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Regge Effects in Weak Amplitudes*

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Recently Regge effects have been studied in weak amplitudes by describing the final-state strong interaction with a Regge pole at $J=\phi(t)$. In particular, it has been shown that the double-helicity-flip amplitude $A_1(s,t)$ for the process $\gamma\pi\rightarrow\gamma\pi$ has a moving pole at $J=\phi(t)-2$ and a fixed pole at $J=-1$, and that the electromagnetic form factor of the pion, $F_\pi(s)$, can be written as $F_\pi(s)=1+G_P(s)+G_\rho(s)$, where $G_\rho(s)\rightarrow s^{\phi(0)-1}[1+O(1/\ln s)]$. In the present paper, we demonstrate how these results can be obtained by a simple and direct method that draws out an immediate physical understanding of the results obtained. Our method involves the study of the asymptotic behavior of the relevant field-theoretic diagrams with the use of Sudakov variables.

I. FIXED POLES IN WEAK AMPLITUDES

IT has been shown[1–4] that sum rules require that the double-helicity-flip amplitude $A_1(s,t)$ for the process $\gamma\pi\rightarrow\gamma\pi$ must have a fixed pole at $J=1$. This was confirmed[2] in the model of Fig. 1. Here, $T_{\mu\nu}$ is the amplitude for $\gamma\gamma\rightarrow\pi\pi$, I is the sum of irreducible graphs for $\gamma\gamma\rightarrow\pi\pi$, and T is the off-shell $\pi\pi$ scattering amplitude, with

$$T_{\mu\nu}(\Delta,R,P)=I_{\mu\nu}(\Delta,R,P)+\frac{1}{2\pi}\int\frac{d^4X\, I_{\mu\nu}(\Delta,R,X)T(\Delta,X,P)}{[(\frac{1}{2}\Delta+X)^2+\mu^2-i\epsilon][(\frac{1}{2}\Delta-X)^2+\mu^2-i\epsilon]}. \quad (1)$$

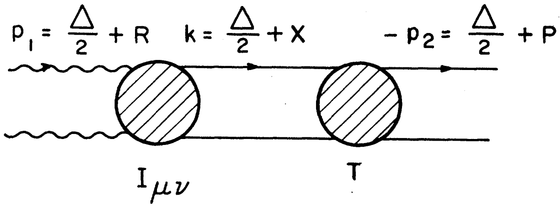

FIG. 1. Model for final-state interactions in the amplitude $T_{\mu\nu}$ of $\gamma\gamma\rightarrow\pi\pi$. Solid lines are pions, wavy lines are photons.

After making the projection

$$T_{\mu\nu}=P_\mu P_\nu A_1+\cdots,\quad I_{\mu\nu}=X_\mu X_\nu I_1+\cdots, \quad (2)$$

and passing to the J plane, it is seen[2] that the J singularities of A_1 are governed by the J singularities of I_1 and T. When the amplitudes I_1 and T are given by the diagrams of Fig. 2, this leads to A_1 having singularities at $J=1$ and $J=\alpha(t)-2$.

We wish to show that this result can be obtained by a simple and direct analysis of Eq. (1) with a technique that displays in an appealing manner a physical understanding for the result. Our results (and much more) were all obtained with the elegant machinery developed in Ref. 2, but we present our technique because it is simple and direct (it does not require any of the P_J'' expansions and Jacob-Wick theory used in Ref. 2), because it displays the precise point at which the effect

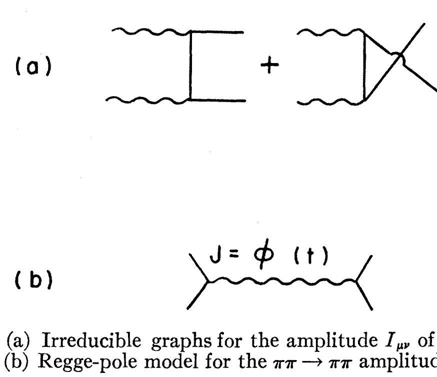

FIG. 2. (a) Irreducible graphs for the amplitude $I_{\mu\nu}$ of $\gamma\gamma\rightarrow\pi\pi$. (b) Regge-pole model for the $\pi\pi\rightarrow\pi\pi$ amplitude.

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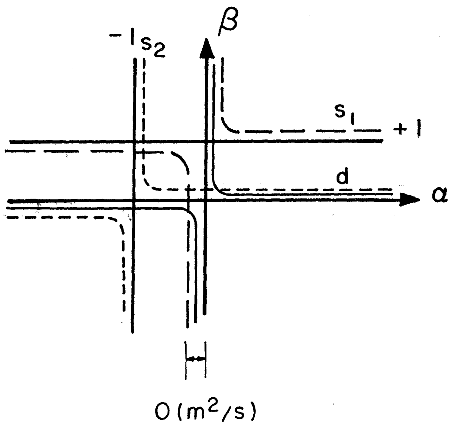

FIG. 3. Region of integration contributing to the leading behavior of $A_1(s,t)$.

of the $P_\mu P_\nu$ projection comes into play, and because it ties in with much of the recent work[5-10] on Regge theory from diagrams.

This technique uses the Sudakov[8,9,11] variables. Lightlike momenta are defined by

$$p_1'^2 = p_2'^2 = 0, \quad 2p_1'\cdot p_2' = s; \tag{3}$$

to lowest order in s,

$$p_1' = p_1 - (M_1^2/s)p_2, \quad p_2' = p_2 - (M_2^2/s)p_1. \tag{4}$$

The internal momenta of integration are replaced by Sudakov variables defined by

$$k = \alpha p_2' + \beta p_1' + K, \quad d^4k = (s/2)d\alpha d\beta dK, \tag{5}$$

where K is a two-dimensional spacelike vector perpendicular to p_1 and p_2. The momentum transfer can be written as

$$\Delta = (t/s)(p_2' p_1') + Q, \quad t = \Delta^2 \simeq Q^2, \tag{6}$$

but for simplicity we will take $t=0$. Then the energy variable of Fig. 1 becomes

$$\begin{aligned} d_1 = d_2 = d = k^2 &= \alpha\beta s + K^2, \\ s_1 = (p_1 - k)^2 &= (1-\beta)(M_1^2/s - \alpha)s + K^2, \\ s_2 = (p_2 + k)^2 &= (1+\alpha)(M_2^2/s + \beta)s + K^2. \end{aligned} \tag{7}$$

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[11] A rigorous formulation of the Sudakov technique has been presented by Negrine (Ref. 6), by studying pinches in the α, β variables that occur when $s \to \infty$. Figure 3 was first studied by Negrine.

We also need the projection of

$$(2k - p_1)_\mu (2k - p_1)_\nu$$

onto $P_\mu P_\nu$. Since

$$p_1 = R, \quad p_2 = -P, \tag{8a}$$

$$k = p_1(\beta - M_1^2\alpha/s) + p_2(\alpha - M_2^2\beta/s) + K, \tag{8b}$$

therefore,

$$2k - p_1 = 2P(-\alpha + M_2^2\beta/s) + R(\beta - M_1^2\alpha/s - 1) + K, \tag{8c}$$

and

$$A(s,0) = s\int d\alpha d\beta dK \frac{(-\alpha + M_2^2\beta/s)^2 I(s_1,0;\, d)}{(d - m^2 + i\epsilon)^2} \times T(s_2,0;\, d). \tag{9}$$

The amplitude I is given by[12]

$$I(s_1,0;\, d) = 1/(s_1 - m^2 + i\epsilon), \tag{10}$$

and the amplitude T satisfies

$$\begin{aligned} T(s_2,0;\, d) &\to s_2^{\phi(0)} g(d) \quad \text{if} \quad s_2 \to \infty \\ &\to 1/d \quad\quad\quad\; \text{if} \quad s_2 \leq d \to \infty\,. \end{aligned}$$

We shall also write T as[13]

$$T(s_2,0;\, d_1) = \int_{4m^2}^{\infty} \frac{dx}{x - s_2} f(x,d). \tag{11}$$

To extract the large-s behavior of $A_1(s,t)$, it is convenient to perform some integrations explicitly. To do this, note[11] that the denominators in Eq. (7) vanish over certain regions in the real $\alpha\beta$ plane, as shown in Fig. 3. In particular, we see that the important region

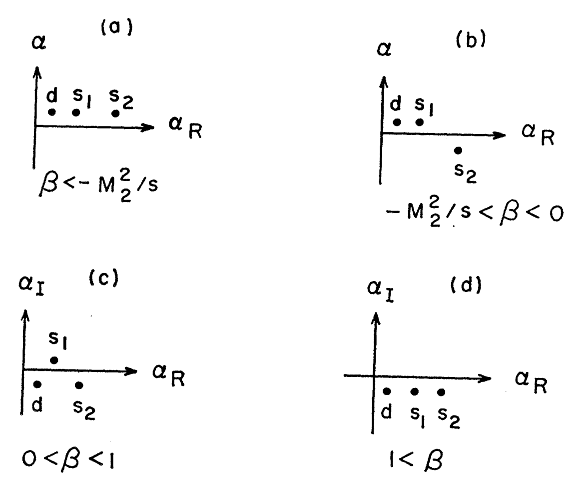

FIG. 4. Distribution of the poles in α of the integrand of Eq. (9) about the real α contour of integration.

[12] The second amplitude of Fig. 2(a) is easily handled by the same technique.

[13] This holds strictly only for $\phi(0) < 0$; the general case can be handled by subtractions.

of integration is the region

$$0<\beta<1, \quad \alpha\sim O(m^2/s), \tag{12a}$$

$$-1<\alpha<0, \quad \beta\sim O(m^2/s). \tag{12b}$$

To express this more precisely, consider the integral (9). As β ranges over the real axis, the terms d, s_1, and s_2 have simple poles in the complex α plane which are prescribed by the $i\epsilon$ prescription:

$$d-m^2+i\epsilon=0, \quad s_1-m^2+i\epsilon=0, \quad s_2-x+i\epsilon=0, \tag{13}$$

and which distribute themselves about the real α contour of integration (Fig. 4). For $\beta>1$ and $\beta<-M_2{}^2/s$, the contribution to Eq. (9) vanishes because the α contour of integration can be closed in the half-plane free of singularities. For $0<\beta<1$, a pole in α from s_1 occurs at

$$\alpha=\frac{M_1{}^2}{s}+\frac{K^2-m^2}{(1-\beta)s}, \tag{14}$$

corresponding to the integration region (12a). Evaluating the residue of (14), we obtain

$$A_1(s,t)\propto\frac{1}{s^2}\int\int dK\int_0^1\frac{d\beta\; h^2T(\beta s+K^2,0;\,d)}{d^2(1-\beta)}, \tag{15a}$$

$$d=\beta\left(M_1{}^2+\frac{K^2-m^2}{1-\beta}\right)+K^2-m^2, \tag{15b}$$

$$h=M_1{}^2+\frac{K^2-m^2}{1-\beta}-\beta M_2{}^2. \tag{15c}$$

Letting $s\to\infty$,

$$A_1(s,t)\to s^{\phi(0)-2}\int\int dK\int_0^1\frac{d\beta\; h^2\beta^{\phi(0)}}{d^2(1-\beta)}g(d). \tag{16}$$

This is the result of the type obtained by Bronzan *et al.*[2] Looking back to Eq. (7), we see that it comes from that region of integration where the energy s_2 flowing through the Reggeon is large (of order s), and the energy s_1 through the left side of the diagram is small. Ordinarily, the presence of the Reggeon would lead to behavior $s^{\phi(0)}$; but the effect of the projection of $X_\mu X_\nu$ onto $P_\mu P_\nu$ introduces an extra factor $1/s^2$ through the coefficient of P in Eq. (8c).

Finally, we turn to the contribution from $\beta\sim O(m^2/s)$ [Fig. 4(b)]. This can be evaluated exactly by picking up the pole in α from the term s_1. However, it is considerably easier to evaluate this contribution if we reverse the roles of α and β by integrating on $-1<\alpha<0$ and picking up the pole in β from s_1 (Fig. 5). One might argue that in so doing we are double-counting the region of integration

$$-1<\alpha<-|O(m^2/s)|, \quad |O(m^2/s)|<\beta<+1 \tag{17}$$

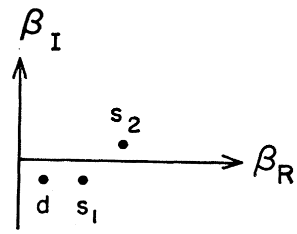

FIG. 5. Poles in β for $-1<\alpha<0$.

(Fig. 3), but it is easily seen that the contributions we are evaluating come from the (nonintersecting) regions of Eqs. (12). The region (17) corresponds to a background integral of Eqs. (12) and gives a lower-order term in s.[14] Evaluating the pole in s_2, then,

$$\beta=-\frac{M_2{}^2}{s}+\frac{x-K^2}{(1+\alpha)s}, \tag{18}$$

and hence

$$A(s,0)\propto\int\int dK\int_{-1}^0\frac{\alpha^2 d\alpha}{(1+\alpha)}\times\int_{4m^2}^\infty\frac{dx\; f(x,d)}{d^2(-\alpha s+K^2+M_1{}^2-m^2)} \tag{19a}$$

$$\to-\frac{1}{s}\int\int dK\int_{-1}^0\frac{\alpha d\alpha}{(1+\alpha)}\int_{4m^2}^\infty\frac{dx\; f(x,d)}{d^2}, \tag{19b}$$

where

$$d=\alpha\left(-M_2{}^2+\frac{x-K^2}{(1+\alpha)}\right)+K^2-m^2. \tag{19c}$$

We see that $A(s,0)$ has a fixed pole at $J=-1$, and this comes from that region of integration where the large energy s flows through the propagator of the amplitude I. Note that the presence of spin, reflected through the term α^2 in Eq. (19a), is essential. Without this term, the integral corresponding to (19b) would diverge. This would mean that $A_1(s,t)$ would behave as $(\ln s)/s$, as a box diagram. The α^2 saves the day, though, and the fixed pole appears.

II. ELECTROMAGNETIC FORM FACTOR OF PION

In Ref. 4 a model was proposed for the electromagnetic form factor of the pion that is based on the diagram of Fig. 6, giving a T matrix proportional to

$$p_\mu-\frac{i}{(2\pi)^4}\int\frac{d^4k(2k+p)_\mu T(s,k^2)}{[(k-p_1)^2-m^2][(k+p_2)^2-m^2]}. \tag{20}$$

[14] This was first noted by Winbow [Eq. (8) of Ref. 10] and more recently expressed rigorously by Negrine (Ref. 6).

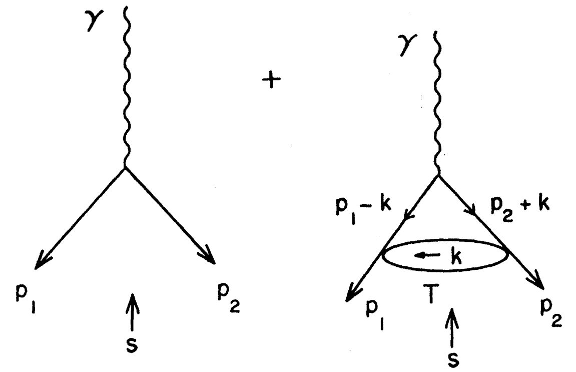

FIG. 6. Model for the electromagnetic form factor of the pion. Solid lines are pions; wavy lines are photons.

This leads to a form factor given by $F_\pi(s)=1+G_P(s)+G_\rho(s)$;

$$G_\rho(s)\propto\int d^4k\left(1+\frac{2p\cdot k}{p^2}\right)\times\frac{A(s,k^2)}{[(k-p_1)^2-m^2][(k+p_2)^2-m^2]}. \quad (21)$$

The amplitude $A(s,k^2)$ was taken to be a Veneziano amplitude.[15] It was expanded in terms of poles, the integrals of Eq. (21) were done directly, and, among other things, the asymptotic behavior of $G_\rho(s)$ was found as $s\to\infty$,

$$G_\rho(s)\to s^{\phi(0)-1}. \quad (22)$$

We would like to show how the result (22) can be obtained directly from Eq. (21) knowing only the asymptotic form of $A(s,k^2)$ and without having to specify its details. Our derivation has two features. First, it uncovers the implied assumption of Ref. 4 that the form factor of $A(s,k^2)$ be neglected. We shall see that form factors would lead to a greatly depressed behavior in (22). Second, it demonstrates that a result like (22) depends only on the asymptotic form of $A(s,k^2)$ and is independent of the particular way (Veneziano model) that this is generated. In particular, the use of a unitarized Veneziano amplitude, say, would not affect the type of result (22).

To extract the asymptotic behavior of $G_\rho(s)$, we again set

$$k=\alpha p_2'+\beta p_1'+K, \quad (23a)$$

$$P=p_1-p_2=p_1'(1-m^2/s)+p_2'(1-m^2/s). \quad (23b)$$

Then

$$G_\rho(s)\propto\int\frac{sd\alpha d\beta dK[1+(\alpha+\beta)(1-m^2/s)^{-1}]A(s,\alpha\beta s+K^2)}{[(1-\beta)(m^2/s-\alpha)s+K^2-m^2+i\epsilon][(1+\alpha)(m^2/s+\beta)s+K^2-m^2+i\epsilon]}. \quad (24)$$

Again performing the integration on α explicitly,

$$\alpha=\frac{1}{(1-\beta)s}(-\beta m^2+K^2) \quad (25)$$

and

$$G_\rho(s)\propto\int d\beta dK\,\frac{1+\beta}{1-\beta}\times\frac{A(s,\beta(-\beta m^2+K^2)/(1-\beta)+K^2)}{\beta s+K^2-m^2}. \quad (26)$$

The main contribution to the β integration comes from the region $0<\beta<\epsilon$ (see Fig. 7), where ϵ is a small fixed number[6]:

$$G_\rho(s)\to\frac{\ln s}{s}\int dK\,A(s,K^2)\propto\frac{\ln s}{s}\int dt'\,A(s,t'). \quad (27)$$

This is the general type of result we wished to obtain. For the special case when $A(s,t')$ is a simple Regge pole,

$$A(s,t')=e^{at'}s^{\phi(t')},\quad \phi(t')=\phi(0)+\phi't', \quad (28)$$

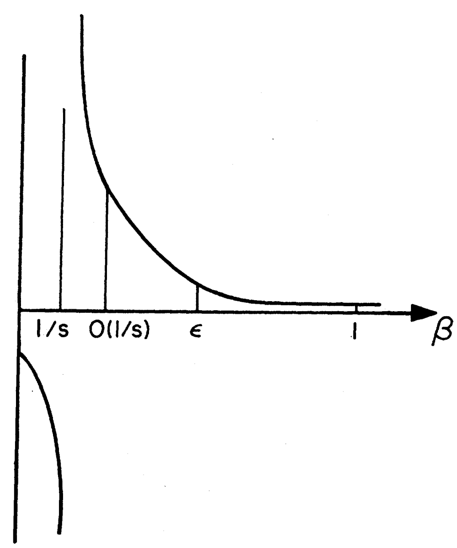

FIG. 7. The term $1/(\beta s-1)$. The main contribution to the integral is $(\ln s)/s$ and comes from the region $O(1/s)<\beta<\epsilon$. The range $0<\beta<O(1/s)$ gives the principal value $1/s$, while the range $\epsilon<\beta<1$ gives a contribution $1/s$.

[15] The term $G_P(s)$ is associated with a Pomeranchon amplitude $B(s,k^2)$ having no resonances.

then

$$G_\rho(s) \to \frac{\ln s}{s} \frac{s^{\phi(0)}}{a+\phi' \ln s} \to s^{\phi(0)-1}\left[1+O\left(\frac{1}{\ln s}\right)\right]. \quad (29)$$

This is the result of Ref. 4. In particular, we see that the leading contribution to $G_\rho(s)$ comes from that region of integration where the line $(k-p_1)^2$ is on the mass shell and the line $(k+p_2)^2$ goes off the mass shell linearly in s. This means that if form factors had been included in the amplitude $A(s,t')$, the behavior (29) would have been depressed to a form $1/s^2$.

III. CONCLUSIONS

The Sudakov technique allows one to evaluate in a simple and direct manner amplitudes arising in weak amplitudes. In particular, effects of the photon spin are displayed, and the asymptotic behaviors obtained are easily associated with regions of integration where the relevant internal energy variables are large.

For scattering processes, the amplitude $A_1(s,0)$ has a fixed pole at $J=-1$ and a moving pole at $J=\alpha(0)-2$. For vertex processes, the form-factor term $G_\rho(s)$ behaves as $s^{\phi(0)-1}[1+O(1/\ln s)]$ when Veneziano $\pi\pi$ amplitudes are used and the pion form factor is neglected; the asymptotic behavior will be depressed if the pion form factor is included.

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Veneziano-Like Model for the Axial-Vector-Current Three-Pion Amplitude*

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In this paper we explore the general features of non-vector-dominance Veneziano models for $A^\mu\pi \to \pi\pi$. We assume that the axial-vector current is conserved (zero-mass pions) and write our invariant amplitudes as products of beta functions and arbitrary functions of the axial-vector-current momentum squared. By using a soft-pion theorem, we are able to make statements about the pion form factor. In a particular model of the above type (the spurion model applied to the five-point function) we find that

$$F(q^2)=\pi^{-1}\Gamma(\tfrac{1}{2})\Gamma(-q^2+\tfrac{1}{2})\Gamma(-\gamma_2+\tfrac{1}{2})/\Gamma(-q^2-\gamma_2+\tfrac{1}{2}).$$

RECENTLY there has been a burst of papers on $A^\mu\pi \to \pi\pi$.[1–6] Most of these papers try to combine field-current identities with the Veneziano-model amplitudes for $\pi\pi \to \pi\pi$ and $\pi\pi \to \pi A_1$ in order to obtain the weak amplitude, or else they assume infinite pole dominance but assume that the A_1 daughters couple to the three-pion system in the same manner as the A_1.[4] It is our philosophy that the generalized Veneziano model which includes knowing the $2n$-point pion amplitudes and which is consistent with factorization is incompatible with the above two assumptions.[7] Specifically, the author has shown in a previous paper that by using spurion techniques on the five-point function for $\sigma+\pi \to \pi\pi\pi$, we find that the A_1 daughters couple differently from the A_1.[8] Also, he found that in a spurion model for $S^\mu\pi \to \pi\pi$ (where S_μ is an isoscalar current), the amplitude for $\omega\pi \to \pi\pi$ is represented by one beta function whereas that for $\gamma\pi \to \pi\pi$ is represented by an infinite number of beta functions, corresponding to the coupling of the ω daughters to the three-pion system.[9] If we assume that the five-point function is given by a single Bardakci-Ruegg (BR)[10] amplitude, and we use spurion techniques, then we obtain an amplitude for

* Work supported in part by the U. S. Office of Naval Research.

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