Experimental Determination of the Dalitz-Plot Distribution of the Decays $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow \pi^+\pi^-\gamma$, and the Branching Ratio $\eta \rightarrow \pi^+\pi^-\gamma/\eta \rightarrow \pi^+\pi^-\pi^0$†

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We have analyzed the Dalitz-plot distribution of the two dominant charged decays of the η meson. From 7250 examples of $\eta \rightarrow \pi^+\pi^-\gamma$, we find that the γ-ray energy spectrum disagrees with the simplest gauge-invariant matrix element corresponding to the dipion in the $J=1$ state. However, it agrees well with a simple ρ-dominant model. The π^0 energy spectrum of 30 000 examples of $\eta \rightarrow \pi^+\pi^-\pi^0$ was fitted to a power-series expansion of the matrix element in the usual Dalitz coordinates. We find that a matrix element linear in the π^0 kinetic energy does not fit our data. We measure the branching ratio $\eta \rightarrow \pi^+\pi^-\gamma/\eta \rightarrow \pi^+\pi^-\pi^0$ to be 0.201 ± 0.006.

HERE we report experimental determination of the Dalitz-plot distributions in the decays $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow \pi^+\pi^-\gamma$ based on 30 000 and 7250 events, respectively, as well as of the branching ratio $\eta \rightarrow \pi^+\pi^-\gamma/\eta \rightarrow \pi^+\pi^-\pi^0$.

The experimental techniques and methods of analysis have already been reported,[1] and we describe them only briefly here. η mesons were produced in the reaction $\pi^- p \rightarrow \eta n$. The incident π^- beam had a momentum of 720 MeV/c with a spread of $\pm 0.75\%$. The neutron momentum was measured by time of flight and the momenta of the charged pions from the decay of the η were measured by a set of sonic chambers in a magnetic field.

The events which could be analyzed were assumed to have originated from one of the following reactions:

$$
\begin{aligned}
&\text{(a)} \quad \pi^- p \rightarrow n\pi^+\pi^- \\
&\text{(b)} \quad \rightarrow n\pi^+\pi^-\gamma \\
&\text{(c)} \quad \rightarrow n\pi^+\pi^-\pi^0.
\end{aligned}
$$

To eliminate most of reaction (a), we required $M_y^2 \geq 1.1 M_n^2$, where M_y^2 is the mass squared of missing neutrals in the reaction $\pi^- p \rightarrow \pi^+\pi^- y$ and M_n is the neutron mass. Using a Monte Carlo simulation of the experiment, we estimate that this restriction eliminates approximately 6% of the decay $\eta \rightarrow \pi^+\pi^-\gamma$ and none of the $\eta \rightarrow \pi^+\pi^-\pi^0$ decays. Note that this cut removes even those examples of reaction (a) in which a neutron scatters after production.

The remaining sample consists primarily of reactions (b) and (c), but includes some examples of reaction (a) in which a pion has scattered or decayed in the spectrometer.[2] To eliminate these events, we impose restrictions upon the quantity $M_{z^\pm}^2$, where $M_{z^\pm}^2$ is the mass squared in the reaction $\pi^- p \rightarrow n\pi^\mp z^\pm$. We require both $M_{z^+}^2$ and $M_{z^-}^2$ to be larger than $2m_{\pi^+}^2$. This requirement removes most of reaction (a) which remained in the sample while simultaneously eliminating approximately 7% of the decay mode $\eta \rightarrow \pi^+\pi^-\gamma$. None of the $\eta \rightarrow \pi^+\pi^-\pi^0$ decays are eliminated by this cut.

The only remaining examples of reaction (a) which could be confused with reaction (b) are those in which the calculated energy of the γ ray is small. To eliminate this possibility, we require that the laboratory energy of the undetected neutral be larger than 60 MeV. This restriction removes an additional 5% of the $\eta \rightarrow \pi^+\pi^-\gamma$ and no $\eta \rightarrow \pi^+\pi^-\pi^0$.

In order to separate reactions (b) and (c), we examine the distribution M_x^2, where M_x^2 is the mass squared of the undetected neutral in the reaction $\pi^- p \rightarrow n\pi^+\pi^- x$. We assign events in the interval $-0.40 m_{\pi^0}^2 \leq M_x^2 \leq 0.55 m_{\pi^0}^2$ to reaction (b) and those in the interval

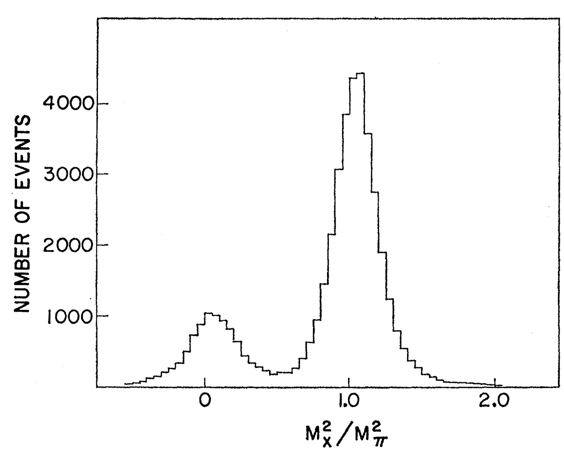

FIG. 1. Histogram of missing mass squared of x in the reaction $\pi^- p \rightarrow n\pi^+\pi^- x$.

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[1] M. Gormley *et al.*, Phys. Rev. Letters **21**, 399 (1968); **21**, 402 (1968); Stephen Stein, Ph.D. thesis, Nevis Report No. 177, Columbia University (unpublished); Michael Gormley, Ph.D. thesis, Nevis report, Columbia University (unpublished). Our sample of $\eta \rightarrow \pi^+\pi^-\pi^0$ decays is smaller than that reported in Gormley *et al.* because (1) we have imposed the restriction that the component of each pion's momentum in the direction transverse to the magnetic field be larger than 60 MeV/c, and (2) we have performed a more careful background subtraction.

[2] The geometric cuts described in Gormley *et al.* eliminate most scattered events.

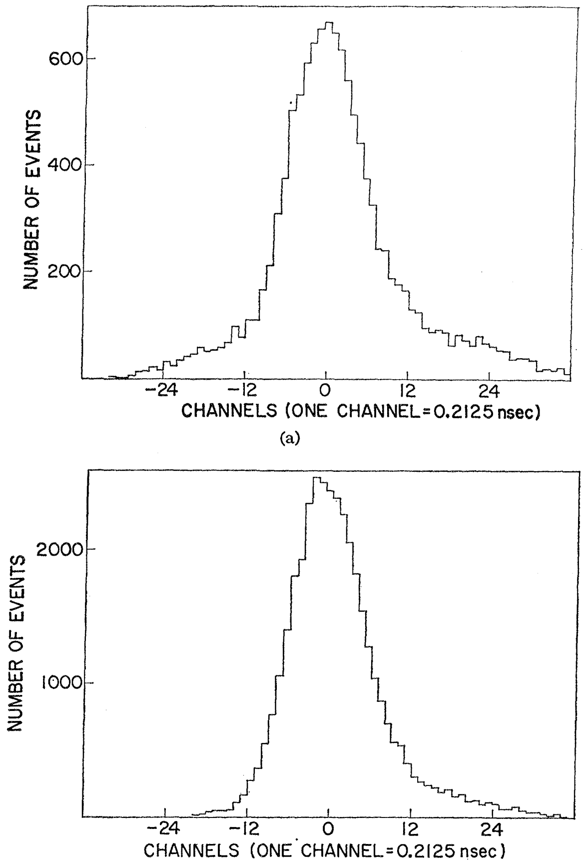

FIG. 2. Histogram of time of flight of neutron. (a) $\pi^- p \to n\pi^+\pi^-\gamma$; (b) $\pi^- p \to n\pi^+\pi^-\pi^0$.

$0.55 m_{\pi^0}{}^2 \leq M_x{}^2 \leq 1.55 m_{\pi^0}{}^2$ to reaction (c). On the basis of the distribution of $M_x{}^2$ shown in Fig. 1, we estimate that events assigned to reaction (b) contain less than a few percent contamination from reaction (c), and those assigned to reaction (c) contain much less than 1% reaction (b).

To eliminate remaining nonresonant background events, we have performed a background subtraction by fitting the neutron time-of-flight spectrum to the sum of a Gaussian and a background polynomial. Figure 2 shows the time-of-flight spectrum for events assigned to reactions (b) and (c).

To correct our data for the variation in detection efficiency over the Dalitz plot, we have used a high-statistics Monte Carlo calculation to calculate the detection efficiency. We have eliminated from both the data and the Monte Carlo simulation any event in which either of the charged pions has less than 60 MeV/c of momentum in the direction transverse to the magnetic field. We have checked the calculation of the detection efficiency by varying the fiducial area in the Monte Carlo simulation. None of the results to be presented are affected by such variations.

I. ANALYSIS OF $\eta \to \pi^+\pi^-\gamma$ DALITZ PLOT

In calculating the experimental detection efficiency, we have subdivided the Dalitz plot into 20 intervals of k (the γ-ray momentum in the η rest frame) and 20 intervals of $\cos\theta$, where θ is the angle between the π^+ and the γ in the dipion center-of-mass system. After correcting for background by performing a flat background subtraction in each bin, we divide the number of events in each bin by its average detection efficiency.

The distribution of $\cos\theta$ is sensitive to the presence of a large C-violating amplitude in $\eta \to \pi^+\pi^-\gamma$ decay. In the absence of a C-violating amplitude, we expect the angular distribution to be represented by

$$\frac{dN}{d(\cos\theta)} = n \sin^2\theta\,, \tag{1}$$

where n is a normalization constant.

The experimental angular distribution, folded about $\cos\theta = 0$, is shown in Fig. 3, together with the results of fitting the distribution to Eq. (1). For nine degrees of freedom, we find $\chi^2 = 4.3$, indicating that the data are consistent with a pure p-wave amplitude. In an attempt to set an upper limit on the magnitude of a possible d-wave amplitude, we have fitted the angular distribution to

$$\frac{dN}{d(\cos\theta)} = n(1 + A\cos^2\theta)\sin^2\theta\,. \tag{2}$$

For eight degrees of freedom, we find $\chi^2 = 3.8$ and $A = -0.060 \pm 0.065$.

The simplest gauge-invariant matrix element for the decay $\eta \to \pi^+\pi^-\gamma$ with the dipion in a p state is

$$|M|^2 \sim |\mathbf{q} \times \mathbf{k}|^2 = q^2 k^2 \sin^2\theta\,, \tag{3}$$

where $\mathbf{k}$ is the photon momentum in the η rest system and $\mathbf{q}$ is the momentum of either pion in the dipion

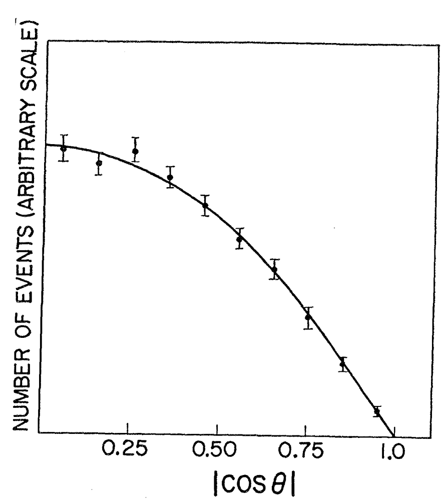

FIG. 3. Folded angular distribution for $\eta \to \pi^+\pi^-\gamma$ corrected for background and detection efficiency.

center-of-mass system.[3] In Fig. 4(a), we compare the experimental γ-ray energy spectrum to the predictions of this matrix element determined by a Monte Carlo calculation. The results indicate that this matrix element is completely inadequate in representing the data. The experimental distribution is shifted to lower energies. For 13 degrees of freedom, we obtain $\chi^2=145$.

In an attempt to find a theoretical model that matches our data better, we include the effect of a π-π final-state interaction assuming that the $I=1$, $J=1$ dipion phase shift is dominated by the ρ meson. Equation (1) now becomes[4]

$$|M|^2 \sim k^2 \sin^2\theta\left(\frac{m_{\pi\pi}}{q}\right)\frac{\Gamma}{(m_\rho^2-m_{\pi\pi}^2)^2+m_\rho^2\Gamma^2}, \quad (4)$$

where

$$\Gamma=(q/q_0)^3\gamma, \quad (5)$$

γ is the reduced width of the ρ, $m_{\pi\pi}$ is the dipion mass, m_ρ is the ρ mass, and q_0 is the value of q at resonance. We use[5]

$$m_\rho=765 \text{ MeV}, \quad \gamma=124 \text{ MeV}, \quad q_0=357 \text{ MeV}. \quad (6)$$

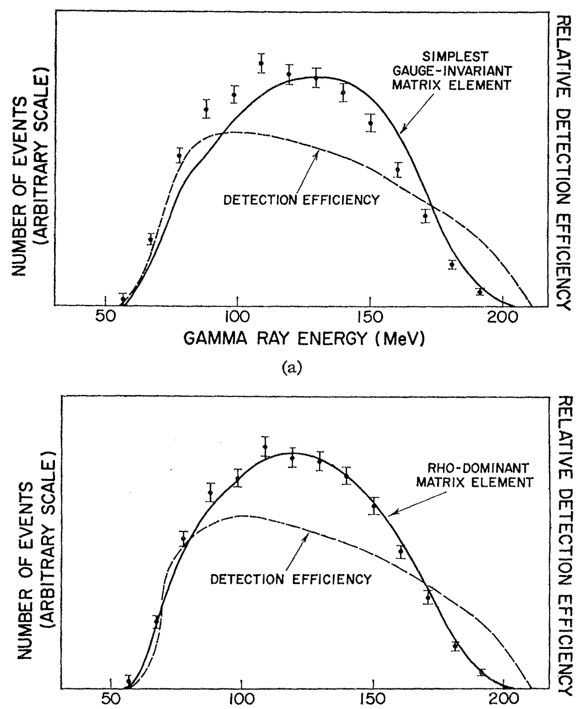

FIG. 4. Histograms of γ-ray energy $\eta \rightarrow \pi^+\pi^-\gamma$ compared with (a) simplest gauge-invariant matrix element and (b) ρ-dominant matrix element. Solid line in both figures shows experimental detection efficiency.

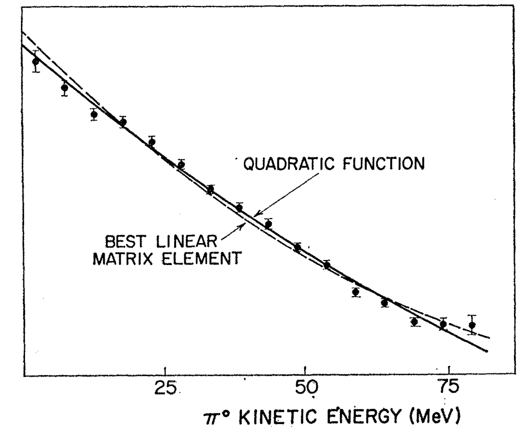

FIG. 5. Histogram of π^0 kinetic energy in $\eta \rightarrow \pi^+\pi^-\pi^0$. Dashed line shows best fit with a linear matrix element to be compared with best fit with a quadratic function (solid line).

In Fig. 4(b) we compare the experimental γ-ray energy spectrum to the predictions of Eq. (4).[6] The agreement is greatly improved over the predictions of Eq. (3) shown in Fig. 4(a). For 13 degrees of freedom, we find $\chi^2=11.5$. To test the sensitivity of our results to the values of the ρ parameters, we have also fitted the γ-ray energy spectrum for a variety of values of the mass and width of the ρ. We find that the experimental spectrum is well described by almost any value for the width of the ρ. For example, using a zero-width resonance (instead of 124 MeV) produces a change in χ^2 of only 0.2. This is to be expected, since in η decay the mass of the two pions is well below the mass of the ρ and the width is only of significance near resonance. The data are, however, considerably more sensitive to the position of the resonance. In order to test this sensitivity, we have fitted the γ-ray energy spectrum assuming the mass of the ρ to range from 600 to 1000 MeV. With 13 degrees of freedom, we find that we obtain a χ^2 smaller than 13 for 660 MeV$\leq m_\rho \leq$775 MeV.

II. DALITZ-PLOT ANALYSIS FOR $\eta \rightarrow \pi^+\pi^-\pi^0$

In analyzing the Dalitz-plot distribution of $\eta \rightarrow \pi^+\pi^-\pi^0$ events, we use the conventional coordinates

$$x=\sqrt{3}\left(\frac{T_+-T_-}{Q}\right), \quad y=3\left(\frac{T_0}{Q}-1\right), \quad (7)$$

where T_{+-0} is the kinetic energy of the π^{+-0} in the three-pion rest system and $Q=M_\eta-m_{\pi^+}-m_{\pi^-}-m_{\pi^0}$. To correct the measured distribution for background and detection efficiency, we proceed as we did for the decay

[3] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[4] J. D. Jackson, Nuovo Cimento **34**, 1944 (1964).

[5] These are the best values of the ρ parameters listed by the Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

[6] Previous experiments which have attempted to observe this effect were either severely limited by statistics or else employed a γ-ray energy cut considerably larger than is used here. Note that a large fraction of our events have γ-ray energies in the interval 50–100 MeV [see Fig. 3(b)]. Frank S. Crawford, Jr., and Leroy R. Price, Phys. Rev. Letters **16**, 333 (1966); P. J. Litchfield *et al.*, Phys. Letters **24B**, 486 (1967); C. Baltay *et al.*, Phys. Rev. Letters **19**, 1498 (1967); A. M. Cnops *et al.*, Phys. Letters **26B**, 398 (1968).

TABLE I. Results of fitting $\eta \to \pi^+\pi^-\pi^0$ Dalitz-plot density to $|M(x,y)|^2 = 1+ay+by^2+cx+dx^2+exy$. $(\cdots)$ means that the corresponding term of the expansion was not included in the fit.

Degrees of freedom	χ^2	a	b	c	d	e
236	216	-1.18 ± 0.02	0.20 ± 0.03	$\cdots$	$\cdots$	$\cdots$
235	211	-1.18 ± 0.02	0.19 ± 0.03	0.05 ± 0.02	$\cdots$	$\cdots$
234	211	-1.18 ± 0.02	0.20 ± 0.03	0.05 ± 0.02	0.04 ± 0.04	$\cdots$
233	211	-1.18 ± 0.02	0.20 ± 0.03	0.05 ± 0.02	0.04 ± 0.04	0.02 ± 0.07
[a] 117	127	-1.17 ± 0.02	0.21 ± 0.03	$\cdots$	0.06 ± 0.04	$\cdots$

[a] The results in this row are obtained by folding the Dalitz plot about $x=0$ and fitting to $|M(|x|,y)|^2 = 1+ay+by^2+dx^2$.

$\eta \to \pi^+\pi^-\gamma$. We subdivide the range of x and y into discrete intervals and group the data by bins. Using the measured neutron time-of-flight spectra, we correct the number of events in each bin by performing a flat-background subtraction. Finally, we divide the number of events in each bin by the average detection efficiency, determined from a high-statistics Monte Carlo simulation.

Theoretical discussions of the decay $\eta \to \pi^+\pi^-\pi^0$ have resorted to a power-series expansion of the matrix element in Dalitz-plot coordinates. We follow such a procedure here, using our experimental data to determine the coefficient of each term in the expansion. We will include as many terms in the expansion as is necessary to represent our data.

Our initial choice of the number of terms to include in the power-series expansion of the matrix element is guided by two experimental observations.

1. The matrix element for the decay $\eta \to \pi^+\pi^-\pi^0$ is known to exhibit a strong dependence upon the π^0 kinetic energy, or, equivalently, the coordinate y.[7]

2. A measurement of the charge asymmetry in this experiment indicated the existence of a small left-right asymmetry which manifests itself in the matrix element in a term linear in x.[1]

Consequently, after correcting the measured Dalitz plot for background and efficiency, we have chosen to make our initial fit of the Dalitz-plot density to a function of the form

$$|M(x,y)|^2 = 1+ay+by^2+cx+dx^2+exy. \quad (8)$$

In Table I, we collect the results obtained by fitting the experimental distribution to a variety of functions of this general form. We summarize the significant features of the table.

1. The coefficients a, b, and c are nonzero. The fitted values for these parameters and their errors are the same for each of the functions used in Table I.

2. The coefficients c and e agree with the results obtained by fitting the Dalitz-plot asymmetry versus x and y, a procedure which did not require a knowledge of the experimental detection efficiency.[8] This agreement provides additional confidence in our Monte Carlo calculation of the detection efficiency.

The results of Table I indicate that we can integrate the matrix element over the Dalitz x coordinate and obtain a function which depends only upon y. To study the y dependence of the Dalitz-plot density, we have fitted the π^0 energy spectrum to

$$M(y) = 1+\alpha y. \quad (9)$$

We find that

$$\mathrm{Re}\alpha = -0.58\pm0.01, \quad \mathrm{Im}\alpha = 0.00\pm0.08,$$

and $\chi^2=51$ for 29 degrees of freedom.

Although these values of Reα and Imα agree with the results of previous experiments,[7] the value of χ^2 suggests that a higher-order expansion of the matrix element is required to represent our data.

The simplest Dalitz-plot density resulting from a nonlinear matrix element is

$$|M(y)|^2 = 1+ay+by^2, \quad (10)$$

where a and b are independent real coefficients. Fitting the π^0 energy spectrum to Eq. (10) yields

$$a = -1.15\pm0.02, \quad b = 0.16\pm0.03,$$

with $\chi^2=36.8$ for 29 degrees of freedom. The π^0 energy spectra predicted by the linear matrix element, Eq. (9), and by the quadratic function, Eq. (10), are shown in Fig. 5.

III. BRANCHING RATIO $\eta \to \pi^+\pi^-\gamma/\eta \to \pi^+\pi^-\pi^0$

We express the branching ratio as the product of two terms:

$$R\left(\frac{\eta \to \pi^+\pi^-\gamma}{\eta \to \pi^+\pi^-\pi^0}\right) = \frac{\epsilon(\eta \to \pi^+\pi^-\pi^0)}{\epsilon(\eta \to \pi^+\pi^-\gamma)} \frac{N(\eta \to \pi^+\pi^-\gamma)}{N(\eta \to \pi^+\pi^-\pi^0)}, \quad (11)$$

where the first term is the ratio of the detection efficiency for $\eta \to \pi^+\pi^-\pi^0$ to that of $\eta \to \pi^+\pi^-\gamma$, and the

[7] A. M. Cnops *et al.* (Ref. 6) find Re$\alpha = -0.55\pm0.02$ and Im$\alpha = 0\pm0.11$; J. Kim *et al.* [Bull. Am. Phys. Soc. **13**, 16 (1968)] give Re$\alpha = -0.54\pm0.04$ and Im$\alpha = 0\pm0.22$; Columbia-Berkeley-Purdue-Wisconsin-Yale Collaboration [Phys. Rev. **149**, 1044 (1966)] find Re$\alpha = -0.48\pm0.04$ and Im$\alpha = 0.05\pm0.39$.

[8] The error associated with the fitted parameters includes statistical contributions from both the data and the Monte Carlo calculation of the efficiency. Our previous fit of the Dalitz-plot asymmetry (see Ref. 1) yielded $\delta = 0.050\pm0.016$ and $\epsilon = -0.015 \pm0.044$. Since these latter values are independent of the Monte Carlo calculation of the efficiency, the errors assigned to them are slightly smaller.

second term is the ratio of the number of observed events. Having determined the matrix elements which describe our data, we can calculate the relative detection efficiency for the two decay modes.

For the decay $\eta \rightarrow \pi^+\pi^-\gamma$, Fig. 4 indicates that our detection efficiency is zero when the energy of the γ ray is less than 50 MeV. In this region our correction is based upon an extrapolation of the matrix element. The fraction of the γ-ray energy spectrum which lies below 50 MeV is, however, very small and quite insensitive to the $\eta \rightarrow \pi^+\pi^-\gamma$ matrix element. For the simplest p-wave matrix element, Eq. (3), only 1.3% of the γ-ray energy spectrum lies below 50 MeV, and

$$\frac{\epsilon(\eta \rightarrow \pi^+\pi^-\pi^0)}{\epsilon(\eta \rightarrow \pi^+\pi^-\gamma)} = 0.861. \qquad (12)$$

For the ρ-dominant matrix element, 3.0% of the spectrum lies below 50 MeV and

$$\frac{\epsilon(\eta \rightarrow \pi^+\pi^-\pi^0)}{\epsilon(\eta \rightarrow \pi^+\pi^-\gamma)} = 0.854. \qquad (13)$$

The difference between Eqs. (12) and (13) indicates the sensitivity of the calculated detection efficiency to the assumed behavior of the matrix element below 50 MeV. Since our data can distinguish clearly between the two matrix elements, our uncertainty in the calculated detection efficiency is undoubtedly smaller than the difference between Eqs. (12) and (13).

We have also investigated the sensitivity of the calculated detection efficiency to an uncertainty in the fiducial area of the spark chambers and the location of the experimental trigger counters. Incorporating all these uncertainties, our best estimate of the calculated detection efficiencies is

$$\frac{\epsilon(\eta \rightarrow \pi^+\pi^-\pi^0)}{\epsilon(\eta \rightarrow \pi^+\pi^-\gamma)} = 0.854 \pm 0.010. \qquad (14)$$

The greater part of the error in this number results from the statistical error of the Monte Carlo calculation.

To determine the ratio of the number of observed events [the second factor in Eq. (11)], we have fitted the experimental time-of-flight spectra for the two decay modes to the sum of a Gaussian and a polynomial background function.

After correcting for background, we are left with 7257 ± 180 $\eta \rightarrow \pi^+\pi^-\gamma$ events and 30 905$\pm$387 $\eta \rightarrow \pi^+\pi^-\pi^0$ events, yielding a ratio

$$\frac{N(\eta \rightarrow \pi^+\pi^-\gamma)}{N(\eta \rightarrow \pi^+\pi^-\pi^0)} = 0.235 \pm 0.007. \qquad (15)$$

We have investigated the sensitivity of the background subtraction to both the assumed form of the background function and to variations in the center and width of the Gaussian. None of these variations yields a result which lies outside the error estimates given in Eq. (15). In general, any function which provides a good fit to the data seems to produce a result in agreement with Eq. (15).

Combining Eqs. (13) and (15) yields a branching ratio

$$R\left(\frac{\eta \rightarrow \pi^+\pi^-\gamma}{\eta \rightarrow \pi^+\pi^-\pi^0}\right) = 0.201 \pm 0.006. \qquad (16)$$

Our measurement of this branching ratio[9] can be combined with the published branching ratios for other decay modes of the η to obtain

$$R = \frac{\eta \rightarrow \pi^+\pi^-\gamma}{\eta \rightarrow \gamma\gamma} = 0.116 \pm 0.015.$$

Here we used the results of a compilation made by Baltay.[10] The result may be compared with the prediction of vector-dominance theory,[11]

$$R = 0.25.$$

In making this prediction, it was assumed in addition to the vector-dominance model that exact $SU(3)$ symmetry holds for the coupling constants and that the η is a pure member of the octet. Presumably, the discrepancy between the prediction and the experimental number can be removed by doing away with one or more assumptions. For example, Chan, Clavelli, and Torgerson have calculated this branching ratio without assuming exact $SU(3)$ symmetry, with the result[12]

$$R \simeq \frac{24}{1000}\left(\frac{g_\rho}{4\pi}\right)^2 \approx 0.1$$

for $g_\rho/4\pi \approx 2.0$. On the other hand, Jacob has suggested that a mixing of η and η' would result in $R \approx 0.1$ if the mixing angle is ~ -0.2 rad.[13]

[9] M. Foster *et al.* [Phys. Rev. **138**, B652 (1965)] and M. Foster [Ph.D. thesis, University of Wisconsin, 1965 (unpublished)] find $R = 0.20 \pm 0.04$; F. Crawford and L. Price [Phys. Rev. Letters **16**, 333 (1966)] find $R = 0.30 \pm 0.06$; C. Baltay *et al.* [*ibid.* **19**, 1498 (1967)] give $R = 0.28 \pm 0.04$; P. J. Litchfield *et al.* [Phys. Letters **24B**, 486 (1967)] find $R = 0.25 \pm 0.035$.

[10] We have used $R(\eta \rightarrow 3\pi^0/\eta \rightarrow \gamma\gamma) = 0.72 \pm 0.05$ and $R(\eta \rightarrow 3\pi^0/\eta \rightarrow \pi^+\pi^-\pi^0) = 1.25 \pm 0.13$. C. Baltay, in *Meson Spectroscopy*, edited by C. Baltay and A. Rosenfeld (Benjamin, New York, 1968).

[11] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[12] L. H. Chan, L. Clavelli, and R. Torgerson, Phys. Rev. **185**, 1754 (1969). We thank Dr. Chan for an illuminating discussion.

[13] M. Jacob, in Proceedings of High-Energy Physics Meeting, Pisa, 1967, p. 233 (unpublished).