Pomeranchuk Theorem and the Serpukhov Data on Total Cross Sections*

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The recent Serpukhov data for K^-p and earlier data for K^+p collisions appear to contradict the Pomeranchuck theorem on the asymptotic equality of particle-target and antiparticle-target total cross sections. Experimental and theoretical consequences of the invalidity of the Pomeranchuk theorem are discussed. These include the result that forward amplitudes for K^-p and K^+p become asymptotically real.

I. INTRODUCTION

THE authors[1] of the IHEP-CERN experiment on total cross sections at high energy have noted a significant difference of about 3 mb between the newly measured K^-p cross sections,[1] which are nearly constant above 20 GeV/c, and the extrapolated values of the previously measured K^+p cross sections.[2] which already seem to be energy-independent below 20 GeV/c. The authors[1] remark that it is not obvious how this discrepancy between seemingly asymptotic values (i.e., constants) of the K^-p and the K^+p cross sections can be reconciled with the Pomeranchuk theorem,[3,4] which asserts the asymptotic equality of particle and antiparticle total cross sections. In this note, I will discuss the experimental and theoretical consequences of the Serpukhov data from the viewpoint that the Pomeranchuk theorem is not valid for K^-p and K^+p cross sections. In particular I wish to emphasize the following three points.

(a) The proofs of the Pomeranchuk theorem[3,4] involve an *ad hoc* assumption about the phase of forward scattering amplitudes whose validity has not been established from any basic axioms.

(b) The Serpukhov results and the axioms of quantum field theory indicate that the phase of the forward amplitudes for K^-p and for K^+p scattering should tend to zero or π, instead of $\frac{1}{2}\pi$ as has usually been assumed in the past.

(c) Several important assumptions made in the use of Regge theory are placed in doubt by the Serpukhov data, and it appears that until these questions are resolved it will be impossible to retain any confidence in our present understanding of phenomenology at high energies. A major step towards their resolution would be achieved by measurement of the phase of forward amplitudes, especially for the difficult cases of K^-p and K^+p, but also for π^-p, π^+p, and for $\bar{p}p$, beyond the energies used in the Brookhaven experiments.[5]

II. DEDUCTIONS HAVING A GENERAL THEORETICAL BASIS

The Pomeranchuk theorem can be deduced from two rather different types of approach, one involving analyticity properties plus an assumption about the phase of the forward amplitude.[3] The second approach involves the dominance of elastic scattering over exchange scattering (originally proposed by Okun and Pomeranchuk) plus an invariance assumption.[6] We will consider both approaches.[4]

Given the analyticity and boundedness that has been derived from quantum field theory, a forward scattering amplitude will satisfy a dispersion relation with no more than two subtractions. If one also assumes that

$$\sigma_1(\text{total } A+B) \to C_1, \quad \sigma_2(\text{total } \bar{A}+B) \to C_2, \quad (1)$$

where F_1 and F_2 denote the particle-target and antiparticle-target amplitudes, respectively. From Eq. (1) their imaginary parts behave like E as $E \to \infty$. It follows that, unless $C_1 = C_2$,

$$\text{Re}[F_i(E)]/\text{Im}[F_i(E)] \to \infty \quad \text{as} \quad E \to \infty. \quad (2)$$

This possibility is excluded by assumption in the proofs of the Pomeranchuk theorem. Although these proofs vary in mathematical content and in the choice of forms for asymptotic cross sections, they all contain some assumption equivalent to saying that the result in Eq. (2) is not allowed. Such assumptions have not been derived from basic axioms and may therefore be false. However, the "counter-theorem" is important for our later discussion:

Counter-theorem. If particle and antiparticle total cross sections are asymptotically constant but not equal, then the phase of the corresponding forward amplitudes will tend to zero or π.

Variations on this counter-theorem may readily be derived if one wishes to make alternative assumptions about the asymptotic form of the cross sections.[4]

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[1] J. V. Allaby *et al.*, IHEP-CERN Collaboration, Phys. Letters **30B**, 500 (1969).

[2] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965).

[3] I. Ya. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. **34**, 725 (1958) [Soviet Phys. JETP **7**, 499 (1958)].

[4] General methods relating to the Pomeranchuk theorem are described by R. J. Eden, *High Energy Collisions of Elementary Particles* (Cambridge U. P., New York, 1967), where further references are given.

[5] S. J. Lindenbaum, in *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energy*, edited by A. Perlmutter, C. A. Hurst, and B. Kursunoglu (Benjamin, New York, 1968); see also K. J. Foley *et al.* Phys. Rev. Letters **19**, 193 (1967).

[6] L. B. Okun and I. Ya. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. **30**, 307 (1956) [Soviet Phys. JETP **3**, 307 (1956)].

The second approach to the Pomeranchuk theorem makes use of the Okun-Pomeranchuk rule and invariance properties.[6] For π^+p and π^-p scattering, if one assumes isospin invariance at high energy and asymptotic dominance of elastic over exchange scattering, it follows that π^-p and π^+p total cross sections must be asymptotically equal. Thus the Pomeranchuk theorem may be deduced when the particle and its antiparticle belong to the same multiplet. As noted in the IHEP-CERN collaboration,[1] their new results are compatible with this conclusion for pions and nucleons.

However, for K^-p and K^+p scattering, one cannot deduce the Pomeranchuk theorem from invariance arguments and the assumption of dominance of elastic over exchange scattering. Neither can it be deduced for proton-proton and antiproton-proton scattering. Although K^- and K^+ belong to the same $SU(3)$ multiplet, it is one in which the symmetry is rather clearly broken. There does not appear to be any very sound reason for assuming that a broken symmetry becomes asymptotically equivalent to exact symmetry. Indeed if this were the case, one might also be forced to expect that Kp and πp total cross sections become asymptotically equal, which would also contradict the data from Serpukhov, though no more strongly than is the case for K^-p and K^+p. From the viewpoint of general theory (as opposed to special models like Regge theory), one can also doubt the asumption that elastic scattering dominates over exchange scattering. The proofs[7] that vacuum exchange is the only type that can possibly dominate (i) do not assert that this dominance must occur, (ii) assume that an exact invariance principle can be applied, and (iii) make an assumption equivalent to asserting that Eq. (2) does not happen. Since it is just this latter assumption that is in doubt, the question of dominance of vacuum exchange must also remain in doubt as far as general theory is concerned.

We see, therefore, that if one accepts the Serpukhov data as evidence for the asymptotic (constant) inequality of K^-p and K^+p cross sections, then the real-to-imaginary ratio of the forward amplitudes must increase logarithmically. One can also conclude either that elastic scattering does not dominate over exchange scattering in the $SU(3)$ group, or that symmetry breaking in $SU(3)$ persists asymptotically in its effect on cross sections, or both possibilities could occur.

If Eq. (1) holds with $C_1 \neq C_2$, it is necessary[4] that the forward peak shrinks like $(\ln E)^{-2}$. This follows from the unitarity requirement that $\sigma(\text{total}) > \sigma(\text{elastic})$.

III. TROUBLE FOR REGGE THEORY

The conclusions noted above must also hold in Regge theory. If the conditions in Eq. (1) hold for total cross sections, then Eq. (2) must also hold. Although such asymptotic behavior can be achieved by introducing a Regge dipole at $\alpha(t)=1$ for $t=0$, this would not give the shrinkage $(\ln E)^{-2}$ for the forward peak as noted above, unless, for example, the dipole becomes a double Regge cut for $t\neq 0$ as in the Finkelstein model.[8] The possibility of this type of behavior is alarming for Regge theory since it would mean that there would be hardly anything left of the original idea of simple dominance from leading Regge poles giving a simple asymptotic form for the amplitude.

A more acceptable possibility from the viewpoint of Regge theory could be obtained by replacing the Pomeranchuk pole by a branch cut in the J plane and assuming a similar cut to be present also for the difference of the amplitudes $F(K^-p)-F(K^+p)$. This would still yield a result analogous to Eq. (2). However, a simple assumption of dominance by the leading term would suggest cross sections that decrease like $(\ln E)^{-2}$, which is not compatible with the data.[1] By balancing contributions from different branch cuts in the J plane, it would be possible to fit the experimental data, but such a procedure would have little theoretical content unless it was based on a reasonable model.[9]

Finally, it should be noted that the previous arguments about the effects of symmetry breaking in $SU(3)$ would apply also in Regge theory. For K^-p and K^+p, the symmetry breaking should lead to dominant real terms in a forward scattering amplitude at high energy that are logarithmically larger than the leading imaginary terms.

IV. CONCLUDING REMARKS

Some results have been noted that follow from the Serpukhov data on total cross sections and the assumption that the Pomeranchuk theorem is not valid for K^-p and K^+p. This is discussion contrasts with the possibilities considered by the IHEP-CERN group,[1] who note that the validity of the Pomeranchuk theorem would mean the asymptotic energy region is still a long way off.

It is apparent that a number of conventional assumptions about strong interactions at high energy are unproven from basic principles. These include (a) the phase of a forward scattering amplitude tends to $\frac{1}{2}\pi$, (b) elastic scattering dominates over exchange scattering near the forward direction, and (c) the symmetry breaking in $SU(3)$ becomes unimportant at high energies. If it turns out that the symmetry breaking of $SU(3)$ causes the cross sections for K^-p and K^+p to be unequal at high energy, one might expect those for antiprotons and protons also to be unequal, unless a

[7] L. F. Foldy and R. F. Peierls, Phys. Rev. **130**, 1585 (1963); see also Ref. 4 and D. Amati *et al.*, Nuovo Cimento **32**, 1685 (1964).

[8] J. Finkelstein, Phys. Rev. Letters **24**, 172 (1970); **24**, 432 (1970).

[9] S. Frautschi and B. Margolis, Nuovo Cimento **56**, 1155 (1968), have shown how branch cuts can make striking changes at subasymptotic energies in conventional Regge theory. See also V. Barger and R. J. Phillips, Phys. Rev. Letters **24**, 291 (1970).

symmetry group is found that contains both $\bar{p}$ and p in the same multiplet.

Further measurements of the phase of forward amplitudes would help reduce the theoretical uncertainties that have been mentioned, and they would assist in resolving the most interesting dilemma raised by the Serpukhov data on total cross sections.

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Dispersive Sum-Rule Approach to π-K Scattering*†

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The dispersive sum-rule method, originally developed by Fubini and Furlan, is applied to π-K elastic scattering. Sum rules are derived for the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ scattering amplitudes, and the isospin-antisymmetric combination of the s-wave scattering lengths is calculated. These expressions contain terms involving the off-mass-shell kappa-kaon-pion coupling constant. By following a procedure introduced by Dashen and Weinstein, and assuming that the $SU(3)\times SU(3)$ symmetry-breaking part of the strong-interaction Hamiltonian transforms according to the $(\mathbf{3},\mathbf{3}^*)+(\mathbf{3}^*,\mathbf{3})$ representation of $SU(3)\times SU(3)$, we evaluate the off-shell corrections. In the evaluation of the off-shell corrections, we obtain expressions for the postulated κ-meson mass and decay width, consistent with a recent experimental indication. The s-wave scattering lengths are consistent with other current-algebra and phenomenological-Lagrangian calculations, but smaller than those recently reported from calculations based on the leading term Veneziano model.

I. INTRODUCTION

SINCE its initial proposal,[1] current algebra has had a great deal of success in dealing with low-energy processes involving the weak and electromagnetic interactions.[2] However, in most of its applications to processes involving mesons, for example, one is forced, through ignorance of certain terms, to take a soft-meson limit. One must then make certain smoothness arguments in order to relate the final result to the real world. Through a great many current-algebra calculations, the idea has generally evolved that the soft-meson limit gives reasonable results when the mass of the meson is small in comparison to other masses in the process. In the case of π-N scattering, for example, the good agreement with the calculated and experimental scattering lengths is presumably due to the fact that the neglected terms are of order $(m_\pi/M)^2$ and, therefore, small. However, for processes in which this is not true, such as π-π scattering where, the soft-meson limit is not valid, a recourse to other methods is necessary.[3] In this connection it has been recently pointed out[4] that there are also processes, such as $A_1\rightarrow\rho+\pi$ decay, in which the pion is "hard" rather than "soft," and in which the use of the soft-meson limit leads to results which are in severe disagreement with experiment. From this initial observation there has grown a vast literature on "hard-meson" processes[4,5] leading to good agreement with experiment. Recently, Fubini and Furlan[6] have developed a dispersive sum-rule formulation within which the results of current algebra stated for zero-mass pions can be extrapolated to those for real pions, in addition to giving conditions under which the uncorrected soft-pion results are valid. In the author's opinion this approach represents an alternative, yet simpler, method than the previously mentioned hard-meson methods.

In this paper we shall apply the method of Fubini and Furlan to elastic π-K scattering. In Sec. II we derive the sum rules for π-K scattering and briefly illustrate the method of Fubini and Furlan. In Sec. III we evaluate the sum rules retaining the connected and semidisconnected contributions, where the continuum contributions are approximated by retaining

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[2] See, e.g., S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).

[3] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[4] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[5] Here we quote only some particular references, in addition to Ref. 4. A more complete list can be found by consulting these papers. I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **175**, 1876 (1968); R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, *ibid.* **175**, 1820 (1968).

[6] S. Fubini and G. Furlan, Ann. Phys. (N. Y.) **48**, 322 (1968).