Constraint on the Universal Function in Electroproduction

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We show that analyticity and asymptotic behavior impose a severe contraint on the scale-invariant function widely suggested for the behavior of $\nu W_2(q^2\nu)$ as $q^2 \to \infty$. If the asymptotic behavior of the off-mass-shell Compton scattering amplitude is dominated by the usual (nonfixed) Regge trajectories, then our constraint is violated by the experimental candidates for the νW_2 curve. On the other hand, the introduction of a fixed pole corresponding to the one used by Damashek and Gilman alters the constraint, which is then consistant with the 18° data.

THE electroproduction structure functions $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$ (in the usual notation)[1] have been the subject of extensive investigation lately.[2–5] The interest centers on the possibility of scale invariance, i.e., that

$$\lim_{q^2\to\infty} W_1(q^2,\nu)=f_1(\omega)$$

and (1)

$$\lim_{q^2\to\infty} \nu W_2(q^2,\nu)=f_2(\omega)\,.$$

We show in this article that the usual requirements of analyticity, along with a very acceptable assumption about Regge residues, leads to a strong constraint on the scale-invariant function $f_2(\omega)$. If one assumes that the off-mass-shell Compton scattering amplitudes involved are dominated only by the usual moving Regge poles (as distinct from fixed poles), then we show that neither the 6° and 10° data of Breidenbach *et al.*[4,5] nor the more recent 18° data[6] satisfy this constraint. Under these conditions, either one must abandon nontrivial scale invariance, or one must hope that these data do not represent fully the $q^2 \to \infty$ limit.

Alternatively, if one considers fixed poles in addition to the usual Regge trajectories, then the above constraint is altered. We discuss the most favorable candidate for such a fixed pole, and show that the 18° curve is consistent with the residue of such a fixed pole as determined by Damashek and Gilman.[7]

Let us start with the case when there are no fixed poles. Consider the forward Compton scattering amplitude (off mass shell), averaged over the nucleon spin,

$$T_{\mu\nu}=4\pi^2\alpha\left[\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)A_1 +\left(p_\mu-\frac{m\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{m\nu}{q^2}q_\nu\right)\frac{A_2}{m^2}\right]. \quad (2)$$

Here W_1 and W_2 are the absorptive parts of A_1 and A_2. The functions A_1 and A_2 are analytic functions of ν in the cut ν plane, and may be written as a sum of leading Regge contributions in the large-ν region. Consider then the function

$$\tilde{A}_2(q^2,\nu)=A_2(q^2,\nu)-\sum_i \beta_i(q^2)|\nu|^{\alpha_i-2}\frac{e^{i\pi\alpha_i}+\eta_i}{\sin\pi\alpha_i}. \quad (3)$$

The summation $\sum_i$ is over the contributions of Regge poles with $\alpha_i \geqslant 0$, in this case presumably just the P, P', and A_2 trajectories. Then $\nu\tilde{A}_2$ falls off faster than $1/\nu$ as $\nu\to\infty$, and therefore

$$\int_{-\infty}^{\infty}\mathrm{Im}(\nu\tilde{A}_2)d\nu=2\int_0^{\infty}[\nu W_2-\sum_i \beta_i(q^2)\nu^{\alpha_i-1}]d\nu =0. \quad (4)$$

Here the negative crossing property of $\nu A_2(q^2,\nu)$ has been used. The integral consists of the nucleon-pole contribution to νW_2 involving the elastic form factors $F_1(q^2)$ and $F_2(q^2)$ and a continuum contribution. Thus,

$$-\frac{q^2}{2m}\left[|F_1(q^2)|^2+\frac{q^2\kappa_p{}^2}{4m^2}|F_2(q^2)|^2\right] =-\int_{\nu_0}^{\infty}[\nu W_2-\sum_i \beta_i(q^2)\nu^{\alpha_i-1}]d\nu. \quad (5)$$

The threshold ν_0 for the continuum is clearly $[(m+\mu_\pi)^2-m^2+q^2]/2m$.

Now let us go to the limit of $q^2\to\infty$. In this limit, scale invariance requires νW_2 to be just $f_2(\omega)$. Further, let each Regge contribution, Eq. (5), be written as $\gamma_i(q^2)(2m\nu/q^2)^{\alpha_i-1}$. The following possibilities exist for the behavior of $\gamma_i(q^2)$ as $q^2\to\infty$:

(a) $\lim_{q^2\to\infty}\gamma_i(q^2)=\text{const}$, where the constant may or may not be zero. In this case, the sum of the three Regge contributions on the right-hand side of Eq. (5)

[1] We use the standard notation, as found e.g., in F. J. Gilman, Phys. Rev. **167**, 1365 (1968). Our metric is $q^2=(\mathbf{q})^2-q_0^2$, and $\nu=-p\cdot q/m$.
[2] J. D. Bjorken Phys. Rev. **179**, 1547 (1969); J. D. Bjorken and E. A. Paschos, *ibid.* **185**, 1975 (1969).
[3] H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters **22**, 500 (1969).
[4] E. D. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969).
[5] M. Breidenbach *et al.*, Phys. Rev. Letters **23**, 935 (1969).
[6] L. W. Mo, in *Proceedings of the Third International Conference on High Energy Collisions, Stony Brook, 1969* (Gordon and Breach, New York, 1969), p. 127.
[7] M. Damashek and F. Gilman, Phys. Rev. D **1**, 1319 (1970); see also C. A. Dominguez *et al.*, Phys. Letters **31B**, 365 (1970).

also "scales" in the $q^2 \to \infty$ limit, i.e., it is a function only of ω. Note that a similar conjecture was made by Abarbanel, Goldberger, and Treiman[3] for the leading Regge term.

(b) $\gamma_i(q^2)$ either diverges or oscillates as $q^2 \to \infty$. In order for νW_2 to be finite and depend only on ω in this limit, this divergent or oscillatory contribution must be cancelled by the rest of the terms in the Regge expansion. Such a cancellation for all values of ω requires a conspiracy between infinite terms. We assume that this delicate conspiracy does not occur.

This leaves possibility (a), in which case the entire integrand in Eq. (5) or Eq. (4) depends only on ω in the $q^2 \to \infty$ limit. Further, the threshold is

$$\omega_0 = \frac{2m\nu_0}{q^2} = 1 + \frac{(m+\mu_\pi)^2 - m^2}{q^2}.$$

Thus, as $q^2 \to \infty$,

$$\lim_{q^2\to\infty}\left[\,|F_1(q^2)|^2 + \frac{q^2\kappa_p{}^2}{4m^2}|F_2(q^2)|^2\right] = \int_1^\infty [f_2(\omega) - R(\omega)]d\omega \equiv I, \tag{6}$$

where $R(\omega)$ is the scale-invariant contribution of the leading Regge poles in the $q^2 \to \infty$ limit and is the asymptotic form of $f_2(\omega)$ for large ω. Note that $R(\omega)$ need not be calculated theoretically and can be obtained from the experimental curve for $f_2(\omega)$ by fitting it at large ω to the form $\sum_i \gamma_i \omega^{\alpha_i - 1}$. Equation (6) therefore gives a severe constraint on the universal function $f_2(\omega)$ determined by the behavior of $F_1{}^2 + (q^2\kappa_p{}^2/4m^2)F_2{}^2$ as $q^2 \to \infty$. In particular, if $F_1(q^2)$ and $F_2(q^2)$ go to zero faster than $1/\sqrt{(q^2)}$ as the present data on form factors strongly indicate, then the integral I in Eq. (6) must vanish.

Let us examine the experimental situation in the light of this constraint. [Note that with our definition of A_1 and A_2 as in Eq. (2), νW_2 is dimensionless and so is the integral I in Eq. (6).] The curves at 6° and 10° in Figs. 2(a) and 2(b) of Breidenbach *et al.*[5] strongly violate the constraint in Eq. (6). The value of the integral I is about -1.2 for these curves, whereas the left-hand side of Eq. (6) is most likely to vanish as $q^2 \to \infty$. Even at $q^2 \simeq 4$ GeV^2, the form factors on the left-hand side are several orders of magnitude less than unity and are falling.

The scale-invariant curve for νW_2 at 18° presented by Mo[6] also violates Eq. (6). Here the curve is much steeper for large ω than at 6° and 10°. Consequently, even if the curve levelled off at the present value of its tail, the integral is definitely nonzero, having a value of about 0.8.

Now let us introduce the possibility of fixed poles. Recently Damashek and Gilman[7] have suggested the existence of a $J=0$ fixed pole in the amplitude A_1. [They suggest a real constant $c \simeq -3$ μb GeV in the asymptotic behavior of the on-mass-shell Compton amplitude $f_1(\nu)$. Note that in our notation $A_1(\nu) = (1/\pi\alpha)f_1(\nu)$.] However, it is well known that $A_2(q^2,\nu)$ must satisfy

$$A_2(q^2,\nu) \to 0 \quad \text{as} \quad q^2 \to 0$$

and

$$\lim_{q^2\to 0} \nu A_2(q^2,\nu) = \lim_{q^2\to 0} \frac{q^2}{\nu} A_1(q^2,\nu). \tag{7}$$

Corresponding to the $J=0$ fixed pole in $A_1(\nu)$, Eq. (7) suggests a term of the form $(c/\pi\alpha)q^2/\nu$ in the amplitude $\nu A_2(q^2,\nu)$, where the constant $c = -3$ μb GeV according to Damashek and Gilman. In the J plane, this is a $J=-1$ fixed pole of residue $cq^2/\pi\alpha$ in the amplitude νA_2. We have taken a linear q^2 dependence because this is the only possibility among residues polynomial in q^2.[8] A higher-order polynomial in q^2 would not give finite scale invariance for νW_2 as $q^2 \to \infty$, whereas a lower-order polynomial would violate Eq. (7).

Introducing such a fixed pole in νA_2 in addition to the usual Regge trajectories in Eq. (3), and repeating the arguments presented earlier, one can easily see that Eq. (6) is altered to

$$\begin{aligned} I &\equiv \int_1^\infty [f_2(\omega) - R(\omega)]d\omega \\ &= -cm/\alpha + (\text{vanishing elastic form factors}) \\ &\simeq 0.9. \end{aligned} \tag{8}$$

Thus, the integral I now need not be zero, and is related to the fixed-pole residue.[9] The value of 0.9 is of course based on the Damashek-Gilman value of $c \simeq -3$ μb GeV. Going back to the experimental value of I as discussed above, we see that $I \simeq 0.9$ is in very good agreement with the trend of the 18° data, although it disagrees with the earlier 6° and 10° data.

We conclude, therefore, that if the off-mass-shell Compton amplitudes are dominated only by the usual Regge trajectories, then scale invariance contradicts the present data at 6°, 10°, and 18°. On the other hand, if a suitable fixed pole is present, the contradiction is removed. The residue of such a fixed pole and its dependence are in remarkable agreement with the similar fixed pole suggested by earlier authors from the on-mass-shell Compton scattering point of view.

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[8] T. P. Cheng and W. K. Tung, Phys. Rev. Letters **24**, 851 (1970).

[9] The above equation has essentially been arrived at from quite a different point of view by J. M. Cornwall *et al.*, Phys. Rev. Letters **24**, 1141 (1970). Note that their definition of ω is the reciprocal of ours.