\mu p^\nu q^\lambda\int d^4x d^4y\, e^{ik\cdot x}e^{-ip\cdot y}\\&\times\langle 0|T(A_\mu{}^\beta(x),\,V_\nu{}^\gamma(y),\,A_\lambda{}^\alpha(0))|0\rangle\\&-if^{\alpha\beta\rho}k^\mu p^\nu\int d^4y\, e^{-ip\cdot y}\langle 0|T(V_\mu{}^\rho(0),\,V_\nu{}^\gamma(y))|0\rangle\\&+if^{\alpha\sigma\gamma}k^\mu p^\nu\int d^4x\, e^{ik\cdot x}\langle 0|T(A_\nu{}^\sigma(0),\,A_\mu{}^\beta(x))|0\rangle\\&+M_1+M_2+M_3+M_4,\end{aligned} \tag{4.9}$$

where in the above we have defined

$$\begin{aligned}M_1\equiv-\int d^4xd^4y\, e^{ik\cdot x}e^{-ip\cdot y}\langle 0|T(\delta(x_0-y_0)\\\times[A_0{}^\beta(x),D_V{}^\gamma(y)],D_A{}^\alpha(0))|0\rangle,\end{aligned} \tag{4.10}$$

$$\begin{aligned}M_2\equiv-\int d^4xd^4y\, e^{ik\cdot x}e^{-ip\cdot y}\langle 0|T(\delta(x_0)\\\times[A_0{}^\beta(x),D_A{}^\alpha(0)],D_V{}^\gamma(y))|0\rangle,\end{aligned} \tag{4.11}$$

$$\begin{aligned}M_3\equiv ik^\mu\int d^4xd^4y\, e^{ik\cdot x}e^{-ip\cdot y}\langle 0|T(\delta(y_0-x_0)\\\times[V_0{}^\gamma(y),A_\mu{}^\beta(x)],D_A{}^\alpha(0))|0\rangle,\end{aligned} \tag{4.12}$$

and

$$\begin{aligned}M_4\equiv ik^\mu\int d^4xd^4y\, e^{ik\cdot x}e^{-ip\cdot y}\langle 0|T(\delta(y_0)\\+[V_0{}^\gamma(y),D_A{}^\alpha(0)],A_\mu{}^\beta(x))|0\rangle.\end{aligned} \tag{4.13}$$

In order to evaluate the contributions of the M's, we need to know the various equal-time commutators indicated in (4.10)–(4.13). As is well known, one can obtain information on current-divergence commutators if one makes a definite assumption on the form of the symmetry breaking. Here we follow Gell-Mann *et al.*[1,8] and assume that the Hamiltonian describing the strong interactions can be decomposed into two pieces, one of which is invariant under the chiral $SU(3)\times SU(3)$ group and the other of which transforms according to the $(\mathbf{3},\mathbf{3}^*)+(\mathbf{3}^*,\mathbf{3})$ representation of the group. Thus, we assume

$$H=H_0+u^0+cu^8, \tag{4.14}$$

where H_0 is invariant under the transformations which generate $SU(3)\times SU(3)$, and the u's are scalar densities defined in a quark model as

$$u^\alpha(x)=\tfrac{1}{2}\bar\psi(x)\lambda^\alpha\psi(x),\quad \alpha=0,1,\ldots,8. \tag{4.15}$$

By analogy with the above, we also introduce a set of pseudoscalar densities as

$$v^\alpha(x)=i\tfrac{1}{2}\bar\psi(x)\lambda^\alpha\gamma_5\psi(x). \tag{4.16}$$

From (4.14)–(4.16), assuming free-field anticommutation relations for the quark fields, $\psi(x)$, and specializing to $\alpha,\ \beta\neq 0,8$ relevant to the present case, we find

$$\delta(x_0)[V_0{}^\alpha(x),u^\beta(0)]=if^{\alpha\beta\gamma}u^\gamma(x)\delta^4(x),\quad \alpha,\beta,\gamma=1,\ldots,7 \tag{4.17}$$

$$\delta(x_0)[V_0{}^\alpha(x),v^\beta(0)]=if^{\alpha\beta\gamma}v^\gamma(x)\delta^4(x), \tag{4.18}$$

$$\delta(x_0)[A_0{}^\alpha(x),u^\beta(0)]=id^{\alpha\beta\gamma}v^\gamma(x)\delta^4(x), \tag{4.19}$$

and

$$\delta(x_0)[A^\alpha(x),v^\beta(0)]=-id^{\alpha\beta\gamma}u^\gamma(x)\delta^4(x), \tag{4.20}$$

with the divergences of the vector and axial-vector currents given, respectively, by

$$\partial^\mu V_\mu{}^\alpha(x)=cf^{\alpha\beta\gamma}u^\gamma(x) \tag{4.21}$$

and

$$\begin{aligned}\partial^\mu A_\mu{}^\alpha(x)&=(1/\sqrt{3})(\sqrt{2}+c)v^\alpha(x),\quad \alpha=1,2,3\\\partial^\mu A_\mu{}^\alpha(x)&=(1/\sqrt{3})(\sqrt{2}-\tfrac{1}{2}c)v^\alpha(x),\quad \alpha=4,5,6,7.\end{aligned} \tag{4.22}$$

From (4.17)–(4.22) we obtain, for example, the commutation relations[19]

$$\begin{aligned}\delta(x_0)&[A_0{}^\beta(x),D_V{}^\gamma(0)]\\&=-if^{\beta\gamma\rho}(F_\kappa m_\kappa{}^2/F_\pi m_\pi{}^2)D_A{}^\rho(x)\delta^4(x),\\&\qquad\beta=4,5(6,7),\quad\gamma=6,7(4,5),\quad\rho=1,2\end{aligned} \tag{4.23}$$

$$\begin{aligned}\delta(x_0)&[A_0{}^\beta(x),D_A{}^\alpha(0)]\\&=-if^{\beta\alpha\gamma}(F_\pi m_\pi{}^2/F_\kappa m_\kappa{}^2)D_V{}^\rho(x)\delta^3(x),\\&\qquad\beta,\rho=4,5,6,7,\quad\alpha=1,2,3\end{aligned} \tag{4.24}$$

$$\begin{aligned}\delta(x_0)&[A_0{}^\alpha(x),D_V{}^\gamma(0)]\\&=if^{\alpha\gamma\rho}(F_\kappa m_\kappa{}^2/F_K m_K{}^2)D_A{}^\rho(x)\delta^4(x),\\&\qquad\alpha=1,2,3\quad\rho,\gamma=4,5,6,7\end{aligned} \tag{4.25}$$

and

$$\begin{aligned}\delta(x_0)&[A_0{}^\alpha(x),D_A{}^\beta(0)]\\&=if^{\alpha\beta\gamma}(F_K m_K{}^2/F_\kappa m_\kappa{}^2)D_V{}^\rho(x)\delta^4(x),\\&\qquad\alpha=1,2,3\quad\beta,\rho=4,5,6,7\end{aligned} \tag{4.26}$$

where we identify

$$F_\kappa m_\kappa{}^2/F_\pi m_\pi{}^2=-3c/2(\sqrt{2}+c) \tag{4.27}$$

and

$$F_\kappa m_\kappa{}^2/F_K m_K{}^2=-3c/2(\sqrt{2}-\tfrac{1}{2}c). \tag{4.28}$$

Equations (4.27) and (4.28) can be combined to predict c as

$$c=\frac{-\sqrt{2}[(F_K/F_\pi)m_K{}^2-m_\pi{}^2]}{(F_K/F_\pi)m_K{}^2+\frac{1}{2}m_\pi{}^2}, \tag{4.29}$$

[19] The commutation relations (4.23)–(4.26) may at first glance seem strange in that no $d^{\alpha\beta\gamma}$ constants appear. They are, however, strictly equivalent to those of Ref. 8 provided the identifications (4.27) and (4.28) are made. We find it convenient to write the commutators in this form for obvious reasons (see below).

which for $F_K/F_\pi=1.28$, $m_\kappa=494$ MeV, and $m_\pi=140$ MeV gives $c=-1.28$, in good agreement with $c=-1.25$ as obtained by Gell-Mann *et al.*[8] Relation (4.29) and similar relations to (4.27) and (4.28) have recently been noted by several authors.[20] Substitution of (4.23)–(4.26) into (4.9), going to the limit $p^2=0=k^2$,[21] and comparing with (4.3) gives

$$f^{\alpha\beta\gamma}(F_\kappa m_\kappa^2 F_\pi - F_\pi m_\pi^2 F_\kappa) + f^{\alpha\beta\gamma}q^2(\tfrac{1}{2}F_\kappa^2-\tfrac{1}{2}F_K^2-\tfrac{1}{2}F_\pi^2+F_\kappa m_\kappa^2 F_K/m_\pi^2) = -F_\kappa F_K(a_0+a_1q^2+\cdots). \quad (4.30)$$

From the relation

$$a_0=-f^{\alpha\beta\gamma}(m_\kappa^2-m_K^2), \quad (4.31)$$

and by equating coefficients in (4.30), we have

$$F_\pi(m_\kappa^2-m_\pi^2)=F_K(m_\kappa^2-m_K^2) \quad (4.32)$$

and

$$a_1=f^{\alpha\beta\gamma}\left(\frac{F_K^2-F_\kappa^2-F_\pi^2}{2F_KF_\kappa}-\frac{F_\pi m_\kappa^2}{F_K m_\pi^2}\right). \quad (4.33)$$

From (4.4), (4.5), (4.31), and (4.33) we then find

$$g_0=(m_\kappa^2-m_K^2)/F_\pi \quad (4.34)$$

and

$$g_1=-\frac{1}{F_\pi}\left(\frac{F_K^2-F_\kappa^2-F_\pi^2}{2F_KF_\kappa}-\frac{F_\pi m_\kappa^2}{F_K m_\pi^2}+\frac{(m_\kappa^2-m_K^2)}{m_\pi^2}\right). \quad (4.35)$$

Equation (4.32) leads to the κ-mass prediction of (3.17). From (4.35), together with

$$F_\pi=F_K-F_\kappa \quad (4.36)$$

and (4.32), it follows that $g_1=0$. The relation (4.36) can be shown to follow from the assumed form of the symmetry breaking that we are making here,[20] but it can also be shown to hold independently of any assumed form of symmetry breaking provided PCAC, PCVC, pole dominance, and certain smoothness assumptions are made.[16,22]

To proceed, we assume that $g(q^2)$ can be represented as

$$g(q^2)=g_0+g_2q^4. \quad (4.37)$$

As previously mentioned, (4.37) is analogous to the proper-vertex expansion in the hard-meson calculations first introduced by Schnitzer and Weinberg.[4] Keeping higher terms would make it difficult to understand the successes of these calculations where one retains only the lowest-order terms. Under the assumption that $g(q^2)$ can be adequately represented by (4.37), we can easily determine g_2 by requiring that (4.37) satisfy (3.13). We then find

$$g_2=2g_0/(3m_\kappa^4+10m_\kappa^2m_K^2+3m_K^2), \quad (4.38)$$

so that for $g(q^2)$ we have

$$g(q^2)=\frac{(m_\kappa^2-m_K^2)}{F_\pi}\times\left[1+\frac{2q^4}{3m_\kappa^4+10m_\kappa^2m_K^2+3m_K^4}\right], \quad (4.39)$$

which will allow us to evaluate the off-shell corrections to the sum rules (3.9) and (3.10).

V. DISCUSSION OF RESULTS

In order to compare our results with other approaches we recall the general definitions

$$M_s^{(+)}\equiv\tfrac{1}{3}[M_s^{(1/2)}+M_s^{(3/2)}], \quad (5.1)$$

$$M_s^{(-)}\equiv\tfrac{1}{3}[M_s^{(1/2)}-M_s^{(3/2)}], \quad (5.2)$$

where the superscripts $\frac{1}{2}$ and $\frac{3}{2}$ refer to the isospin and subscripts s refer to the s channel. These are related to the t-channel amplitudes as

$$M_s^{(+)}=(1/\sqrt{6})M_t^{(0)}. \quad (5.3)$$

$$M_s^{(-)}=\tfrac{1}{2}M_t^{(1)}. \quad (5.4)$$

Thus, from (3.9) and (3.10) we have

$$M_s^{(+)}=-\frac{2S}{F_\pi^2\sqrt{6}}+\frac{m_\pi^2}{4m_\kappa}\left\{\frac{g^2((m_\kappa-m_K)^2)}{(m_\kappa-m_K)[(m_\kappa-m_K)^2-m_\pi^2]}+\frac{g^2((m_\kappa+m_K)^2)}{(m_\kappa+m_K)[(m_\kappa+m_K)^2-m_\pi^2]}\right\} \quad (5.5)$$

and

$$M_s^{(-)}=\frac{m_Km_\pi}{F_\pi^2}+\frac{m_\pi^3}{4m_\kappa}\left\{\frac{g^2((m_\kappa-m_K)^2)}{(m_\kappa-m_K)^2[(m_\kappa-m_K)^2-m_\pi^2]}-\frac{g^2((m_\kappa+m_K)^2)}{(m_\kappa+m_K)^2[(m_\kappa+m_K)^2-m_\pi^2]}\right\}. \quad (5.6)$$

In terms of the M amplitudes, the scattering lengths are given by

$$a^{(\pm)}=\frac{M^{(\pm)}m_\pi}{8\pi(m_K+m_\pi)}\frac{1}{m_\pi}, \quad (5.7)$$

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[20] Y. Y. Lee, Nuovo Cimento **64A**, 474 (1969); N. H. Fuchs and T. K. Kuo *ibid.* **64A**, 382 (1969); and J. Cleymans, *ibid* **65A**, 72 (1970).

[21] No soft-meson limit is implied by setting $p^2=k^2=0$. This is done since we are interested in terms involving q^2 (see Ref. 9). In this limit, the first term on the right-hand side of (4.9) and M_4 defined in (4.13) do not contribute.

[22] In this connection we note the work of L. K. Pande [Phys. Rev. Letters **23**, 353 (1969)], who finds that if pole dominance is applied to commutators involving currents and current divergences, in addition to those involving currents, then one is led automatically to the **(3,3*)+(3*,3)** form of the symmetry breaking.

where in direct analogy with (5.1) and (5.2),

$$a^{(+)}=\tfrac{1}{3}(a^{1/2}+2a^{3/2}),\quad a^{(-)}=\tfrac{1}{3}(a^{1/2}-a^{3/2}). \quad (5.8)$$

The first term on the right-hand side of (5.6) is the usual current-algebra result and for $m_K=494$ MeV, $m_\pi=140$ MeV, and $F_\pi^2=0.47\ m_\pi^2$ yields

$$a^{(-)}=0.066(1/m_\pi). \quad (5.9)$$

By using $F_K/F_\pi=1.28$, which predicts a reasonable κ mass and width,[15] (3.17) and (4.39) can be used to evaluate the off-shell corrections in (5.6), and we find

$$a^{(-)}=0.071(1/m_\pi), \quad (5.10)$$

which is consistent with several other predictions based on current-algebra and phenomenological-Lagrangian calculations,[23] but smaller than values obtained by calculations based on the leading term Veneziano model.[24] Owing to the unknown constant S, we are unable, however, to compute $a^{(+)}$ within the present framework.

As a final indication of the consistency of our approach, we can estimate F_K/F_π from our sum rules. If we take the pion as the target and the kaon as the incoming particle, we derive the analog of (5.6),

$$M_s^{(-)}=\frac{m_K m_\pi}{F_K^2}+\frac{m_K^3}{4m_\kappa}\left\{\frac{\tilde{g}^2((m_\kappa-m_\pi)^2)}{(m_\kappa-m_\pi)^2[(m_\kappa-m_\pi)^2-m_K^2]}-\frac{\tilde{g}^2((m_\kappa+m_\pi)^2)}{(m_\kappa+m_\pi)^2[(m_\kappa+m_\pi)^2-m_K^2]}\right\}. \quad (5.11)$$

Using $m_\kappa=1020$ MeV and equating (5.6) and (5.11) then leads to the prediction $F_K/f_\pi=1.24$, compatible with the approximations involved.

In summary, we have applied the dispersive sum-rule method of Fubini and Furlan to a study of π-K elastic scattering. By assuming current algebra, PCAC applied to both kaon and pion, PCVC, single-particle pole dominance, and the assumption that the $SU(3)\times SU(3)$-symmetry-breaking part of the strong-interaction Hamiltonian transforms according to the $(\mathbf{3},\mathbf{3}^*)+(\mathbf{3}^*,\mathbf{3})$ representation of $SU(3)\times SU(3)$, we have derived sum rules for the s-wave $I=\frac{1}{2}$ and $I=\frac{3}{2}$ scattering amplitudes. By appealing to a method introduced by Dashen and Weinstein, we have evaluated the off-shell corrections and have found scattering lengths consistent with previous calculations based on current-algebra and phenomenological-Lagrangian methods, but smaller than those predicted from the leading term Veneziano model. In addition, we have found the κ-meson mass and width consistent with a recent experimental indication. All of the above results depend essentially only on three fairly well-known parameters: F_K/F_π, m_K, and m_π. If $m_\kappa\simeq1020$ MeV is taken, the sum rules for the amplitudes predict $F_K/F_\pi=1.24$.

In conclusion, it should be stressed that the Fubini-Furlan dispersive sum-rule method provides a much simpler method for performing hard-meson calculations, and serves as a good framework for testing various models of $SU(3)\times SU(3)$ symmetry breaking. In the particular case considered here, we find that the $(\mathbf{3},\mathbf{3}^*)+(\mathbf{3}^*,\mathbf{3})$ form is consistent with experiment.[25] We also see that when the off-shell corrections can be evaluated, by pole-dominance methods, for example, one is led to interesting relations between masses and coupling constants.

Note added in proof. After this paper was submitted for publication, the following papers involving multiple-term Veneziano models came to the author's attention: D. Corrigan, Phys. Rev. **188**, 2465 (1969); and D. W. McKay, W. F. Palmer, and W. W. Wada, Phys. Rev. D (to be published). The models of these authors lead to $a^{(-)}=0.073$ $(1/m_\pi)$ and $a^{(-)}=0.069$ $(1/m_\pi)$, respectively. These results are in excellent agreement with Eq. (5.10).

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[23] The consistency of (5.10) with other results depends upon whom one compares with. For example, S. Weinberg (Ref. 3) obtains $a^{(-)}=0.086(1/m_\pi)$ including the reduced-mass correction; J. A. Cronin [Phys. Rev. **161**, 1483 (1967)] obtains $a^{(-)}=0.066$ $(1/m_\pi)$; H. Yabuki [Phys. Rev. **170**, 1410 (1968)] obtains $a^{(-)}=0.129(1/m_\pi)$ including off-shell corrections; and R. W. Griffith [Phys. Rev. **176**, 1705 (1968)] obtains $a^{(-)}=0.086(1/m_\pi)$.

[24] See, e.g., K. Kawarabayashi, S. Kitakado, and H. Yabuki [Phys. Letters **28B**, 432 (1969)], who obtain $a^{(-)}=0.093(1/m_\pi)$; and R. Arnowitt, P. Nath, Y. Srivastava, and M. H. Friedman [Phys. Rev. Letters **22**, 1158 (1969)], who obtain $a^{(-)}=0.136(1/m_\pi)$.

[25] F. von Hippel and J. K. Kim, Phys. Rev. Letters **22**, 740 (1969).