Aside from symmetry breaking, an important question concerns the relation of our work to duality. Since both schemes are based on identical quark graphs, it seems likely that they may be fused in a unified approach. After completing our work on the manner in which $SU(6)_W$ leads to fixed Regge cuts, we learned that Bardakci and Halpern[21] have, in fact, constructed a dual amplitude containing fixed cuts and have proposed that this amplitude be utilized in the quark model. The leading trajectory in their model couples according to $SU(6)_W$ in the manner we have described. Ellis[22] has also investigated this problem, which we expect to open a fruitful area of new research.

[21] K. Bardakci and M. B. Halpern, Phys. Rev. Letters **24**, 428 (1970).

ACKNOWLEDGMENTS

We would like to thank S. C. Frautschi for his advice and encouragement in our research. R. C. thanks M. Gell-Mann for a particularly helpful discussion. M. K. thanks R. P. Feynman for suggesting investigating this problem and for numerous helpful discussions.

[22] S. Ellis (unpublished).

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Some Properties of a Hamiltonian Model of Broken $SU(3)\times SU(3)$ Symmetry. II*

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An inherent ambiguity of broken $SU(3)\times SU(3)$ symmetry is discussed. It is shown to arise from a discrete unitary transformation in the $SU(3)\times SU(3)$ space. For Hamiltonian models for which the symmetry-breaking term transforms like the $(3,\bar{3})+(\bar{3},3)$ representation, we find that, in general, there are two such terms which describe the same physical system. Some consequences of this result are discussed.

I. INTRODUCTION AND GENERAL DISCUSSION

RECENTLY the significance of the chiral $SU(3)\times SU(3)$ symmetry has been clarified greatly by Glashow and Weinberg,[1] and by Gell-Mann, Oakes, and Renner.[2] They proposed that the strong-interaction Hamiltonian density should be written as

$$H=H_0+H', \tag{1}$$

where H_0 is invariant under $SU(3)\times SU(3)$ rotations,[3] and the symmetry-breaking term H' is considered to conserve the $U(1)\times SU(2)$ symmetry and to have definite transformation properties under the $SU(3)\times SU(3)$ group. In particular, GOR suggest that the simplest form for H' is

$$H'=\alpha(u_0+\sqrt{2}ru_8), \tag{2}$$

where α and r are real parameters, and u_i, $i=0,1,\ldots,8$, together with v_i, transform like the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$. There may also be terms transforming like $(1,8)+(8,1)$, $(8,8)$, etc. However, so far, very little is known about these other possibilities. In the following we will concentrate on Eq. (2), but will comment on the other choices when appropriate.

Now, as has been emphasized by Cabibbo and Maiani,[4] the directions in the $SU(3)\times SU(3)$ space are not fixed *a priori*. Indeed, there are an infinite number of Hamiltonians which describe the same hadronic world. These systems are connected by arbitrary rotations R in the $SU(3)\times SU(3)$ space. Let us denote by S the system described by Eq. (2). Then the system $\tilde{S}$, in which a state $|\alpha\rangle$ in S becomes $R|\alpha\rangle$ and an operator O becomes ROR^{-1}, is completely equivalent to S. Physically, the unitary transformation from S to $\tilde{S}$ means that we must redefine internal quantum numbers of the hadronic states, etc.

What are the effects of the electromagnetic and weak interactions? As far as the hadrons are concerned, they may be regarded as external fields. In considering rotations in the hadronic world, we should leave the directions defined by the electromagnetic and the weak[5] interactions unchanged. However, insofar as the direction of the hypercharge is not fixed, there is no *a priori* value for the Cabibbo angle either. Thus we may regard the direction of the weak currents as arbitrary in con-

* Supported in part by the U. S. Atomic Energy Commission.
[1] S. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).
[2] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968). This paper will be referred to as GOR.
[3] The notation used in this paper will be as follows. For the "charges" of the vector and axial vector currents, we write $F_{1,\ldots,8}$ and $F_{1,\ldots,8}{}^5$. The generators of the "left" and "right" $SU(3)$'s are then $\frac{1}{2}(F_i\pm F_i{}^5)$. Also, we use Y_5, $I_3{}^5$, etc., to denote the axial counterpart of Y,I_3, etc.
[4] N. Cabibbo and L. Maiani, Phys. Rev. D **1**, 707 (1970). This paper will be referred to as CM. In this connection, see also the related papers by R. Gatto, G. Santoni, and M. Tonin [Phys. Letters **28B**, 128 (1968)] and N. Cabibbo and L. Maiani [*ibid.* **28B**, 131 (1968)].
[5] By the direction of the weak interaction, we are referring to the Cabibbo angle. Nonleptonic weak interaction may be considered to arise from the current-current interaction.

sidering the rotation. In a previous paper,[6] which is a summary of the present work, we have assumed that the hypercharge direction is fixed by the medium strong interaction. We now realize that this simplifying assumption is unnecessary, and we will drop it in the following discussions.

Out of all the possible rotations in $SU(3)\times SU(3)$, one class is of special interest. They are those rotations that leave the form of H' invariant. In particular, for H' given in Eq. (2), we would like to find an R such that

$$RHR^{-1}=\tilde{H}'=\tilde{\alpha}(u_0+\sqrt{2}\tilde{r}u_8). \quad (3)$$

If this is possible, our discussion in the previous paragraphs would establish the equivalence of the theories described by the parameters r and $\tilde{r}$. In Sec. II we show that there is a discrete rotation W which does that. We find further that W leaves the electromagnetic and weak currents invariant. Thus, the strong, electromagnetic, and weak interactions do not distinguish theories described by r or $\tilde{r}$. The generators transform according to

$$(F_{1,2,3,8};F_{4,5,6,7})\xrightarrow{W}(F_{1,2,3,8};\ -F_5{}^5,\ F_4{}^5,\ -F_7{}^5,\ F_6{}^5).$$

Either set gives rise to an $SU(3)$ group. We shall call the first $SU(3)$ [or the "ordinary" $SU(3)$] and the second $\tilde{S}U(3)$ [or the "hybrid" $SU(3)$ [7,8]]. Now, when we consider the $SU(3)\times SU(3)$ symmetry broken down to $U(1)\times SU(2)$, we still keep the $SU(3)$ notation if only for the sake of labeling the hadronic states. As we are going to show in Secs. II and III, the unitary transformation W amounts to a change of coordinate systems in $SU(3)\times SU(3)$ space. We shall denote the two systems by S and $\tilde{S}$. Specifically, W has the following properties.

(1) It sends $SU(3)$ into $\tilde{S}U(3)$.

(2) It leaves the electromagnetic and weak currents invariant.

(3) Viewed in S, W changes the relative parity of states with $\Delta Y=1$. For instance, the transformed pion and kaon states would have different parities with respect to the untransformed parity operator in S. (Of course, in $\tilde{S}$, the transformed pion and kaon will have the same parity.)

Now the relative parity of states with $\Delta Y=1$ is but a convention. What we are saying, then, is that, in the $SU(3)\times SU(3)$ space, descriptions in terms of $SU(3)$ and $\tilde{S}U(3)$ are completely equivalent, provided that we also change the relative parity of states with $\Delta Y=1$. This result applies to any Hamiltonian model of the form of Eq. (1). It is an inherent ambiguity in the breaking of $SU(3)\times SU(3)$ down to $U(1)\times SU(2)$. We would like to point out, however, that the change of parity is not the only thing being done on the hadronic states.[9] In general, W, when applied to, say, a pion, would generate a pion plus many pions, kaons, etc. Also, the vacuum state is not invariant under W.

What is the effect on H' when we apply W? Our discussion in the previous paragraph suggests (but does not explicitly prove) that, independent of its detailed transformation properties, $WH'W^{-1}$ would have the same form as H'. In the specific case $H'\approx(1,8)+(8,1)$, H' turns out to be acutally invariant under W. (See Sec. III.) For $H'\sim(3,\bar{3})+(\bar{3},3)$, we find that the form of H' is invariant, but the parameter r defined in Eq. (2) is changed into $\tilde{r}$. [See Eq. (12)]. Therefore, in this special case, there is no way to tell the difference between theories specified by r or $\tilde{r}$. This is the main result of this work.

The plan of this paper is as follows. In Sec. II we establish the existence of a nontrivial finite rotation R according to Eq. (3). We then discuss its properties in Sec. III, proving the statements made in this general discussion. Applications and conclusions are contained in Secs. IV and V. Finally, mathematical details of how to find the actual rotations are studied in the Appendices.

II. ROTATION IN $SU(3)\times SU(3)$ SPACE

In this section we wish to find a rotation R according to Eq. (3). The most general rotation which commutes with the charge operator is a rotation generated by a $U(2)\times U(2)$ subgroup of $SU(3)\times SU(3)$. Let us denote by $(Q_L,\mathbf{U}_L)\otimes(Q_R,\mathbf{U}_R)$ the charge and U-spin operators in $SU(3)_L$ and $SU(3)_R$ spaces, respectively. Then we must consider R generated by

$$(Q_L,\mathbf{U}_L)\otimes(Q_R,\mathbf{U}_R). \quad (4)$$

Before we compute explicitly the effect of R on H', it is useful to introduce the notation given by CM. Corresponding to any general operator $\sum_i(a_iu_i+b_iv_i)$, we define a 3×3 matrix M:

$$M=\sum(a_i+ib_i)\lambda_i. \quad (5)$$

A rotation in the $SU(3)\times SU(3)$ space, when applied to the quark states, is given in terms of a pair of 3×3 matrices (U,V). They are just the 3×3 representation of R in $SU(3)_L$ and $SU(3)_R$, respectively. Under the rotation (U,V), then, the transformation of M is

$$M\to V^\dagger MU, \quad M^\dagger\to U^\dagger M^\dagger V. \quad (6)$$

In this notation, H' would be represented by

$$\mathcal{H}'=\begin{bmatrix} a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & b\end{bmatrix}, \quad (7)$$

where a and b are real so that no v_i's will come in.

[6] T. K. Kuo, Nuovo Cimento Letters **3**, 803 (1970). This work will be denoted as I.

[7] The existence of this "hybrid" $SU(3)$ was first pointed out to the author by S. P. Rosen. It is precisely what Okubo and Mathur (see Ref. 8) called "chimeral" $\tilde{S}U(3)$.

[8] S. Okubo and V. Mathur, Phys. Rev. Letters **23**, 1412 (1969); Phys. Rev. D **1**, 2046 (1970).

[9] In this paper we take the view that degenerate vacuum and Goldstone bosons appear for the realization of chiral symmetry.

In Appendix A, we shall prove that the only nontrivial transformation on $\mathcal{H}'$ which leaves the charge direction unchanged is given by

$$\mathcal{H}' \to \tilde{\mathcal{H}}' = W_R{}^\dagger \mathcal{H}' W_L = \begin{pmatrix} -a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & b \end{pmatrix}. \qquad (8)$$

The transformation W is a finite rotation in the $SU(3) \times SU(3)$ space:

$$W = \exp(i\tfrac{3}{2}\pi Y_5)\,, \qquad (9)$$

or, in 3×3 representation,

$$W_L = \begin{pmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -1 \end{pmatrix} = W_R{}^\dagger. \qquad (10)$$

We could have considered more general rotations

$$W(\theta,\Phi) = \exp(i\theta Y)\exp(i\Phi I_3) W\,, \qquad (11)$$

which gives the same $\tilde{\mathcal{H}}'$ as W. We will see in Sec. III that it is sometimes more convenient to use $W(\theta,\Phi)$ instead of W.

In terms of the parameter r defined in Eq. (2), we can easily compute the transformed $\tilde{r}$ from Eq. (8). We have, under W,

$$r \to \tilde{r}\,, \quad \tilde{r} = \frac{2-r}{1+4r}\,, \quad r = \frac{2-\tilde{r}}{1+4\tilde{r}}. \qquad (12)$$

We will discuss the numerical aspects of this transformation in Sec. IV.

III. PROPERTIES OF W OPERATOR

In the previous section we found an operator W which preserves the form of H', and induces the transformation $r \to \tilde{r}$ according to Eq. (12). We will now discuss the transformation properties of the $SU(3) \times SU(3)$ generators and the physical state vectors.

The generators transform like the $(1,8)+(8,1)$ representation of $SU(3)\times SU(3)$. Let us denote by (g_i,h_i), $i=1,\ldots,8$ a set of operators transforming like $(1,8)+(8,1)$. Then, corresponding to the operator $\sum_i(c_i g_i + d_i h_i)$, we may define the 3×3 matrices

$$M^{\pm} = \sum_i (c_i \pm d_i)\lambda_i. \qquad (13)$$

Under an $SU(3)\times SU(3)$ rotation defined by (U,V), $M^\pm$ transform according to

$$M^+ \to U^\dagger M^+ U\,, \quad M^- \to V^\dagger M^- V\,, \qquad (14)$$

Using Eq. (10), we find that

$$(F_{1,2,3,8}; F_{1,2,3,8}{}^5) \xrightarrow{W} (F_{1,2,3,8}; F_{1,2,3,8}{}^5)\,, \qquad (15)$$

$$(F_{4,5,6,7}; F_{4,5,6,7}{}^5) \xrightarrow{W} (-F_5{}^5\, F_4{}^5,\, -F_7{}^5,\, F_6{}^5;\; -F_5,\, F_4,\, -F_7,\, F_6). \qquad (16)$$

It is easy to verify that the operators $(F_{1,2,3,8}; -F_5{}^5, F_4{}^5, -F_7{}^5, F_6{}^5)$ form the generators of $\widetilde{SU}(3)$. We note than an equivalent set of generators of $\widetilde{SU}(3)$ is $(F_{1,2,3,8}; F_{4,5,6,7}{}^5)$. In fact, the two sets are related by a rotation around the 8th axis. Another way of saying this is that we may use the operator $W(\theta,\Phi)$ defined in Eq. (11). Using $\theta=\frac{1}{2}\pi$, $\Phi=0$, we have

$$(F_{1,2,3,8}; F_{1,2,3,8}{}^5) \xrightarrow{W(\frac{1}{2}\pi,0)} (F_{1,2,38}; F_{1,2,3,8}{}^5)\,, \qquad (17)$$

$$(F_{4,5,6,7}; F_{4,5,6,7}{}^5) \xrightarrow{W(\frac{1}{2}\pi,0)} (F_{4,5,6,7}{}^5; F_{4,5,6,7}). \qquad (18)$$

With the help of Eqs. (15) and (16), we may immediately obtain the transformation properties of the electromagnetic and the weak currents. They are

$$J_\mu{}^{\text{e.m.}} \xrightarrow{W} J_\mu{}^{\text{e.m.}}\,, \qquad (19)$$

$$(J_\mu{}^{\text{weak}})_{\Delta S=0} \xrightarrow{W} (J_\mu{}^{\text{weak}})_{\Delta S=0}\,, \qquad (20)$$

$$(J_\mu{}^{\text{weak}})_{\Delta S=1} \xrightarrow{W} i(J_\mu{}^{\text{weak}})_{\Delta S=1}. \qquad (21)$$

Since an over-all phase factor between the $\Delta S=0$ and $\Delta S=1$ weak currents is unobservable (it corresponds to adding a phase to states with $\Delta S=1$), we conclude that the electromagnetic and the weak currents are invariant under W. [An equivalent way of getting rid of the i in Eq. (21) is to use the operator $W(\frac{1}{2}\pi,0)$ as in Eq. (18)].

We turn next to the question of the transformation of physical states. Let us define, corresponding to a state $|\alpha\rangle$, the transformed state $|\tilde{\alpha}\rangle$:

$$|\tilde{\alpha}\rangle = W|\alpha\rangle. \qquad (22)$$

In the original coordinate system S, $|\tilde{\alpha}\rangle$ is in general a very complicated state owing to the nonlinear nature of the chiral symmetry. In particular, the vacuum state $|0\rangle$ is transformed into a state with two pions, four pions, etc. However, the state

$$|\hat{\alpha}\rangle = W^2|\alpha\rangle \qquad (23)$$

is very simple. For, the operator W^2, in addition to being unitary, is also Hermitian with the eigenvalues ± 1. This follows from the property

$$W^4 = I\,, \qquad (24)$$

as can be deduced from Eq. (10). Thus,

$$(W^2)^\dagger = W^{-2} = W^4 W^{-2} = W^2. \qquad (25)$$

Further,

$$[W^2,H]=[W^2,Y]=[W^2,\mathbf{I}]=0. \tag{26}$$

Also, under parity,

$$PWP^{-1}=W^{-1}, \tag{27}$$

$$PW^2P^{-1}=W^{-2}=W^2. \tag{28}$$

Thus eigenstates of $(H,Y,\mathbf{I},P)$ should be diagonal with respect to W^2. Barring degeneracy, then

$$|\hat{\alpha}\rangle=W^2|\alpha\rangle=\zeta_\alpha|\alpha\rangle, \tag{29}$$

$$\zeta_\alpha=\pm 1. \tag{30}$$

Similarly,

$$W^2|\tilde{\alpha}\rangle=\zeta_{\tilde{\alpha}}|\tilde{\alpha}\rangle \tag{31}$$

$$=W^3|\alpha\rangle=WW^2|\alpha\rangle=\zeta_\alpha|\tilde{\alpha}\rangle; \tag{32}$$

hence

$$\zeta_\alpha=\zeta_{\tilde{\alpha}}. \tag{33}$$

One further useful relation is

$$\tilde{P}=WPW^{-1}, \tag{34}$$

$$\tilde{P}P=W^{-2}=W^2. \tag{35}$$

The choice $\zeta\alpha=\pm 1$ is not arbitrary, since

$$\begin{aligned}\langle\alpha|F_i|\beta\rangle&=\langle\alpha|W^2(W^2F_iW^2)W^2|\beta\rangle\\&=\zeta_\alpha\zeta_\beta\langle\alpha|(W^2F_iW^2)|\beta\rangle.\end{aligned} \tag{36}$$

But, from Eq. (16),

$$W^2(F_{1,2,3,8})W^2=+(F_{1,2,3,8}), \tag{37}$$

$$W^2(F_{4,5,6,7})W^2=-(F_{4,5,6,7}), \tag{38}$$

it follows that the relative signs of ζ_α for two states with $\Delta Y=0$ and $\Delta Y=1$ must be the same and opposite, respectively. We shall adopt the convention that

$$\begin{aligned}\zeta_\alpha&=+1 \quad \text{for} \quad \alpha=\pi,\Lambda,\ldots,\\ \zeta_\alpha&=-1 \quad \text{for} \quad \alpha=K,p,\ldots.\end{aligned} \tag{39}$$

Let us now go back to Eq. (22) and consider

$$\begin{aligned}P|\tilde{\alpha}\rangle&=PW|\alpha\rangle\\&=(PWP^{-1})(P|\alpha\rangle)\\&=W^{-1}(\eta_\alpha|\alpha\rangle)\\&=\eta_\alpha W^2(W|\alpha\rangle)\\&=\eta_\alpha\zeta_\alpha|\tilde{\alpha}\rangle,\end{aligned} \tag{40}$$

where η_α is the parity of the state $|\alpha\rangle$. We can see then that $W|\alpha\rangle=|\tilde{\alpha}\rangle$ is an eigenstate of P with the eigenvalue $\eta_\alpha\zeta_\alpha$. According to Eq. (39), $|\tilde{\alpha}\rangle$ will have the same or opposite parities as $|\alpha\rangle$ depending on whether $\alpha=\pi,\Lambda,\ldots$ or $\alpha=K,p,\ldots$. Thus, the W transformation changes $SU(3)$ into $\widetilde{SU}(3)$, and simultaneously it changes the parities of the $Y=\pm1,\pm3,\ldots$ states. It leaves the electromagnetic and the weak currents invariant. The two coordinate systems S and $\tilde{S}$ connected by W are physically indistinguishable. As a consequence, theories defined by r and $\tilde{r}$ as in Eq. (12) are physically equivalent.

Before we go on, we remark that, according to Eq. (15), if H' has a term transforming like g_8, where $(g_i,h_i)\sim(1,8)+(8,1)$, then this term is invariant under W. In this work we shall not examine in detail the cases when H' contains terms transforming like (8,8), $(6,\bar{6})+(\bar{6},6)$, etc.

IV. APPLICATIONS

We wish now to concentrate on the numerical results that follow from Eq. (12). Equation (12) represents a hyperbola in the $r\tilde{r}$ plane. It has the asymptotes $r=-\frac{1}{4}$ and $\tilde{r}=-\frac{1}{4}$. It is symmetric under the interchange $r\leftrightarrow\tilde{r}$. The two symmetric points are $r=\tilde{r}=-1$ and $r=\tilde{r}=\frac{1}{2}$. For $r=-1$, H' defines a $U(1)\times SU(2)\times SU(2)$ symmetry. Thus W preserves the $SU(2)\times SU(2)$ symmetry.[10] For $r=\frac{1}{2}$, unless one assumes that H_0 is invariant under the $U(3)\times U(3)$ group, there is no simple physical interpretation for this particular r value. However, as has been emphasized by GOR, owing to the large η' mass (≈ 960 MeV), this assumption is not very useful. So we will not attach too much significance to the point $r=\frac{1}{2}$.[11]

GOR suggested that in reality $r\approx -1$. In fact, they obtained

$$r=-1+\epsilon,\quad \epsilon\approx\tfrac{1}{10}. \tag{41}$$

Under W, according to Eq. (12), we would have

$$r\rightarrow\tilde{r}\approx -1-\epsilon. \tag{42}$$

The two solutions[12] r and $\tilde{r}$ do not differ very much in this case, and are perhaps not outside the error of their determination, which was estimated to be about 25%. On the other hand, there are other theories which depend crucially on ϵ itself. For instance, in CM, H' is assumed to be determined by self-consistency requirements. While we have nothing against this principle, their final result turns out to be a relation between the Cabibbo angle and the parameter ϵ:

$$\tan^2\theta\approx\epsilon/(3-2\epsilon), \tag{43}$$

which cannot be reconciled with Eq. (42). In this connection we would like to call attention to a basic assumption used in CM, namely,

$$\mathcal{H}^{\text{e.m.}}=\begin{bmatrix}\alpha&0&0\\0&\beta&0\\0&0&\beta\end{bmatrix}. \tag{44}$$

We notice that this $\mathcal{H}^{\text{e.m.}}$ is not invariant under W, although the electromagnetic current [also, approxi-

[10] That the $SU(2)\times SU(2)$ symmetry is invariant under W was found some time ago in collaboration with N. Fuchs.

[11] Compare with Ref. 8, where a different viewpoint is taken.

[12] It may be asked why GOR obtained only one solution for r. If we write $\tilde{u}_i=Wu_iW^{-1}$, then there are two solutions to the matrix element $\langle\alpha|u_i|\beta\rangle$, corresponding to the inherent ambiguity between using u_i or $\tilde{u}_i$.

mately, $H \approx H_0 - (u_0 - \sqrt{2} u_8)]$ is. For this reason, we believe that the validity of Eq. (44) is rather doubtful. In another interesting paper Oakes,[13] using a different approach, obtained yet another relation between the Cabibbo angle and ϵ. It is

$$\tan^2\theta = 2\epsilon/(3-2\epsilon)\,, \tag{45}$$

which is again incompatible with the transformation $\epsilon \to -\epsilon$.

We would also like to comment briefly on the recent work of Okubo and Mathur.[8] They proposed to treat the parameter r in Eq. (2) as a continuous parameter, so that physical quantities will be functions of r. These functions $f(r)$ are required to satisfy constraints at particular values of r, when the Hamiltonian exhibits certain symmetries. For instance, they wrote $m_\pi^2 = (1+r)m_0^2(r)$, since for $r=-1$, we have $SU(2)\times SU(2)$ symmetry and hence zero pion mass. The transformation induced by W affects this argument to the extent that the functions used must satisfy $f(r) = f(\tilde{r})$.

So far we have confined our discussion to Hamiltonians of the form $H' = \alpha(u_0 + \sqrt{2} r u_8)$. It was first proposed[14] by the authors of Ref. 4 that H' be written as

$$H' = \alpha[u_0 + \sqrt{2} r u_8 + (\sqrt{\tfrac{2}{3}}) s u_3]\,. \tag{46}$$

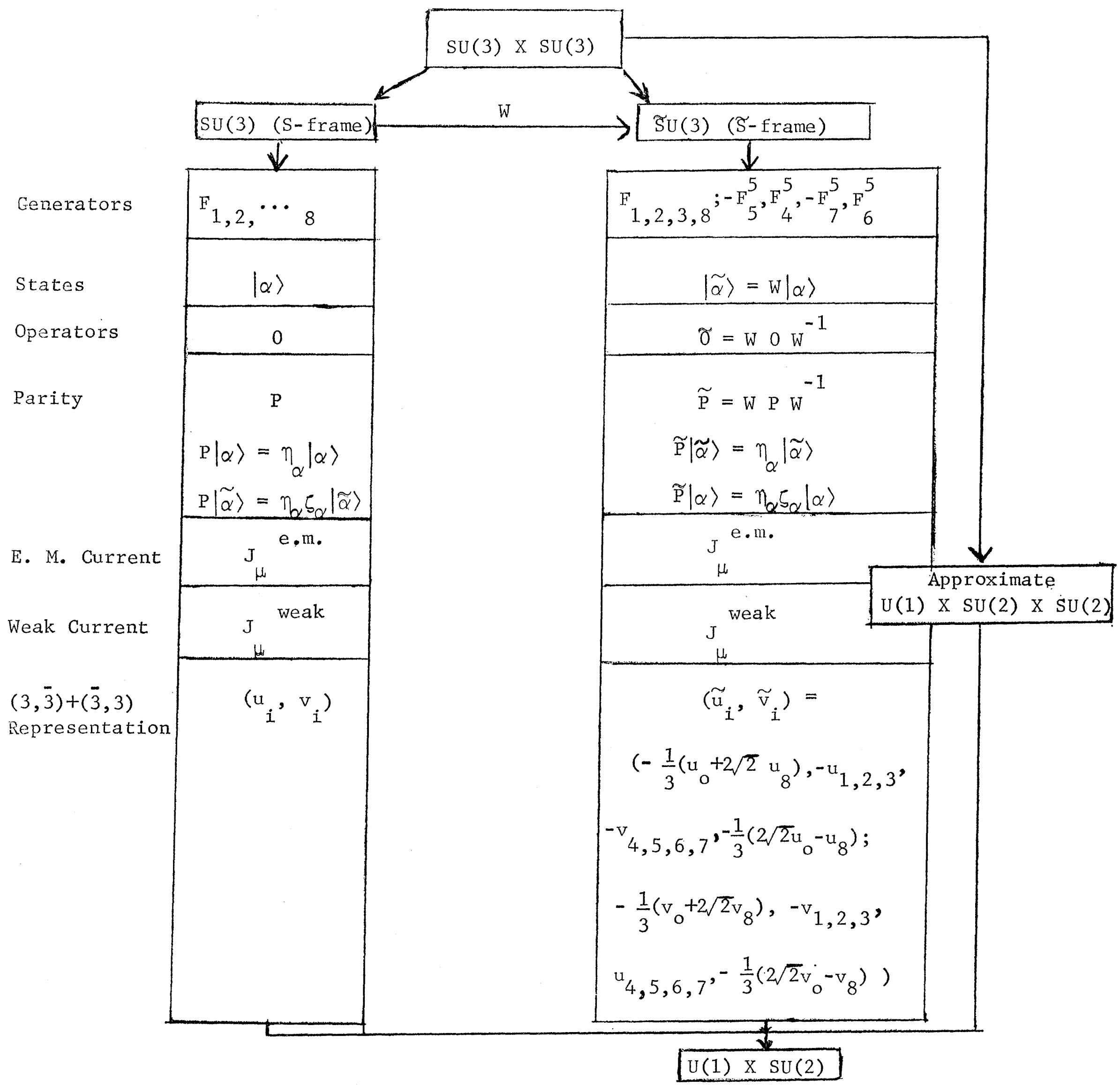

FIG. 1. Schematic representation of $SU(3)\times S(3)$ symmetry breaking. The notation follows that in Sec. III.

[13] R. Oakes, Phys. Letters **29B**, 683 (1969).

[14] See also R. Oakes, Ref. 13, and Phys. Letters **30B**, 262 (1969), where $\eta \to 3\pi$ is analyzed using a Hamiltonian of the form of Eq. (46).

The real coefficient s, which represents a genuine breaking of isospin symmetry in strong interaction, should be of the order of 10^{-2}. Now if we admit Hamiltonians of the type given in Eq. (46), we should consider rotations which give

$$RH'R^{-1}=\tilde{H}'=\tilde{\alpha}[u_0+\sqrt{2}\tilde{r}u_8+(\sqrt{\tfrac{2}{3}})\tilde{s}u_3]. \qquad (47)$$

Again we should limit R to rotations that commute with the charge operator. In Appendix B we find that there are altogether three possible choices for R.[15] The transformation law $(r,s)\to(\tilde{r},\tilde{s})$ is contained in Eq. (B22). If we take the "physical" values $r\approx -1+10^{-1}$, $s\approx 10^{-2}$, then, corresponding to the three solutions, we have

$$\begin{pmatrix}\tilde{r}\\ \tilde{s}\end{pmatrix}\approx\begin{pmatrix}-1\\ -10^{-2}\end{pmatrix},\ \begin{pmatrix}\frac{1}{2}\\ -\frac{3}{2}\end{pmatrix},\ \begin{pmatrix}-1\\ 10^{-1}\end{pmatrix}. \qquad (48)$$

It seems clear that no invariant meaning can be assigned to a "small" isospin breaking term in H'. Although all $(\tilde{r},\tilde{s})$ values in Eq. (48) give necessarily the same physical consequences as (r,s), the ambiguties exhibited make it very difficult to accept H' in the form of Eq. (46).

V. CONCLUDING REMARKS

In this paper we have discussed an inherent ambiguity of the $SU(3)\times SU(3)$ symmetry when broken down to the $U(1)\times SU(2)$ symmetry. This ambiguity corresponds to two equivalent choices of coordinates in the $SU(3)\times SU(3)$ space. Namely, in the intermediate step between $SU(3)\times SU(3)$ and $U(1)\times SU(2)$, either $SU(3)$ or $\widetilde{SU}(3)$ can be used as a reference frame to specify physical states. We showed that the electromagnetic and weak currents remain invariant under this transformation. In the particular case when the symmetry is broken by H' as in Eq. (2), we have seen that this ambiguity leads to the equivalence of r and $\tilde{r}$ as related in Eq. (12). The possibility that $SU(3)\times SU(3)$ is actually broken down to $U(1)\times U(1)$ (hypercharge and charge) is also examined. We find that such a breakdown, although possible, would be very difficult to understand from an esthetic point of view.

Finally, we schematically summarize our results in Fig. 1. The items there are self-explanatory. The possibility of an approximate $U(1)\times SU(2)\times SU(2)$ "intermediate symmetry" is symbolically represented by a separate "route," since this is invariant under the transformation from S to $\tilde{S}$.

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[15] In I, because of the more restrictive conditions, there are only two possible solutions.

APPENDIX A

In this appendix we wish to examine rotations R which preserve the form of H'. [See Eq. (3)]. As we discussed in Sec. I, R must leave the charge direction invariant. They are therefore generated by the operators [Eq. (4)]

$$(Q_L,\mathbf{U}_L)\times(Q_R,\mathbf{U}_R). \qquad (A1)$$

Now, in the 3×3 matrix representation, a rotation of angle Φ_L generated by Q_L is

$$\exp(i\Phi_L Q_L)=\begin{bmatrix}e^{i\frac{2}{3}\Phi_L} & 0 & 0\\ 0 & e^{-\frac{1}{3}\Phi_L} & 0\\ 0 & 0 & e^{-i\frac{1}{3}\Phi_L}\end{bmatrix}, \qquad (A2)$$

while a rotation of angle θ_L along the direction $\hat{n}$ in the U_L-spin space is

$$\exp(i\theta_L\hat{n}\cdot\mathbf{U}_L)=\begin{bmatrix}1 & 0\ 0\\ 0 & \\ 0 & \exp(i\frac{1}{2}\theta_L\hat{n}\cdot\boldsymbol{\sigma})\end{bmatrix}. \qquad (A3)$$

Similar expressions hold for the rotations Φ_R and θ_R. The most general rotation on H' is then

$$H'=\exp(-i\theta_R\hat{n}'\cdot\mathbf{U}_R)\exp(-i\Phi_R Q_R)H' \times\exp(i\Phi_L Q_L)\exp(i\theta_L\hat{n}\cdot\mathbf{U}_L). \qquad (A4)$$

In terms of 3×3 matrix, $\tilde{\mathcal{H}}'$ is required to take the form

$$\tilde{\mathcal{H}}'=\begin{bmatrix}\tilde{a} & 0 & 0\\ 0 & \tilde{a} & 0\\ 0 & 0 & \tilde{b}\end{bmatrix}, \qquad (A5)$$

where $\tilde{a}$ and $\tilde{b}$ are real. [Compare Eq. (7)]. Let us first consider the rotation

$$\bar{H}'=\exp(-i\Phi_R Q_R)\mathcal{H}'\exp(i\Phi_L Q_L)$$
$$=\begin{bmatrix}ae^{i\frac{2}{3}(\Phi_L-\Phi_R)} & 0 & 0\\ 0 & ae^{-i\frac{1}{3}(\Phi_L-\Phi_R)} & 0\\ 0 & 0 & be^{-i\frac{1}{3}(\Phi_L-\Phi_R)}\end{bmatrix}. \qquad (A6)$$

Since, according to Eq. (A3), the matrix element $ae^{i\frac{2}{3}(\Phi_L-\Phi_R)}$ will not be affected by the rotations θ_L and θ_R, we have

$$ae^{i\frac{2}{3}(\Phi_L-\Phi_R)}=\tilde{a}. \qquad (A7)$$

The reality condition on $\tilde{a}$ yields two possible solutions:

$$\text{(I)}\quad \Phi_L=\Phi_R,\quad a=\tilde{a}. \qquad (A8)$$

$$\text{(II)}\quad \Phi_L=\Phi_R+\tfrac{3}{2}\pi,\quad a=-\tilde{a}. \qquad (A9)$$

Now the transformations (A4) leave the determinant of $\mathcal{H}'$ invariant, independent of the angles of rotation. It follows from Eqs. (A8) and (A9) that there are only two possibilities:

$$\text{(I)}\quad \tilde{\mathcal{H}}'=\mathcal{H}'=\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & b\end{bmatrix}. \qquad (A10)$$

$$\text{(II)}\quad \tilde{\mathcal{H}}'=\begin{bmatrix}-a & 0 & 0\\ 0 & -a & 0\\ 0 & 0 & b\end{bmatrix}. \qquad (A11)$$

From Eq. (A11), it is very easy to convince oneself that the only rotation which makes $\mathcal{H}' \rightarrow \tilde{\mathcal{H}}'$ is the finite rotation W given in Eq. (9), or the equivalent ones given in Eq. (11).

APPENDIX B

We now consider symmetry-breaking Hamiltonians of the form [Eq. (46)]

$$H' = \alpha[u_0 + \sqrt{2}ru_8 + (\sqrt{\tfrac{2}{3}})su_3] \tag{B1}$$

and the general rotation R [Eq. (47)]

$$RH'R^{-1} = \tilde{H}' = \tilde{\alpha}[u_0 + \sqrt{2}\tilde{r}u_8 + (\sqrt{\tfrac{2}{3}})\tilde{s}u_3]. \tag{B2}$$

Again R is generated by the operators which commute with charge and are given in Eq. (A1). In terms of 3×3 matrices, H' and $\tilde{H}'$ can be written as

$$\mathcal{H}' = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}, \tag{B3}$$

$$\tilde{\mathcal{H}}' = \begin{bmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{b} & 0 \\ 0 & 0 & \tilde{c} \end{bmatrix}. \tag{B4}$$

Using Eq. (A6), we find again two solutions from rotations generated by Q_L and Q_R:

$$\text{(I)} \quad \Phi_L = \Phi_R, \qquad a = \tilde{a}, \tag{B5}$$

$$\text{(II)} \quad \Phi_L = \Phi_R + \tfrac{3}{2}\pi, \quad a = -\tilde{a}. \tag{B6}$$

Let us concentrate on case I, when $a = \tilde{a}$. The rotations generated by $\mathbf{U}_L$ and $\mathbf{U}_R$ can now be restricted to 2×2 matrices. We have

$$\begin{aligned} \begin{pmatrix} \tilde{b} & 0 \\ 0 & \tilde{c} \end{pmatrix} &= \exp(-i\theta_R \hat{n}' \cdot \mathbf{U}_R) \begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix} \exp(i\theta_L \hat{n} \cdot \mathbf{U}_L) \\ &= (\cos\tfrac{1}{2}\theta_R - i\sin\tfrac{1}{2}\theta_R \sigma_{n'}) \begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix} \\ &\quad \times (\cos\tfrac{1}{2}\theta_L + i\sin\tfrac{1}{2}\theta_L \sigma_n), \end{aligned} \tag{B7}$$

where $\sigma_n = \boldsymbol{\sigma}\cdot\hat{n}$, $\sigma_{n'} = \boldsymbol{\sigma}\cdot\hat{n}'$. Let us write

$$\begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix} = \tfrac{1}{2}(b+c)\sigma_0 + \tfrac{1}{2}(b-c)\sigma_3. \tag{B8}$$

Then, since the form of Eq. (B7) is true for arbitrary b and c, we may discuss the transformation of σ_0 and σ_3 separately. Considering σ_0, we obtain the condition

$$\hat{n} \times \hat{n}' = 0; \tag{B9}$$

further, either

$$\theta_L = \theta_R, \quad \sigma_0 \rightarrow \sigma_0, \tag{B10}$$

or

$$\theta_L = \theta_R + 2\pi, \quad \sigma_0 \rightarrow -\sigma_0. \tag{B11}$$

Corresponding to $\theta_L = \theta_R$, we may study the transformation of σ_3. We obtain, in terms of the definition

$$\sigma_n = \lambda\sigma_1 + \xi\sigma_2 + \eta\sigma_3, \quad \lambda^2 + \xi^2 + \eta^2 = 1, \tag{B12}$$

two possible solutions:

$$\text{(1)} \quad \lambda = \xi = 0, \quad \sigma_3 \rightarrow \sigma_3 \tag{B13}$$

$$\text{(2)} \quad \eta = 0, \quad \theta_L = \theta_R = \pi, \quad \sigma_3 \rightarrow -\sigma_3. \tag{B14}$$

Corresponding to $\theta_L = \theta_R + 2\pi$, we have similarly

$$\text{(1)} \quad \eta = 0, \quad \sigma_3 \rightarrow \sigma_3 \tag{B15}$$

$$\text{(2)} \quad \lambda = \xi = 0, \quad \theta_L = -\theta_R = \pi, \quad \sigma_3 \rightarrow -\sigma_3. \tag{B16}$$

Summarizing, we see that, for the case $\Phi_L = \Phi_R$, there are three nontrivial solutions:

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \rightarrow \begin{bmatrix} a & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & -c \end{bmatrix}, \quad \begin{bmatrix} a & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & b \end{bmatrix}, \quad \begin{bmatrix} a & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & -b \end{bmatrix}. \tag{B17}$$

Turning to Eq. (B6), where $\Phi_L = \Phi_R + \frac{3}{2}\pi$, we have

$$\bar{H}' = e^{-i\Phi_R Q_R} \mathcal{H}' e^{i\Phi_L Q_L} = \begin{bmatrix} -a & 0 & 0 \\ 0 & -ib & 0 \\ 0 & 0 & -ic \end{bmatrix}. \tag{B18}$$

Since

$$(-i)\begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix} = \begin{pmatrix} b & 0 \\ 0 & -c \end{pmatrix} e^{-i\pi\sigma_3/2}, \tag{B19}$$

the discussions following Eq. (B7) can be taken over completely. There are then four nontrivial solutions for the case $\Phi_L = \Phi_R + \frac{3}{2}\pi$:

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \rightarrow \begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & c \end{bmatrix}, \quad \begin{bmatrix} -a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & -c \end{bmatrix}, \quad \begin{bmatrix} -a & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & b \end{bmatrix}, \quad \begin{bmatrix} -a & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & -b \end{bmatrix}. \tag{B20}$$

Actually, out of the seven solutions in Eqs. (B17) and (B20), only three are independent, which may be taken as

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \rightarrow \begin{bmatrix} -a & 0 & 0 \\ b & -b & 0 \\ 0 & 0 & c \end{bmatrix}, \quad \begin{bmatrix} a & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & b \end{bmatrix}, \quad \begin{bmatrix} -a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & -c \end{bmatrix}. \tag{B21}$$

Note that the first is W and the third is the X transformation defined in Paper I. All the other transformations can be obtained by successive applications of those listed in Eq. (B21).

The solutions (B21) lead to the following relations between $(\tilde{r},\tilde{s})$ and (r,s):

$$\begin{pmatrix} \tilde{r} \\ \tilde{s} \end{pmatrix} = \begin{pmatrix} \dfrac{2-r}{1+4r} \\ \dfrac{3s}{1+4r} \end{pmatrix}, \quad \begin{pmatrix} \frac{1}{2}(-r+s) \\ \frac{1}{2}(3r+s) \end{pmatrix}, \quad \begin{pmatrix} \dfrac{1-2r-s}{-1+2r-2s} \\ \dfrac{3(1+r)}{1-2r+2s} \end{pmatrix}. \quad \text{(B22)}$$

We have already discussed the numerical results that follow from Eq. (B22) in Sec. IV in the text.

Finally, we give the transformation properties of the $SU(3)\times SU(3)$ generators under Eq. (B21). For W, this was already done in Sec. III. It is easy to verify that the second transformation in Eq. (B21) corresponds to $\exp(i\pi F_7)$, which is the finite rotation in $SU(3)$ space that carries the isospin into the V spin. This shows very clearly how the relative weights of U_3 and U_8 in Eq. (B1) get changed. Note that, in this case, the Cabibbo angle undergoes the transformation: $\theta \rightarrow \frac{1}{2}\pi+\theta$. We emphasize that an "intrinsic" tadpole model at the $SU(3)$ level, if it contains a "small" isospin-breaking term, is rendered ambiguous by this transformation. Physically, this says that if $F_{1,2,4,5}$ are not conserved, then the statement that $F_{4,5}$ are "less" conserved than $F_{1,2}$ does not have an invariant meaning. Lastly, under X, the generators transform according to

$$(F_{3,8,4,5};F_{3,8,4,5}{}^5) \xrightarrow{X} (F_{3,8,4,5};F_{3,8,4,5}{}^5),$$

$$(F_{1,2,6,7};F_{1,2,6,7}{}^5) \xrightarrow{X} (-F_2{}^5, F_1{}^5, F_7{}^5, -F_6{}^5, -F_2, F_1, F_7, -F_6). \quad \text{(B23)}$$

This means that if F_1 and F_2 are not conserved, then we may yet construct a third $SU(3)$ group with the generators $(-F_2{}^5, F_1{}^5, F_3, F_4, F_5, F_7{}^5, -F_6{}^5, F_8)$. In the framework in which only the $U(1)\times U(1)$ symmetry is preserved, this $SU(3)$ is not distinguishable from the "ordinary" $SU(3)$.

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Spontaneous Breakdown of Chiral Symmetry with Linear Realizations for Asymptotic Fields

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A method for considering the spontaneous breakdown of chiral symmetry without recourse to a particular Lagrangian model is presented. The physical or asymptotic fields are assigned to linear representations of chiral $SU_3\times SU_3$ together with appropriate c-number addition to the scalar fields. This c-number addition manifests the spontaneous breakdown of the symmetry. Expansion of the interpolating fields in terms of asymptotic fields is introduced, which, together with the spontaneously broken chiral symmetry and the arbitrariness in the choice of interpolating fields, produces sum rules relating leptonic, semileptonic, and strong-coupling constants of $0^{\pm}$ and $1^{\pm}$ mesons. Results similar to those of current algebra are obtained with some notable differences. Among the most important is the appearance of the soft-meson amplitude as a consequence of our mechanism of spontaneous breakdown. We are thereby led, in an exceedingly simple way, to generalized soft-meson sum rules. In particular, new results are given for the semileptonic decays of the K meson, and generalized relations among strong-coupling constants and leptonic decays of $1^{\pm}$ mesons are derived.

I. INTRODUCTION

It is widely recognized that any local operator having the same quantum numbers as an asymptotic field can be chosen as an interpolating, Heisenberg field for that asymptotic field.[1] Although the transition matrix may take different forms off the mass shell (depending upon the choice of the interpolating field), it is unique on the mass shell. Thus, for example, any isovector, pseudoscalar, local operator can be used as an interpolating field for the pion. Specifically, one may choose the divergence of the axial-vector current as its interpolating field. This so-called partially conserved axial-vector current (PCAC) condition[2] has been extensively employed in the current-algebra approach[3] to chiral

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