Dispersion Methods and the Two-Nucleon Interaction in Charged Scalar Theory*

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The charged scalar static model is studied from the viewpoint of dispersion theory with the aim of including two-meson effects in the binding energy between two nucleons at zero separation. The method of analysis is that due to Blankenbecler and Cook, who advocate the use of vertex functions for investigating bound-state properties. In this case a deuteron vertex is examined in terms of Bronzan's one-nucleon two-meson solution which possesses two- and three-particle unitarity and a crossing-symmetric scattering amplitude. On the one hand, matrix elements containing the deuteron state at one end and two- or three-particle states at the other are contracted with respect to mesons. In this approach a summation over intermediate states involving two nucleons and zero, one, or more mesons is neglected. On the other hand, this summation may be avoided by contracting nucleons, but it appears that this approach leads to an intractable system of coupled integral equations. For this reason, only the simplest matrix element is treated this way, and even then some two- and three-particle contributions are ignored. Under these conditions, it is found that a factorization property of the one-nucleon two-meson connected scattering matrix leads to inhomogeneous Omnès equations having solutions which transform the two-meson dispersion expansion of the deuteron vertex into a condition for the determination of the binding energy.

I. INTRODUCTION

THE well-known charged scalar static-source model[1] has recently been studied in the one- and two-meson approximations via the methods of dispersion theory,[2] and a modified Tamm-Dancoff approach.[3] The two-meson solution presented in the former treatment is distinguished by its two- and three-particle unitarity and a crossing-symmetric scattering amplitude. This work precludes dynamical bound states by assuming a sufficiently small meson-nucleon coupling. Another version[4,5] which retains these states tends to be unreliable for large values of the coupling.[6] The latter formulation mentioned above, although subject to the complete absence of crossing symmetry, demonstrates the possibility of bound states in the spectrum of the theory on the basis of conventional Lee-model results.

The system under consideration in these investigations consists of charge-conserving s-wave interactions between relativistic charged scalar mesons and a spin-zero static nucleon source which can exist in either positive or neutral charge states. Electromagnetic forces are neglected. The present paper is concerned with incorporating two-meson effects in the interaction between two such nucleons. For this purpose, we employ vertices of the type pointed out by Blankenbecler and Cook[7] which allow the use of the techniques of dispersion theory. For simplicity, we examine the two-nucleon problem in the zero-range limit while assuming that both heavy and light particles are bosons.

It is hardly necessary to say that a model field-theory calculation of this kind is rather incapable of yielding a realistic two-nucleon potential, and we shall not dwell on the many unsatisfactory aspects that make this so. Indeed, a detailed determination of the corresponding interaction energy is not the main reason for this treatise. Instead, we take the attitude that static models containing two sources provide an interesting opportunity to examine bound states and dispersion techniques in a simplified but nontrivial framework of ideas reminiscent of fully relativistic theories. Numerous papers have already pursued the question of a meson-theoretic derivation of a static two-nucleon potential. In particular, we mention the nonrelativistic fourth-order (two-meson-exchange) perturbation calculation of Gartenhaus,[8] who used the Chew-Low[9] extended-source (cut-off) p-wave model to develop a potential which was successfully applied to the deuteron problem. This potential does not seem to be reliable at inter-nucleon distances of the order of half the pion Compton wavelength since many-pion exchange, relativistic effects, and heavy-meson exchange are predominant in that region.[10] Furthermore, it is also not unexpected that the absence of nucleon recoil rules out the possibility of a spin-orbit interaction.[11] In spite of these misgivings, it should be instructive, in a future paper, to explore a dispersion-theory description of the interaction between two sources in the Chew-Low model.

In an earlier Lee-model work,[12] we succeeded in carrying out a dispersion analysis of the bound state

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formed by the interaction between two V particles. Even though this interaction entails the simultaneous exchange of two mesons, a contraction scheme was devised which used only the states found in the one-meson sectors, V and VN, as intermediate states. In this way the more complicated two- and three-particle states of the $V\theta$ sector were avoided. It is a result of the selection rules in the Lee model that the possibility of solution with dispersion theory is traced to the emergence of a finite set of coupled integral equations. To obtain a similar situation in charged scalar theory, we choose a two-meson approximation leading to equations which owe their solution to a factorization property of the one-nucleon two-meson scattering matrix.[2] The scattering matrix in the $V\theta$ and $2V$ sectors of the Lee model also has this property.[13–15]

In Sec. II, we introduce an expression for the deuteron vertex $\langle n|f_p|d\rangle$, henceforth called Γ. Because of the symmetry in the theory Γ is taken equal to $\langle p|f_n|d\rangle$. In these matrix elements, $|p\rangle$, $|n\rangle$, and $|d\rangle$ denote the physical "proton," "neutron," and "deuteron" states, while f_p and f_n are the proton and neutron current operators at zero time. We see at once, in the two-meson approximation, that two pairs of matrix elements arise for consideration. It turns out however, that there is only one independent function in each pair. Since one of these functions is contracted in two different ways, we must solve three Omnès integral equations.

In Sec. III, we calculate the matrix elements containing the vacuum state at one end and two- or three-particle states at the other in terms of the Omnès functions appearing in the one- and two-meson solutions. The first procedure in Sec. IV for evaluating the matrix elements with the deuteron state at one end and two- or three-particle states at the other is essentially the same as before. The Omnès equation in this case differs from the previous one only in the inhomogeneous term. Another contraction scheme aimed at recovering information on the binding energy lost in the first approach is also explored. If all contributions in the two-meson approximation are retained, then one faces difficult mathematical complications which frustrate further progress. To continue the calculation, we use the most convenient integral equation for one of these matrix elements while maintaining the earlier integral equation for the other. On collecting results in Sec. V, we see that Γ cancels out of its original expression and there remains an eigenvalue condition for the determination of the binding energy including two-meson effects. We conclude with some general observations about the static two-nucleon potential problem.

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II. DEUTERON VERTEX

In terms of conventional notation, such as that used in Ref. 2, the current operators in the model are given by

$$
\begin{aligned}
f_p(t) &\equiv \left(-i\frac{d}{dt}+m\right)\psi_p(t) = -\delta m\psi_p(t) - \frac{g}{Z}\psi_n(t)\sum_k X(\omega)[a_k(t)+b_k^\dagger(t)],\\
f_n(t) &\equiv \left(-i\frac{d}{dt}+m\right)\psi_n(t) = -\delta m\psi_n(t) - \frac{g}{Z}\psi_p(t)\times\sum_k X(\omega)[a_k^\dagger(t)+b_k(t)],\\
j(t) &\equiv X^{-1}(\omega)\left(-i\frac{d}{dt}+\omega\right)a_k(t)\\
&= X^{-1}(\omega)\left(i\frac{d}{dt}+\omega\right)b_k^\dagger(t) = -g\psi_n^\dagger(t)\psi_p(t).
\end{aligned}
\tag{1}
$$

The operators ψ_p ($\psi_p^\dagger$) and ψ_n ($\psi_n^\dagger$) are the renormalized annihilation (creation) operators for the internal p and n states of a static source. For convenience, both p and n are assigned the energy m; Z is the source wave-function renormalization constant, and δm is the mass renormalization counter term. It is assumed that the renormalized coupling constant g is not large enough to form meson-nucleon bound states. The operators a_k ($a_k^\dagger$) and b_k ($b_k^\dagger$) annihilate (create) positive and negative mesons, respectively, of three-momentum k and relativistic energy $\omega=(k^2+\mu^2)^{1/2}$; μ is the rest mass. The quantity $X(\omega)$ is an abbreviation for the ratio $f(\omega)/(2\omega\Omega)^{1/2}$, where Ω is the volume of quantization, while $f(\omega)$ is a real and positive-definite cutoff function that vanishes in the high-energy limit. The equal-time commutation relations are

$$
\begin{aligned}
[\psi_p(t),\psi_p^\dagger(t)] &= [\psi_n(t),\psi_n^\dagger(t)] = Z^{-1},\\
[a_k(t),a_{k'}^\dagger(t)] &= [b_k(t),b_{k'}^\dagger(t)] = \delta_{kk'}.
\end{aligned}
\tag{2}
$$

We see from Eq. (1) that the theory is invariant under the simultaneous replacements $p \rightleftarrows n$ and $\pi^+ \rightleftarrows \pi^-$.

On applying the contraction technique to the n particle in Γ, we obtain the integral form

$$\Gamma = i\int_{-\infty}^{\infty} e^{imt}\langle 0|[f_n(t),f_p]\theta(t)|d\rangle dt, \tag{3}$$

where $\theta(t)$ is the usual step function. We insert intermediate states $|s\rangle$ to get the representation

$$\Gamma = \sum_s \langle 0|f_n|s\rangle\langle s|f_p|d\rangle \times\left(\frac{1}{E_s-m-i\epsilon}+\frac{1}{E_s+m-E_d+i\epsilon}\right), \tag{4}$$

where ϵ is a positive number to be treated as infinitesimally small. The deuteron energy E_d will be written as $2m+\omega_d$, thus defining ω_d as the interaction energy. Assume that ω_d is negative. In arriving at Eq. (4), we have invoked the invariance mentioned above. Keeping in mind that $\langle 0|f_n|n\rangle$ vanishes, we next express Eq. (4) in continuous space as

$$\Gamma=\frac{1}{\pi}\int_\mu^\infty \rho(\omega)K_1(\omega)D_1(\omega)\left(\frac{1}{\omega}+\frac{1}{\omega-\omega_d}\right)d\omega +\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty \rho(\omega_1)\rho(\omega_2)K_2(\omega_1,\omega_2)D_2(\omega_1,\omega_2) \times\left(\frac{1}{\omega_{12}}+\frac{1}{\omega_{12}-\omega_d}\right)d\omega_1 d\omega_2\,, \quad (5)$$

where $\omega_{12}\equiv\omega_1+\omega_2$ and $\rho(\omega)\equiv kf^2(\omega)/4\pi$. The functions appearing in Eq. (5) are defined by

$$K_1(\omega)=X^{-1}(\omega)\langle 0|f_n|p\pi_k^-\rangle\,, \quad (6a)$$

$$K_2(\omega_1,\omega_2)=X^{-1}(\omega_1)X^{-1}(\omega_2)\langle 0|f_n|n\pi_{k_1}^+\pi_{k_2}^-\rangle\,, \quad (6b)$$

$$D_1(\omega)=X^{-1}(\omega)\langle p\pi_k^-|f_p|d\rangle\,, \quad (7a)$$

$$D_2(\omega_1,\omega_2)=X^{-1}(\omega_1)X^{-1}(\omega_2)\langle n\pi_{k_1}^+\pi_{k_2}^-|f_p|d\rangle\,. \quad (7b)$$

Although the customary "out" designation will not be displayed, all two- and three-particle states are taken to be out states. It will be shown that K_2 and D_2 are directly related to K_1 and D_1, respectively, and that the latter obey Omnès-type singular integral equations having a common kernel. The origin of this kernel is traced to a factorization property of the one-nucleon two-meson connected scattering matrix.[2] The relationship between D_1 and D_2 referred to here comes about only by adopting the simplifications mentioned previously. Because the inhomogeneous terms in the D_1 equations contain a factor of Γ, it follows that the solutions for D_1 and D_2 also carry this factor. Thus Γ must eventually cancel out of Eq. (5) which then converts into an eigenvalue condition for ω_d. This procedure differs from that adopted in the $2V$ problem, where the elimination of two vertices from two simultaneous algebraic relations yields the desired condition.

III. K FUNCTIONS

In this section we first obtain a coupled pair of singular integral equations for the vertex functions K_1 and K_2. These equations are then simplified by kernel transformations of the type discussed in the Appendix of Ref. 13 and then combined into showing that K_2 is, except for a constant, a product of K_1, an energy factor, and Omnès functions characteristic of the one-nucleon one-meson approximations. The problem of finding K_1 is solved after a brief summary of the diagonalization of the connected two-meson scattering matrix.

Contraction of the meson in K_1 yields

$$K_1(\omega)=-\frac{g}{Z}-i\int_{-\infty}^\infty e^{-i\omega t}\langle 0|[f_n,j(t)]\theta(t)|p\rangle dt\,, \quad (8)$$

where the first term on the right-hand side comes from an equal-time commutator. We insert the appropriate intermediate states and manipulate Eq. (8) into the form

$$K_1(\omega)=-\frac{g}{Z}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')K_1(\omega')[T_-(\omega')-M_+(\omega')]d\omega'}{\omega'-\omega+i\epsilon}+\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega_1)\rho(\omega_2)P_-(\omega_1,\omega_2)K_2(\omega_1,\omega_2)d\omega_1 d\omega_2}{\omega_{12}-\omega+i\epsilon} +\frac{1}{\pi}\int_\mu^\infty\frac{e^{i\delta_+(\omega')}\sin\delta_+(\omega')K_1(\omega')d\omega'}{\omega'-\omega+i\epsilon}\,, \quad (9)$$

where we have added and subtracted the last term on the right-hand side. The amplitudes $M_+(\omega)$ and $M_-(\omega)$ describe the scattering of $p\pi^+$ (and $n\pi^-$) and $p\pi^-$ (and $n\pi^+$), respectively, in the one-meson approximation.[2] These amplitudes are expressed in terms of their corresponding real phase shifts $\delta_\pm(\omega)$ by

$$\rho(\omega)M_\pm(\omega)=e^{i\delta_\pm(\omega)}\sin\delta_\pm(\omega)\,, \quad (10)$$

and satisfy the crossing relation $M_+(\omega)=M_-(-\omega)$. The phase shifts can be chosen to vanish at the lower (μ) and upper (∞) limits. Expressions are given in Ref. 2 for $M_\pm(\omega)$ and their two-meson approximation analogs $T_\pm(\omega)$. Also found there are the associated production amplitudes $P_\pm$. For future reference we note that

$$P_-(\omega_1,\omega_2)=\frac{g}{\omega_1}[T_-(\omega_{12})-M_+(\omega_{12})]\frac{\Delta_-(\omega_1)\Delta_+(\omega_2)}{\Delta_+(\omega_{12})}\,, \quad (11)$$

where the Omnès functions $\Delta_\pm(\omega)$ are defined by

$$\Delta_\pm(\omega)=\exp\left[\frac{\omega}{\pi}\int_\mu^\infty\frac{d\omega'\delta_\pm(\omega')}{\omega'(\omega'-\omega-i\epsilon)}\right]. \quad (12)$$

In the high-energy limit, $\Delta_\pm(\omega)$ approach a constant.

To secure the other equation interrelating K_1 and K_2, we contract the positive meson in K_2. This leads to

$$K_2(\omega_1,\omega_2)=-iX^{-1}(\omega_2)\int_{-\infty}^{\infty}e^{-i\omega_1 t}\langle 0|[f_n,j^\dagger(t)]\theta(t)]n\pi_{k_2}^-\rangle dt. \tag{13}$$

Proceeding in the usual manner, we then find

$$K_2(\omega_1,\omega_2)=\frac{g}{\omega_1}K_1(\omega_2)+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega)Q_-^*(\omega,\omega_2)K_1(\omega)d\omega}{\omega-\omega_{12}+i\epsilon}+\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega)\rho(\omega')R_-^*(\omega_2,\omega',\omega)K_2(\omega,\omega')d\omega d\omega'}{\omega+\omega'-\omega_{12}+i\epsilon}$$
$$+\frac{1}{\pi}\int_\mu^\infty\frac{e^{-i\delta_-(\omega)}\sin\delta_-(\omega)K_2(\omega,\omega_2)d\omega}{\omega-\omega_1+i\epsilon}. \tag{14}$$

This equation incorporates the following definitions:

$$Q_-^*(\omega_1,\omega_2)=X^{-1}(\omega_1)X^{-1}(\omega_2)[\langle p\pi_{k_1}^-|j^\dagger|n\pi_{k_2}^-\rangle+g\delta_{k_1k_2}], \tag{15}$$

$$R_-^*(\omega_1,\omega_2,\omega_3)=X^{-1}(\omega_1)X^{-1}(\omega_2)X^{-1}(\omega_3)[\langle n\pi_{k_3}^+\pi_{k_2}^-|j^\dagger|n\pi_{k_1}^-\rangle-\delta_{k_1k_2}X(\omega_3)M_-(\omega_3)]. \tag{16}$$

In Ref. 2 it is proved that

$$Q_-^*(\omega_1,\omega_2)=\frac{g}{\omega_1-\omega_2+i\epsilon}[T_-(\omega_1)-M_+(\omega_1)]\frac{\Delta_+^*(\omega_2)\Delta_-(\omega_1-\omega_2)}{\Delta_+(\omega_1)}, \tag{17}$$

$$R_-^*(\omega_1,\omega_2,\omega_3)=\frac{gP_-(\omega_3,\omega_2)\Delta_+^*(\omega_1)\Delta_-(\omega_{23}-\omega_1)}{(\omega_{23}-\omega_1+i\epsilon)\Delta_+(\omega_{23})}. \tag{18}$$

In the next step we eliminate the last terms on the right-hand sides of Eqs. (9) and (14) by the kernel transformations referred to above. This procedure immediately yields a new pair of equations which provide the relation

$$K_2(\omega_1,\omega_2)=\frac{g\Delta_-^*(\omega_1)\Delta_+^*(\omega_2)K_1(\omega_{12})}{\omega_1\Delta_+^*(\omega_{12})}. \tag{19}$$

Using this result in the transformed version of Eq. (9), and introducing the function $K(\omega)$ defined as the ratio of $K_1(\omega)\Delta_+(\infty)$ to $\Delta_+^*(\omega)$, we obtain

$$K(\omega)=-\frac{g}{Z}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')[T_-(\omega')-M_+(\omega')]e^{-2i\delta_+(\omega')}K(\omega')d\omega'}{\omega'-\omega+i\epsilon}+\frac{1}{\pi}\int_{2\mu}^\infty\frac{d\omega'[T_-(\omega')-M_+(\omega')]I(\omega')K(\omega')d\omega'}{(\omega'-\omega+i\epsilon)[\Delta_+(\omega')]^2}, \tag{20}$$

where the integral $I(\omega)$ is given by

$$I(\omega)=\frac{g^2}{\pi}\int_\mu^{\omega-\mu}\frac{\rho(\omega')\rho(\omega-\omega')|\Delta_-(\omega')\Delta_+(\omega-\omega')|^2d\omega'}{\omega'^2}. \tag{21}$$

It can be shown that Eq. (20) has the form of an Omnès equation. Similar demonstrations have been given elsewhere.[13–15] In the present context the problem is that of solving the eigenvalue equation of the connected S matrix S_c in the $p\pi^-$ channel. There exists of course a similar eigenvalue problem in the $p\pi^+$ channel. The result is that the eigenvalues λ of S_c are determined by the equation

$$\lambda^2-2\lambda\left[1+i\rho(\omega)T_-(\omega)+i\frac{[T_-(\omega)-M_+(\omega)]I(\omega)}{[\Delta_+(\omega)]^2}\right]+2ie^{2i\delta_+(\omega)}\frac{[T_-(\omega)-M_+(\omega)]I(\omega)}{[\Delta_+(\omega)]^2}+2i\rho(\omega)T_-(\omega)+1=0. \tag{22}$$

Calling the two nontrivial solutions of this equation $\lambda_1=e^{2i\theta_1}$ and $\lambda_2=e^{2i\theta_2}$, and noting that their product $\lambda_1\lambda_2$ equals the determinant of S_c, while also equaling the λ-independent part of Eq. (22), we find with the use of Eqs. (46) and (48) in Ref. 2 that

$$\det S_c=\lambda_1\lambda_2=\left[\frac{T_-(\omega)}{M_+(\omega)}-1\right]\left[\frac{T_-^*(\omega)}{M_+^*(\omega)}-1\right]^{-1}. \tag{23}$$

It is equally noteworthy that

$$\frac{\lambda_1\lambda_2 e^{-2i\delta_+}-1}{2i}=e^{i(\theta_1+\theta_2-\delta_+)}\sin(\theta_1+\theta_2-\delta_+)=\rho(\omega)[T_-(\omega)-M_+(\omega)]e^{-2i\delta_+(\omega)}+\frac{[T_-(\omega)-M_+(\omega)]I(\omega)}{[\Delta_+(\omega)]^2}. \quad (24)$$

By virtue of the last result, we may now rewrite Eq. (20) in the standard form

$$K(\omega)=-\frac{g}{Z}+\frac{1}{\pi}\int_\mu^\infty \frac{e^{i[\theta_1(\omega')+\theta_2(\omega')-\delta_+(\omega')]}\sin[\theta_1(\omega')+\theta_2(\omega')-\delta_+(\omega')]K(\omega')d\omega'}{\omega'-\omega+i\epsilon}. \quad (25)$$

The solution to this equation can be written down at once in terms of the Omnès functions $\Delta_+(\omega)$ and $\Delta(\omega)$, where the latter is defined by

$$\Delta(\omega)=\exp\left[\frac{\omega}{\pi}\int_\mu^\infty \frac{d\omega'[\theta_1(\omega')+\theta_2(\omega')]}{\omega'(\omega'-\omega-i\epsilon)}\right]. \quad (26)$$

In view of the relation $\ln\lambda_1\lambda_2=2i(\theta_1+\theta_2)$, the determinant of S_c also equals $\Delta(\omega)/\Delta^*(\omega)$. This fact, and the equality $\Delta_+(\omega)/\Delta_+^*(\omega)=\exp[2i\delta_+(\omega)]$, are useful in verifying the solution of Eq. (25), namely,

$$K(\omega)=-\frac{g\Delta^*(\omega)\Delta_+(\infty)}{Z\Delta_+^*(\omega)\Delta(\infty)}. \quad (27)$$

Finally, it follows that

$$K_1(\omega)=-\frac{g}{Z}\frac{\Delta^*(\omega)}{\Delta(\infty)}, \quad (28)$$

$$K_2(\omega_1,\omega_2)=-\frac{g^2\Delta^*(\omega_{12})\Delta_-^*(\omega_1)\Delta_+^*(\omega_2)}{Z\Delta(\infty)\omega_1\Delta_+^*(\omega_{12})}. \quad (29)$$

The pion-nucleon coupling constant g is defined by the matrix elements

$$\langle n|j|p\rangle=-g,\quad K_1(0)=-g. \quad (30)$$

With this normalization $Z\Delta(\infty)$ equals unity, as shown by Eq. (28). To show the equality between $\langle n|j|p\rangle$ and $K_1(0)$ contract the n particle, make a two-meson expansion, and compare with Eq. (9).

Diagonalization of the one-nucleon two-meson connected S matrix continues its vital role in the solution of Omnès integral equations for D_1 and D_2 formulated in accordance with the provisos indicated earlier.

IV. D FUNCTIONS

We now undertake an evaluation of the remaining functions D_1 and D_2 by employing the simplest procedures enabling us to relate these functions as in the previous case. In one development, we completely neglect summations carrying intermediate states with the quantum numbers of two nucleons and zero, one, or more mesons. We see by another approach that it is possible to avoid these states and presumably a more accurate two-meson binding energy would result; however, the ensuing coupled integral equations for D_1 and D_2 are quite formidable from the standpoint of a complete solution. To continue along the path of least mathematical resistance, we omit certain terms from the D_1 equation and return to the D_2 equation used previously.

If we think of retaining the one-meson approximation in the summations mentioned above, then it appears that we have the additional problem of determining vertex functions and scattering amplitudes involving meson-deuteron scattering states. Actually, we have already learned how to contract the bound state of two static particles. For example, the $2V$ state $|B\rangle$, with normalization constant Z_B, can be contracted in terms of the operator ψ_B defined by $\psi_B=(Z_B/\sqrt{2})^{-1}\psi_V\psi_V$.[12] Analogously, the deuteron state $|d\rangle$ can be represented, in the usual definition of an asymptotic state, by the operator $\psi_d=Z_d^{-1}\,\psi_p\psi_n$, where Z_d is the corresponding normalization constant. This prescription will be useful in a discussion of meson-deuteron scattering to be examined elsewhere.

In Ref. 12 we were able to secure the binding energy of one two-nucleon bound state ($2V$) in terms of that of another (VN). The latter is a one-meson problem and is established independently of the former. This one-meson approximation which is automatic in the Lee model would require the existence of a dineutron or diproton in charged scalar theory.

Assuming that we wish to treat D_1 and D_2 symmetrically and excluding contraction of the deuteron itself, we have then the option of contracting mesons or nucleons. The latter approach is concerned with an attempt to remain within one-nucleon channels, whereas the former introduces states with the quantum numbers of two nucleons. In the final part of this section we combine D_1 and D_2 obtained from the nucleon and meson contractions, respectively.

We proceed by contracting mesons in D_1 and D_2. Let us first consider the former. We have

$$D_1(\omega)=\frac{g\Gamma}{Z\omega_d}+i\int_{-\infty}^{\infty} e^{i\omega t}\langle p|[j^\dagger(t),f_p]\theta(t)|d\rangle dt. \quad (31)$$

The first term on the right-hand side of Eq. (31) comes from the equal-time commutator

$$X^{-1}(\omega)\langle p|[b_k,f_p]d\rangle=-(g/Z)\langle p|\psi_n|d\rangle=g\Gamma/Z\omega_d. \quad (32)$$

Inserting a sum over a complete set of states in each term of the commutator and performing the time integrations as before, we get

$$D_1(\omega)=\frac{g\Gamma}{Z\omega_d}+\sum_s\frac{\langle p|j^\dagger|s\rangle\langle s|f_p|d\rangle}{E_s-m-\omega-i\epsilon}-\sum_s\frac{\langle p|f_p|s\rangle\langle s|j^\dagger|d\rangle}{E_d-E_s-\omega-i\epsilon}. \quad (33)$$

The s states in the first summation on the right-hand side of Eq. (33) have the quantum numbers of an n particle, while those in the second summation have quantum numbers of two p particles. In confining ourselves to the first sum and then to the s states as before, we continue to operate within the framework of functions already introduced. Note that in doing this we lose information carried by terms containing the binding energy explicitly. Under these conditions, the above expression for $D_1(\omega)$ becomes

$$D_1(\omega)=g\Gamma\left(\frac{1}{Z\omega_d}+\frac{1}{\omega}\right)+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')[T_-^*(\omega')-M_+^*(\omega')]D_1(\omega')d\omega'}{\omega'-\omega-i\epsilon}$$
$$+\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega_1)\rho(\omega_2)P_-^*(\omega_1,\omega_2)D_2(\omega_1,\omega_2)d\omega_1d\omega_2}{\omega_{12}-\omega-i\epsilon}+\frac{1}{\pi}\int_\mu^\infty\frac{e^{-i\delta_+(\omega')}\sin\delta_+(\omega')D_1(\omega')d\omega'}{\omega'-\omega-i\epsilon}, \quad (34)$$

where, once again, we have added and subtracted a term.

Next, we contract the π^+ meson in D_2 to obtain

$$D_2(\omega_1,\omega_2)=iX^{-1}(\omega_2)\int_{-\infty}^{\infty}e^{i\omega_1t}\langle n\pi_{k_2}^-|[j(t),f_p]\theta(t)|d\rangle dt. \quad (35)$$

By a familiar process, this equation expands into

$$D_2(\omega_1,\omega_2)=X^{-1}(\omega_2)\sum_s\frac{\langle n\pi_{k_2}^-|j|s\rangle\langle s|f_p|d\rangle}{E_s-m-\omega_{12}-i\epsilon}-X^{-1}(\omega_2)\sum_s\frac{\langle n\pi_{k_2}^-|f_p|s\rangle\langle s|j|d\rangle}{E_d-E_s-\omega_1-i\epsilon}. \quad (36)$$

The s states in the first and second summations on the right-hand side of Eq. (36) have the quantum numbers of one n particle and two n particles, respectively. Repeating the procedure adopted above for D_1, we find

$$D_2(\omega_1,\omega_2)=-\frac{\Gamma T_+(\omega_2)}{\omega_{12}}+\frac{gD_1(\omega_2)}{\omega_1}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega)Q_-(\omega,\omega_2)D_1(\omega)d\omega}{\omega-\omega_{12}-i\epsilon}+\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega)\rho(\omega')R_-(\omega_2,\omega',\omega)D_2(\omega,\omega')d\omega d\omega'}{\omega+\omega'-\omega_{12}-i\epsilon}$$
$$+\frac{1}{\pi}\int_\mu^\infty\frac{e^{-i\delta_-(\omega)}\sin\delta_-(\omega)D_2(\omega,\omega_2)d\omega}{\omega-\omega_1-i\epsilon}. \quad (37)$$

In arriving at the first term on the right-hand side of this equation we have used the two-meson approximation amplitude T_+ instead of its one-meson counterpart M_+. Eliminating the third integrals in Eqs. (34) and (37) by means of kernel transformations, we again obtain a new pair of equations which lead to

$$D_2(\omega_1,\omega_2)=\Delta_-(\omega_1)\Delta_+(\omega_2)\left[\frac{\Gamma\Phi(\omega_{12})}{\omega_{12}}+\frac{gD_1(\omega_{12})}{\omega_1\Delta_+(\omega_{12})}\right], \quad (38)$$

where $\Phi(\omega)$ is given by

$$\Phi(\omega)=\frac{g^2}{\omega}-\frac{T_+(\omega)}{\Delta_+(\omega)\Delta_-(-\omega)}. \quad (39)$$

The function Φ vanishes if T_+ is replaced by M_+. Next, we obtain an Omnès equation for the function $D(\omega)$ defined by

$$D(\omega)=D_1(\omega)/\Delta_+(\omega). \quad (40)$$

From Eqs. (11) and (24), the transformed version of Eqs. (34) and (38), we have

$$D(\omega)=\Gamma\left[\frac{g}{Z\omega_d\Delta_+(\infty)}+\frac{g}{\omega}+\frac{1}{\pi}\int_{2\mu}^{\infty}\frac{J(\omega')[T_-^*(\omega')-M_+^*(\omega')]d\omega'}{\omega'(\omega'-\omega-i\epsilon)[\Delta_+^*(\omega')]^2}\right]$$
$$+\frac{1}{\pi}\int_{\mu}^{\infty}\frac{e^{-i[\theta_1(\omega')+\theta_2(\omega')-\delta_+(\omega')]}\sin[\theta_1(\omega')+\theta_2(\omega')-\delta_+(\omega')]D(\omega')d\omega'}{\omega'-\omega-i\epsilon},\quad(41)$$

where

$$J(\omega)=\frac{g}{\pi}\int_{\mu}^{\omega-\mu}\frac{\rho(\omega')\rho(\omega-\omega')|\Delta_-(\omega')\Delta_+(\omega-\omega')|^2\Phi(\omega-\omega')d\omega'}{\omega'}.\quad(42)$$

The solution of Eq. (41) yields

$$D_1(\omega)=\Gamma\Delta(\omega)\left[\frac{g}{\omega_d}+\frac{g}{\omega}+\frac{1}{\pi}\int_{2\mu}^{\infty}\frac{J(\omega')[T_-^*(\omega')-M_+^*(\omega')]d\omega'}{\omega'(\omega'-\omega-i\epsilon)\Delta^*(\omega')\Delta_+^*(\omega')}\right],\quad(43)$$

while $D_2(\omega_1,\omega_2)$ is specified by Eq. (38). We have now completed a derivation which approximates the matrix elements appearing in the deuteron vertex expansion, Eq. (5), in terms of the functions arising in the one- and two-meson solutions of the model with a single static nucleon. Note that the D functions carry Γ as an overall factor. This characteristic causes Eq. (5) to become an eigenvalue condition for the binding energy.

To continue this discussion of the D functions, we would like to consider briefly the situation brought about by contracting nucleons in D_1 and D_2. Specifically, we shall be content with examining here the consequences of contracting the p particle in D_1, since the contraction of the n particle in D_2 leads to many more complications. In this approach we circumvent intermediate states with the quantum numbers of two nucleons and zero, one, or more mesons. On the other hand, the resulting coupled integral equations do not suggest a method of solution consistent with the two-meson approximation. For D_1 we get

$$D_1(\omega)=-\Gamma X^{-1}(\omega)\langle\pi_k^-|f_p|n\rangle\left(\frac{1}{\omega}+\frac{1}{\omega_d}\right)$$
$$+X^{-1}(\omega)\sum_{k'}X(\omega')\langle\pi_k^-|f_p|p\pi_{k'}^-\rangle D_1(\omega')$$
$$\times\left(\frac{1}{\omega'-\omega-i\epsilon}+\frac{1}{\omega'-\omega_d}\right)$$
$$+X^{-1}(\omega)\sum_{k_1}\sum_{k_2}X(\omega_1)X(\omega_2)\langle\pi_k^-|f_p|n\pi_{k_1}^+\pi_{k_2}^-\rangle$$
$$\times D_2(\omega_1,\omega_2)\left(\frac{1}{\omega_{12}-\omega-i\epsilon}+\frac{1}{\omega_{12}-\omega_d}\right).\quad(44)$$

To proceed further, we must enquire after the three remaining matrix elements in Eq. (44). For the purpose of completely disclosing the ω dependence on the right-hand side of this equation, it is appropriate to contract the mesons on the left-hand side in each of these.

In the first place, we get

$$X^{-1}(\omega)\langle\pi_k^-|f_p|n\rangle$$
$$=-\frac{g}{Z}-i\int_{-\infty}^{\infty}e^{i\omega t}\langle 0|[f_p,j^\dagger(t)]\theta(t)|n\rangle dt.\quad(45)$$

Since the theory is invariant under the simultaneous interchanges $p\rightleftarrows n$, $j\rightleftarrows j^\dagger$, it follows by comparing Eqs. (45) and (8) that

$$X^{-1}(\omega)\langle\pi_k^-|f_p|n\rangle=K_1(-\omega).\quad(46)$$

Next, we consider the matrix element in the first summation on the right-hand side of Eq. (44). It is found that

$$X^{-1}(\omega)\langle\pi_k^-|f_p|p\pi_{k'}^-\rangle$$
$$=X(\omega')T_-^*(\omega')+\sum_s\langle 0|f_p|s\rangle\langle s|j^\dagger|p\pi_{k'}^-\rangle$$
$$\times\left(\frac{1}{E_s+\omega-\omega'-m+i\epsilon}-\frac{1}{E_s-m-i\epsilon}\right).\quad(47)$$

Notice that the summation in Eq. (47) vanishes when the meson energies ω and ω' are equal. That this is not unexpected follows from a comparison between the two forms of the scattering matrix element $\langle p\pi_{k'}^-\text{ out}\times|p\pi_k^-\text{ in}\rangle$ obtained by contracting the in-meson and the in-proton.

Lastly, we turn to the matrix element in the double summation on the right-hand side of Eq. (44). In this case we get

$$X^{-1}(\omega)\langle\pi_k^-|f_p|n\pi_{k_1}^+\pi_{k_2}^-\rangle$$
$$=\delta_{kk_2}X^{-1}(\omega)X(\omega_1)K_1(\omega_1)+X(\omega_1)X(\omega_2)P_-^*(\omega_1,\omega_2)$$
$$+\sum_s\langle 0|f_p|s\rangle\langle s|j^\dagger|n\pi_{k_1}^+\pi_{k_2}^-\rangle$$
$$\times\left(\frac{1}{E_s+\omega-\omega_{12}-m+i\epsilon}-\frac{1}{E_s-m-i\epsilon}\right).\quad(48)$$

Note that both the first term and the summation on the right-hand side of Eq. (48) give no contribution when ω_{12} and ω are equal. Again, this is predicted by the two forms of $\langle n\pi_{k_1}{}^{+}\pi_{k_2}{}^{-}\text{ out}|p\pi_k{}^{-}\text{in}\rangle$ obtained by separately contracting the particles on the right-hand side.

The dispersion relation resulting from the substitution of the above expressions into Eq. (44) will not be displayed here in its entirety. It is a consequence of the first term on the right-hand side of Eq. (48) that ω appears not only in simple denominator factors, and a known function, but also at one point, as the second variable in the other unknown function D_2. Similarly, in trying to write an integral equation for D_2 in one of its variables, we find that that variable cannot be confined only to denominator factors and known functions, but is also present in D_1 and D_2 on the right-hand side of the equation. It is not at all transparent that one can solve this complicated system of coupled equations. It is convenient, although an obvious mutilation of matrix elements, to use $-g$, $X(\omega')\ T_-{}^*(\omega')$, and $X(\omega_1)X(\omega_2)\times P_-{}^*(\omega_1,\omega_2)$ in place of the left-hand sides of Eqs. (46), (47), and (48), respectively. In that case we get the interesting equation

$$D_1(z)=\frac{D_1(\omega_d)}{2}+\frac{g\Gamma}{z}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')[T_-{}^*(\omega')-M_+{}^*(\omega')]D_1(\omega')d\omega'}{\omega'-z}$$
$$+\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega_1)\rho(\omega_2)P_-{}^*(\omega_1,\omega_2)D_2(\omega_1,\omega_2)d\omega_1d\omega_2}{\omega_{12}-z}+\frac{1}{\pi}\int_\mu^\infty\frac{e^{-i\delta_+(\omega')}\sin\delta_+(\omega')D_1(\omega')d\omega'}{\omega'-z},\quad(49)$$

written in a form which stresses the analytic properties of D_1. As we shall see, the solution of Eq. (49) provides a condition for the determination of $D_1(\omega_d)$. The definition of $D_1(\omega_d)$, which may be read from Eq. (49) evaluated at $z=\omega_d$, could also be used for this purpose but not as easily. A simple version of the steps taken here is given in the Appendix, where we re-examine the equation characteristic of the VN potential problem. The last integral in Eq. (49) may be eliminated in the usual way. Maintaining the earlier transformed equation for D_2, but now combining it with the revised and transformed equation for D_1, we find that D_2 again has the form of Eq. (38). This is expected since the two D_1's differ only in their ω_d dependence. As before, D_1 satisfies an Omnès integral equation, the solution of which is

$$D_1(z)=\Delta(z)\left[\frac{D_1(\omega_d)}{2\Delta(\infty)}+\frac{g\Gamma}{z}+\frac{\Gamma}{\pi}\int_{2\mu}^\infty\frac{J(\omega')[T_-{}^*(\omega')-M_+(\omega')]d\omega'}{\omega'(\omega'-z)[\Delta_+{}^*(\omega')]^2}\right].\quad(50)$$

The remaining unknown quantity is $D_1(\omega_d)$. It is obvious that Eq. (50) itself provides the means for securing this quantity. Simply evaluate it at $z=\omega_d$ and solve for $D_1(\omega_d)$. We get

$$D_1(\omega_d)=\frac{2\Gamma\Delta(\omega_d)\Delta(\infty)}{2\Delta(\infty)-\Delta(\omega_d)}\left[\frac{g}{\omega_d}+\frac{1}{\pi}\int_{2\mu}^\infty\frac{J(\omega')[T_-{}^*(\omega')-M_+{}^*(\omega')]d\omega'}{\omega'(\omega'-\omega_d)[\Delta_+{}^*(\omega')]^2}\right].\quad(51)$$

Note that this result contains an over-all factor of Γ.

In this section we have considered two possible contraction schemes for the determination of the D functions. One of these leads to an encounter with intermediate states containing two nucleons and zero, one, or more mesons. In neglecting these states we close off some information on the binding energy. The other approach circumvents these intermediate states by remaining in channels containing one nucleon, but leads to an intractable system of coupled integral equations. It is not without interest to neglect certain two- and three-particle contributions in one of these equations and to solve it in conjunction with an earlier equation. In this way we recover part of the information referred to above.

V. FINAL RESULTS AND CONCLUDING REMARKS

For the purpose of illustration and for the sake of completeness we proceed now to present eigenvalue conditions for the determination of the binding energy ω_d. Because we are not concerned here with a detailed examination of these conditions, and because of the complexity of the functions, it will be convenient to express the result in general form. This complexity serves as a hint to the mathematical effort that would be demanded by a realistic two-meson dispersion calculation of binding energy.

From Eqs. (5), (28), (29), and (38) we find

$$\Gamma=-\frac{g}{\pi}\int_\mu^\infty\rho(\omega)\Delta^*(\omega)D_1(\omega)\left(\frac{1}{\omega}+\frac{1}{\omega-\omega_d}\right)d\omega$$
$$-\frac{g}{\pi}\int_{2\mu}^\infty\frac{\Delta^*(\omega)}{\omega|\Delta_+(\omega)|^2}[\omega D_1(\omega)+\Gamma\Delta_+(\omega)J(\omega)]\times\left(\frac{1}{\omega}+\frac{1}{\omega-\omega_d}\right)d\omega.\quad(52)$$

In Eq. (52) we must insert either Eq. (43) or Eq. (50). The cancellation of Γ and thus the emergence of an eigenvalue equation from Eq. (52) rests on the presence of Γ as an over-all factor in D_1. In the case of Eq. (50) we recall Eq. (51). Of course, we have not established the existence of a two-nucleon bound state in the theory. In order for this to be so, Eq. (52) must have at least one appropriate root. From the beginning we have assumed that $|d\rangle$ is one such state. Besides giving the eigenvalue equation, dispersion-theory formalism can also yield the detailed structure of the bound state. An example of this is given in Ref. 12. Note that Eq. (52) has been derived without appealing to the bare state expansion of $|d\rangle$. This is also a convenient point at which to note that one can go back and easily make the appropriate simplifications leading to the one-meson analog of Eq. (52).

In this paper, which is intended to be a sequel to earlier ones,[12,16] we have been interested in studying the properties of composite particles in terms of matrix elements which may be examined by the methods of dispersion theory. We have restricted these considerations to the realm of two static nucleons in interaction with each other at zero range through the exchange of relativistic scalar mesons. On the one hand, dispersion calculations in the Lee model have led to eigenvalue conditions due to the exchange of one meson in the VN case, and to two mesons in the $2V$ case. Because of the special nature of this model, it was possible in the latter problem to avoid the complicated states of the $V\theta$ sector. If the contractions are such as to implicate these states, then one faces technical problems akin to those found in the present case when nucleons are contracted in D_1 and D_2. On the other hand, one of the many shortcomings of the Lee model is its lack of crossing symmetry. In the present work we have turned to the charged scalar theory in order to include this aspect in our dispersion calculations of bound-state parameters. In this connection, we have incorporated the two-meson solution developed by Bronzan which has both two- and three-particle unitarity and a crossing-symmetric scattering amplitude.

A factorization property of the two-meson scattering matrix enables us to write dispersion relations for various matrix elements as Omnès equations with different inhomogeneous terms. Thus, the vertex functions K_1 and K_2 involving the vacuum state at one end are obtained in the two-meson approximation, without omitting any terms. The more complicated functions D_1 and D_2 containing the bound state at one end are first treated by dropping terms which implicate intermediate states with two nucleons and one, two, or more mesons. This treatment is somewhat unsatisfactory in that the neglected terms possess further information on the binding energy. In another approach we improve on this situation by calculating a new D_1, but all contributions in the two-meson approximation are not taken into account. It seems that the most satisfactory situation would be to include these terms and to calculate D_2 in the same way. However, it is not evident that the resulting integral equations for D_1 and D_2 are soluble, and in fact one may be forced into compromises such as that made in obtaining Eq. (49).

As mentioned before, we do not address our attention to the roots of Eq. (52). Instead we have been interested in pursuing various contraction possibilities presented by the methods of dispersion theory for including higher-order effects in a meson-theoretic description of the interaction between two static nucleons within a fairly tractable context. As far as we know, the present work is the first instance of a two-meson dispersion calculation of a composite particle in a theory with crossing symmetry. We have looked for these possibilities by following Blankenbecler and Cook, who advocate the use of vertex functions as a means of examining bound-state properties. Their program aims at providing "a potential which is chosen to yield the bound-state properties, not low-energy scattering properties of field theory." In another dispersion-relation approach,[17] an effort is made to calculate nucleon-nucleon scattering directly in terms of one-meson- and two-meson-exchange contributions. Taketani and his collaborators[10] have proposed on the basis of pion theory that the nucleon potential be divided into three regions, namely, classical ($x>1.5$), dynamical ($0.7<x<1.5$), and phenomenological ($x<0.7$), where x is the internucleon distance in units of the pion Compton wavelength. In the dynamical region the two-pion-exchange potential competes with and exceeds the one-pion-exchange potential. The former depends very much on recoil effects, the type of coupling (p wave or other), the nucleon form factor, and the higher-energy pion field cutoff procedure, not to mention multiple-scattering effects and radiative corrections. Therefore, it would be very presumptuous to assert that our static-model considerations have prepared us for a realistic dispersion calculation of two-meson effects in the two-nucleon potential. Since we have been bound to the two-meson solution of the charged scalar theory developed by Bronzan, the limitations of his solution and methods are also present in our work. As a result, it appears doubtful that we could proceed in the same way to include two-meson effects in the interaction between two nucleons in static models such as symmetric scalar[18] and neutral pseudoscalar[18] theories.

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APPENDIX

Here we reexamine a vertex function treatment of the bound state $|B\rangle$ formed by the interaction between a V and N particle, both considered to be bosons situated at the origin of coordinates. In Ref. 16 it is shown that the vertex

$$\Gamma=\langle V|f_N|B\rangle \tag{A1}$$

expands, on contracting the V particle, into the expression

$$\Gamma=\sum_k \frac{1}{\omega}\langle 0|f_V|N\theta_k\rangle\langle N\theta_k|f_N|B\rangle\,, \tag{A2}$$

where the out-state $|N\theta_k\rangle$ describes the scattering of a θ particle by an N particle. The matrix element $X^{-1}(\omega)\langle 0|f_V|N\theta_k\rangle$ is the complex conjugate of the Goldberger-Treiman[19] function $K(\omega)$. Hence, we have

$$X^{-1}(\omega)\langle 0|f_V|N\theta_k\rangle=-\frac{g}{1-\beta^*(\omega)}\,, \tag{A3}$$

where the integral function $\beta(\omega)$ is given by

$$\beta(\omega)=-\frac{g^2}{\pi}\omega\int_\mu^\infty \frac{\rho(\omega')d\omega'}{\omega'(\omega'-\omega-i\epsilon)}\,; \tag{A4}$$

$\rho(\omega)$ and $X(\omega)$ have the same meaning as in the text.

The other matrix element in Eq. (A2) is defined as

$$R(\omega)=X^{-1}(\omega)\langle N\theta_k|f_N|B\rangle. \tag{A5}$$

On contracting the N particle, one finds the Omnès equation

$$R(z)=\frac{R(\omega_0)}{2}+\frac{g\Gamma}{z}+\frac{1}{\pi}\int_\mu^\infty \frac{e^{-i\delta(\omega')}\sin\delta(\omega')R(\omega')d\omega'}{\omega'-z}\,, \tag{A6}$$

written in a form which stresses the analytic properties of R. In this equation $\delta(\omega)$ is the phase shift for $N\theta$ scattering, while $R(\omega_0)$ is given by

$$R(\omega_0)=\frac{2g\Gamma}{\omega_0}+\frac{2}{\pi}\int_\mu^\infty \frac{e^{-i\delta(\omega')}\sin\delta(\omega')R(\omega')d\omega'}{\omega'-\omega_0}\,; \tag{A7}$$

ω_0 is the negative interaction energy between V and N. The quantity $R(\omega_0)$ is an unknown constant which can be obtained in two different ways. One of these is simply to evaluate the solution of Eq. (A6) at $z=\omega_0$ and solve for $R(\omega_0)$. The other is to insert the solution of Eq. (A6) into the right-hand side of Eq. (A7), which then reduces, after integration, to an algebraic condition for $R(\omega_0)$. Having found $R(\omega_0)$, we then put it back into the solution of Eq. (A6), thus determining R. These procedures differ from that used in Ref. 16, where Eq. (A6) was first reduced to the quantity $g\Gamma'/\omega_0$ with Γ' representing the vertex $\langle N|f_V|B\rangle$.

The solution of Eq. (A6) is

$$R(z)=\frac{1}{1-\beta(z)}\left[Z\frac{R(\omega_0)}{2}+\frac{g\Gamma}{z}\right]. \tag{A8}$$

In arriving at this result we have used the asymptotic value $1-\beta(\infty)=Z$, where Z is the V-particle wave-function renormalization constant. Carrying out the procedures described above, we find[20]

$$R(\omega)=\frac{g\Gamma}{1-\beta(\omega)}\left[\frac{1}{\omega}+\frac{Z}{2\omega_0[1-\beta(\omega_0)]-Z\omega_0}\right]. \tag{A9}$$

As expected, Γ appears as an over-all factor in $R(\omega)$ and subsequently cancels out of Eq. (A2). The remaining integration leads to the binding-energy condition

$$1-\beta(\omega_0)=-\lambda(\omega_0)\,, \tag{A10}$$

where $\lambda(\omega_0)$ is defined by

$$\lambda(\omega_0)=\frac{g^2}{\pi\omega_0}\int_\mu^\infty \frac{\rho(\omega)d\omega}{\omega-\omega_0}. \tag{A11}$$

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