the reliable dynamical group for particles in relativistic quantum mechanics. A further unpleasant feature of this model is concerned with the fact that to each mass state we have infinitely many particles with different spins varying from some lowest spin value j_0 and going up to infinity or that there are only spin-zero particles in the world.

All these unphysical features of the presented theory have their origin in postulating the naive and simplest covariant generalization of the commutator (2.21) between position and momentum operators. To obtain some reasonable physical results within the framework of this kind of theory, we must give up the aforementioned commutator and replace it by some other relativistically covariant form.[13] This replacement will induce, however, a radical change in the structure of the dynamical group here investigated.

To conclude this discussion, it should be stressed that relativistic quantum mechanics using the naive commutator (2.21) leads to completely unphysical results, and thus the dynamical group $\tilde{G}_5$, which is mainly based on this commutator, has no chance of giving any reasonable physical predictions.

ACKNOWLEDGMENTS

The author is grateful for many useful conversations with Professor S. P. Rosen and Dr. J. Katz. He also thanks Professor R. H. Capps and Professor M. Sugawara as well as the other members of the theoretical group for the kind hospitality extended to him at Purdue University. He is also indebted to Professor Aghassi, Professor Roman, and Professor Santilli for helpful discussions and correspondence.

PHYSICAL REVIEW D VOLUME 2, NUMBER 2 15 JULY 1970

Compositeness Criterion for Unstable Particles

J. N. Passi
Saha Institute of Nuclear Physics, Calcutta-9, India

(Received 30 March 1970)

The problem of making unstable elementary particles equivalent to resonances is investigated in a soluble model field theory. It is demonstrated that the irreducible part of the scattering amplitude develops a pair of complex conjugate poles on the second sheet of the energy plane, one of them corresponding to an S-wave resonance. The poles in the vertex and inverse propagator induced by these complex poles of the irreducible part of the scattering amplitude are such that the so-called Jin-MacDowell cancellation holds good. We show that under the conditions $Z_1=0$ and $Z_3=0$, the S-wave resonance completely replaces the unstable particle.

I. INTRODUCTION

It has been demonstrated by several authors[1] that the stable elementary particles become composite under vanishing of the wave function and the vertex-function renormalization constants. In the models employed by these authors, the poles of the reducible part of the scattering amplitude are made to cancel each other, and the pole of the irreducible part which corresponds to the dynamical state is made to move to the elementary-particle pole position. In this paper, we examine this type of compositeness mechanism for an unstable elementary particle.

We define unstable particles according to the suggestion by Peierls[2] that they correspond to the complex poles of the propagator analytically continued to unphysical sheets. For our purpose, we consider a model field theory which consists of V, N, and θ particles, where N and θ are stable, but V is unstable. We show that the irreducible part of the $N\theta$ scattering amplitude analytically continued onto the second sheet develops a complex conjugate pair of poles corresponding to resonance and antiresonance of N and θ. We demonstrate that these poles of the irreducible part get canceled by the corresponding induced poles in the reducible part, i.e., the Jin-MacDowell[3] cancellation holds good for resonant states also. Conditions are found under which the pair of poles due to the unstable V particle gets canceled with the induced poles of the reducible part and the $N\theta$ resonance pole replaces the unstable V-particle pole.

---

[1] P. E. Kaus and F. Zachariasen, Phys. Rev. **138**, B1304 (1965); I. S. Gerstein and N. G. Deshpande, *ibid.* **140**, B1643 (1965); T. Saito, *ibid.* **152**, 1339 (1966); T. Pradhan and J. N. Passi, *ibid.* **160**, 1336 (1967); J. M. Cornwall and D. J. Levy, *ibid.* **178**, 2356 (1968). See also M. Lévy, Nuovo Cimento **13**, 115 (1959); J. Gunson and J. G. Taylor, Phys. Rev. **119**, 1121 (1960).

[2] R. E. Peierls, in *Proceedings of the Glasgow Conference on Nuclear and Meson Physics* (Pergamon, New York, 1954), p. 296.

[3] Y. S. Jin and S. W. MacDowell, Phys. Rev. **137**, B688 (1965).

II. MODEL FIELD THEORY

We shall work within the framework of a soluble model field theory whose Hamiltonian is given by

$$H=m_V^{(0)}\int d^3p\,\psi_V^\dagger(\mathbf{p})\psi_V(\mathbf{p})+m_N\int d^3q\,\psi_N^\dagger(\mathbf{q})\psi_N(\mathbf{q})+\int d^3k\left(\frac{k^2}{2\mu}+\mu\right)\phi^\dagger(\mathbf{k})\phi(\mathbf{k})$$
$$+\left[\frac{g_0}{(2\pi)^{3/2}}\int\int d^3p d^3k\,\psi_V^\dagger(\mathbf{p})\psi_N(\mathbf{p}-\mathbf{k})f(k)\phi(\mathbf{k})+\text{H.c.}\right]$$
$$+\frac{\lambda_0}{(2\pi)^3}\int d^3q_1d^3q_2d^3k_1d^3k_2 f(k_1)f(k_2)\delta^{(3)}(q_1+k_1-q_2-k_2)\psi_N^\dagger(\mathbf{q}_1)\psi_N(\mathbf{q}_2)\phi^\dagger(\mathbf{k}_1)\phi(\mathbf{k}_2)\,. \quad (2.1)$$

The $N\theta\to N\theta$ scattering amplitude can be written as

$$T(E)=\frac{g_0^2f^2(p)}{(2\pi)^3}\Gamma^2(E)\Delta'_V(E)+T_1(E)\,, \quad (2.2)$$

where $\Gamma(E)$ is the $VN\theta$ vertex function, $\Delta'_V(E)$ is the V-particle complete propagator, and $T_1(E)$ is the $N\theta\to N\theta$ scattering amplitude without contributions of the V particle. The expressions for $\Gamma(E)$, $\Delta'_V(E)$, and $T_1(E)$ are

$$T_1(E)=\frac{\lambda_0f^2(p)}{(2\pi)^3}\frac{1}{1-[\lambda_0/(2\pi)^3]I(E)}\,, \quad (2.3)$$

$$\Gamma(E)=\frac{1}{1-[\lambda_0/(2\pi)^3]I(E)}\,, \quad (2.4)$$

$$\Delta'_V(E)=\frac{1-[\lambda_0/(2\pi)^3]I(E)}{(E-m_V^{(0)})\{1-[\lambda_0/(2\pi)^3]I(E)\}-[g_0^2/(2\pi)^3]I(E)}\,, \quad (2.5)$$

where

$$I(E)=\int\frac{d\omega\,F^2(\omega)}{E-m_N-\omega}\,,$$

with

$$\omega=k^2/2\mu+\mu\,,$$
$$F^2(\omega)=4\pi\mu[2\mu(\omega-\mu)]^{1/2}f^2(k)\,.$$

To consider the $N\theta$ resonance and unstable V particle in the theory, we must analytically continue these functions onto the second sheet of the E plane. This analytic continuation of the $\Gamma(E)$, $\Delta'_V(E)$, and $T_1(E)$ onto the second sheet of the E plane can be done through the cut from $\mu+m_N$ to ∞, and we get

$$\Gamma_{\text{II}}(E)=\frac{\Gamma(E)}{\chi(E)}\,, \quad (2.6)$$

$$T_{1,\text{II}}(E)=\frac{\lambda_0f^2(p)}{(2\pi)^3}\frac{\Gamma(E)}{\chi(E)}\,, \quad (2.7)$$

$$\Delta'_{V,\text{II}}(E)=\frac{\chi(E)}{(E-m_V^{(0)})\chi(E)-(g_0^2/\lambda_0)[\Gamma(E)-\chi(E)]}\,, \quad (2.8)$$

where

$$\chi(E)=1+[i\lambda_0/(2\pi)^2]\Gamma(E)F^2(E-m_N)\,.$$

By analogy with the stable-particle mass and coupling-constant renormalizations, we define the mass renormalization, the wave-function renormalization constant of the unstable V particle, and the $VN\theta$ vertex-function renormalization constant as follows:

$$m_V-m_V^{(0)}=\frac{g_0^2}{\lambda_0}\frac{\Gamma(m_V)-\chi(m_V)}{\chi(m_V)}\,, \quad (2.9)$$

$$Z_V^{-1}=1-\frac{g_0^2}{\lambda_0}\frac{\partial}{\partial E}W(E)\bigg|_{E=m_V}\,, \quad (2.10)$$

$$Z_1^{-1}=\Gamma(m_V)/\chi(m_V)\,, \quad (2.11)$$

where

$$W(E)=[\Gamma(E)-\chi(E)]/\chi(E)\,.$$

We note that Z_V and Z_1 are complex and hence lead to a complex renormalized coupling constant (see Lévy[2])

$$g=g_0^2Z_1^{-2}Z_V\,. \quad (2.12)$$

III. POLES OF $T_{1,\text{II}}(E)$, $\Gamma_{\text{II}}(E)$, AND $\Delta'_{V,\text{II}}(E)$

The expression for $T_{1,\text{II}}(E)$,

$$T_{1,\text{II}}(E)=\frac{\lambda_0 f^2(p)}{(2\pi)^3}\times\frac{\Gamma(E)}{1+[i\lambda_0/(2\pi)^2]\Gamma(E)F^2(E-m_N)},\qquad(3.1)$$

indicates that for a sufficiently strong coupling constant, $T_{1,\text{II}}(E)$ can have a resonance, i.e.,

$$1+[i\lambda_0/(2\pi)^2]\Gamma(m)F^2(m-m_N)=0.\qquad(3.2)$$

The resonance occurs at $E=m$ (m is complex). Equation (3.1) indicates that $T_{1,\text{II}}(E)$ satisfies the reality condition

$$T_{1,\text{II}}^*(E)=T_{1,\text{II}}(E^*),\qquad(3.3)$$

implying that $T_{1,\text{II}}(E)$ has a pole at $E=m^*$ also. From expressions (2.6) and (2.8), we see that these poles of $T_{1,\text{II}}(E)$ induce in $\Gamma_{\text{II}}(E)$ and in $[\Delta'_{V,\text{II}}(E)]^{-1}$ poles at $E=m$ and $E=m^*$. Separating out the pole terms of $T_{1,\text{II}}(E)$, $\Gamma_{\text{II}}(E)$, and $[\Delta'_{V,\text{II}}(E)]^{-1}$, we find

$$T_{1,\text{II}}(E)=\frac{R_1}{E-m}+\frac{R_1^*}{E-m^*}+\cdots,\qquad(3.4)$$

$$\Gamma_{\text{II}}(E)=\frac{R_2}{E-m}+\frac{R_2^*}{E-m^*}+\cdots,\qquad(3.5)$$

$$[\Delta'_{V,\text{II}}(E)]^{-1}=\frac{R_3}{E-m}+\frac{R_3^*}{E-m^*}+\cdots,\qquad(3.6)$$

where

$$R_1=\frac{\lambda_0 f^2(p)}{(2\pi)^3}\frac{\Gamma(m)}{(m-m^*)A(m)},\qquad(3.7)$$

$$R_2=\frac{\Gamma(m)}{(m-m^*)A(m)},\qquad(3.8)$$

$$R_3=-\frac{(m-m^*)A(m)}{(g_0^2/\lambda_0)\Gamma(m)},\qquad(3.9)$$

and

$$A(E)=\chi(E)/(E-m)(E-m^*).\qquad(3.10)$$

Thus the pole structure of the complete scattering amplitude for $N\theta\rightarrow N\theta$ is given by

$$T_{\text{II}}(E)=\frac{g^2f^2(p)/(2\pi)^3}{E-m_V}+\frac{g^{*2}f^2(p)/(2\pi)^3}{E-m_V^*}+\frac{R_2^2}{R_3}\frac{g_0^2f^2(p)/(2\pi)^3}{E-m}+\frac{R_2^{*2}}{R_3^*}\frac{g_0^2f^2(p)/(2\pi)^3}{E-m^*}+\frac{R_1}{E-m}+\frac{R_1^*}{E-m^*}+\cdots.\qquad(3.11)$$

It can be checked that

$$\frac{g_0^2f^2(p)}{(2\pi)^3}\frac{R_2^2}{R_3}=-R_1,\qquad(3.12)$$

implying thereby that the poles of $T_{1,\text{II}}(E)$ cancel the induced poles of $\Gamma_{\text{II}}^2\Delta'_{V,\text{II}}$ and do not appear in $T_{\text{II}}(E)$. Thus the Jin-MacDowell cancellation holds for unstable particles also.

IV. UNSTABLE V PARTICLE AS $N\theta$ RESONANCE

In this section, we find the conditions under which the $N\theta$ resonance of $T_{1,\text{II}}(E)$ completely replaces the unstable V particle. This is equivalent to demanding that

$$m\rightarrow m_V\qquad(4.1)$$

and

$$R_1\rightarrow g^2f^2(p)/(2\pi)^3.\qquad(4.2)$$

We note that under these conditions the poles of the reducible part cancel each other, since $R_1=-[g_0^2f^2(p)/(2\pi)^3](R_2^2/R_3)$. It can easily be checked that under these requirements the corresponding conjugate poles of the reducible part cancel each other, and the pole at $E=m_V^*$ moves to the antiresonance pole at $E=m^*$. Making use of Eq. (3.10) in Eq. (2.11), we get

$$Z_1^{-1}=\frac{\Gamma(m_V)}{(m_V-m)(m_V-m^*)A(m_V)}.\qquad(4.3)$$

This indicates that the requirement $m\rightarrow m_V$ is equivalent to demanding $Z_1\rightarrow 0$. The other requirement $R_1\rightarrow g^2f^2(p)/(2\pi)^3$ can be written as

$$\frac{\lambda_0}{g^2}\frac{\Gamma(m)}{(m-m^*)A(m)}\rightarrow 1.\qquad(4.4)$$

Equation (2.10) indicates that under $Z_1=0$,

$$Z_V=1-\frac{g^2}{\lambda_0}\frac{(m-m^*)A(m)}{\Gamma(m)},\qquad(4.5)$$

showing that the requirement (4.2) is satisfied if Z_V vanishes.

Under $Z_1=0$ and $Z_V=0$, we find that the reducible part vanishes. Thus we see that if $Z_1=0$ and $Z_V=0$, then $T_{\text{II}}(E)$ has no contribution from the unstable V particle, and $N\theta$ resonance completely replaces the unstable V particle.

ACKNOWLEDGMENT

I am extremely grateful to Professor T. Pradhan for many helpful discussions.