Calculation of the Mass Shifts of the K_S Meson*

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The mass shifts of the K_S meson are calculated in a multichannel ND^{-1} formalism by considering coupling of the K_S meson to both the $I=0$ and $I=2$ $(J=0)$ $\pi\pi$ states. A pole approximation is used for the weak coupling of the K_S meson to the $\pi\pi$ channel. The results for the coupling of K_S to the $I=J=0$ $\pi\pi$ partial wave gives a value of $\Delta m_s^{(0)}/(\frac{1}{2}\gamma_s^{(0)})$ of the order of -1, in agreement with previous calculations. The ratio $\Delta m_s^{(2)}/(\frac{1}{2}\gamma_s^{(2)})$ is sensitive to the high-energy behavior of the $I=2, J=0$ phase shift and may be larger than previously expected.

I. INTRODUCTION

DISPERSION techniques were used by Barger and Kazes[1] to discuss K_L and K_S mass differences qualitatively. Nishijima[2] later did a more quantitative study of the mass differences by making the following assumptions: (a) The mass shift of the K_S meson relative to the K_L meson is due to weak interactions because $\Gamma(K_S\to \text{all})\gg\Gamma(K_L\to\text{all})$. (b) The inequality $\Gamma_S\gg\Gamma_L$ persists over a wide energy range. (c) Since the K_S meson decays primarily into the 2π channel, we can replace Γ_S by the partial width $\Gamma_{2\pi}$ at all energies. Then we can write (W is the total c.m. energy)

$$\Delta m_S=\frac{1}{\pi}P\int_0^\infty \frac{W^2\Gamma_{2\pi}(W)}{m_S(m_S^2-W^2)}dW, \qquad (1)$$

where Δm_S is the mass shift of K_S, m_S is the mass of K_S, and P means principal-value integral. A similar expression can be written for Δm_L. Because of assumption (b), we can write $|\Delta m_S|\gg|\Delta m_L|$, which leads one to expect that the K_L-K_S mass difference is given in large part by $\Delta m\approx\Delta m_S$. The width $\Gamma_{2\pi}$ is determined from $(s=W^2)$

$$\Gamma_{2\pi}(W)=\text{const}\frac{(s-4m_\pi^2)^{1/2}}{s}\times\exp\left[\frac{2}{\pi}(s-m_s^2)P\int_{4m_\pi^2}^\infty ds'\frac{\delta_0(s')}{(s'-m_s^2)(s'-s)}\right], \qquad (2)$$

where δ_0 is the $I=J=0$ phase shift in $\pi\pi$ scattering. If we assume that δ_0 is known, we can compute $\Gamma_{2\pi}$ and hence Δm. Nishijima assumed a form for δ_0 such that $\delta_0\to\pi$ as $s\to\infty$. With this asymptotic behavior, no cutoff or subtraction is required to obtain a finite result by using Eqs. (1) and (2). A cutoff is required if $\delta_0\to 0$ as $s\to\infty$.

Another well-known attempt to compute Δm was made by Truong.[3] He used the same assumptions listed above along with a single-channel N/D approach to derive the formula

$$\Delta m=-\tfrac{1}{2}\Gamma_{2\pi}\cot\delta_0, \qquad (3)$$

after neglecting terms which were assumed small. This result was examined and criticized by several authors.[4,5]

More recently, a dispersion-theoretic approach was used by Kamal and Kenny[6] to compute the mass shift of K_S due to the decay of K_S into the $I=2$, $J=0$ 2π channel. If this mass shift and ϵ' are known, then the contribution of the 2π $I=2$, $J=0$ channel to $\text{Re}\epsilon$ can be computed.[7] The most optimistic value of ϵ' and the mass shift resulted in a value of $\text{Re}\epsilon$ too small by a factor of 2. In the calculation it was assumed that $\delta_2\to -k\pi$ as $s\to\infty$, so that a cutoff must be used to compute the mass shift by using Eqs. (1) and (2).

In this paper we present a slightly different model to compute the mass shifts based on a multichannel ND^{-1} formalism. We assume that the K_S meson is produced as a bound state of some high-mass channel which is then coupled to the 2π channel via the weak interactions. The expression we obtain for the mass shift reduces to Eq. (3) in a certain limit. The formalism is presented in Sec. II. In Sec. III we show the results of the calculations, and we discuss the results in Sec. IV.

II. FORMALISM

We assume that the Born matrix is given by

$$B=\begin{pmatrix} B_{11} & g_{12}/(s+s_0) \\ g_{12}/(s+s_0) & B_{22}\end{pmatrix}. \qquad (4)$$

The K_S meson is assumed to be produced as a bound state in the second channel by B_{22} while B_{11} can be computed for physical s from a knowledge of the $\pi\pi$ phase shifts. For example, if channel 1 is the $I=0$, $J=0$ $\pi\pi$ channel

$$B_{11}(s)=\frac{\sin\delta_0(s)\cos\delta_0(s)}{\rho_1(s)}-\frac{1}{\pi}P\int_4^\infty\frac{\sin^2\delta_0(s')}{\rho_1(s')}\frac{ds'}{s'-s}, \qquad (5)$$

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[1] V. Barger and E. Kazes, Phys. Rev. **124**, 279 (1961).
[2] K. Nishijima, Phys. Rev. Letters **12**, 39 (1964).
[3] T. Truong, Phys. Rev. Letters **17**, 1102 (1966).
[4] R. Rockmore and T. Yao, Phys. Rev. Letters **18**, 501 (1967).
[5] K. Kang and D. Land, Phys. Rev. Letters **18**, 503 (1967).
[6] A. Kamal and B. Kenny, Phys. Rev. **186**, 1473 (1969).
[7] For a discussion of ϵ and ϵ', see N. Byers, S. McDowell, and C. N. Yang, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

where ρ_1 is a kinematical factor which we take to be $\rho_1(s)=[(s-4)/s]^{1/2}(s-4)$.[8] We assume the weak coupling between the 2π channel and the K_S channel can be approximated by a single pole term $g_{12}/(s+s_0)$, where s_0 is related to the range of the weak interactions.

If we make a subtraction in D at $s=-s_0$, we obtain

$$N_{ii}(s)=B_{ii}(s)+\frac{1}{\pi}\int_{s_i}^{\infty}\frac{B_{ii}(s')-[(s+s_0)/(s'+s_0)]B_{ii}(s)}{s'-s}\times\rho_i(s')N_{ii}(s')ds', \quad (6)$$

where s_i is the threshold for the ith channel. For $i\neq j$,

$$N_{ij}(s)=g_{12}F_i, \quad (7)$$

where

$$F_i=\frac{1}{s+s_0}+\frac{1}{\pi}\int_{s_i}^{\infty}\frac{B_{ii}(s')-[(s+s_0)/(s'+s_0)]B_{ii}(s)}{s'-s}\times\rho_i(s')F_i(s')ds'. \quad (8)$$

D_{ij} can be computed from

$$D_{ij}(s)=\delta_{ij}-\frac{s+s_0}{\pi}\int_{s_i}^{\infty}\frac{\rho_i(s')N_{ij}(s')}{s'+s_0}\frac{ds'}{s'-s-i\epsilon}. \quad (9)$$

We write

$$D_{12}=-g_{12}(C_1+i\rho_1F_1) \quad (10)$$

and

$$D_{21}=-g_{12}\Phi_2, \quad (11)$$

where Φ_2 is real in the region of the K^0 meson. We assume that near the K_S meson we may write

$$D_{22}(s)=d(m_K^2-s), \quad (12)$$

where m_K^2 is the mass which the K_S meson would have if it were not coupled to the 2π channel.

After making the approximation in Eq. (12), we find that the S-matrix element for the first channel may be written as

$$S_{11}=e^{2i\delta}(s_R-s+\tfrac{1}{2}i\Gamma)/(s_R-s-\tfrac{1}{2}i\Gamma), \quad (13)$$

where[9]

$$N_{11}/D_{11}=e^{i\delta}\sin\delta, \quad (14)$$

$$\Gamma=\Gamma_1(C\sin\delta+\cos\delta)^2, \quad (15)$$

$$C=C_1/\rho_1F_1, \quad (16)$$

$$\Gamma_1=2\rho_1g_{12}^2\Phi_2F_1^2(r+r_0)/d, \quad (17)$$

$$s_R=m_K^2-\tfrac{1}{2}\alpha\Gamma, \quad (18)$$

$$\alpha=(C\cot\delta-1)/(C+\cot\delta). \quad (19)$$

[8] We use units $\hbar=c=m_\pi=1$.

[9] These results were quoted in previous work by P. Coulter and G. Shaw (unpublished).

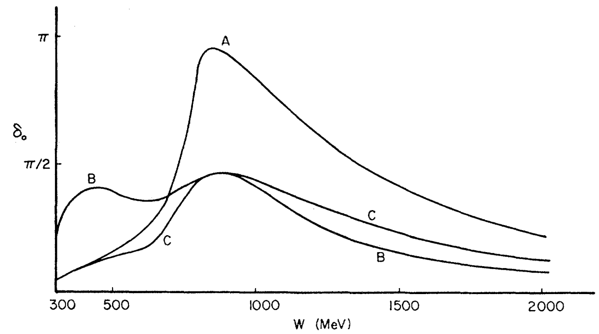

FIG 1. δ_0 as a function of W. Below 1 GeV curves A, B, and C are possible fits to the $I=J=0$ $\pi\pi$ phase shifts. Above 1 GeV we let $\delta\to 0$ smoothly.

Thus in this model the mass shift is given by

$$\Delta m_S=-\tfrac{1}{2}\gamma\frac{C\cot\delta-1}{C+\cot\delta}=-\tfrac{1}{2}\gamma\alpha, \quad (20)$$

where γ is the width written in energy units ($\gamma=\Gamma/2m_K$). Truong's result, Eq. (3), is obtained if $C\gg\cot\delta$.

In order to compute $\Delta m_S/\gamma$ we must know C and δ. C can be computed provided δ is known for all energies and provided that we have some way to determine s_0. We will see below that the dependence on s_0 is not very strong.

III. CALCULATIONS

A. Coupling of K_S to $I=J=0$ $\pi\pi$ Partial Wave

In Fig. 1 we show some of the possible results of the partial-wave analysis of $\pi\pi$ scattering for the $I=J=0$ partial wave.[10] The phase shifts are known up to about 1 GeV. The phase shifts at higher energies are a smooth continuation assumed to approach zero for large s. The mass shifts corresponding to the phase shifts shown in Fig. 1 are presented in Table I as a function of s_0. We see that the variation as a function of s_0 is slow. For the "down-up" phase shifts A,[11] we find[12] $\Delta m_S^{(0)}/(\frac{1}{2}\gamma_S^{(0)})$ in the range -0.95 to -1.5 as s_0 varies from 50 to 400.[8] For the "up-down" solution B, $\Delta m_S^{(0)}/$

TABLE I. Values of $\Delta m_S^{(0)}/(\frac{1}{2}\gamma_S^{(0)})$ for different values of s_0 and the phase shifts shown in Fig. 1.

s_0 \ Phase shifts	A	B	C
50	−0.95	1.17	−0.27
100	−1.23	0.96	−0.46
200	−1.42	0.83	−0.57
400	−1.50	0.75	−0.64

[10] J. H. Scharenguivel *et al.*, Phys. Rev. **186**, 1387 (1969). References to other phase-shift analyses of the $\pi\pi$ system can be found in this paper.

[11] E. Malamud and P. Schlein, Phys. Rev. Letters **19**, 1056 (1967).

[12] We use superscripts 0 and 2 to refer to the $I=0$, 2 $\pi\pi$ channels, respectively. $\gamma_S^{(0)}$ is the partial width for $K_S\to\pi\pi$ ($I=J=0$).

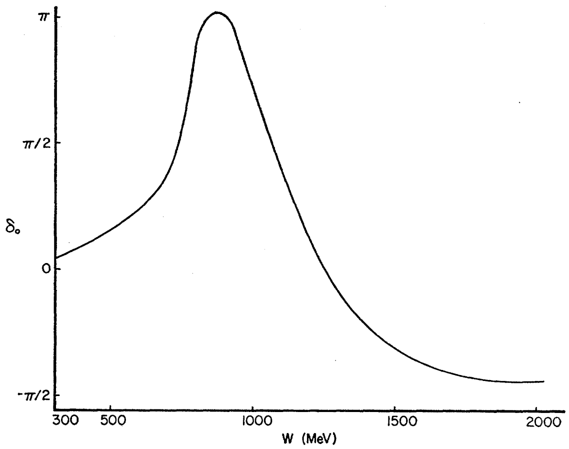

FIG. 2. δ_0 as a function of W. This is a variation of curve A in Fig. (1) with different asymptotic behavior.

$(\frac{1}{2}\gamma_S{}^{(0)})$ is positive. For solution C we find that $\Delta m_S{}^{(0)}/(\frac{1}{2}\gamma_S{}^{0})$ is negative, but smaller in magnitude than for solution A. Solution C is apparently too small to account for Δm unless Δm_L is positive. Solution A can account for Δm if Δm_L is not too large negatively. Thus this analysis tends to favor solution A, which is also favored by experimental fits to the data.[10]

We also considered different high-energy phase shifts from those shown in Fig. 1. For most variations from Fig. 1 the results quoted in Table I do not change greatly. However, for the phase shift shown in Fig. 2 we find $\Delta m_S{}^{(0)}/(\frac{1}{2}\gamma_S{}^{(0)})=-4.4$ for $s_0=400$, so the high-energy behavior can be important.

B. Coupling of K_S to $I=2$, $J=0$ $\pi\pi$ Partial Wave

If we want to talk about K_S coupling to both $I=0$ and $I=2$ partial waves, we must clearly use a three-channel model. However, if the decay $K_S\rightarrow\pi\pi(I=2)$ is due to a direct coupling between K_S and the $\pi\pi(I=2)$ state and if there is no direct coupling between the $I=0$ and $I=2$ $\pi\pi$ states, then we can still use the formalism in Sec. II.[13] In this case the total mass shift will be the sum of two terms of the form of Eq. (20) coming from coupling K_S to the $I=0$ and $I=2$ $\pi\pi$ channels $\Delta m_S=\Delta m_S{}^{(0)}+\Delta m_S{}^{(2)}$. We will assume that this is the case.

Figure 3 shows two different forms we assumed for δ_2. Again the phase shifts are only known up to about 1 GeV.[14] If we use solution I, we find values of $\Delta m_S{}^{(2)}/(\frac{1}{2}\gamma_S{}^{(2)})$ ranging from -0.65 to -1.95 as s_0 varies from 50 to 400.

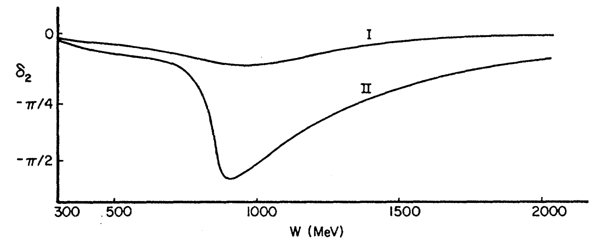

FIG. 3. δ_2 as a function of W. Curve I is an approximation to the phase-shift analysis of Rev. 13 below 1 GeV. Above 1 GeV we let $\delta\rightarrow 0$ smoothly. We obtain $\Delta m_S{}^{(2)}/(\frac{1}{2}\gamma_S{}^{(2)})=-0.64$, -1.03, -1.46, and -1.95 for $s_0=50$, 100, 200, and 400, respectively. Curve II shows a different high-energy behavior above 700 MeV, giving $\Delta m_S{}^{(2)}/(\frac{1}{2}\gamma_S{}^{(2)})=-17.5$ for $s_0=200$.

At low energies C tends to be a positive number regardless of the sign of δ. Thus for coupling to the $I=J=0$ partial wave, both C and $\cot\delta$ have the same sign and the value of α tends to be fairly stable as the high-energy behavior of δ changes. For the $I=2$, $J=0$ partial wave, C and $\cot\delta$ generally have opposite signs and the value of α is more sensitive to the high-energy behavior of the phase shift. For curve II in Fig. 3 we find $-\alpha=17.5$ for $s_0=200$ because $C+\cot\delta$ is small.

IV. DISCUSSION

By using the model presented here, we can compute $\Delta m_s/\gamma_S$ for K_S $[\gamma_S=\gamma_S{}^{(0)}+\gamma_S{}^{(2)}]$ provided the $\pi\pi$ phase shifts and the pole approximation for the weak nondiagonal forces are known. The variation as a function of the pole parameter s_0 is slow. We find that the results are dependent on the (unknown) high-energy behavior of δ. This dependence is not strong for coupling to the $I=J=0$ $\pi\pi$ state. The values obtained for $\Delta m_s{}^{(0)}/\gamma_s{}^{(0)}$ for this case favor solution A for the phase shifts shown in Fig. 1.

The values of $\Delta m_S{}^{(2)}/\gamma_S{}^{(2)}$ for coupling to the $I=2$, $J=0$ $\pi\pi$ partial waves are more sensitive to the high-energy phase-shift behavior. The maximum value of $|\Delta m_S{}^{(2)}/\gamma_S{}^{(2)}|$ found by Kamal and Kenny was 5.76; we can easily obtain larger values for this ratio for some high-energy phase-shift behavior.

The calculations presented here confirm previous calculations that the coupling of K_S to the $I=J=0$ $\pi\pi$ partial wave is sufficient give values of $\Delta m_S{}^{(0)}/(\frac{1}{2}\gamma_S{}^{(0)})$ of the order of -1. It is possible, depending on the high-energy behavior of δ_2, that the magnitude of $\Delta m_S{}^{(2)}/(\frac{1}{2}\gamma_S{}^{(2)})$ can be larger than previously calculated.

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[13] See Ref. 9 for a more complete discussion of the formalism for more than two channels.

[14] J. P. Baton, G. Laurens, and J. Reignier, Nucl. Phys. **B3**, 349 (1967).