Regge Amplitude Arising from $SU(6)_W$ Vertices*†

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The spin structure of three-particle vertices may be determined from the quark model. Using these $SU(6)_W$ vertices in the Van Hove model, we derive a Reggeized scattering amplitude. In addition to Regge poles, there are necessarily fixed Regge cuts in both fermion and boson exchange amplitudes. The magnitudes of the pole and cut terms in an entire class of $SU(6)$-related reactions are determined by their magnitudes in a single reaction. As an example, we explain the observed presence or absence of wrong-signature nonsense dips in a class of reactions involving vector-meson exchange.

INTRODUCTION

THE $SU(3)$ symmetry of the quark model has been extremely useful in classifying strongly interacting particles and in predicting the relative strengths of their couplings. Spin has been incorporated in the model to give a successful classification of hadron states under $SU(6)$.[1] The most natural way of treating spin at three-particle vertices yields the symmetry $SU(6)_W$.[2,3] While this symmetry correctly describes many vertices,[3] there has previously been no successful application to scattering[4] amplitudes.

In this paper we determine the form of the Regge amplitude which results from assuming $SU(6)_W$ as a vertex symmetry. Knowledge of vertices involving a spin-J resonance enables us to construct the Feynman amplitude for the exchange of the resonance. Then, using the Van Hove model,[5] we can express the Regge amplitude as a formal sum (on J) of such resonance exchanges. Hence in this model the form of the Regge amplitude is determined by the assumption of $SU(6)_W$-symmetric vertices. We find that Regge poles are, in general, accompanied by fixed Regge cuts, with branch points at the zero-energy intercept of the trajectory. These fixed cuts are similar to those suggested previously[6] for fermion exchange amplitudes as a consequence of the absence of parity-doubled fermion states. The fixed cuts found here for meson (quark-antiquark) exchange amplitudes may be viewed as a consequence of the absence of parity-doubled quarks.[7]

The presence of significant fixed-cut terms has important experimental consequences. The shrinkage characteristic of a Regge trajectory with normal slope will be absent in those amplitudes which have fixed cuts, and there will be no dips at wrong-signature nonsense points along the trajectory. Given the magnitudes of pole and cut terms in some reaction, we are able to predict the magnitudes of these terms for a whole class of $SU(6)$-related reactions. Applying our approach to a set of vector-meson-exchange processes, we find that the numerical importance of cut terms—as indicated by the presence or absence of wrong-signature nonsense dips in differential cross sections—is in accord with our predictions.

An outline of the paper is as follows. In Sec. I we show that $SU(6)_W$ is the natural vertex symmetry arising from the quark model and remind the reader how to calculate $SU(6)$ vertices. Construction of a Regge amplitude from the $SU(6)_W$ vertices is carried out in Sec. II. Some consequences of our approach are given in Sec. III, with particular attention to the question of wrong-signature nonsense dips. A discussion of our work is given in Sec. IV with some suggestions for further research on this problem.

I. QUARK MODEL AND $SU(6)_W$

The classification of baryons as qqq composites and mesons as $q\bar{q}$ composites implies that any $SU(3)$-invariant vertex may be pictorially represented by quark graphs. (See Fig. 1.) These graphs are drawn according to the following rules. (a) Each quark or antiquark is represented by a directed line. (b) A baryon or antibaryon is represented by three lines running in the same direction. (c) A meson is represented by two lines running in opposite directions. Zweig[8] suggested an additional rule: (d) The quark and

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† This paper supersedes R. Carlitz, Caltech Report No. CALT-68-202, 1969 (unpublished). In that paper, construction of a Regge amplitude from $SU(6)_W$ vertices was proposed, but the important role of Regge cuts was not recognized.

[1] See, e.g., rapporteur talks of B. French and A. Donnachie, in *Proceedings of the Fourteenth International Conference on High Energy Physics* (CERN, Geneva, 1968).

[2] $SU(6)_W$ was first proposed by H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965).

[3] A good review of $SU(6)_W$ is given by H. Harari, in *Lectures in Theoretical Physics* (Colorado U.P., Boulder, Colo., 1965), Vol. VIII-B.

[4] $SU(6)_W$ is, of course, meaningless for noncollinear processes. If we admit the idea of Regge-pole dominance and factorization, we can see why it does not work for collinear processes either. This is because $SU(6)_W$ relates spin-nonflip amplitudes to flip-flip amplitudes, and the latter vanish as a consequence of factorization.

[5] L. Van Hove, Phys. Letters **24B**, 183 (1967); R. P. Feynman, Caltech lecture, 1967 (unpublished).

[6] R. Carlitz and M. Kislinger, Phys. Rev. Letters **24**, 186 (1970).

[7] See R. Delbourgo and H. Rashid, Phys. Rev. **176**, 2074 (1968). They give a model with $SU(6)_W$-symmetric vertices and no Regge cuts, but find it necessary to use parity-doubled quarks to eliminate $\sqrt{t}$ singularities. In our model, these singularities are canceled by the cut terms.

[8] G. Zweig, CERN Report No. Th-402, 1964 (unpublished).

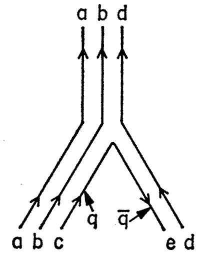

FIG. 1. Quark graph for meson-baryon vertex.

antiquark lines of a single meson should not be connected. (This rule accounts for the absence of the decay $\phi \rightarrow \rho\pi$ and the weak coupling of the ϕ to nucleons.) Note that the rules (a)–(d) are precisely those used by Harari[9] and by Rosner[10] to construct their duality diagrams.

We wish to incorporate spin into the quark-graph picture in the simplest possible fashion. We choose a Lorentz frame in which the particle momenta are collinear along the z axis. In such a frame, we assume that the spins of quarks a, b, and d (in Fig. 1) are unchanged in the reaction. How, then, must the spins of the annihilating quarks c and e be related? The parity of a $q\bar{q}$ pair is $-(-1)^L$, so if parity is to be conserved, q_c and $\bar{q}_e$ must annihilate in an odd angular momentum state. Then angular momentum conservation requires that $L=1$ and that the quark spins form a triplet. If, furthermore, the transverse motions of the annihilating quarks may be neglected, then the quark momenta lie along the z axis, so $L_z=0$ and hence $S_z=0$.

Thus we see that the $q\bar{q}$ state annihilates with the quark spins in a triplet state, $S=1$, $S_z=0$. This is exactly the result given by the collinear symmetry $SU(2)_W$.[2] Taking into account the $SU(3)$ quantum numbers of the quarks, we obtain $SU(6)_W$ as the natural vertex symmetry of the quark model.[11] Note that the derivation above is independent of what collinear frame we choose, since $SU(2)_W$ states are invariant under boosts along the z axis.

Choosing some collinear frame, it is easy to calculate the $SU(6)_W$-symmetric vertex functions. Let each quark be represented by a pair of indices (α,a), where α specifies its $SU(3)$ nature and a gives its spin orientation along the z axis. In a collinear frame, the meson-baryon vertex (Fig. 1) has the form

$$\bar{B}_{(\alpha a)(\beta b)(\gamma c)} B^{(\alpha a)(\beta b)(\delta d)} M^{(\gamma e)}{}_{(\delta d)} D_{ec}. \tag{1}$$

The matrix

$$D=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \tag{2}$$

in (1) specifies that q_c and $\bar{q}_e$ annihilate in a spin state $S=1$, $S_z=0$.

[9] H. Harari, Phys. Rev. Letters **22**, 562 (1969).
[10] J. Rosner, Phys. Rev. Letters **22**, 689 (1969).
[11] Normal $SU(6)$ is clearly not an appropriate symmetry for the vertices, for it requires a $q\bar{q}$ pair to annihilate in a singlet spin state—contrary to angular momentum and parity conservation.

TABLE I. Baryon-baryon-meson vertices.

Vertex	Spins[a]	Value[b]
BBP	$\frac{1}{2}, \frac{1}{2}, 0$	$(1/18)(5\langle\bar{B}PB\rangle+\langle\bar{B}BP\rangle)$
BBV	$\frac{1}{2}, -\frac{1}{2}, 1$	$(1/9\sqrt{2})(5\langle\bar{B}VB\rangle+\langle\bar{B}BV\rangle-\langle\bar{B}B\rangle\langle V\rangle)$
	$\frac{1}{2}, \frac{1}{2}, 0$	$\frac{1}{6}(\langle\bar{B}VB\rangle-\langle\bar{B}BV\rangle+\langle\bar{B}B\rangle\langle V\rangle)$
DBP	$\frac{1}{2}, \frac{1}{2}, 0$	$-\frac{1}{3}(\sqrt{\frac{2}{3}})(\bar{D}_{\alpha\beta\gamma}\epsilon^{\beta\delta\tau}B^{\alpha}{}_{\tau}P^{\gamma}{}_{\delta})$
DBV	$\frac{3}{2}, \frac{1}{2}, 1$	$\frac{1}{3}(\bar{D}_{\alpha\beta\gamma}\epsilon^{\beta\delta\tau}B^{\alpha}{}_{\tau}V^{\gamma}{}_{\delta})$
	$\frac{1}{2}, \frac{1}{2}, 0$	0
	$\frac{1}{2}, -\frac{1}{2}, 1$	$(1/3\sqrt{3})(\bar{D}_{\alpha\beta\gamma}\epsilon^{\beta\delta\tau}B^{\alpha}{}_{\tau}V^{\gamma}{}_{\delta})$
DDP	$\frac{3}{2}, \frac{3}{2}, 0$	$\frac{1}{2}(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}P^{\gamma}{}_{\delta})$
	$\frac{1}{2}, \frac{1}{2}, 0$	$\frac{1}{6}(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}P^{\gamma}{}_{\delta})$
DDV	$\frac{3}{2}, \frac{3}{2}, 0$	$\frac{1}{2}(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}V^{\gamma}{}_{\delta})$
	$\frac{3}{2}, \frac{1}{2}, 1$	$(\sqrt{\frac{1}{6}})(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}V^{\gamma}{}_{\delta})$
	$\frac{1}{2}, \frac{1}{2}, 0$	$\frac{1}{2}(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}V^{\gamma\beta})$
	$\frac{1}{2}, -\frac{1}{2}, 1$	$\frac{1}{3}\sqrt{2}(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}V^{\gamma}{}_{\delta})$
	$\frac{1}{2}, \frac{3}{2}, -1$	$(\sqrt{\frac{1}{6}})(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}V^{\gamma}{}_{\delta})$

[a] Particle 1 is outgoing; particles 2 and 3 are incoming.
[b] $\langle\bar{B}PB\rangle$ denotes $\bar{B}^{\alpha}{}_{\beta}P^{\beta}{}_{\gamma}B^{\gamma}{}_{\alpha}$.

Let us recall the form of the $SU(6)$ wave functions[12] for the **56** baryons, $B^{(\alpha a)(\beta b)(\gamma c)}$, and the **35** mesons, $M^{(\alpha a)}{}_{(\beta b)}$:

$$B^{(\alpha a)(\beta b)(\gamma c)m}=\tfrac{1}{3}[\epsilon^{\alpha\beta\sigma}B^{\gamma}{}_{\sigma}C_{ab}\chi_c{}^{(m)}+\epsilon^{\beta\gamma\sigma}B^{\alpha}{}_{\sigma}C_{bc}\chi_a{}^{(m)} +\epsilon^{\gamma\alpha\sigma}B^{\beta}{}_{\sigma}C_{ca}\chi_b{}^{(m)}]+D^{\alpha\beta\gamma}(\sigma^i C)_{ab}\xi_{ci}{}^{(m)}. \tag{3}$$

The superscript $m=a+b+c$ gives the spin projection of the baryon. χ is a two-component spinor,

$$\chi^{(+1/2)}=\begin{pmatrix}1\\0\end{pmatrix},\quad \chi^{(-1/2)}=\begin{pmatrix}0\\1\end{pmatrix}, \tag{4}$$

and $\xi_a{}^{(m)}$ is a vector spinor,

$$\xi_a{}^{(m)}=\sum_{\sigma\rho}(\tfrac{1}{2}\sigma,1\rho\,|\,\tfrac{3}{2}m)\chi_a{}^{(\sigma)}\epsilon^{(\rho)}, \tag{5}$$

where

$$\epsilon^{\pm 1}=\tfrac{1}{2}(\mp 1,-i,0), \quad \epsilon^0=(0,0,1). \tag{6}$$

The matrix C is given by

$$C=i\sigma_y=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}. \tag{7}$$

$B^{\alpha}{}_{\beta}$ and $D^{\alpha\beta\gamma}$ are the $SU(3)$ matrices for the octet and decuplet:

$$B^{\alpha}{}_{\beta}=\begin{bmatrix}\frac{1}{\sqrt{2}}\Sigma^0+\frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p\\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0+\frac{1}{\sqrt{6}}\Lambda^0 & n\\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0\end{bmatrix}, \tag{8}$$

[12] See B. W. Lee, in *Particle Symmetries* (Gordon and Breach, New York, 1966).

$$\begin{aligned} &D^{111}=\Delta^{++}, && D^{112}=(1/\sqrt{3})\Delta^{+},\\ &D^{113}=(1/\sqrt{3})\Sigma^{*+}, && D^{122}=(1/\sqrt{3})\Delta^{0},\\ &D^{123}=(1/\sqrt{6})\Sigma^{*0}, && D^{133}=(1/\sqrt{3})\Xi^{0},\\ &D^{222}=\Delta^{-}, && D^{223}=(1/\sqrt{3})\Sigma^{*-},\\ &D^{233}=(1/\sqrt{3})\Xi^{-}, && D^{333}=\Omega^{-}. \end{aligned} \tag{9}$$

The meson wave function is

$$M^{(\alpha a)}{}_{(\beta b)}{}^{m}=P^{\alpha}{}_{\beta}C_{ab}+V^{\alpha}{}_{\beta}\epsilon_i{}^{(m)}(\sigma^i C)_{ab}, \tag{10}$$

where

$$P^{\alpha}{}_{\beta}=\begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta^0 & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{bmatrix}, \tag{11}$$

and

$$V^{\alpha}{}_{\beta}=\begin{bmatrix} \frac{1}{\sqrt{2}}(\omega^0+\rho^0) & \rho^+ & K^{*+}\\ \rho^- & \frac{1}{\sqrt{2}}(\omega^0-\rho^0) & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \phi \end{bmatrix}. \tag{12}$$

The superscript m specifies polarization for the vector-meson nonet.

Using (3) and (10) in (1), we obtain the vertex functions given in Table I. The **56**-**56**-**35** vertices are all determined to within a single constant factor by the $SU(6)_W$ symmetry. Similarly, the **35**-**35**-**35** vertices may be computed from the coupling

$$M^{1(\alpha a)}{}_{(\beta b)}M^{2(\beta b)}{}_{(\gamma c)}M^{3(\gamma d)}{}_{(\alpha a)}D_{cd}. \tag{13}$$

The results are summarized in Table II.

Using the information in Tables I and II, it is a straightforward matter to calculate the invariant vertex functions which, in a collinear frame, reduce to those given in the tables. These invariant vertex functions are listed in Table III.[13] Common mass factors have been absorbed in the constants c and d to give the entries a simple form.

TABLE II. Meson-meson-meson vertices.

Vertex	Spins	Value
P_1P_2V	0, 0, 0	$\frac{1}{2}(\langle P_1P_2V\rangle-\langle P_1VP_2\rangle)$
V_1V_2P	1, 1, 0	$\frac{1}{2}(\langle V_1V_2P\rangle+\langle V_1PV_2\rangle)$
$V_1V_2V_3$	0, 0, 0	$\frac{1}{2}(\langle V_1V_2V_3\rangle-\langle V_3V_2V_1\rangle)$
	1, 1, 0	$\frac{1}{2}(\langle V_1V_2V_3\rangle-\langle V_3V_2V_1\rangle)$
	1, 0, 1	$\frac{1}{2}(\langle V_1V_2V_3\rangle-\langle V_3V_2V_1\rangle)$
	0, 1, −1	$\frac{1}{2}(\langle V_1V_2V_3\rangle-\langle V_3V_2V_1\rangle)$

The extension of the couplings in Table III to vertices involving Regge recurrences of the listed states is easily made. For example, the $SU(6)_W$-symmetric coupling of two pseudoscalar mesons to a V recurrence (quark spin 1) of spin $J=L+1$ will be

$$-d(J)M_3(J)\epsilon_{3\nu\mu_1\cdots\mu_L}(p_1+p_2)_\nu(p_1+p_2)_{\mu_1}\cdots(p_1+p_2)_{\mu_L}\times(\langle P_1P_2V\rangle-\langle P_1VP_2\rangle).$$

(Here ϵ denotes the polarization of the V recurrence.) The higher-spin indices simply couple to appropriate momentum factors. In general, couplings for excited states with a given quark-spin assignment may be constructed by decomposing the spin of the states into quark spin and orbital angular momentum and coupling the quark spin according to $SU(6)_W$. This will give a unique result whenever two of the states have no orbital excitation. In other cases there will be more than a single coupling for each class of $SU(6)$-related relations.

Note the presence in Table III of factors involving the masses M_1, M_2, and M_3. These factors have a simple kinematic origin. Some of these, e.g., $(M_1+M_2)^2-M_3{}^2$, arise because the $SU(6)_W$ symmetry relates vertices involving different angular momenta. Other factors, e.g., M_3 in the P_1P_2V vertex, arise because the symmetry relates vertices involving vector mesons of different helicities. The polarization vector for a vector meson of zero helicity is proportional to $1/M_V$.[14] Hence, within a class of vertices related by $SU(6)_W$, those vertices involving a zero-helicity vector meson will contain an extra factor of M_V relative to those not involving a zero-helicity vector meson. This is the only source of odd powers of meson masses; the angular momentum factors will always contain factors of (meson mass)^2.

II. CONSTRUCTION OF REGGE AMPLITUDE

The presence of extra "kinematic" factors in the vertex functions has important consequences when we construct a Regge amplitude. In general, we find the presence of fixed Regge cuts with branch points coinciding with the zero-energy intercepts of the Regge trajectories. This phenomenon has been discussed previously for fermion exchange processes; here we find cuts for boson or fermion exchange processes and predict the relative strengths of the cuts in different processes.

The Van Hove model[15] expresses a Regge amplitude

[13] We have calculated the entires in Table III by selecting a complete set of invariant vertex functions, evaluating these functions in a collinear frame, equating these values to those in Tables I and II, and solving the resulting set of linear equations. An alternative method would be to construct relativistic wave functions and write down a covariant version of (1). See B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965).

[14] For a meson with four-momentum $(E;0,0,q)$, $E^2-q^2=M_V{}^2$, the normalized polarization vector is $\epsilon^0=(q;0,0,E)/M_V$.

[15] Details of the Van Hove construction are discussed by R. Blankenbecler and R. L. Sugar, Phys. Rev. **168**, 1597 (1968); see also Ref. 6.

TABLE III. Invariant vertex functions.

Vertex	Function[a]
BBP	$\frac{1}{9}c[(M_1+M_2)^2-M_3^2](\bar{u}_1\gamma_5u_2)(5\langle\bar{B}PB\rangle+\langle\bar{B}BP\rangle)$
BBV	$\frac{1}{9}c\{[\epsilon_3\cdot(p_1+p_2)(\bar{u}_2u_1)][(M_1+M_2)(5\langle\bar{B}VB\rangle+\langle\bar{B}BV\rangle-\langle\bar{B}B\rangle\langle V\rangle)-3M_3(\langle\bar{B}VB\rangle-\langle\bar{B}BV\rangle+\langle\bar{B}B\rangle\langle V\rangle)]$ $-(\bar{u}_1\gamma\cdot\epsilon_3u_2)[(M_1+M_2)^2-M_3^2](5\langle\bar{B}VB\rangle+\langle\bar{B}BV\rangle-\langle\bar{B}B\rangle\langle V\rangle)\}$
DBP	$+\frac{4}{3}cM_1p_{2\mu}(\bar{u}_{1\mu}u_2)(D_{\alpha\beta\gamma}\epsilon^{\beta\delta\tau}B^{\alpha}{}_{\tau}P^{\gamma}{}_{\delta})$
DBV	$-\frac{2}{3}c\{[(M_1+M_2)^2-M_3^2]\epsilon_{3\mu}(\bar{u}_{1\mu}\gamma_5u_2)-p_{1\mu}(\bar{u}_{1\mu}\gamma_5u_2)\epsilon_3\cdot(p_1+p_2)$ $+2M_1p_{2\mu}(\bar{u}_{1\mu}\gamma\cdot\epsilon_3\gamma_5u_2)\}(\bar{D}_{\alpha\beta\gamma}\epsilon^{\beta\delta\tau}B^{\alpha}{}_{\tau}V^{\gamma}{}_{\delta})$
DDP	$c\{2p_{2\mu}(\bar{u}_{1\mu}\gamma_5u_{2\nu})p_{1\nu}-[(M_1+M_2)^2-M_3^2](\bar{u}_{1\mu}\gamma_5u_{2\mu})\}(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}P^{\gamma}{}_{\delta})$
DDV	$c\{[(M_1+M_2)^2-M_3^2](\bar{u}_{1\mu}\gamma\cdot\epsilon_3u_{2\mu})-(M_1+M_2-M_3)(\bar{u}_{1\mu}u_{2\mu})\epsilon_3\cdot(p_1+p_2)$ $-2p_{2\mu}(\bar{u}_{1\mu}\gamma\cdot\epsilon_3u_{2\nu})p_{1\nu}+2p_{2\mu}(u_{1\mu}u_{2\nu})p_{1\nu}\epsilon_3\cdot(p_1+p_2)/(M_1+M_2+M_3)\}(\bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\delta}V^{\gamma}{}_{\delta})$
P_1P_2V	$-dM_3\epsilon_3\cdot(p_1+p_2)(\langle P_1P_2V\rangle-\langle P_1VP_2\rangle)$
V_1V_2P	$-id\epsilon^{\mu\nu\rho\sigma}p_{3\mu}(p_1+p_2)_{\nu}\epsilon_{2\rho}\epsilon_{1\sigma}^*(\langle V_1V_2P\rangle+\langle V_1PV_2\rangle)$
$V_1V_2V_3$	$d[M_3(\epsilon_1^*\cdot\epsilon_2)(\epsilon_3\cdot(p_1+p_2))-M_2(\epsilon_1^*\cdot\epsilon_3)[\epsilon_2\cdot(p_3+p_1)]-M_1(\epsilon_2\cdot\epsilon_3)[\epsilon_1^*\cdot(p_2-p_3)]$ $+\epsilon_1^*\cdot(p_2-p_3)\epsilon_2\cdot(p_3+p_1)\epsilon_3\cdot(p_1+p_2)/2(M_1+M_2+M_3)](\langle V_1V_2V_3\rangle-\langle V_3V_2V_1\rangle)$

[a] M_i and p_i refer to the mass and momentum of the ith particle; $i=1, 2, 3$. Particle 1 is outgoing; particles 2 and 3 are incoming.

as a formal sum of Feynman diagrams for the exchange of all resonances along a given trajectory.[16] Consider, for example, the reaction

$$\pi^+ + \pi^- \to \pi^0 + \omega^0,$$

mediated by ρ exchange (see Fig. 2). The coupling at a $\omega\rho^*\pi$ vertex is

$$-i\sqrt{2}d(J)\epsilon^{\alpha\mu_1\gamma\delta}\epsilon_{(\omega)\alpha}{}^*p_{(\rho^*)\gamma}p_{(\pi)\delta}p_{(\pi)\mu_2}\cdots p_{(\pi)\mu_J}\times 2^J\epsilon_{(\rho^*)\mu_1\mu_2\cdots\mu_J}, \quad (14)$$

for a ρ^* of spin J. The $\pi\pi\rho^*$ coupling is

$$-\sqrt{2}d(J)m_{\rho^*}(J)p_{(\pi)\mu_1}p_{(\pi)\mu_2}\cdots p_{(\pi)\mu_J}2^J\epsilon_{(\rho^*)\mu_1\mu_2\cdots\mu_J}. \quad (15)$$

The Feynman diagram for the exchange of a ρ^* of spin J is therefore

$$\mathfrak{M}(J)=2id^2(J)m(J)4^J\frac{p_{\mu_1}{}^+\cdots p_{\mu_J}{}^+T_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}{}^{(J)}p_\delta{}^-p_{\nu_2}{}^-\cdots p_{\nu_2}{}^-Q_\gamma\epsilon_{(\omega)\alpha}{}^*\epsilon^{\alpha\nu_1\gamma\delta}}{t-m^2(J)}$$

$$=\frac{2id^2(J)m(J)4^J}{t-m^2(J)}\epsilon_{(\omega)\alpha}{}^*Q_\gamma p_\delta{}^-\epsilon^{\alpha\beta\gamma\delta}\frac{1}{J}\frac{\partial}{\partial p_\beta{}^-}\mathcal{P}_J(p^+,p^-), \quad (16)$$

where $T_{\mu;\nu}{}^{(s)}/[t-m^2(J)]$ is the Feynman propagator for a particle of spin J,

$$\mathcal{P}_J(p^+,p^-)=4^J(|p^+||p^-|)^JP_J\left(\frac{p^+\cdot p^-}{|p^+||p^-|}\right), \quad (17)$$

and $t=Q^2$. We assume that $m_\pi=m_\omega$ to simplify the kinematics. Summing over J and transforming the sum into a contour integral gives

$$\mathfrak{M}=\sum_J\mathfrak{M}(J)$$
$$=\tfrac{1}{2}iZ_\beta\int_C dJ\frac{d^2(J)m(J)}{t-m^2(J)}\frac{1}{J}\frac{\partial}{\partial p_\beta{}^-}\frac{\mathcal{P}_J(p^+,-p^-)}{\sin\pi J}, \quad (18)$$

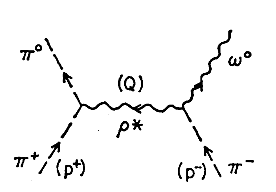

FIG. 2. Kinematics for $\pi\pi\to\pi\omega$.

using the abbreviation

$$Z_\beta=2i\epsilon_{(\omega)\alpha}{}^*Q_\gamma p^-{}_\delta\epsilon^{\alpha\beta\gamma\delta}. \quad (19)$$

The contour C is indicated in Fig. 3. If we assume that the m^2 is a linear function of J,

$$m^2(J)=(J-\alpha_0)/\alpha', \quad (20)$$

and that $d^2(J)$ is analytic, then we can open the contour in the J plane obtaining contributions from the pole at $m^2(J)=t$ and the cut with branch point $J=\alpha_0$. This

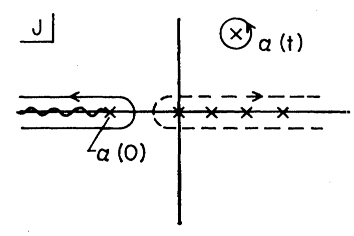

FIG. 3. Dashed line—initial contour; solid line—opened contour.

[16] There is an ambiguity in the Van Hove model of Reggeization corresponding to the ambiguity in the behavior of the propagator and vertex functions off the mass shell. We eliminate this ambiguity by choosing to Reggeize t-channel helicity amplitudes and including in the Van Hove sum (16) terms whose only t dependence is in the denominator $t-m(J)^2$.

gives

$$\mathfrak{M}=\frac{\pi d^2(\alpha)(\sqrt{t})\alpha'}{\sin\pi\alpha}Z_\beta\frac{1}{J}\frac{\partial}{\partial p_\beta^-}\mathcal{P}_J(p^+,-p^-)-Z_\beta\int_{-\infty}^{\alpha_0}dJ$$
$$\times\frac{d^2(J)[-m^2(J)]^{1/2}}{[t-m^2(J)]\sin\pi J}\frac{1}{J}\frac{\partial}{\partial p_\beta^-}\mathcal{P}_J(p^+,-p^-),\quad(21)$$

where

$$\alpha(t)=\alpha_0+\alpha' t.\quad(22)$$

In the limit $s=(p^++p^-)^2\to\infty$,

$$\mathfrak{M}\to\frac{\pi d^2(\alpha(t))b(\alpha(t))\alpha' s^{\alpha(t)-1}}{\sin\pi\alpha(t)}Z_\beta p_\beta^+\sqrt{t}$$
$$-Z_\beta p_\beta^+\int_{-\infty}^{\alpha_0}dJ\frac{d^2(J)[-m^2(J)]^{1/2}b(J)s^{J-1}}{[t-m^2(J)]\sin\pi J},\quad(23)$$

where

$$b(J)=\Gamma(2J+1)/[\Gamma(J+1)]^2.\quad(24)$$

It is clear that (23) consists of a moving Regge-pole term and a fixed Regge cut. Nonsense couplings along the trajectories are presumably eliminated in the usual way by zeros of $d^2(J)$ at $J=0, -1, -2, \ldots$. Therefore, for negative t, we can approximate

$$\frac{d^2(\alpha(t))b(\alpha(t))\alpha'}{\sin\pi\alpha(t)}=d_0.\quad(25)$$

Then (23) becomes

$$\mathfrak{M}\simeq\pi d_0Z_\beta p_\beta^+[(\sqrt{t})\,\mathrm{erf}((\alpha' t\ln s)^{1/2})s^{\alpha(t)-1}$$
$$+(\alpha'\pi\ln s)^{-1/2}s^{\alpha_0-1}]$$
$$\simeq\pi d_0Z_\beta p_\beta^+\left\{(\sqrt{t})s^{\alpha(t)-1}+\frac{s^{\alpha_0-1}}{2t(\sqrt{\pi})(\alpha'\ln s)^{3/2}}\right.$$
$$\left.\times\left[1+O\left(\frac{1}{\alpha' t\ln s}\right)\right]\right\}.\quad(26)$$

In Eq. (18) it is clear that the fixed cut arises from the presence of an odd power of $m(J)$ and the assumption that $d^2(J)$ is even in $m(J)$, i.e., analytic in J. If, for example, $d^2(J)=m(J)d_1(J)$, with $d_1(J)$ analytic, then the $\pi\pi\to\pi\omega$ amplitude would have no fixed cuts. In this case, however, there would be fixed cuts in the amplitudes $\pi\pi\to\pi\pi$ and $\pi\omega\to\pi\omega$. Hence the presence of fixed cuts in some amplitudes is inescapable.

III. EXPERIMENTAL CONSEQUENCES

The presence of fixed Regge cuts has three important experimental consequences. (a) Asymptotic behavior: At sufficiently high energies, the Regge-cut term gives an energy falloff independent of t.[17] (b) Polarization: When signature is incorporated in the model by the replacement

$$\mathfrak{M}(t,z_t)\to\tfrac{1}{2}[\mathfrak{M}(t,z_t)+\tau\mathfrak{M}(t,-z_t)],\quad(27)$$

Regge-pole terms will acquire the usual Regge phase, but cut terms will have some complicated varying phase. Thus there can be interference between the pole and cut terms, and exchange of a single Regge trajectory will be able to give nonzero polarization. (c) Wrong-signature nonsense dips: The contribution from a single Regge-pole term vanishes when the trajectory passes through a nonsense value of the wrong signature. The cut term does not vanish, however, so an amplitude with a significant cut term will show no dip at wrong-signature nonsense points.[18]

A qualitative discussion of the third point is easy to make. We will restrict our attention to processes involving vector-meson exchange. In Sec. I we gave a simple criterion for determining the presence of odd powers of M_V in vector-meson vertices. The argument required examining the vertex in a collinear frame. Note that the t-channel c.m. frame is collinear for both vertices, so we can apply the argument of Sec. I directly to t-channel helicity amplitudes. The factors of $m(J)$ which will appear in t-channel helicity amplitudes involving various ρ^* or ω^* vertices are given in Table IV. Assuming the absence of a cut term in some particular helicity amplitude, this table allows us to predict the presence or absence of cut terms in other helicity amplitudes. In general, a reaction will have no cuts in some amplitudes and cuts in others. We must pay attention to the relative magnitudes of the different amplitudes in order to assess the importance of cut terms in any given process.

In Table V we tabulate the magnitudes of the helicity amplitudes and the factors of $m(J)$ which lead to Regge cuts for a number of reactions[19] involving the vertices

TABLE IV. Odd powers of $m(J)$ in vector-meson vertices.

Vertex	V^* helicity	$m(J)$ factors
$NN\rho^*$	±1	$c(J)$
	0	$m(J)c(J)$
$\Delta N\rho^*$	$\pm1^{a}$	$c(J)$
$NN\omega^*$	±1	$c(J)$
	0	$m(J)c(J)$
$\pi\pi\rho^*$	0^{b}	$m(J)d(J)$
$\pi\rho\omega^*$	$\pm1^{b}$	$d(J)$
$\pi\omega\rho^*$	$\pm1^{b}$	$d(J)$
$\eta\rho\rho^*$	$\pm1^{b}$	$d(J)$

[a] Only helicities allowed by $SU(6)_W$.
[b] Only helicities allowed by parity and angular momentum.

[17] This may not occur until quite high energies. From Eq. (26) we see that the leading term in the asymptotic expansion of the cut dominates only when $\alpha' t\ln s\gg1$.

[18] Note that our approach to wrong-signature nonsense dips is compatible with the argument that the exchange of a pair of exchange-degenerate trajectories will produce no dip.

[19] Photoproduction processes are included in Table V, assuming vector dominance. This is an unambiguous procedure for the reactions listed because the VVP vertex is automatically gauge invariant.

TABLE V. Wrong-signature nonsense dips for vector-meson exchange (see Ref. 19).

Reaction	Exchange	Helicities	Relative magnitude	Cut	Dip
$\pi^- p \to \pi^0 n$	ρ	1, 0	$5\sqrt{2}(\sqrt{t})g(J)$	none	yes
		0, 0	$3\sqrt{2}m(J)g(J)$	weak	
$\pi^- p \to \omega n$	ρ	1, 1	$5tg(J)/m(J)$	strong	no
		0, 1	$3(\sqrt{t})g(J)$	none	
$\pi^+ p \to \pi^0 \Delta^{++}$	ρ	$1(\frac{3}{2}, \frac{1}{2})$, 0	$6(\sqrt{t})g(J)$	none	yes
		$1(\frac{1}{2}, -\frac{1}{2})$, 0	$2\sqrt{3}(\sqrt{t})g(J)$	none	
$\pi^+ p \to \omega \Delta^{++}$	ρ	$1(\frac{3}{2}, \frac{1}{2})$, 1	$3\sqrt{2}tg(J)/m(J)$	strong	no
		$1(\frac{1}{2}, -\frac{1}{2})$, 1	$(\sqrt{6})tg(J)/m(J)$	strong	
$\pi^0 p \to \rho^0 p$	ω	1, 1	$(3/\sqrt{2})tg(J)/m(J)$	strong	yes[a]
		0, 1	$(9/\sqrt{2})(\sqrt{t})g(J)$	none	
$\rho^0 p \to \pi^0 p$	ω	1, 1	$(3/\sqrt{2})tg(J)/m(J)$	strong	yes[b]
		0, 1	$(9/\sqrt{2})(\sqrt{t})g(J)$	none	
$\rho^0 p \to \eta p$	ρ	1, 1	$(5/\sqrt{6})tg(J)/m(J)$	strong	no[c]
		0, 1	$(3/\sqrt{6})(\sqrt{t})g(J)$	none	
$\omega p \to \pi^+ n$	ρ	1, 1	$5tg(J)/m(J)$	strong	no[d]
		0, 1	$3(\sqrt{t})g(J)$	none	

[a] $(d\sigma/dt)(\pi^+ p \to \rho^+ p) + (d\sigma/dt)(\pi^- p \to \rho^- p) - (d\sigma/dt)(\pi^- p \to \rho^0 n)$.
[b] $(d\sigma/dt)(\gamma p \to \pi^0 p)$.
[c] $(d\sigma/dt)(\gamma p \to \eta p)$.
[d] $(d\sigma/dt)(\gamma p \to \pi^+ n) - (d\sigma/dt)(\gamma n \to \pi^- p)$.

of Table IV. The relative magnitudes of the contribution of a spin-J V^* exchange to the various t-channel helicity amplitudes are given by

$$f_{\lambda\mu}(t=m^2(J),\, s) = V_1^{(\lambda)} V_2^{(\mu)} \epsilon_1^{(\lambda)*} \cdot \epsilon_2^{(\mu)} G(J). \quad (28)$$

$V_1^{(\lambda)}$ and $V_2^{(\mu)}$ are the $SU(6)_W$ vertex coefficients from Tables I and II, and $\epsilon_1^{(\lambda)}$ and $\epsilon_2^{(\mu)}$ specify the orientation of the quark spin of the V^* at vertices 1 and 2, respectively, in the t-channel c.m. system. Equation (28) is obvious for $J=1$ and is valid for arbitrary J because the higher-spin indices always couple to additional momentum factors at each vertex. In the limit $s \to \infty$,

$$|\epsilon_1^{(\lambda)} \cdot \epsilon_2^{(\mu)}| \to |\epsilon_1^{(0)*} \cdot \epsilon_2^{(0)}| / \sqrt{2}^{|\lambda|+|\mu|}.$$

In Table V, then, we tabulate simply

$$|V_1^{(\lambda)} V_2^{(\mu)} c(J) d(J) / \sqrt{2}^{|\lambda|+|\mu|}|.$$

For convenience, we have defined

$$g(J) = c(J)d(J)/36m(J).$$

Genuine kinematic factors (kinematics of $\pi N \to \pi N$ are assumed for all reactions listed) are tabulated as $\sqrt{t}$ or t. The factors of $m(J)$ induced by $SU(6)_W$ are determined from Table IV and tabulated in the appropriate helicity amplitudes.

The qualitative features of the data for the reaction $\pi^- p \to \pi^0 n$ indicate that the reaction is dominated by a Regge pole in the t-channel helicity-1 amplitude. Therefore, the function $g(J)$ in Table V must be approximately even in $m(J)$, i.e., analytic in J, so that the helicity-1 amplitude is purely a Regge-pole term while the helicity-0 amplitude contains a Regge cut as well. This cut arises from the presence in the amplitude of a factor $m(J)$ and is referred to in Table V as a weak cut.

As can be seen in (26), the contribution to the scattering amplitude of a weak cut $[m(J)g(J)]$ at $t=0$ is suppressed by a factor $(\pi\alpha' \ln s)^{-1/2}$ relative to a pole term $[g(J)]$. A cut arising from the presence of a factor $1/m(J)$ is referred to as a strong cut. The magnitude of a strong cut $[g(J)/m(J)]$ is larger than that of a weak cut by a factor of $2\alpha' \ln s$.

Now in Table V we see that the helicity-1 amplitude in $\pi^- p \to \omega n$ contains a strong cut with a numerical coefficient larger than that for the pole term in the helicity-0 amplitude. Therefore, in this reaction, the cut effects should be appreciable and we expect no wrong-signature nonsense dip. Proceeding in this manner, we may make the other predictions given in the last column of Table V. These predictions agree with experiment for all the reactions listed.[20]

IV. DISCUSSION

The qualitative discussion above should be largely unaffected by the manner in which the $SU(6)$ symmetry of our theory is broken. Symmetry breaking will alter the $SU(3)$ factors and numerical coefficients in Table III, but will not affect the mass factors, which arise solely from kinematic considerations. In Table V it is apparent that the question of dip or no dip depends primarily on these mass factors. In a quantitative fit of differential cross sections and polarization phenomena, symmetry-breaking effects will be important and a more detailed theory will be necessary.

[20] A summary of the data and an empirical rule for the presence of wrong-signature nonsense dips in vector exchange processes is given in a review talk on photoproduction by H. Harari, in Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969 (unpublished).

Aside from symmetry breaking, an important question concerns the relation of our work to duality. Since both schemes are based on identical quark graphs, it seems likely that they may be fused in a unified approach. After completing our work on the manner in which $SU(6)_W$ leads to fixed Regge cuts, we learned that Bardakci and Halpern[21] have, in fact, constructed a dual amplitude containing fixed cuts and have proposed that this amplitude be utilized in the quark model. The leading trajectory in their model couples according to $SU(6)_W$ in the manner we have described. Ellis[22] has also investigated this problem, which we expect to open a fruitful area of new research.

[21] K. Bardakci and M. B. Halpern, Phys. Rev. Letters **24**, 428 (1970).

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[22] S. Ellis (unpublished).

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Some Properties of a Hamiltonian Model of Broken $SU(3)\times SU(3)$ Symmetry. II*

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An inherent ambiguity of broken $SU(3)\times SU(3)$ symmetry is discussed. It is shown to arise from a discrete unitary transformation in the $SU(3)\times SU(3)$ space. For Hamiltonian models for which the symmetry-breaking term transforms like the $(3,\bar{3})+(\bar{3},3)$ representation, we find that, in general, there are two such terms which describe the same physical system. Some consequences of this result are discussed.

I. INTRODUCTION AND GENERAL DISCUSSION

RECENTLY the significance of the chiral $SU(3)\times SU(3)$ symmetry has been clarified greatly by Glashow and Weinberg,[1] and by Gell-Mann, Oakes, and Renner.[2] They proposed that the strong-interaction Hamiltonian density should be written as

$$H=H_0+H', \tag{1}$$

where H_0 is invariant under $SU(3)\times SU(3)$ rotations,[3] and the symmetry-breaking term H' is considered to conserve the $U(1)\times SU(2)$ symmetry and to have definite transformation properties under the $SU(3)\times SU(3)$ group. In particular, GOR suggest that the simplest form for H' is

$$H'=\alpha(u_0+\sqrt{2}ru_8), \tag{2}$$

where α and r are real parameters, and u_i, $i=0,1,\ldots,8$, together with v_i, transform like the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$. There may also be terms transforming like $(1,8)+(8,1)$, $(8,8)$, etc. However, so far, very little is known about these other possibilities. In the following we will concentrate on Eq. (2), but will comment on the other choices when appropriate.

Now, as has been emphasized by Cabibbo and Maiani,[4] the directions in the $SU(3)\times SU(3)$ space are not fixed *a priori*. Indeed, there are an infinite number of Hamiltonians which describe the same hadronic world. These systems are connected by arbitrary rotations R in the $SU(3)\times SU(3)$ space. Let us denote by S the system described by Eq. (2). Then the system $\tilde{S}$, in which a state $|\alpha\rangle$ in S becomes $R|\alpha\rangle$ and an operator O becomes ROR^{-1}, is completely equivalent to S. Physically, the unitary transformation from S to $\tilde{S}$ means that we must redefine internal quantum numbers of the hadronic states, etc.

What are the effects of the electromagnetic and weak interactions? As far as the hadrons are concerned, they may be regarded as external fields. In considering rotations in the hadronic world, we should leave the directions defined by the electromagnetic and the weak[5] interactions unchanged. However, insofar as the direction of the hypercharge is not fixed, there is no *a priori* value for the Cabibbo angle either. Thus we may regard the direction of the weak currents as arbitrary in con-

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[1] S. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).

[2] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968). This paper will be referred to as GOR.

[3] The notation used in this paper will be as follows. For the "charges" of the vector and axial vector currents, we write $F_1,\ldots,_8$ and $F_1,\ldots,_8{}^5$. The generators of the "left" and "right" $SU(3)$'s are then $\frac{1}{2}(F_i\pm F_i{}^5)$. Also, we use Y_5, $I_3{}^5$, etc., to denote the axial counterpart of Y,I_3, etc.

[4] N. Cabibbo and L. Maiani, Phys. Rev. D **1**, 707 (1970). This paper will be referred to as CM. In this connection, see also the related papers by R. Gatto, G. Santoni, and M. Tonin [Phys. Letters **28B**, 128 (1968)] and N. Cabibbo and L. Maiani [*ibid.* **28B**, 131 (1968)].

[5] By the direction of the weak interaction, we are referring to the Cabibbo angle. Nonleptonic weak interaction may be considered to arise from the current-current interaction.