

is redefined by

$$V_0(\alpha, s, k) = (\omega^{1/2}/k^\alpha) \bar{V}(\alpha, s, k), \quad (14)$$

we obtain

$$\bar{V}(\alpha, s, k) = k^\alpha + \frac{2G^2}{\pi} \times \int \frac{dq_0 q dq Q_\alpha(x) \bar{V}(\alpha, s, q)}{[(q+iE)^2 + \mu^2][(q-iE)^2 + \mu^2]}. \quad (15)$$

Except for the inhomogeneous term, (15) is the partial-wave Bethe-Salpeter equation.¹ Since the positions of the poles in $\bar{V}(\alpha, s, k)$ do not depend on the inhomogeneous term, we have our desired result. The Regge trajectories, secondary as well as leading, obtained from (2) by means of (6) are identical to those found by solving the Bethe-Salpeter equation, as expected.

Whether (6) is to be preferred to the Bethe-Salpeter equation depends on the questions being investigated. The kernel of (6) is easier to handle than that in (15), but a price is paid in terms of the continuous-dimensional integration. A simple separable approximation to (6) gives quantitatively good results which can be continued above the elastic threshold,⁶ while (15) is to be preferred if exact numerical solutions are desired.² As a method of deriving weak-coupling solutions for secondary trajectories, a sequence of separable approximations to (6) proves to be simpler than either perturbation theory applied directly to ladder diagrams to isolate poles at $\alpha = -N$,¹¹ or weak-coupling approximations to the Bethe-Salpeter equation.¹²

¹¹ I. G. Halliday and P. V. Landshoff, *Nuovo Cimento* **56A**, 983 (1968); A. R. Swift, *J. Math. Phys.* **8**, 2420 (1967).

¹² M. Fontannaz, *Nuovo Cimento* **59A**, 215 (1969).

Asymptotic Symmetry, Current Algebra, and the Veneziano Ansatz

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The universality of the slopes of meson trajectories is established in the context of the Veneziano model from the postulate of asymptotic $SU(3) \times SU(3)$ symmetry and without reference to the Adler consistency conditions.

IT has been suggested by Mandelstam¹ that a Regge-pole model with linearly rising $\alpha(s)$ must have all trajectories parallel: $\alpha_i(s) = a_i + bs$, where b is a universal constant. This conjecture is fairly well supported by experiment, as is evident from an inspection of the Chew-Frautschi plot for meson and baryon trajectories. Recently, Ademollo, Veneziano, and Weinberg² (AVW) have successfully employed this idea of a universal slope in conjunction with the Veneziano representation³ and the Adler partially conserved axial-vector current (PCAC) condition⁴ for a soft pion to predict several mass relations between hadrons. The work of AVW and others⁵ was motivated by the work of Lovelace,⁶ who first pointed out the importance of the Veneziano model and its possible connection with chiral symmetry. The equality of slopes of various Regge trajectories (assumed linear) of either normality can be derived within

the Veneziano framework by appealing to the Adler partial-conservation conditions for π , K , and κ mesons.⁷ There is also the closely related question of the universality of coupling of the ρ meson to other hadrons such as π , K , A_1 , etc. It has been shown⁸ that a universal ρ coupling is a consequence of the requirement that the minimal Veneziano forms for various amplitudes involving the pion be consistent with the low-energy theorems of Adler and Weisberger⁹ (AW). The concepts of a universal slope of trajectories and a universal ρ coupling have emerged, therefore, as consistency conditions imposed on the Veneziano amplitudes by PCAC (PCVC) and charge algebra, respectively.

In the present note, we wish to make an exploratory study of the possible high-energy constraints, if any, on the Veneziano amplitudes for meson systems. Since the structure of the Veneziano amplitude is motivated to a large extent by asymptotic considerations, it appears quite natural to look for constraints (on the amplitude)

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¹ S. Mandelstam, *Phys. Rev.* **166**, 1539 (1968).

² M. Ademollo, G. Veneziano, and S. Weinberg, *Phys. Rev. Letters* **22**, 83 (1969).

³ G. Veneziano, *Nuovo Cimento* **57A**, 190 (1968).

⁴ S. L. Adler, *Phys. Rev.* **137**, B1022 (1965).

⁵ H. J. Schnitzer, *Phys. Rev. Letters* **22**, 1154 (1969); R. Arnowitt, P. Nath, Y. Srivastava, and M. H. Friedman, *ibid.* **22**, 1158 (1969); C. J. Goebel, M. L. Blackmon, and K. C. Wali, *Phys. Rev.* **182**, 1487 (1969).

⁶ C. Lovelace, *Phys. Letters* **28B**, 265 (1968).

⁷ K. Kawarabayashi, S. Kitakado, and H. Yabuki, *Phys. Letters* **28B**, 432 (1968).

⁸ D. McKay and W. Wada, *Phys. Rev. Letters* **23**, 619 (1969); **23**, 1008(E) (1969).

⁹ S. L. Adler, *Phys. Rev. Letters* **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965).

arising from asymptotic symmetry in the sense of Okubo.¹⁰ We find the following results: (a) The universality of the slopes of linear meson Regge trajectories follows from the no-satellite Veneziano forms with the usual added assumption of the absence of exotic resonances without reference to the Adler PCAC constraints by postulating asymptotic $SU(3)$ and $SU(2) \times SU(2)$ symmetries [i.e., $SU(3) \times SU(3)$]. (b) The equalities of the ρ coupling $f_{\rho\pi\pi} = f_{\rho KK}$ and $f_{\rho\rho\rho} = f_{\rho AA}$ follow from asymptotic $SU(3)$ and $SU(2) \times SU(2)$, respectively, without invoking the AW theorems. (c) The universality of the ρ couplings $f_{\rho\pi\pi} = f_{\rho KK} = f_{\rho\rho\rho} = f_{\rho AA}$ can be achieved in the presence of a nonvanishing s -wave $A_{1\rho\pi}$ coupling by imposing asymptotic symmetry on the minimal Veneziano forms for $\pi\pi$, πK , $\pi\rho$, and πA_1 systems in contrast to the situation found in Ref. 8, where it is shown that the AW theorems lead to a universal ρ coupling *only* if $G_S^A = 0$. Finally, we comment on the mutual consistency between the low-energy constraints imposed by PCAC and current algebra, on the one hand, and the high-energy restrictions arising from asymptotic symmetry on the other. We argue that it is unlikely that such a mutual consistency can be achieved within the conventional Veneziano framework unless the no-satellite ansatz is relaxed.

We begin with the following one-term Veneziano ansatz for $\pi\pi$ and πK elastic scattering with the u channel assumed to be exotic:

$$M_{\pi^- \pi^+}(s, t) = 2f_{\rho\pi\pi}^2 \frac{\Gamma(1 - \alpha_\rho(t))\Gamma(1 - \alpha_\rho(s))}{\Gamma(1 - \alpha_\rho(t) - \alpha_\rho(s))}, \quad (1)$$

$$M_{\pi^- K^+}(s, t) = f_{\rho\pi\pi} f_{\rho KK} \frac{\Gamma(1 - \alpha_\rho(t))\Gamma(1 - \alpha_{K^*}(s))}{\Gamma(1 - \alpha_\rho(t) - \alpha_{K^*}(s))}. \quad (2)$$

The amplitudes in Eqs. (1) and (2) have been normalized in the standard fashion.⁸ The Adler conditions⁴ for pion and kaon PCAC require that

$$\alpha_\rho(m_\pi^2) = \frac{1}{2} \quad (3)$$

and

$$\alpha_\rho(m_\pi^2) + \alpha_{K^*}(m_K^2) = \alpha_\rho(m_K^2) + \alpha_{K^*}(m_\pi^2) = 1. \quad (4)$$

Equation (4) with a linear $\alpha_{\rho, K^*}(s)$ implies that

$$\alpha_{\rho'} = \alpha_{K^*}. \quad (5)$$

Equations (3) and (4) also yield

$$\alpha_{K^*}(m_K^2) = \frac{1}{2}. \quad (6)$$

The AW theorems for the $\pi\pi$ and πK systems then imply the coupling relation⁸

$$f_{\rho\pi\pi}^2 = f_{\rho\pi\pi} f_{\rho KK} = \frac{1}{\pi\alpha'} \frac{1}{2F_\pi^2} = \frac{2}{\pi} \frac{m_\rho^2}{2F_\pi^2}. \quad (7)$$

¹⁰ T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967).

We now show that Eq. (5) and the first equality in Eq. (7) follow from asymptotic $SU(3)$ symmetry without reference to the Adler condition and the AW theorems for $\pi\pi$ and πK systems.

First of all, we observe that although exact $SU(3)$ does not ordinarily relate the $\pi\pi$ and πK amplitudes in a simple way, there does exist such a relation within the simple Veneziano framework.

For exact $SU(3)$, we must have

$$\alpha_\rho(s) = \alpha_{K^*}(s) \quad \text{for all } s \quad (8)$$

and

$$f_{\rho\pi\pi} = f_{\rho KK}. \quad (9)$$

Therefore, we have for exact $SU(3)$

$$M_{\pi^- \pi^+}(s, t) = 2M_{\pi^- K^+}(s, t) \quad \text{for all } s \text{ (and } t). \quad (10)$$

We formulate asymptotic $SU(3)$ as follows:

$$\lim_{s \rightarrow \infty} [M_{\pi^- \pi^+}(s, t) - 2M_{\pi^- K^+}(s, t)] = 0, \quad (11)$$

with t arbitrarily fixed.

From the asymptotic behavior of the ratio of the Γ functions, it follows that

$$(M_{\pi^- \pi^+} - 2M_{\pi^- K^+}) \underset{s \rightarrow \infty}{\sim} \Gamma(1 - \alpha_\rho(t)) (-s)^{\alpha_\rho(t)} \times [f_{\rho\pi\pi}^2 (\alpha_{\rho'})^{\alpha_\rho(t)} - f_{\rho\pi\pi} f_{\rho KK} (\alpha_{K^*})^{\alpha_\rho(t)}], \quad (12)$$

where we have used $\alpha(s) \sim \alpha's$.

Equations (11) and (12) imply that

$$f_{\rho\pi\pi} f_{\rho KK}^{-1} = (\alpha_{K^*} / \alpha_{\rho'})^{\alpha_\rho(t)}. \quad (13)$$

Equation (13) must hold for arbitrary t [and hence $\alpha_\rho(t)$]. The f 's do not depend on t and hence

$$\alpha_{K^*} = \alpha_{\rho'}, \quad f_{\rho\pi\pi} = f_{\rho KK}.$$

The above derivation makes it quite clear that the equality of the slopes is closely related to the equality of the ρ couplings. It is also clear that exact $SU(3)$ invariance has survived only in the coupling relation. This procedure may be simply extended to $\pi\pi$ and KK systems.¹¹ In KK ($K\bar{K}$) scattering, the exchange-degenerate ρ as well as the exchange-degenerate ϕ can be exchanged and the ϕ trajectory is assumed to differ from the ρ . Moreover, the (ϕ, ϕ) and (ρ, ρ) terms are excluded in the Veneziano structure. It is then easy to show from asymptotic $SU(3)$ that $\alpha_{\rho'} = \alpha_\phi'$ and $f_{\rho K\bar{K}}^2 = \frac{1}{2} f_{\phi K\bar{K}}^2$. Similarly, one may apply asymptotic $SU(2) \times SU(2)$ symmetry to πK and $\pi\kappa$ elastic scatterings. One writes the minimal forms

$$A_{\pi K^\pm}(s, t, u) = \beta_{\pi K} [V_{st}(K^*, \rho) \pm V_{ut}(K^*, \rho)] \quad (14)$$

and

$$A_{\pi\kappa^\pm}(s, t, u) = \beta_{\pi\kappa} [V_{st}(K_A, \rho) \pm V_{ut}(K_A, \rho)], \quad (15)$$

¹¹ For a review, see M. Jacob, Lecture Notes at Schladming Winter School in Physics, 1969 (unpublished).

where

$$V_{st}(x,y) = \frac{\Gamma(1-\alpha_x(s))\Gamma(1-\alpha_y(t))}{\Gamma(1-\alpha_x(s)-\alpha_y(t))}.$$

Asymptotic $SU(2) \times SU(2)$ in the form

$$\lim_{s \rightarrow \infty; t \text{ fixed}} [A_{\pi K}{}^{3/2}(u,t) - A_{\pi \bar{K}}{}^{3/2}(u,t)] = 0, \quad (16)$$

where $A^{3/2} = A^+ - A^-$, immediately yields $\alpha_{K^*} = \alpha_{K_A'}$ and $\beta_{\pi K} = \beta_{\pi \bar{K}}$. A similar procedure when applied to the KK and $K\bar{K}$ systems yields the equalities $\alpha_{\rho'} = \alpha_{A_1'}$ and $\alpha_{\phi'} = \alpha_{D'}$, where the α_D trajectory at $\alpha_D = 1$ may be identified with the D meson ($I=0, 1^{++}; 1285 \text{ MeV}$).

Thus, the validity of asymptotic $SU(3) \times SU(3)$ implies the universal slope relation

$$\alpha_{\rho'} = \alpha_{K^*} = \alpha_{\phi'} = \alpha_{A_1'} = \alpha_{K_A'} = \alpha_{D'} \quad (17)$$

and the equality of the couplings

$$f_{\rho\pi\pi} = f_{\rho K\bar{K}} = \frac{1}{\sqrt{2}} \sqrt{2} f_{\phi K\bar{K}}, \text{ etc.} \quad (18)$$

We next briefly comment on the $\pi\rho$ and πA_1 systems. The most complete and careful discussions of these two processes have been given by Abers and Teplitz¹² ($\pi\rho$) and by Carruthers and Cooper¹³ (πA_1). As has been emphasized by these authors, there exists a considerable amount of nonuniqueness in the selection of the Veneziano structures appropriate to these scatterings. In view of this arbitrariness inherent in the construction of Veneziano amplitudes for particles with spin, it seems clear that any attempt at applying asymptotic symmetry to such systems will, in general, lead to model-dependent results. There is, however, one point to be made in this connection: the considerations of Carruthers and Cooper do not support the conclusion of Ref. 8 that the s -wave $A_1\rho\pi$ coupling G_S^A is zero. The application of asymptotic $SU(2) \times SU(2)$ to the invariant amplitude $D(s,t)$ brings to light the inadequacy of the somewhat arbitrary choice made in Ref. 8 for the form of this amplitude, thereby pointing once again to the necessity of a more careful treatment of the problem such as that outlined in Ref. 13.

To see this explicitly, let us consider the expressions for $D(s,t)$ given in Ref. 8 seriously and impose asymptotic $SU(2) \times SU(2)$:

$$\begin{aligned} D^{\pi^- \rho^+}(s,t) = & 2f_{\rho\pi\pi} f_{\rho\rho\rho} \frac{\Gamma(1-\alpha_\rho(t))\Gamma(2-\alpha_\pi(s))}{\Gamma(2-\alpha_\rho(t)-\alpha_\pi(s))} \\ & - f_{\rho\pi\pi} f_{\rho\rho\rho} \frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\pi(s))}{\Gamma(2-\alpha_\rho(t)-\alpha_\pi(s))} \\ & + \alpha' g_{\omega\rho\pi}^2 \left\{ \frac{1}{2}st - \frac{1}{4}[s-(m-\mu)^2][s-(m+\mu)^2] \right\} \\ & \times \frac{\Gamma(2-\alpha_\rho(t))\Gamma(1-\alpha_\omega(s))}{\Gamma(3-\alpha_\rho(t)-\alpha_\omega(s))} \quad (19) \end{aligned}$$

¹² E. Abers and V. L. Teplitz, Phys. Rev. Letters **22**, 909 (1969); Phys. Rev. D **1**, 624 (1970).

¹³ P. Carruthers and F. Cooper, Phys. Rev. D **1**, 1223 (1970); M. L. Whippman, *ibid.* **1**, 701 (1970).

and

$$\begin{aligned} D^{\pi^- A_1^+}(s,t) = & 2f_{\rho\pi\pi} f_{\rho A_1 A} \frac{\Gamma(1-\alpha_\rho(t))\Gamma(2-\alpha_\rho(s))}{\Gamma(2-\alpha_\rho(t)-\alpha_\rho(s))} \\ & - (G_S^A)^2 \alpha' \frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\rho(s))}{\Gamma(2-\alpha_\rho(t)-\alpha_\rho(s))}, \quad (20) \end{aligned}$$

where the notation is that of Ref. 8.

Asymptotic $SU(2) \times SU(2)$ in the form

$$\lim_{s \rightarrow \infty; t \text{ fixed}} [D^{\pi^- \rho^+}(s,t) - D^{\pi^- A_1^+}(s,t)] = 0 \quad (21)$$

yields

$$f_{\rho\rho\rho} = f_{\rho A_1 A}, \quad (22)$$

$$g_{\omega\rho\pi} = 0, \quad (23)$$

$$f_{\rho\pi\pi} f_{\rho\rho\rho} = (G_S^A)^2 \alpha'. \quad (24)$$

Equations (22) and (23) are the chiral-symmetric results.¹⁴ Equation (24) tells us that the s -wave $A_1\rho\pi$ coupling constant is nonvanishing. Furthermore, universal ρ coupling would lead to

$$G_S^A = \sqrt{2} m_\rho f_{\rho\pi\pi}. \quad (25)$$

This conclusion is at variance with that of Ref. 8 and thus we see that asymptotic $SU(2) \times SU(2)$ does not imply that $G_S^A = 0$ even if one takes Eqs. (19) and (20) of Ref. 8 seriously.

Turning to the crucial question of mutual consistency among asymptotic $SU(2) \times SU(2)$, the minimal forms Eqs. (19) and (20), and the AW theorems for $\pi\rho$ and πA_1 systems, we observe with Ref. 8 that the AW theorem for πA_1 imposes the following constraint on Eq. (20):

$$2 = 2F_\pi^2 [2\pi\alpha' (G_S^A)^2 \alpha' + 2\pi\alpha' f_{\rho A_1 A} f_{\rho\pi\pi}]. \quad (26)$$

Equations (25) and (26) yield (assuming universality of ρ coupling)

$$f_{\rho\pi\pi}^2 = (1/2\pi) m_\rho^2 / F_\pi^2. \quad (27)$$

This value for $f_{\rho\pi\pi}^2$ is exactly one-half of the value [Eq. (7)] obtained by comparing the minimal Veneziano $\pi\pi$ amplitude with Weinberg's soft-pion current algebra expression.¹⁵ In other words, the contribution of Eq.

¹⁴ S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. **41**, 531 (1969). We observe that although the term in $g_{\omega\rho\pi}$ has the same leading large- s behavior as the first terms on the right-hand side of Eqs. (19) and (20), respectively, there is, however, an *extra* factor of $[1-\alpha_\rho(t)]$ multiplying it and hence this term cannot be combined with the latter in the asymptotic limit; thus we must set $g_{\omega\rho\pi} = 0$ for consistency. This is in accord with the conclusion that asymptotic symmetry leads to exact symmetry results as far as coupling constants are concerned (e.g., $f_{\rho\pi\pi} = f_{\rho K\bar{K}}$ and $f_{\rho\rho\rho} = f_{\rho A_1 A}$).

¹⁵ S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

(20) to the AW sum rule is too large by a factor of 2, if one demands asymptotic $SU(2) \times SU(2)$ and a universal ρ coupling. The situation is reminiscent of the case of πK scattering discussed by Corrigan.¹⁶ It is found there that the simplest Veneziano structure for πK scattering [Eq. (2)] leads to a violation of the AW sum rule for this process by a factor of 2, if the $K^*(890)$ width is fixed at its experimental value. It has been suggested in Ref. 16 that barring 100% unitarity corrections, the only alternative available is to admit the existence of satellite Veneziano structures, and a simple modification [see Eq. (29) below] in this direction is found to be in good agreement with experiment. In the present case, a similar procedure suggests itself. It is appropriate to recall here that asymptotic $SU(2) \times SU(2)$ has led, in other connections, to both the AW sum rules¹⁷ and the Weinberg spectral-function sum rules¹⁸ and therefore there is every reason to take asymptotic $SU(2) \times SU(2)$ quite seriously. Since the choice made in Ref. 8 for $D(s,t)$ is thus clearly inadequate, one must necessarily fall back on the more general Veneziano forms suggested by Carruthers and Cooper.¹³ It is perhaps worth emphasizing at this point that asymptotic $SU(3) \times SU(3)$ will in general provide severe constraints (on the coupling parameters) owing to the presence of several invariant amplitudes; for example, the $\pi\rho$ and πA_1 mass-shell amplitudes involve four invariant amplitudes, namely, $A(s,t)$, $B(s,t)$, $C(s,t)$, and $D(s,t)$.¹⁴ Therefore, asymptotic $SU(2) \times SU(2)$ imposes four different constraints:

$$\lim_{s \rightarrow \infty; t \text{ fixed}} [F^{\pi\rho}(s,t) - F^{\pi A_1}(s,t)] = 0, \quad (28)$$

where $F(s,t) = A(s,t)$, etc. The question of mutual consistency of these constraints as well as applications of asymptotic $SU(3) \times SU(3)$ to inelastic Veneziano amplitudes¹⁹ (e.g., $\pi K \rightarrow A_1 K$ and $\pi K \rightarrow K_A \pi$, $K\pi \rightarrow K\rho$, and $\pi\pi \rightarrow \pi A_1$) will be discussed in an article under preparation. We only wish to remark here that asymptotic symmetry provides a valuable criterion for the introduction (or otherwise) of satellite terms in processes which are related by this symmetry. To see this explicitly, let us return for a moment to the πK problem and take the modification introduced in Ref. 16

seriously:

$$M_{K^+\pi^-}(s,t,u) = \beta_1 \frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_{K^*}(s))}{\Gamma(1-\alpha_\rho(t)-\alpha_{K^*}(s))} + \beta_2 \frac{\Gamma(2-\alpha_\rho(t))\Gamma(1-\alpha_{K^*}(s))}{\Gamma(2-\alpha_\rho(t)-\alpha_{K^*}(s))} + \beta_3 \frac{\Gamma(1-\alpha_\rho(t))\Gamma(2-\alpha_{K^*}(s))}{\Gamma(2-\alpha_\rho(t)-\alpha_{K^*}(s))}, \quad (29)$$

where $\beta_2 + \beta_3 = 0$, so that the Adler condition is satisfied.

Let us impose asymptotic $SU(3)$ on the minimal form Eq. (1) for $\pi\pi$ and Eq. (29). The condition is met provided $\beta_2 = 0$ because the term in β_2 has a large s behavior of $(-s)^{\alpha_\rho(t)-1}$ which cannot be matched against a similar term in Eq. (1). The vanishing of β_2 implies that β_3 is also zero and hence the modified form Eq. (29) reduces to the minimal form Eq. (2). It is therefore clear that the modification introduced in Ref. 16 for the πK case cannot be maintained unless a similar modification is effected in the $\pi\pi$ case.

We end with a few comments.

(a) Although the empirical data appear to favor the no-satellite Veneziano ansatz as a very good first approximation, it appears quite likely that the "true physical" amplitudes should involve satellite terms in order to satisfy both the low-energy AW theorems and the high-energy postulate of asymptotic $SU(3) \times SU(3)$ symmetry.

(b) It is interesting that the universality of slopes of meson trajectories can be established without invoking the Adler consistency conditions. Even in those cases in which the hard-meson PCAC conditions have enabled one to build up models for the inelastic amplitudes in terms of the corresponding elastic ones,²⁰ the postulate of asymptotic symmetry may prove useful in reducing the number of parameters. We suggest that the use of charge algebra, PCAC, and asymptotic $SU(3) \times SU(3)$ symmetry in a complementary fashion and in conjunction with the Veneziano ansatz could provide a useful tool in the correlation of experimental information.

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¹⁶ D. Corrigan, Phys. Rev. **188**, 2465 (1969).

¹⁷ Fayyazuddin and Husain, Phys. Rev. **164**, 1864 (1967); R. Acharya, P. Narayanawamy, and H. H. Aly, *ibid.* **186**, 1567 (1969).

¹⁸ S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

¹⁹ G. Costa (unpublished) has applied asymptotic symmetry arguments to processes involving the η meson.

²⁰ See Ref. 5 and P. Nath, R. Arnowitt, and M. Friedman, Phys. Rev. D **1**, 1813 (1970).