The solutions (B21) lead to the following relations between $(\tilde{r},\tilde{s})$ and (r,s):

$$\begin{pmatrix}\tilde{r}\\ \\ \tilde{s}\end{pmatrix}=\begin{pmatrix}\dfrac{2-r}{1+4r}\\ \\ \dfrac{3s}{1+4r}\end{pmatrix},\quad\begin{pmatrix}\frac{1}{2}(-r+s)\\ \\ \frac{1}{2}(3r+s)\end{pmatrix},\quad\begin{pmatrix}\dfrac{1-2r-s}{-1+2r-2s}\\ \\ \dfrac{3(1+r)}{1-2r+2s}\end{pmatrix}. \tag{B22}$$

We have already discussed the numerical results that follow from Eq. (B22) in Sec. IV in the text.

Finally, we give the transformation properties of the $SU(3)\times SU(3)$ generators under Eq. (B21). For W, this was already done in Sec. III. It is easy to verify that the second transformation in Eq. (B21) corresponds to $\exp(i\pi F_7)$, which is the finite rotation in $SU(3)$ space that carries the isospin into the V spin. This shows very clearly how the relative weights of U_3 and U_8 in Eq. (B1) get changed. Note that, in this case, the Cabibbo angle undergoes the transformation: $\theta\to\frac{1}{2}\pi+\theta$. We emphasize that an "intrinsic" tadpole model at the $SU(3)$ level, if it contains a "small" isospin-breaking term, is rendered ambiguous by this transformation. Physically, this says that if $F_{1,2,4,5}$ are not conserved, then the statement that $F_{4,5}$ are "less" conserved than $F_{1,2}$ does not have an invariant meaning. Lastly, under X, the generators transform according to

$$(F_{3,8,4,5};F_{3,8,4,5}{}^5)\xrightarrow{X}(F_{3,8,4,5};F_{3,8,4,5}{}^5),$$

$$(F_{1,2,6,7};F_{1,2,6,7}{}^5)\xrightarrow{X}(-F_2{}^5,F_1{}^5,F_7{}^5,-F_6{}^5,\; -F_2,F_1,F_7,-F_6). \tag{B23}$$

This means that if F_1 and F_2 are not conserved, then we may yet construct a third $SU(3)$ group with the generators $(-F_2{}^5, F_1{}^5, F_3, F_4, F_5, F_7{}^5, -F_6{}^5, F_8)$. In the framework in which only the $U(1)\times U(1)$ symmetry is preserved, this $SU(3)$ is not distinguishable from the "ordinary" $SU(3)$.

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Spontaneous Breakdown of Chiral Symmetry with Linear Realizations for Asymptotic Fields

L. Bessler, T. Muta,* H. Umezawa, and D. Welling
Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201
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A method for considering the spontaneous breakdown of chiral symmetry without recourse to a particular Lagrangian model is presented. The physical or asymptotic fields are assigned to linear representations of chiral $SU_3\times SU_3$ together with appropriate c-number addition to the scalar fields. This c-number addition manifests the spontaneous breakdown of the symmetry. Expansion of the interpolating fields in terms of asymptotic fields is introduced, which, together with the spontaneously broken chiral symmetry and the arbitrariness in the choice of interpolating fields, produces sum rules relating leptonic, semileptonic, and strong-coupling constants of $0^{\pm}$ and $1^{\pm}$ mesons. Results similar to those of current algebra are obtained with some notable differences. Among the most important is the appearance of the soft-meson amplitude as a consequence of our mechanism of spontaneous breakdown. We are thereby led, in an exceedingly simple way, to generalized soft-meson sum rules. In particular, new results are given for the semileptonic decays of the K meson, and generalized relations among strong-coupling constants and leptonic decays of $1^{\pm}$ mesons are derived.

I. INTRODUCTION

It is widely recognized that any local operator having the same quantum numbers as an asymptotic field can be chosen as an interpolating, Heisenberg field for that asymptotic field.[1] Although the transition matrix may take different forms off the mass shell (depending upon the choice of the interpolating field), it is unique on the mass shell. Thus, for example, any isovector, pseudoscalar, local operator can be used as an interpolating field for the pion. Specifically, one may choose the divergence of the axial-vector current as its interpolating field. This so-called partially conserved axial-vector current (PCAC) condition[2] has been extensively employed in the current-algebra approach[3] to chiral

* Permanent address: Department of Physics, Kyoto University, Japan.

[1] K. Nishijima, Phys. Rev. **133**, B204 (1964), and references therein; see also K. Nishijima, *High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), p. 137.

[2] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

[3] S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968); B. Renner, *Current Algebras and Their Applications* (Pergamon, Oxford, England, 1968).

symmetry. The current-algebra approach has presented an efficient way of deriving sum rules for the soft-pion amplitudes by making use only of the asymptotic condition (the PCAC assumption) and algebraic properties of currents.

It was nevertheless tempting to construct a Lagrangian which satisfies the PCAC condition and manifests chiral symmetry. This approach is used in chiral dynamics. As a simple example, consider the chiral $SU_2 \times SU_2$ symmetry of the π-σ system. If the π and σ fields are assigned to the basic (quartet) representation of $SU_2 \times SU_2$, then $\boldsymbol{\pi}^2+\sigma^2$ is an invariant. An interesting situation develops in the case where this invariant acts as a c number: The σ field is not independent of the π field and can be eliminated from the Lagrangian. When derivatives are excluded from the interaction term, the Lagrangian invariant under chiral $SU_2 \times SU_2$ is unique in the sense that all the possible invariant interaction terms are c-number constants of the form $(\boldsymbol{\pi}^2+\sigma^2)^N$, with $N=2, 3, \ldots$. The only meaningful interaction term comes from the kinetic-energy part of the Lagrangian when we replace the σ field by $(\text{const}-\boldsymbol{\pi}^2)^{1/2}$. Then the Lagrangian becomes nonlinear in the π field:

$$\mathcal{L}=\tfrac{1}{2}[(\partial_\mu \boldsymbol{\pi})^2+(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})^2/(R^2-\boldsymbol{\pi}^2)],$$

where $R^2=\boldsymbol{\pi}^2+\sigma$. This Lagrangian is nothing but the π-meson part of the invariant nonlinear σ model of Gell-Mann and Lévy,[2] which reproduces many of the current-algebra results when a certain symmetry-breaking term is introduced. Weinberg[4,5] and others[6] have developed a general method of constructing nonlinear Lagrangians. The invariant Lagrangians for π mesons so obtained are all equivalent to the nonlinear σ model in the sense that they reduce to it by suitable redefinitions of the π field. Now, if two Lagrangians are related by field redefinition, they give the same physical results,[7] although this redefinition does change the chiral transformation property of the field. Thus the nonlinear σ model is the most general Lagrangian for the pion system which reproduces current-algebra results.

What we have learned from the Lagrangian version of current algebra, then, is that what is important is not the specific form of the Lagrangian, but rather its physical uniqueness. In other words, it is not the transformation property of the interpolating field but that of the asymptotic field which plays an essential role. We are thus faced with the important question as to how the asymptotic fields behave under the chiral transformation. The transformation of the asymptotic fields in the mass symmetry limit must be one which keeps their free-field equations invariant. Now a nonlinear transformation generates higher powers of the fields, but none of the powers of the asymptotic fields can satisfy the free-field equations. So the possibility of nonlinear transformations of the asymptotic fields is highly implausible.[8] In the following we shall require that the asymptotic fields transform linearly under the chiral transformation in the invariant limit. We note that, in the case of $SU_2 \times SU_2$ chiral dynamics mentioned above, we eliminated the σ field from the Lagrangian. We see now, however, that it should reappear to complete the chiral $SU_2 \times SU_2$ quartet for the asymptotic fields. Hence the σ particle should appear as a bound state of the pions in the invariant limit.[8] The σ could, however, become highly unstable as the breaking mechanism is turned on.

In this paper we shall assign, in the mass symmetry limit, all the physical (asymptotic) fields to linear representations of the chiral $SU_3 \times SU_3$ algebra. It is important to observe that we can freely add c-number constants to the scalar physical fields without changing the canonical commutation relations. However, if the free-field equations are to remain invariant under the c-number transformation, then the masses of such fields must vanish in this invariant limit. But actually the physical particles have nonvanishing masses, and one might think that the mass breaking may cancel out the effects coming from the c-number addition. We, however, assume that the c number remains even when the explicit mass-breaking mechanism is turned on. Hence to complete the $SU_3 \times SU_3$ multiplet of scalar fields we include the possible addition of c numbers.[9] This addition of c numbers to the scalar multiplet is the manifestation of the spontaneous breakdown[10] of chiral $SU_3 \times SU_3$ symmetry. In particular, we shall add c numbers only to the $I=Y=0$ members of the physical scalar multiplet; we shall add no c numbers which would prevent the conservation of isospin and hypercharge. It should be remarked that the addition of c numbers to

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[4] S. Weinberg, Phys. Rev. Letters **18**, 188 (1967); J. Schwinger, Phys. Letters **24B**, 473 (1967).

[5] S. Weinberg, Phys. Rev. **166**, 1568 (1968).

[6] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, Jr., S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2247 (1969).

[7] S. Kamefuchi, L. O'Raifeartaigh, and A. Salam, Nucl. Phys. **28**, 529 (1961); see also L. Bessler, Phys. Rev. **184**, 1523 (1969).

[8] H. Umezawa, University of Wisconsin-Milwaukee Report No. UWM-4867-68-6, 1968 (unpublished).

[9] The present paper is mainly a generalization to $SU_3 \times SU_3$ of our previous paper: T. Muta and H. Umezawa, University of Wisconsin-Milwaukee Report No. UWM-4867-69-4, 1969 (unpublished). We would also like to point out that the c numbers considered here as a manifestation of spontaneous breakdown have some similarity to the c number which measures the explicit symmetry breaking in the model of M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968). See also S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).

[10] See, e.g., G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, in *Advances in Particle Physics*, edited by P. L. Cool and R. E. Marshak (Interscience, New York, 1968), Vol. 2. The possibility of the spontaneous breakdown of the chiral $SU_2 \times SU_2$ symmetry has already been suggested by P. B. Kantor and J. L. Pientenpol, Phys. Rev. Letters **21**, 241 (1968); A. Salam and J. Strathdee, International Center for Theoretical Physics Report No. IC/68/109, 1968 (unpublished). Certain models exhibiting the spontaneous breakdown of chiral $SU_3 \times SU_3$ symmetry have been discussed by G. Cicogna, F. Strocchi, and R. Vergara Caffarelli, Phys. Rev. Letters **22**, 497 (1969), and by J. Honerkamp, Universität Bonn Report, 1969 (unpublished).

any fields other than the scalar field would spontaneously break parity conservation, Lorentz invariance, etc. The c numbers added to the scalar fields are fundamental constants in our theory and will be determined in terms of the leptonic decay constants of the pseudoscalar mesons.

We shall use the technique of expanding Heisenberg operators in terms of the physical fields[11] and shall apply the transformation rules of chiral $SU_3 \times SU_3$ to the expansions in order to derive soft-meson sum rules. We shall derive sum rules on the leptonic and semileptonic decays of mesons. By taking advantage of the freedom of choice of interpolating fields, we shall also obtain the generalized Adler consistency condition[12] and relations involving the strong coupling constants of mesons. We consider mesons only, but it is not difficult to introduce baryons into our scheme.

In Sec. II we present the formulation of the theory and determine the constants C_0 and C_8 in terms of the leptonic decays of spinless mesons. Particular results are given in Sec. III. In Sec. IV we make some concluding remarks about the nature of the spontaneous breakdown of chiral symmetry and illustrate the difference between this approach and current algebra.

II. THEORY

A. Commutation Relations

As was mentioned, we shall be concerned with $0^\pm$ and $1^\pm$ spin-parity physical (asymptotic) fields, together with appropriate c numbers, assigned to linear representations of the chiral $SU_3 \times SU_3$ algebra in the invariant limit. This algebra is defined by the 16 generators

$$Q_A^{\pm} = \tfrac{1}{2}(T_A \pm X_A) \tag{1}$$

($A = 1, \ldots, 8$) with nonvanishing commutation relations

$$[Q_A^{\pm}, Q_B^{\pm}] = i f_{ABC} Q_C^{\pm}, \tag{2}$$

where the f_{ABC} are the structure constants of SU_3. The action of the parity operator P is

$$P T_A P^{-1} = +T_A, \tag{3a}$$

$$P X_A P^{-1} = -X_A. \tag{3b}$$

The T_A and X_A are the space-integrated charge densities of the vector $V_{\mu,A}$ and axial-vector $A_{\mu,A}$ currents, respectively:

$$T_A = \int d^3x\, V_{0,A}(x), \tag{4a}$$

$$X_A = \int d^3x\, A_{0,A}(x). \tag{4b}$$

[11] R. Hagedorn, *Introduction to Field Theory and Dispersion Relations* (Pergamon, Oxford, England, 1964), p. 27; see also L. Leplae, R. N. Sen, and H. Umezawa, Progr. Theoret. Phys. (Kyoto) Suppl. Extra No., 637 (1965).

[12] S. L. Adler, Phys. Rev. **137**, B1022 (1965).

The local commutation relations are

$$[T_A, V_{\mu,B}(x)] = i f_{ABC} V_{\mu,C}(x), \tag{5a}$$

$$[T_A, A_{\mu,B}(x)] = i f_{ABC} A_{\mu,C}(x), \tag{5b}$$

$$[X_A, V_{\mu,B}(x)] = i f_{ABC} A_{\mu,C}(x), \tag{5c}$$

$$[X_A, A_{\mu,B}(x)] = i f_{ABC} V_{\mu,C}(x). \tag{5d}$$

We shall choose the $(3,\bar{3})+(\bar{3},3)$ representation for the 18 0^+, 0^- mesons S_i, P_j, where $i, j = 0, \ldots, 8$. The appropriate transformation properties for the *asymptotic* fields are then given by

$$[T_A, S_i(x)] = i f_{Aij} [S_j(x) + C_j], \tag{6a}$$

$$[T_A, P_i(x)] = i f_{Aij} P_j(x), \tag{6b}$$

$$[X_A, S_i(x)] = -i d_{Aij} P_j(x), \tag{6c}$$

$$[X_A, P_i(x)] = i d_{Aij} [S_j(x) + C_j], \tag{6d}$$

where, following the argument in the Introduction, we have added c numbers to S_j without violating isospin or hypercharge conservation; that is,

$$C_j = \delta_{j0} C_0 + \delta_{j8} C_8,$$

with C_0 and C_8 to be determined. The nonvanishing of C_0 and C_8 produces the spontaneous breakdown of chiral symmetry and SU_3 symmetry, respectively. It is interesting to note that if the linear representation $(1,8)+(8,1)+(1,1)$ had been chosen instead of $(3,\bar{3}) + (\bar{3},3)$, the pion would not decay. Indeed, in that case, $\langle 0 | [X_A, P_i(x)] | 0 \rangle = i f_{Aij} C_j = 0$, for $i = 1, 2, 3$.

The 1^+, 1^- mesons $a_{\mu,j}$, $v_{\mu,j}$ are chosen to belong to the $(1,8)+(8,1)+(1,1)$ representation of $SU_3 \times SU_3$. This representation is chosen so that opposite charge parity is guaranteed for $v_{\mu,3}$ and $a_{\mu,3}$. Here we have

$$[T_A, v_{\mu,i}(x)] = i f_{Aij} v_{\mu,j}(x), \tag{7a}$$

$$[T_A, a_{\mu,i}(x)] = i f_{Aij} a_{\mu,j}(x), \tag{7b}$$

$$[X_A, v_{\mu,i}(x)] = i f_{Aij} a_{\mu,j}(x), \tag{7c}$$

$$[X_A, a_{\mu,i}(x)] = i f_{Aij} v_{\mu,j}(x), \tag{7d}$$

where $i, j = 0, \ldots, 8$ and where $v_{\mu,i}(x)$ and $a_{\mu,i}(x)$ are *asymptotic* fields.

B. Determination of C_0 and C_8

The fundamental constants in our theory, C_0 and C_8, will be determined in terms of the pure leptonic decays of the pseudoscalar and scalar mesons.

Consider $P_i \to$ leptons. We obtain, using (6d),

$$\langle 0 | [X_A, P_i(x)] | 0 \rangle = i d_{Aij} C_j, \tag{8a}$$

and, using (4b) and the fact that $P_i(x)$ is a physical field,

$$\langle 0|[X_A,P_i(x)]|0\rangle = \{\langle 0|A_{0,A}(0)|P_i(k)\rangle[(2\pi)^3 2k_0]^{1/2}/k_0\}_{\mathbf{k}=0}, \quad (8b)$$

where $\mathbf{k}$ is the three-momentum.

Thus,

$$id_{Aij}C_j = \{\langle 0|A_{0,A}(0)|P_i(k)\rangle[(2\pi)^3 2k_0]^{1/2}/k_0\}_{\mathbf{k}=0}. \quad (9)$$

Similarly, for $S_i \to$ leptons, we employ (6a) and (4a) to obtain

$$if_{Aij}C_j = \{\langle 0|V_{0,A}(0)|S_i(k)\rangle[(2\pi)^3 2k_0/k_0]^{1/2}\}_{\mathbf{k}=0}. \quad (10)$$

It should be noted that, because of the f-type combination in (10), only strangeness-changing decays through the vector current are allowed.

Relations (9) and (10) then enable us to express the C_j in terms of F_π, F_K, and F_κ, the pion, kaon, and κ leptonic decay constants. We define

$$\tfrac{1}{2}\sqrt{2}\langle 0|A_\mu{}^{1-i2}(0)|\pi^+(k)\rangle = ik_\mu F_\pi/[(2\pi)^3 2k_0]^{1/2}, \quad (11a)$$

$$\tfrac{1}{2}\sqrt{2}\langle 0|A_\mu{}^{4-i5}(0)|K^+(k)\rangle = ik_\mu F_K/[(2\pi)^3 2k_0]^{1/2}, \quad (11b)$$

$$\tfrac{1}{2}\sqrt{2}\langle 0|V_\mu{}^{4\mp i5}(0)|\kappa^\pm(k)\rangle = \mp k_\mu F_\kappa/(2\pi)^3 2k_0]^{1/2}, \quad (11c)$$

where $A_\mu{}^{1-i2}(0) \equiv A_{\mu,1}(0) - iA_{\mu,2}(0)$, etc. Substitution of (9) and (10) into (11) yields

$$F_\pi = (\sqrt{2}C_0 + C_8)/\sqrt{3}, \quad (12a)$$

$$F_K = (2\sqrt{2}C_0 - C_8)/2\sqrt{3}, \quad (12b)$$

$$F_\kappa = \tfrac{1}{2}\sqrt{3}C_8 = F_\pi - F_K. \quad (12c)$$

The experimental values for F_K and F_π imply that neither C_0 nor C_8 can vanish, but that C_8 is rather small $|C_8/C_0| \simeq 0.20$.

C. Currents in Terms of Asymptotic Fields

We now construct the expansions of the currents in terms of the asymptotic fields (see Hagedorn[11]):

$$V_{\mu,A}(x) = f_A v_{\mu,A}(x) - f_{Abd}C_d\partial_\mu S_b(x) + \frac{1}{2!}\int dydz\, F_{\mu,Abc}{}^{(1)}(xyz)P_b(y)P_c(z) + \frac{1}{2!}\int dydz\, F_{\mu,Abc}{}^{(2)}(xyz)S_b(y)S_c(z)$$
$$+ \int dydz\, F_{\mu\nu,Abc}{}^{(3)}(xyz)a^\nu{}_{,b}(y)P_c(z) + \int dydz\, F_{\mu\nu,Abc}{}^{(4)}(xyz)v^\nu{}_{,b}(y)S_c(z) + \cdots, \quad (13)$$

$$A_{\mu,A}(x) = g_A a_{\mu,A}(x) - d_{Abd}C_d\partial_\mu P_b(x) + \int dydz\, G_{\mu,Abc}{}^{(1)}(xyz)S_b(y)P_c(z) + \int dydz\, G_{\mu\nu,Abc}{}^{(2)}(xyz)v^\nu{}_{,b}(y)P_c(z)$$
$$+ \int dydz\, G_{\mu\nu,Abc}{}^{(3)}(xyz)a^\nu{}_{,b}(y)S_c(z) + \frac{1}{3!}\int dydzdw\, G_{\mu,Abcd}{}^{(4)}(xyzw)P_b(y)P_c(z)P_d(w) + \cdots, \quad (14)$$

where the products of asymptotic fields are understood to be normal ordered. In order to demonstrate how the coefficients $F_{\mu,Abc}{}^{(1)}(xyz)$, etc., are identified, we take the matrix element of $V_{\mu,A}(x)$ between the pseudoscalar-meson states and use the reduction formula[13] to obtain

$$F_{\mu,Abc}{}^{(1)}(xyz) = i^2(\Box_y + m_b{}^2)(\Box_z + m_c{}^2) \times \langle 0|T[V_{\mu,A}(x)\hat{P}_b(y)\hat{P}_c(z)]|0\rangle,$$

where $\hat{P}_b(y)$ and $\hat{P}_c(z)$ are interpolating fields. The coefficients are, in general, vacuum expectation values of the retarded (or advanced) products of the relevant Heisenberg operators. However, up to the term of third order in the asymptotic fields, these coefficients can be rewritten in the form of the T product. In writing down the second terms in (13) and (14), use was made of the relations (9) and (10).

The constants f_A and g_A which appear in Eqs. (13) and (14) are determined experimentally from the decays $v \to$ leptons and $a \to$ leptons. Since isospin and hypercharge are conserved, we must have

$$f_1 = f_2 = f_3 \equiv f_\rho,$$
$$f_4 = f_5 = f_6 = f_7 \equiv f_{K^*}, \quad (15)$$
$$f_8 \equiv f_\phi, \text{ etc.}$$

D. Derivation of Sum Rules

We first impose the commutation relations (5)–(7) on the expansions (13) and (14) to obtain, for example,

$$f_{ABC}A_{\mu,C}(x) = -i[X_A, V_{\mu,B}(x)]$$
$$= f_B f_{ABc}a_{\mu,c}(x) + d_{Abc}f_{Bbd}C_d\partial_\mu P_c(x)$$
$$+ d_{Acd}C_d\int dydz\, F_{\mu,Bbc}{}^{(1)}(xyz)P_b(y)$$
$$+ d_{Ade}C_e\int dydz\, F_{\mu\nu,Bcd}{}^{(3)}(xyz)a^\nu{}_{,c}(y) + \cdots. \quad (16)$$

We then take various matrix elements of (16).

[13] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento 1, 205 (1955).

Consider, e.g.,

$$\begin{aligned} f_{ABC}\langle 0|A_{\mu,C}(x)|P_a(k)\rangle &= -i\langle 0|[X_A,V_{\mu,B}(x)]|P_a(k)\rangle \\ &= -ik_\mu d_{Aba}f_{Bbd}C_d e^{-ikx}/[(2\pi)^3 2k_0]^{1/2} \\ &\quad + d_{Acd}C_d\int dydz\, F_{\mu,Bac}{}^{(1)}(xyz)\frac{e^{-iky}}{[(2\pi)^3 2k_0]^{1/2}}. \end{aligned} \quad (17)$$

We immediately notice that the last term on the right-hand side of Eq. (17) is nothing but the soft-meson amplitude, i.e.,

$$\{[(2\pi)^3 2k_0']^{1/2}\langle P_c(k')|V_{\mu,B}(x)|P_a(k)\rangle\}_{k'=0} = \int dydz\, F_{\mu,Bac}{}^{(1)}(xyz)\frac{e^{-iky}}{[(2\pi)^3 2k_0]^{1/2}}. \quad (18)$$

The soft-meson limit obtained here is a natural consequence of the addition of c numbers to the physical scalar-meson fields. Whenever we use the reduction technique, we are assuming the symmetric mass for each chiral multiplet. We will break the mass symmetry only in the final stages of our calculation.

Thus we are led to

$$\begin{aligned} f_{ABC}\langle 0|A_{\mu,C}(x)|P_a(k)\rangle &= -ik_\mu d_{Aba}f_{Bbd}C_d e^{-ikx}/[(2\pi)^3 2k_0]^{1/2} \\ &\quad + d_{Acd}C_d\{[(2\pi)^3 2k_0']^{1/2}\langle P_c(k')|V_{\mu,B}(x)|P_a(k)\rangle\}_{k'=0}, \end{aligned} \quad (19)$$

or

$$d_{Acd}C_d\{[(2\pi)^6 4k_0k_0']^{1/2}\langle P_c(k')|V_{\mu,B}(0)|P_a(k)\rangle\}_{k'=0} = ik_\mu(f_{ABC}d_{Cab}C_b + d_{Aba}f_{Bbd}C_d). \quad (20)$$

We have, finally,

$$\langle P_a(0)|V_B|P_a(k)\rangle = ik_\mu f_{aBa}, \quad (21a)$$

where, for convenience, we have used

$$\{[(2\pi)^6 4k_0k_0']^{1/2}\langle P_c(k')|V_{\mu,B}(0)|P_a(k)\rangle\}_{k'=0} \equiv \langle P_c(0)|V_B|P_a(k)\rangle.$$

We note that the c numbers on both sides of (21a) canceled out. By the same procedure, we can also obtain

$$\langle S_a(0)|V_B|S_c(k)\rangle = ik_\mu f_{aBc}, \quad (21b)$$

$$\langle S_a(0)|A_B|P_c(k)\rangle = ik_\mu d_{aBc}, \quad (21c)$$

$$\langle P_a(0)|A_B|S_c(k)\rangle = -ik_\mu d_{aBc}, \quad (21c')$$

$$f_{Aa8}C_8\langle S_a(0)|V_B|v_c(k)\rangle = 0, \quad (21d)$$

$$f_{Aa8}C_8\langle S_a(0)|A_B|a_c(k)\rangle = 0, \quad (21e)$$

$$\langle P_A(0)|A_B|v_C(k)\rangle = \epsilon_\mu(k)f_{ABC}(f_C-g_B)/F_A, \quad (21f)$$

$$\langle P_A(0)|V_B|a_C(k)\rangle = \epsilon_\mu(k)f_{ABC}(g_C-f_B)/F_A, \quad (21g)$$

$$\begin{aligned} \langle P_A(0)P_a(k')|A_B|P_b(k)\rangle &= [f_{ABC}\langle P_a(k')|V_C|P_b(k)\rangle \\ &\quad + d_{Aca}\langle S_c(k')|A_B|P_b(k)\rangle \\ &\quad + d_{Acb}\langle P_a(k')|A_B|S_c(k)\rangle]/F_A, \end{aligned} \quad (21h)$$

where in the last three equations we have used the fact that, when $d\neq 0$,

$$d_{Ade}C_e = F_A\delta_{Ad},$$

with

$$F_1=F_2=F_3=F_\pi,$$
$$F_4=F_5=F_6=F_7=F_K,$$
$$F_8=\tfrac{1}{3}(4F_K-F_\pi).$$

E. Spinless Heisenberg Operators

So far we have employed the expansions of the vector and axial-vector currents in terms of asymptotic fields. In general we can expand any local Heisenberg operator in terms of asymptotic fields. Here we shall make use of the expansion of the interpolating field of the 0^- meson,

$$\begin{aligned} \hat{P}_a(x) &= P_a(x) + \int dydz\, R_{abc}{}^{(1)}(xyz)S_b(y)P_c(z) \\ &\quad + \frac{1}{3!}\int dydzdw\, R_{abcd}{}^{(2)}(xyzw) \\ &\qquad \times P_b(y)P_c(z)P_d(w) + \cdots, \end{aligned} \quad (22)$$

where the coefficients $R_{abc}(xyz)$, etc., are identified by taking suitable matrix elements of $\hat{P}_a(x)$ between relevant states. Applying the commutation relation, we have

$$\begin{aligned} [X_A,\hat{P}_a(x)] &= id_{Aab}C_b - id_{Abd} \\ &\quad \times \int dydz\, R_{abc}{}^{(1)}(xyz)P_d(y)P_c(z) \\ &\quad + \frac{1}{2!}id_{Ade}C_e\int dydzdw\, R_{abcd}{}^{(2)}(xyzw) \\ &\qquad \times P_b(y)P_c(z) + \cdots. \end{aligned} \quad (23)$$

We again use our basic requirement that any local operator which has the same quantum numbers as the asymptotic field can be taken to be its interpolating field. Thus we make the identification

$$\hat{S}_b(x) \equiv iN_{bAd}\{[X_A,\hat{P}_d(x)] - id_{Adc}C_c\}, \quad (24)$$

where N_{bAd} is a suitable renormalization constant. Inserting Eq. (23) into Eq. (24) and taking a matrix element, we get

$$\begin{aligned} \langle P_a(k')|\hat{S}_b(0)|P_c(k)\rangle &= N_{bAd}\{d_{Aea}\langle S_e(k')|\hat{P}_d(0)|P_c(k)\rangle \\ &\quad + d_{Aec}\langle P_a(k')|\hat{P}_d(0)|S_e(k)\rangle \\ &\quad - d_{Aef}C_f\{\langle P_a(k')P_e(k'')|\hat{P}_d(0)|P_c(k)\rangle \\ &\qquad \times [(2\pi)^3 2k_0'']^{1/2}\}_{k''=0}\}. \end{aligned} \quad (25)$$

III. RESULTS

A. Semileptonic Decays of $1^{\pm}$ Mesons

The immediate consequence of Eqs. (21d) and (21e) is that the vector and axial-vector meson decays into κ mesons and leptons are forbidden in the soft-κ-meson limit.

The decays $v\to P+$leptons and $a\to P+$leptons are described by Eqs. (21f) and (21g). We list here the predictions for two interesting $v\to P+$leptons processes[14]:

$$\rho^{\pm}\to\pi^0 l^{\pm}\nu,\quad \pm i\sqrt{2}(f_\rho-g_{A_1})\epsilon_\mu/F_\pi;$$
$$\rho^{-}\to K^0 l^{-}\nu,\quad +i(f_\rho-g_{K_A})\epsilon_\mu/F_K.$$

Let us consider the process $\rho^{\pm}\to\pi^0 l^{\pm}\nu$. In order to compare our result with experiment we must determine f_ρ and g_{A_1} which characterize the amplitudes for the processes $\rho\to l\bar{l},l\nu$ and $A_1\to l\nu$, respectively. The decays $\rho\to e^+e^-$ and $\rho\to\mu^+\mu^-$ have already been observed, but there are no data on the decay $A_1\to l\nu$.

B. Semileptonic Decays of $0^{\pm}$ Mesons

The well-known result for K_{l3} decay derived from current algebra[15] is, in our notation,

$$\langle\pi^0(0)|V^{4-i5}|K^+(k)\rangle=-\tfrac{1}{2}\sqrt{2}(F_K/F_\pi)k_\mu,\qquad(26)$$

while our result from (21a) is

$$\langle\pi^0(0)|V^{4-i5}|K^+(k)\rangle=-\tfrac{1}{2}\sqrt{2}k_\mu.\qquad(27)$$

The origin of this discrepancy is the presence of the scalar nonet in our scheme. Indeed, if in (26) we replace F_K by F_K+F_8 and use (12c), we obtain (27). (See Sec. IV.)

In the case of the decay $\pi^+\to\pi^0 l^+\nu$, the scalar-meson contribution vanishes in (21a) and we obtain exactly the same result as in current algebra,

$$\langle\pi^0(0)|V^{1-i2}|\pi^+(k)\rangle=-\sqrt{2}k_\mu.\qquad(28)$$

The relations (21b), (21c), and (21c′) yield results for the decays $S\to S+$leptons, $P\to S+$leptons, $S\to P$ +leptons. So far these processes have not been observed.

C. Decays $P\to PP+$ Leptons

The K_{l4} decays have also been analyzed by using the current-algebra approach.[16] Our results for these processes, (21h), agree with those of current algebra *except for the scalar-meson contributions*. In particular, we obtain the following relations:

$$F_\pi\langle\pi^+(0)\pi^-(k')|A^{4-i5}|K^+(k)\rangle=(\sqrt{\tfrac{2}{3}})\langle\sigma_0(k')|A^{4-i5}|K^+(k)\rangle+(\sqrt{\tfrac{1}{3}})\langle\sigma_8(k')|A^{4-i5}|K^+(k)\rangle+(\sqrt{\tfrac{1}{2}})\langle\pi^-(k')|A^{4-i5}|\kappa^0(k)\rangle,\qquad(29)$$

$$F_\pi\langle\pi^-(0)\pi^+(k')|A^{4-i5}|K^+(k)\rangle=(\sqrt{\tfrac{2}{3}})\langle\sigma_0(k')|A^{4-i5}|K^+(k)\rangle\ (\sqrt{\tfrac{1}{3}})\langle\sigma_8(k')|A^{4-i5}|K^+(k)\rangle+i(\sqrt{\tfrac{1}{2}})\langle\pi^+(k')|V^{6-i7}|K^+(k)\rangle,\qquad(30)$$

$$F_\pi\langle\pi^0(0)\pi^0(k')|A^{4-i5}|K^+(k)\rangle=(\sqrt{\tfrac{2}{3}})\langle\sigma_0(k')|A^{4-i5}|K^+(k)\rangle+(\sqrt{\tfrac{1}{3}})\langle\sigma_8(k')|A^{4-i5}|K^+(k)\rangle+\tfrac{1}{2}\langle\pi^0(k')|A^{4-i5}|\kappa^+(k)\rangle+\tfrac{1}{2}i\langle\pi^0(k')|V^{4-i5}|K^+(k)\rangle.\qquad(31)$$

It should be noted that (21h) is not symmetric with respect to the two pseudoscalar mesons in the final state and thus the conclusion depends on which meson in the final state is taken to be soft.

We also remark that if we take an additional meson in the final state to be soft, we obtain

$$\langle P_A(0)P_a(0)|A_B|P_b\rangle=0.\qquad(32)$$

If we define the K_{l4} form factors by

$$\langle P_A(k'')P_a(k')|A_B|P_b(k)\rangle=(k'+k'')F_1+(k'-k'')F_2+(k-k'-k'')F_3,\qquad(33)$$

then our result requires, with both final-state mesons soft,

$$F_3=0.\qquad(34)$$

D. Coupling Constants

Since any local operator which has the same quantum numbers as an asymptotic field can be chosen as its interpolating field, we can choose the vector and axial-vector currents, $V_{\mu,A}(x)$ and $A_{\mu,A}(x)$, as interpolating fields for the vector and axial-vector octets up to the normalization[17]

$$V_{\mu,A}(x)=f_A\hat{v}_{\mu,A}(x),\qquad(35a)$$
$$A_{\mu,A}(x)=g_A\hat{a}_{\mu,A}(x),\qquad(35b)$$

where $\hat{v}_{\mu,A}$ and $\hat{a}_{\mu,A}$ denote the interpolating fields and where f_A and g_A are the constants which have appeared in Eqs. (13) and (14) and characterize the leptonic decay amplitudes of the vector and axial-vector mesons.

From (21a) we obtain

$$\langle P_A(0)|J_{\mu,B}{}^{(V)}(0)|P_C(k)\rangle=\langle P_A(k)|J_{\mu,B}{}^{(V)}(0)|P_C(0)\rangle=ik_\mu f_{ABC}(m_B{}^{(V)2}-k^2)/f_{B'},\qquad(36)$$

where $J_{\mu,B}{}^{(V)}(x)=(\Box+m_B{}^{(V)2})\hat{v}_{\mu,B}(x)$. We define the

[14] Our $SU(3)$ classification of the mesons will be as follows. 0^-: π, K, η, X^0. 0^+: δ, $\kappa(\sim1100)$, $\sigma_8(1070)$, $\sigma_0(720)$. 1^-: ρ, K^*, ϕ; ω. 1^+: A_1, K_A, D; E.

[15] C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966); V. S. Mathur, S. Okubo and L. K. Pandit, *ibid.* **16**, 371 (1966).

[16] Callan and Treiman (Ref. 15); S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

[17] These field-current identities were originally obtained in a Lagrangian field theory; see N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967); T. D. Lee and B. Zumino, *ibid.* **163**, 1667 (1967).

VPP form factors by

$$\langle P_A(k')|J_{\mu,B}{}^{(V)}(0)|P_C(k)\rangle = if_{ABC}[(k+k')_\mu g_{V_BP_AP_C}{}^{(1)}(t) + (k-k')_\mu g_{V_BP_AP_C}{}^{(2)}(t)], \quad (37)$$

where $t\equiv(k-k')^2$ and where $g_{V_BP_AP_C}{}^{(2)}(t)=0$ for $B=1, 2, 3$, and 8. If the form factors are smooth, e.g.,

$$g_{\rho\pi\pi}(m_\pi{}^2)\simeq g_{\rho\pi\pi}(m_\rho{}^2)\equiv g_{\rho\pi\pi}\,, \quad (38)$$

then we obtain

$$f_\rho g_{\rho\pi\pi}=m_\rho{}^2-m_\pi{}^2\,, \quad (39a)$$

$$f_\phi g_{\phi K\bar{K}}=m_\phi{}^2-m_K{}^2\,, \quad (39b)$$

$$f_{K^*}(g_{K^*K\pi}{}^{(1)}+g_{K^*K\pi}{}^{(2)})=m_{K^*}{}^2-m_\pi{}^2\,, \quad (39c)$$

$$f_{K^*}(g_{K^*K\pi}{}^{(1)}-g_{K^*K\pi}{}^{(2)})=m_{K^*}{}^2-m_K{}^2\,. \quad (39d)$$

If in Eq. (39a) we neglect $m_\pi{}^2$, which is small compared to $m_\rho{}^2$, then we obtain just the Gell-Mann–Zachariasen relation.[18]

Similarly, it follows from Eqs. (21c) and (21c′) that

$$\langle S_A(0)|J_{\mu,B}{}^{(A)}(0)|P_C(k)\rangle = ik_\mu d_{ABC}(m_B{}^{(A)2}-k^2)/g_B\,, \quad (40a)$$

$$\langle S_A(k')|J_{\mu,B}{}^{(A)}(0)|P_C(0)\rangle = ik_\mu' d_{ABC}(m_B{}^{(A)2}-k'^2)/g_B\,. \quad (40b)$$

Assuming smoothness for g_{APS}, we obtain

$$g_B(g_{A_BP_CS_A}{}^{(1)}+g_{A_BP_CS_A}{}^{(2)})=m_B{}^{(A)2}-m_C{}^{(P)2}\,, \quad (41a)$$

$$g_B(g_{A_BP_CS_A}{}^{(2)}-g_{A_BP_CS_A}{}^{(2)})=m_B{}^{(A)2}-m_A{}^{(S)2}\,. \quad (41b)$$

Using (21f) and (21g), we find that

$$\langle P_A(0)|J_{\mu,B}{}^{(V)}(0)|a_C(k)\rangle = \epsilon_\mu(k)f_{ABC}(g_C/f_B-1)(m_B{}^{(V)2}-m_C{}^{(A)2})/F_A\,, \quad (42a)$$

$$\langle P_A(0)|J_{\mu,C}{}^{(A)}(0)|v_B(k)\rangle = \epsilon_\mu(k)f_{ABC}(f_B/g_C-1)(m_C{}^{(A)2}-m_B{}^{(V)2})/F_A\,. \quad (42b)$$

Expressing these matrix elements in terms of form factors,

$$\langle P_A(k')|J_{\mu,B}{}^{(V)}(0)|a_C(k)\rangle \equiv \epsilon^\nu(k)f_{ABC}[g_{ABC}{}^{(1)}(t)g_{\mu\nu}+g_{ABC}{}^{(2)}(t)k_\mu k_\nu' + g_{ABC}{}^{(3)}(t)k_\mu' k_\nu'], \quad (43a)$$

$$\langle P_A(k')|J_{\mu,C}{}^{(A)}(0)|v_B(k)\rangle \equiv \epsilon^\nu(k)f_{ABC}[h_{ABC}{}^{(1)}(t)g_{\mu\nu}+h_{ACB}{}^{(2)}(t)k_\mu k_\nu' + h_{ACB}{}^{(3)}(t)k_\mu' k_\nu'], \quad (43b)$$

we find

$$g_{ABC}{}^{(1)}(m_C{}^{(A)2}) = (g_C/f_B-1)(m_B{}^{(V)2}-m_C{}^{(A)2})/F_A\,, \quad (44a)$$

$$h_{ACB}{}^{(1)}(m_B{}^{(V)2}) = (f_B/g_C-1)(m_C{}^{(A)2}-m_B{}^{(V)2})/F_A\,, \quad (44b)$$

where ABC is such that $f_{ABC}\neq 0$. The $a_Cv_BP_A$ coupling constant is given by $g_{ABC}{}^{(1)}(m_C{}^{(V)2})=h_{ACB}{}^{(1)}(m_C{}^{(A)2})$. Assuming smoothness of the form factors near the mass shell, we are led to

$$(g_C/f_B-1)(m_B{}^{(V)2}-m_C{}^{(A)2}) \simeq (f_B/g_C-1)(m_C{}^{(A)2}-m_B{}^{(V)2})\,, \quad (45)$$

or simply

$$f_B\simeq g_C\,. \quad (46)$$

In particular, this includes Weinberg's relation $f_\rho\simeq g_{A_1}$,[19] If we now rewrite (44a) and (44b) in the form

$$\frac{g_C}{f_B}=1+\frac{F_Ag_{ABC}{}^{(1)}(m_C{}^{(A)2})}{m_B{}^{(V)2}-m_C{}^{(A)2}}\,, \quad (47a)$$

$$\frac{f_B}{g_C}=1+\frac{F_Ah_{ACB}{}^{(1)}(m_B{}^{(V)2})}{m_C{}^{(A)2}-m_B{}^{(V)2}}\,, \quad (47b)$$

then (46) indicates that the smoothness assumption is equivalent to requiring

$$\left|\frac{F_Ag_{ABC}{}^{(1)}(m_C{}^{(A)2})}{m_B{}^{(V)2}-m_C{}^{(A)2}}\right|\ll 1 \quad (48a)$$

and

$$\left|\frac{F_Ah_{ACB}{}^{(1)}(m_B{}^{(V)2})}{m_C{}^{(A)2}-m_B{}^{(V)2}}\right|\ll 1\,. \quad (48b)$$

E. Generalized Adler Consistency Condition

If the scalar nonet were absent, we could still obtain Eq. (25), but without the terms containing scalar particles; i.e., we find

$$N_{bAd}d_{Aef}C_f\{[(2\pi)^3 2k_0'']^{1/2} \times\langle P_a(k')P_e(k'')|P_d(0)|P_c(k)\rangle\}_{k''=0}=0\,. \quad (49)$$

If we restrict our argument to $SU_2\times SU_2$, then $N_{bAd}d_{Aef}C_f \rightarrow \delta_{ed}F_\pi$ and

$$\{[(2\pi)^3 2k_0'']^{1/2}\langle\pi_a(k')\pi_d(k'')|\pi_d(0)|\pi_c(k)\rangle\}_{k''=0}=0. \quad (50)$$

This is nothing but Adler's[12] consistency condition.

IV. CONCLUSIONS

We have treated chiral dynamics by means of the linear realization for the asymptotic (physical) fields. Many relations were derived among form factors and coupling constants by writing suitable interpolating fields in terms of asymptotic fields and by applying the chiral transformations. It is important to note, however, that the intrinsic mass-breaking effects were taken into account by considering the physical masses of the particles only in the final sum rules. Independent of intrinsic mass breaking, we are assuming a more fundamental breakdown of symmetry, viz., the linear realization of the spinless fields which carry the additional c numbers (C_0,C_8) so that their transformations are of the form (6). Most of our results were due to the existence of these c numbers which we interpret physically

[18] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[19] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

as the *spontaneous breakdown of chiral symmetry*. Indeed, (6) implies that the vacuum is not a chiral-invariant state, i.e., $X|0\rangle \neq 0$, even when we ignore intrinsic mass breaking. Thus, we interpret the spontaneous breakdown as the cause, e.g., of the leptonic decay of the pion. This approach to chiral dynamics was originally motivated by the spontaneous breakdown which occurs in the chiral Lagrangian theory with nonlinear realizations. In general,[20] when the spontaneous breakdown occurs, massless or Goldstone fields appear carrying c numbers and collectively complete a linear representation. This suggests then that the physical spinless particles are, in the mass-symmetry limit, the Goldstone fields. Thus, instead of making use of nonlinear realizations of interpolating fields, we have used the linear representations for physical fields with added c numbers.

We have restricted these c numbers by isospin and strangeness conservation, thus leaving only C_0 and C_8 nonzero. It is possible that other c numbers are extremely small but not zero, measuring such things as the nonleptonic weak interactions. If. e.g., the physical K field would carry a c number, $\Delta I = \frac{1}{2}$ would be introduced and parity would be violated.

We should also like to call attention to the results which differed from those of current algebra. To see how these differences arise, we consider the K_{l3} decay as an example. By using the current commutator, we rewrite (11b) in the form

$$\begin{aligned}\langle 0|A_\mu{}^{4-i5}(0)|K^+(k)\rangle &= -2\int d^4x\,\delta(x_0)\langle 0|[A_{0,3}(x),V_\mu{}^{4-i5}(0)]|K^+(k)\rangle \\ &= +2\int d^4x\,\theta(x_0)\langle 0|[\partial^\nu A_{\nu,3}(x),V_\mu{}^{4-i5}(0)]|K^+(k)\rangle .\end{aligned}$$

Imposing our linear realization requirement and including, in addition to the pion-pole terms, the κ-pole terms, we find

$$\begin{aligned}\{[(2\pi)^3 2q_0]^{1/2}\langle \pi^0(q)|V_\mu{}^{4-i5}(0)|K^+(k)\rangle\}_{q=0} &= -\frac{1}{\sqrt{2}}\frac{F_K}{F_\pi}\frac{k_\mu}{[(2\pi)^3 2k_0]^{1/2}} + i\sqrt{2}\frac{F_\kappa}{F_\pi} \\ &\quad \times\langle 0|[X_3,\partial_\mu \kappa^+(0)]|K^+(k)\rangle ,\end{aligned}$$

where the second term is the κ-pole term. Then, using (6c) and (12c), we obtain (27).

Let us now consider the parallel argument in the ordinary computation of current algebra. In this case we obtain (26),

$$\begin{aligned}\langle 0|A_\mu{}^{4-i5}(0)|K^+(k)\rangle &= 2\int d^4x\,\theta(x_0)\langle 0[\partial^\nu A_{\nu,3}(x),V_\mu{}^{4-i5}(0)]|K^+(k)\rangle \\ &= \frac{-2}{m_\pi{}^2}\lim_{q\to 0}\int d^4x\, e^{+iqx}(q^2-m_\pi{}^2)\theta(x_0) \\ &\qquad \times\langle 0|[\partial^\nu A_{\nu,3}(x),V_\mu{}^{4-i5}(0)]|K^+(k)\rangle \\ &= -2iF_\pi\{[(2\pi)^3 2q_0]^{1/2}\langle \pi^0(q)|V_\mu{}^{4-i5}(0)|K^+(k)\rangle\}_{q=0} .\end{aligned}$$

In the last step, use was made of the reduction formula, *which can be justified only when q is on the mass shell.* Indeed, the matrix element is qualitatively of the form $[a/(q^2-m_\pi{}^2)+b/(q^2-m_\kappa{}^2)]$, so that if the reduction technique is applied consistently, the residue of the pole at $q^2=m_\pi{}^2$ is just a. However, in the limit $q\to 0$ we have $a+(m_\pi{}^2/m_\kappa{}^2)b$, and thus obtain the contribution from the κ pole. Although these remarks may serve us to understand the relation of our results to current algebra, the interpolating expansion method together with spontaneous breakdown proposed here is exceedingly simple and leads one directly to the soft-meson relations.

The inclusion of baryons and photons and the consideration of weak nonleptonic decays will be discussed elsewhere.

Note added in manuscript. When (48a) is applied to the $A_1\rho\pi$ coupling, we find that $g^{(1)}(m_{A_1}{}^2) \ll (m_{A_1}{}^2 - m_\rho{}^2)/F_\pi$, where $g^{(1)}$ is related to the transverse $A_1\rho\pi$ coupling constant by $g^{(1)}(m_\rho{}^2) = \mathbf{q}^3 g_T$, with $\mathbf{q}$ being the three-momentum of ρ in the rest frame of A_1. This result predicts $g_T \ll 8/F_\pi \simeq 0.08$ MeV^{-1}. $g_T = 0$ has been derived by the single-particle saturation of superconvergence relations in ρ-π scattering [see, e.g., F. J. Gilman and H. Harari, Phys. Rev. Letters **18**, 1150 (1967)] and is consistent with experiment [J. Ballam *et al.*, *ibid.* **21**, 934 (1968)]. We should also like to note that Eq. (12) has been derived by other techniques [see, e.g., W. A. Bardeen and B. W. Lee, Phys. Rev. **177**, 2389 (1969)].

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[20] L. Leplae, R. N. Sen, and H. Umezawa, Nuovo Cimento **49**, 1 (1967); R. N. Sen and H. Umezawa, *ibid.* **50**, 53 (1967).