using our earlier requirements on $|0\rangle$ and introducing

$$K=\langle 0|\mathcal{H}_c|0\rangle=\Lambda+C\langle 0|P^{-1}|0\rangle.$$

Now the differential coefficients in (3) are each invariant under left group translations. Let

$$\tilde{U}\equiv U[\tilde{p},\tilde{q},\tilde{t}]=U[p_0,q_0,t_0]U[p,q,t],$$

where p_0, q_0, and t_0 remain unvaried. Then

$$i\hbar\tilde{U}^\dagger d\tilde{U}=i\hbar UdU,$$

which leads to the three differential forms

$$\tilde{p}d\tilde{q}-\tilde{p}\tilde{q}^2d\tilde{t}=pdq-pq^2dt, \tag{4a}$$

$$2\tilde{q}d\tilde{t}+d\tilde{p}/\tilde{p}=2qdt+dp/p, \tag{4b}$$

$$d\tilde{t}/\tilde{p}=dt/p. \tag{4c}$$

Solutions to these differential relations (the Maurer-Cartan equations[11]) exhibit the invariance transformations of the classical action. It is convenient to split up the result into three basic invariance transformations, one each for t_0, p_0, and q_0. The first is the trivial transformation $\tilde{p}=p$, $\tilde{q}=q$, $\tilde{t}=t+t_0$. The second is given by $\tilde{q}=p_0p$, $\tilde{q}=q/p_0$, $\tilde{t}=p_0t$. The third transformation reads

$$\tilde{p}=p(1+q_0t)^{-2},$$
$$\tilde{q}=(1+q_0t)^2q+q_0(1+q_0t),$$
$$\tilde{t}=t(1+q_0t)^{-1}.$$

It is clear that the first transformation applies to any potential $V(p)$. The second and third transformations require that $V(p)=K/p$. Note that any value of K is consistent since that term in the action is *separately* invariant according to (4c). Under the second transformation the two terms $p\dot{q}$ and pq^2 making up the free action are separately invariant—invariance would be maintained even if pq^2 were changed by a scale factor to αpq^2. Under the third transformation, however, there is "mixing" of $p\dot{q}$ and pq^2 and no separate scaling would be possible. The latter transformation has much of the appearance of a coordinate transformation: The "metric" $p(t)$ transforms homogeneously as a "tensor," while the "connection" $q(t)$ possesses an inhomogeneous term as well.

[11] P. M. Cohen, *Lie Groups* (Cambridge U. P., London, 1961).

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Classical Charged Tachyon Self-Energy Problem

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The classical self-energy problem for charged tachyons is more serious than that for charged bradyons. As a result, the theoretical basis for generally expected experimental properties of such objects is shaky.

HISTORICALLY, the problem of a classical electromagnetic charged particle coupled to an external field has been complicated by the self-energy problem associated with the point singularity at the location of the particle. Methods used to deal with this[1] are suitable only for particles whose speeds do not exceed that of light, however, and it appears that for tachyons the problem is rather more severe than usual. This fact may have bearing on theoretical expectations concerning the experimental properties of charged tachyons.[2] The purpose of the present paper is to point out the difficulties involved since they do not appear to be generally recognized and they are fundamental in character.

If a tachyon is not itself the source of an electromagnetic field, its equation of motion in a given external field is most naturally taken to be that following from the action principle based on the Lagrangian

$$L=m_t[x'(\alpha)^2]^{1/2}+qx'(\alpha)\cdot\mathcal{A}(x), \tag{1}$$

where $x'(\alpha)^\mu\equiv dx^\mu(\alpha)/d\alpha$, with α an arbitrary parameter, the tachyon mass m_t is defined to be real, and we have used the space-favoring metric. When the tachyon is the source of a field, the Lorentz force equation following from (1) is expected to contain an additional term for the radiation reaction. So, to determine the full equations of motion for the charge, it is necessary to solve the Maxwell-Lorentz field equation in the presence of a prescribed tachyon source and to compute the energy and momentum of radiation.

For a source with world line prescribed by the equations

$$x^\mu=\xi^\mu=\xi^\mu(\tau), \tag{2}$$

the electromagnetic field equation in the Lorentz gauge

[1] F. Rohrlich, *Classical Charged Particles* (Addison-Wesley, Reading, Mass., 1965); Phys. Rev. Letters **12**, 375 (1964).

[2] O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. **30**, 718 (1962); G. Feinberg, Phys. Rev. **159**, 1089 (1967).

is

$$\Box A^\mu(x)=j^\mu(x)=q\int_{-\infty}^{+\infty}d\tau\, u^\mu(\tau)\delta^{(4)}(x-\xi(\tau)),\quad (3)$$

where $u^\mu(\tau)\equiv d\xi^\mu(\tau)/d\tau$, with τ the source proper time and $u^0>0$, and q is the coupling strength to the field A^μ. The complete solution to Eq. (3) is given by

$$A^\mu(x)=A_{\text{in}}{}^\mu(x)+\int dx'\, D_{\text{ret}}(x-x')j^\mu(x'),\quad (4)$$

where $A_{\text{in}}{}^\mu(x)$ satisfies the homogeneous equation with initial conditions given for $A^\mu(x)$ in the remote past. Taking $A_{\text{in}}{}^\mu(x)=0$, the solution to Eq. (4) is

$$A^\mu(x)=\frac{q}{4\pi}\sum_n\frac{u^\mu(\tau_n)\theta_n}{|[x-\xi(\tau_n)]\cdot u(\tau_n)|},\quad (5)$$

where the $\xi^\mu(\tau_n)$ are roots to the equation

$$[x-\xi(\tau)]^2=0,\quad (6)$$

and θ_n is a unit step function corresponding to the retardation condition.

For bradyons,[3] Eq. (6) always has exactly two solutions, coinciding with the intersections of forward and backward light cones from the field event x with the timelike world line of the source. For the advanced root $\xi(\tau_A)$, θ vanishes because $\xi^0(\tau_A(x))>x^0$ always holds, while for $\xi(\tau_R(x))$, $\theta\equiv 1$. The solution is then the usual Liénard-Wiechert potential. For tachyons there is no such general characterization because the world-line tangent vector $u^\mu(\tau)$ is always spacelike; the number of backward light cone intersections depends on the location of the field event and the shape of the tachyon line.

The simplest example is that of a uniformly moving charge, for which

$$A_b{}^\mu(x)=(qu_b{}^\mu/4\pi)[x^2+(x\cdot u_b)^2]^{-1/2}\quad (7\text{b})$$

if $u^2=-1$, and

$$A_t{}^\mu(x)=\theta\cdot(qu_t{}^\mu/2\pi)[-x^2+(x\cdot u_t)^2]^{-1/2}\quad (7\text{t})$$

if $u^2=+1$. In Eqs. (7), we have used the identities

$$|(x-\xi_R)\cdot u_b|=[x^2+(x\cdot u_b)^2]^{1/2},$$

$$|[x-\xi(\tau_1)]\cdot u_t|=|[x-\xi(\tau_2)]\cdot u_t|=[-x^2+(x\cdot u_t)^2]^{1/2},$$

there being either two roots or no roots to Eq. (6) in the straight-line tachyon case. In Eq. (7t), the value of the θ function is 1 when $-x^2+(x\cdot u_t)^2>0$, that is, for field points in the "shadow of the tachyon,"[4] i.e., behind the Čerenkov front; it is zero outside.

[3] From the Greek βραδυς≡slow.

[4] A. Sommerfeld, Koninkl. Ned. Akad. Wetenshap. Proc. **8**, 346 (1904).

The singularity at the source in Eq. (7b) is of the form

$$A_b{}^\mu\propto r^{-1},\quad (8\text{b})$$

where r is the distance to the charge in its own rest frame. In Eq. (7t),

$$A_t{}^\mu\propto(t^2-\rho^2)^{-1/2},\quad (8\text{t})$$

where ρ is the distance to the tachyon line in the frame where its speed is infinite and t is the time elapsed since its occurrence. In the bradyon case, the self-energy singularity is confined to a *point* $r=0$, whereas in the tachyon case there is a singular *surface* which propagates to the field observation points. The total radiated energy per unit length of the tachyon line involves a time integral of the Poynting vector and it diverges owing to the self-energy contribution on the edge of the propagating front.

The energy loss of a charged bradyon moving rapidly through matter contains a contribution arising from the polarization of distant parts of the medium and which can be identified with Čerenkov radiation.[5] The singularity arising in the classical calculation of the field of such a charge in a medium arises from the phenomenological representation of many-body effects through an index of refraction, and the difficulty is properly not regarded as fundamental because this approximation may be expected to break down for points along the Čerenkov front.

For the case of a charged tachyon in *empty space*, there is no such many-particle effect to remove the difficulty, and the divergence of the integral for the radiated energy must be regarded as *fundamental* (unless we are satisfied by *ad hoc* hypotheses, i.e., theoretical dodges, involving extended charge distributions). Because the radiated energy and momentum from a moving tachyon is infinite, a "radiation reaction" term in the Lorentz-Dirac force law is pathological and it appears that we cannot formulate the equations of the coupled particle-field system in a meaningful way. The procedures of Ref. 1 do not seem to be helpful.

Physically what is happening is that the tachyon, no matter how long its world line may be, after being created goes away entirely,[6] not just to an "asymptotic region," and it does so all in an instant, the only trace being its self-energy *problem* in the form of a propagating singular wave front. It may be that the field approximation to the influence of a charged particle as given by Eq. (3) will not be found useful in theories involving "particles" which move faster than light.

[5] E. Fermi, Phys. Rev. **57**, 485 (1940).

[6] Like a spatially extended event [cf. R. G. Cawley, Naval Ordnance Laboratory report (unpublished)].