We note that the compatibility condition $\Psi_{,21}=\Psi_{,12}$ implies that

$$\Delta\tilde{\lambda}=0. \tag{14}$$

Equations (10) and (11) when expressed in terms of $\tilde{\lambda}$ become

$$\nu_{,1}=r(\tilde{\lambda}_{,1}{}^2-\tilde{\lambda}_{,2}{}^2)+[\ln(r\cosh^2\tilde{\lambda})]_{,1}, \tag{10'}$$

$$\nu_{,2}=2r\tilde{\lambda}_{,1}\tilde{\lambda}_{,2}+[\ln(r\cosh^2\tilde{\lambda})]_{,2}. \tag{11'}$$

We can therefore write $\nu=\tilde{\nu}+\ln(r\cosh^2\tilde{\lambda})$, where $\tilde{\nu}$ is determined from

$$\tilde{\nu}_{,1}=r(\tilde{\lambda}_{,1}{}^2-\tilde{\lambda}_{,2}{}^2) \quad\text{and}\quad \tilde{\nu}_{,2}=2r\tilde{\lambda}_{,1}\tilde{\lambda}_{,2}. \tag{15}$$

Equations (14) and (15) are just the equations that must be solved to generate vacuum Weyl fields.[1] Thus for every vacuum Weyl field there corresponds an electromagnetic vacuum field of the type discussed in this paper.

In closing we note that the two special cases of (14) and (15) corresponding to $\tilde{\lambda}=mz+n$ and $\tilde{\lambda}=m\ln r+n$ have been treated previously by Bonnor.[4]

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Elementary Model for Quantum Gravity

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An elementary model of the gravitational field in the presence of external sources is studied. This model has positivity requirements, invariance under transformations, and nonlinear equations of motion, all of which are analogs of similar properties in the full theory. The classical solutions of the "free" version of this model exhibit singularities, but these are removed in the quantum theory.

I. INTRODUCTION

SINCE the quantum theory of the gravitational field is so complicated, it is fruitful to analyze models incorporating some of the features of the full theory. In the spirit that a study of the harmonic oscillator sheds light on the quantum theory of the free field, we discuss some properties of one of the most elementary models of the quantum theory of gravitation.[1] In the way of motivation, consider a long-wavelength limit and assume that space-time is conformally flat. Then the metric $g_{\mu\nu}(x)$ is characterized by a single function of time,

$$g_{\mu\nu}(x)=p(t)g_{\mu\nu}{}^{\text{Lorentz}},$$

where physical requirements on the signature of space-time demand that

$$p(t)>0.$$

Under these assumptions, there is only one independent component of the connection $\Gamma_{\mu\nu}{}^{\alpha}(x)$, which we represent by the single function of time $q(t)$. Guided by the invariant form of the action functional for gravitational theory[2] (based on the scalar curvature, and the Palatini formulation of independent variations of the metric and connection), we adopt for our model the classical action functional

$$I=\int\{p(t)\dot{q}(t)-p(t)q^2(t)-V[p(t)]\}dt. \tag{1}$$

Here the first two terms correspond to the free gravitational theory, while the last term characterizes a possible interaction with an external source.

Although this simple model has but a single degree of freedom, the requirement that $p(t)>0$ makes quantization not immediately evident. In addition, in the important "free" model ($V=0$), the classical solutions exhibit singularities which are overcome in the quantum solutions. We show how the quantum theory leads to a natural modification of the classical theory—a quantitatively minor but qualitatively major modification—that points the way toward the removal of the classical singularity. It is suggestive that similar

[1] Studies of the present model appear in E. W. Aslaksen, thesis, Lehigh University, 1968 (unpublished); J. R. Klauder, in *Proceedings of the Fifth International Conference on Gravitation and the Theory of Relativity* (Tbilisi University, Tbilisi, U.S.S.R., to be published); *Relativity*, edited by M. Carmeli, S. I. Fickler, and L. W. Witten (Plenum, New York, 1970), p. 1.

[2] R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965).

quantum modifications may remove classical singularities in the full theory of general relativity.

In a separate section, we discuss the transformation group leaving the action (1) invariant. This invariance mimics—in an elementary fashion to be sure—the invariance under coordinate transformations of the full theory of gravitation. Maximal invariance is secured only when $V[p]$ has the special form K/p. This allowed addition is rather like the "cosmological term," which is the only invariant modification of the Einstein action that is possible.

Although our model is extremely simplified, it retains certain features of the full theory. These include positivity requirements $[p(t)>0]$, similar algebraic structure of action and thus of equations of motion, invariance under transformations imitating coordinate transformations, and singular classical solutions of the free theory. Elsewhere,[3] we have discussed field models based on the present example, and we will return to those models in a subsequent paper.

II. QUANTIZATION OF SINGLE-DEGREE-OF-FREEDOM MODEL

At first glance, the natural approach to the quantization of our model is to introduce canonical operators P and Q satisfying the Heisenberg commutation relation

$$[Q,P]=i\hbar,$$

and proceed in familiar fashion. As usual, we would assume that we could adopt either the Q or the P representation. However, the physical requirement $P>0$ leads to the consequence that Q can *not* be diagonalized. For example, if we diagonalize P and represent it as multiplication by k, then all wave functions $\psi(k)$ must vanish for $k<0$; or, in other words, inner products cover only positive k values:

$$\langle\psi|\psi\rangle=\int_0^\infty |\psi(k)|^2dk.$$

In this representation, $Q=i\hbar\partial/\partial k$, so that $e^{-isQ/\hbar}=e^{s\partial/\partial k}$. Thus

$$e^{s\partial/\partial k}\psi(k)=\psi(k+s),$$

which may well translate the function $\psi(k)$ out of the allowed set of states. As a consequence, $e^{-isQ/\hbar}$ is *not* a unitary operator which is necessary if Q is to be diagonalized.

Alternative Kinematical Group

An alternative kinematical operator pair may be considered instead. To determine the appropriate operators we proceed heuristically as follows: Multiply both sides of the Heisenberg commutation relation by P and define

$$B=\tfrac{1}{2}(PQ+QP).$$

Then it follows that

$$[B,P]=i\hbar P, \tag{2}$$

which is recognized as the commutator of a two-parameter Lie algebra. This is the Lie algebra of the so-called affine group, the group of translations and dilations (without reflections) of the real line: $x\to p^{-1}x-q$, $p>0$. Analysis of the representations of this group[4] shows that there is an irreducible representation that respects the requirement $P>0$ and in which both P and B generate unitary transformations. In the representation in which P is multiplication by k, it follows that

$$B=\tfrac{1}{2}i\hbar\left(k\frac{\partial}{\partial k}+\frac{\partial}{\partial k}k\right).$$

Thus

$$e^{irB/\hbar}=\exp\left[-\tfrac{1}{2}r\left(k\frac{\partial}{\partial k}+\frac{\partial}{\partial k}k\right)\right]$$
$$=\exp\left[-r\left(\frac{1}{2}+\frac{\partial}{\partial\ln k}\right)\right],$$

and therefore

$$(e^{irB/\hbar}\psi)(k)=e^{-\frac{1}{2}r}\psi(e^{-r}k),$$

which is clearly a unitary transformation on the appropriate space of functions $\psi(k)$, $k>0$. It is not unreasonable, albeit unconventional, to regard P and B (and not P and Q) as the "basic" variables.[5] This viewpoint is rather like that of adopting currents rather than fields as basic[6] [since Eq. (2) forms a closed Lie algebra], and this similarity will be even more apparent in a subsequent discussion of field models.

Let us introduce the family of unitary operators

$$U[p,q]\equiv e^{-iqP/\hbar}e^{i(\ln p)B/\hbar},$$

which characterizes the affine group. By our hypothesis all other operators are functions of the basic pair P and B, or of the unitary operators $U[p,q]$. Elsewhere we have discussed[7] the nature of general operators constructed in the form

$$T=(2\pi\hbar)^{-1}\int t(p,q)U[p,q]dpdq,$$

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[3] See Ref. 1; also, J. R. Klauder, J. Math. Phys. **11**, 609 (1970).

[4] I. M. Gel'fand and M. A. Naimark, Dokl. Akad. Nauk SSSR **55**, 570 (1947); E. W. Aslaksen and J. R. Klauder, J. Math. Phys. **9**, 206 (1968).

[5] The models constructed by C. W. Misner [Phys. Rev. **186**, 1319 (1969); *Relativity*, edited by M. Carmeli, S. I. Fickler, and L. Witten (Plenum, New York, 1970), p. 55] solve the relation $[B,P]=i\hbar P$ by the equivalent solution, $P=e^\beta$, $B=i\hbar\partial/\partial\beta$.

[6] See, e.g., R. F. Dashen and D. H. Sharp, Phys. Rev. **165**, 1857 (1968); D. H. Sharp, *ibid.* **165**, 1867 (1968).

[7] E. W. Aslaksen and J. R. Klauder, J. Math. Phys. **10**, 2267 (1969).

and have shown that at least formally

$$t(p,q)=\mathrm{Tr}(TPU^{\dagger}[p,q]).$$

While such a viewpoint is completely correct, it is somewhat luxurious in the single-degree-of-freedom model. We may always introduce

$$Q=\tfrac{1}{2}(P^{-1}B+BP^{-1})$$

as an auxiliary operator. This expression demonstrates that Q is Hermitian, i.e., $\langle\psi|Q|\varphi\rangle^*=\langle\varphi|Q|\psi\rangle$, without asserting that Q is self-adjoint as is necessary to generate unitary transformations.

Existence of Quantum Dynamics

Certain symmetric combinations of P and B or of P and Q yield self-adjoint operators suitable to act as dynamical generators. Let us consider the Hamiltonian

$$\mathcal{H}=QPQ+\mathcal{V}(P)\equiv\mathcal{H}_0+\mathcal{V}(P)$$

suggested by our classical model. In the "free" model the evolution operator is given by

$$U_0(t)=\exp(-itQPQ/\hbar)$$
$$=\exp\left(i\hbar t\frac{\partial}{\partial k}k\frac{\partial}{\partial k}\right).$$

It may be shown that

$$\psi_t(k)=\exp\left(i\hbar t\frac{\partial}{\partial k}k\frac{\partial}{\partial k}\right)\psi(k)$$
$$=\int\mathcal{K}_0(k,t;\kappa)\psi(\kappa)d\kappa,$$

in which

$$\mathcal{K}_0(k,t;\kappa)=(i\hbar t)^{-1}J_0(2(k\kappa)^{1/2}/\hbar t)e^{i(k+\kappa)/\hbar t},$$

where J_0 is the usual zero-order Bessel function. With the standard integral representation for J_0, we also find

$$\mathcal{K}_0(k,t;k')=(2\pi i\hbar t)^{-1}\times\int_0^{2\pi}\exp\{(i/\hbar t)[k+k'-2(kk')^{1/2}\cos\theta']\}d\theta'.$$

This relation suggests that we introduce a two-dimensional vector $\boldsymbol{\gamma}=(\gamma_1,\gamma_2)$ and define $\boldsymbol{\gamma}^2=k$. Then $\mathcal{K}_0$ is recognized as the propagator for circularly symmetric functions of $\boldsymbol{\gamma}$. Specifically, if

$$K_0(\boldsymbol{\gamma},t;\boldsymbol{\gamma}')\equiv(\pi i\hbar t)^{-1}\exp[(i/\hbar t)(\boldsymbol{\gamma}-\boldsymbol{\gamma}')^2]$$

and $\boldsymbol{\gamma}\cdot\boldsymbol{\gamma}'\equiv\gamma\gamma'\cos\theta'$, then

$$\psi_t(\gamma)=\int\int K_0(\boldsymbol{\gamma},t;\boldsymbol{\gamma}')\psi(\gamma')\gamma'd\gamma'd\theta'$$
$$=\int\mathcal{K}_0(k,t;k')\psi(k')dk'.$$

The evident unitarity of K_0 for arbitrary square-integrable functions of $\boldsymbol{\gamma}$, and the invariance of the subspace of circularly symmetric functions under K_0, establishes the unitarity of $\mathcal{K}_0$.

If the potential term $\mathcal{V}(P)$ is bounded from below, then general theorems assure us that $\mathcal{H}$ generates unitary transformations. Indeed one has that

$$e^{-it\mathcal{H}/\hbar}=\lim_{n\to\infty}[e^{-it\mathcal{H}_0/\hbar n}e^{-it\mathcal{V}/\hbar n}]^n.$$

In turn, this expression may be combined with our formulas for $\mathcal{K}_0$ to determine a Feynman path integral for our model in much the same fashion as Nelson has done for conventional Schrödinger mechanics.[8]

Classical Singularity Is Removed in Quantum Theory

In the absence of a potential ($V\equiv0$), the classical solutions exhibit a singularity which is in no way evident in the quantum theory. The appropriate classical Hamiltonian $H_0=pq^2$ leads to the two equations of motion $\dot{p}=-2pq$ and $\dot{q}=q^2$. The solutions are given by

$$p(t)=p'(1-q't)^2,$$
$$q(t)=q'(1-q't)^{-1},$$

where $p'=p(0)$ and $q'=q(0)$. The energy of the solution is $E=p'q'^2$, and it is clear that every solution with nonzero energy possesses a singularity at $t=q'^{-1}$. At the singularity, $q(t)$ becomes infinite while $p(t)$ vanishes, violating the physical requirement that $p>0$.

Quantum-mechanically this singularity does not arise. Note first that

$$[P,\mathcal{H}_0]=-2i\hbar B,$$
$$[B,\mathcal{H}_0]=-i\hbar\mathcal{H}_0$$

for $\mathcal{H}_0=QPQ$. Thus P, B, and $\mathcal{H}_0$ form a three-parameter Lie algebra. The time-dependent operator $P(t)$ can therefore be given as a linear sum of the three generators. Specifically,

$$P(t)\equiv e^{i\mathcal{H}_0t/\hbar}Pe^{-i\mathcal{H}_0t/\hbar}=P-2tB+t^2\mathcal{H}_0,$$

as follows from the appropriate commutation relations. The expected value of $P(t)$ is then given by

$$\langle P(t)\rangle\equiv\langle\psi|P(t)|\psi\rangle=\langle P\rangle-2t\langle B\rangle+t^2\langle\mathcal{H}_0\rangle.$$

This expression achieves its minimum at $t=\langle B\rangle/\langle\mathcal{H}_0\rangle$, with the consequence that

$$\langle P(t)\rangle\geq\langle P(t)\rangle_{\min}=\langle P\rangle-\langle B\rangle^2/\langle\mathcal{H}_0\rangle.$$

Since $\langle\psi|\psi\rangle=1$, it follows that

$$\langle B\rangle^2+\tfrac{1}{4}\hbar^2=|\langle\psi|PQ|\psi\rangle|^2=|\langle\psi|P^{1/2}P^{1/2}Q|\psi\rangle|^2$$
$$\leq\langle\psi|P|\psi\rangle\langle\psi|QPQ|\psi\rangle=\langle P\rangle\langle\mathcal{H}_0\rangle.$$

[8] E. Nelson, J. Math. Phys. 5, 332 (1964).

Therefore, we have the time-independent bound

$$\langle P(t)\rangle_{\min}\geq\tfrac{1}{4}\hbar^2/\langle\mathcal{H}_0\rangle,$$

demonstrating that the quantum solutions have no singularity.

Classical Singularity Disappears from Weak-Correspondence-Principle Viewpoint

As a different procedure to study the classical singularity, we adopt the approach of the weak correspondence principle.[9] In this approach, one chooses

$$H(p,q)\equiv\langle p,q|\mathcal{H}|p,q\rangle$$

as the classical Hamiltonian appropriate to the quantum Hamiltonian $\mathcal{H}$. Here the states $|p,q\rangle$ are members of an overcomplete family of states[10] (OFS) and are defined by

$$|p,q\rangle=U[p,q]|0\rangle=e^{-iqP/\hbar}e^{i(\ln p)B/\hbar}|0\rangle,$$

where $|0\rangle$ is a rather general unit vector. We impose the requirements

$$\langle 0|0\rangle=\langle 0|P|0\rangle=1,$$
$$\langle 0|Q|0\rangle=\langle 0|B|0\rangle=0.$$

In virtue of these modest requirements, we find the specific mean values

$$\langle p,q|(\alpha P+\beta Q+\gamma B)|p,q\rangle=\alpha p+\beta q+\gamma pq.$$

Of course, not all mean values mimic their operator construction. Consider the general Hamiltonian

$$\mathcal{H}=QPQ+\mathcal{V}(P).$$

In this case, the mean values become

$$H(p,q)=\langle p,q|\mathcal{H}|p,q\rangle=pq^2+V(p),$$

where

$$\begin{aligned}V(p)&=\langle 0|e^{-i(\ln p)B/\hbar}\mathcal{H}e^{i(\ln p)B/\hbar}|0\rangle\\&=\langle 0|p^{-1}QPQ+\mathcal{V}(pP)|0\rangle\\&\equiv p^{-1}\Lambda+v(p).\end{aligned}$$

Even when $\mathcal{V}=0$, there is a modification of the "free" classical Hamiltonian such that

$$H_0(p,q)=pq^2+\Lambda/p.$$

Here $\Lambda=\langle 0|QPQ|0\rangle$ is a positive constant which can be made arbitrarily small but which vanishes only in the limit $\hbar\rightarrow 0$.

If we adopt $H_0(p,q)$ as our classical Hamiltonian, what changes arise in the classical solutions? The modified equations of motion read

$$\dot p=-2pq,$$
$$\dot q=q^2-p^{-2}\Lambda,$$

which have as their solution

$$p(t)=p^*(1-q^*t)^2+\epsilon,$$
$$q(t)=q^*[(1-q^*t)+(\epsilon/p^*)(1-q^*t)^{-1}]^{-1}.$$

Here $\epsilon=\Lambda/E$, where E is the energy of the solution, and

$$p^*=p'-\epsilon,$$
$$q^*=q'/(1-\epsilon/p')$$

represent "renormalized" initial values at $t=0$. Evidently, the classical singularity is now removed and $p(t)\geq\epsilon=\Lambda/E$. The minimum of $p(t)$ occurs at a renormalized time $t=q^{*-1}=q'^{-1}(1-\epsilon/p')$. These modified classical solutions incorporate the essence of the quantum solutions, and this serves as strong motivation for the adoption of the appropriately modified classical Hamiltonian.

III. COORDINATE TRANSFORMATION INVARIANCE

Here we demonstrate a limited coordinate invariance of certain of the models under study, in particular, those for which $V(p)=K/p$. Let us adopt the quantum Hamiltonian

$$\mathcal{H}_c=QPQ+CP^{-1},$$

where C is a constant. The importance of this class of Hamiltonians stems from the fact that

$$[B,P]=i\hbar P,\quad [P,\mathcal{H}_c]=-2i\hbar B,\quad [B,\mathcal{H}_c]=-i\hbar\mathcal{H}_c,$$

independent of C. Thus for any value of C we still have a closed, three-parameter Lie algebra corresponding to a three-parameter Lie group. Indeed, it is the same abstract group for any value of C. Let

$$U[p,q,t]=e^{it\mathcal{H}_c/\hbar}e^{-iqP/\hbar}e^{i(\ln p)B/\hbar}$$

denote the unitary group operators, and let us consider the differential expression

$$i\hbar U[p,q,t]^\dagger dU[p,q,t]$$

formed from small variations in the variables p, q, and t. This differential is an element of the Lie algebra; specifically,

$$i\hbar U^\dagger dU=(pdq-pq^2dt)P-(2qdt+dp/p)B-(dt/p)\mathcal{H}_c. \quad (3)$$

Note that the expectation in the state $|0\rangle$ integrated over the independent variable yields the classical action:

$$I=\int i\hbar\langle 0|U^\dagger[p(t),q(t),t](d/dt)U[p(t),q(t),t]|0\rangle dt$$

$$=\int(p\dot q-pq^2-K/p)dt,$$

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[9] J. R. Klauder, J. Math. Phys. **4**, 1058 (1963); **8**, 2392 (1967).
[10] J. R. Klauder, J. Math. Phys. **4**, 1055 (1963).

using our earlier requirements on $|0\rangle$ and introducing

$$K=\langle 0|\mathcal{H}_c|0\rangle=\Lambda+C\langle 0|P^{-1}|0\rangle.$$

Now the differential coefficients in (3) are each invariant under left group translations. Let

$$\tilde{U}\equiv U[\tilde{p},\tilde{q},\tilde{t}]=U[p_0,q_0,t_0]U[p,q,t],$$

where p_0, q_0, and t_0 remain unvaried. Then

$$i\hbar\tilde{U}^\dagger d\tilde{U}=i\hbar U dU,$$

which leads to the three differential forms

$$\tilde{p}d\tilde{q}-\tilde{p}\tilde{q}^2d\tilde{t}=pdq-pq^2dt, \tag{4a}$$

$$2\tilde{q}d\tilde{t}+d\tilde{p}/\tilde{p}=2qdt+dp/p, \tag{4b}$$

$$d\tilde{t}/\tilde{p}=dt/p. \tag{4c}$$

Solutions to these differential relations (the Maurer-Cartan equations[11]) exhibit the invariance transformations of the classical action. It is convenient to split up the result into three basic invariance transformations, one each for t_0, p_0, and q_0. The first is the trivial transformation $\tilde{p}=p$, $\tilde{q}=q$, $\tilde{t}=t+t_0$. The second is given by $\tilde{q}=p_0p$, $\tilde{q}=q/p_0$, $\tilde{t}=p_0t$. The third transformation reads

$$\tilde{p}=p(1+q_0t)^{-2},$$
$$\tilde{q}=(1+q_0t)^2q+q_0(1+q_0t),$$
$$\tilde{t}=t(1+q_0t)^{-1}.$$

It is clear that the first transformation applies to any potential $V(p)$. The second and third transformations require that $V(p)=K/p$. Note that any value of K is consistent since that term in the action is *separately* invariant according to (4c). Under the second transformation the two terms $p\dot{q}$ and pq^2 making up the free action are separately invariant—invariance would be maintained even if pq^2 were changed by a scale factor to αpq^2. Under the third transformation, however, there is "mixing" of $p\dot{q}$ and pq^2 and no separate scaling would be possible. The latter transformation has much of the appearance of a coordinate transformation: The "metric" $p(t)$ transforms homogeneously as a "tensor," while the "connection" $q(t)$ possesses an inhomogeneous term as well.

[11] P. M. Cohen, *Lie Groups* (Cambridge U. P., London, 1961).

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Classical Charged Tachyon Self-Energy Problem

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The classical self-energy problem for charged tachyons is more serious than that for charged bradyons. As a result, the theoretical basis for generally expected experimental properties of such objects is shaky.

HISTORICALLY, the problem of a classical electromagnetic charged particle coupled to an external field has been complicated by the self-energy problem associated with the point singularity at the location of the particle. Methods used to deal with this[1] are suitable only for particles whose speeds do not exceed that of light, however, and it appears that for tachyons the problem is rather more severe than usual. This fact may have bearing on theoretical expectations concerning the experimental properties of charged tachyons.[2] The purpose of the present paper is to point out the difficulties involved since they do not appear to be generally recognized and they are fundamental in character.

If a tachyon is not itself the source of an electromagnetic field, its equation of motion in a given external field is most naturally taken to be that following from the action principle based on the Lagrangian

$$L=m_t[x'(\alpha)^2]^{1/2}+qx'(\alpha)\cdot\mathcal{A}(x), \tag{1}$$

where $x'(\alpha)^\mu\equiv dx^\mu(\alpha)/d\alpha$, with α an arbitrary parameter, the tachyon mass m_t is defined to be real, and we have used the space-favoring metric. When the tachyon is the source of a field, the Lorentz force equation following from (1) is expected to contain an additional term for the radiation reaction. So, to determine the full equations of motion for the charge, it is necessary to solve the Maxwell-Lorentz field equation in the presence of a prescribed tachyon source and to compute the energy and momentum of radiation.

For a source with world line prescribed by the equations

$$x^\mu=\xi^\mu=\xi^\mu(\tau), \tag{2}$$

the electromagnetic field equation in the Lorentz gauge

[1] F. Rohrlich, *Classical Charged Particles* (Addison-Wesley, Reading, Mass., 1965); Phys. Rev. Letters **12**, 375 (1964).
[2] O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. **30**, 718 (1962); G. Feinberg, Phys. Rev. **159**, 1089 (1967).