certainly present in the process $0^-(\mathbf{8})+\frac{1}{2}^+(\mathbf{8}) \to 0^-(\mathbf{8}) +\frac{1}{2}^+(\mathbf{8})$, should decouple from the process $0^+(\mathbf{8}) +\frac{1}{2}^+(\mathbf{8}) \to 0^-(\mathbf{8})+\frac{1}{2}^+(\mathbf{8})$. At first sight, this appears to conflict with the factorization, since, according to our scheme, there is a nonvanishing **10** contribution for the process $0^+(\mathbf{8})+\frac{1}{2}^+(\mathbf{8}) \to 0^+(\mathbf{8})+\frac{1}{2}^+(\mathbf{8})$. An obvious solution, consistent with the factorization, will be to assume another **10** trajectory coupled strongly to $0^+(\mathbf{8})$-$\frac{1}{2}^+(\mathbf{8})$ but not to $0^-(\mathbf{8})-\frac{1}{2}^+(\mathbf{8})$. However, discussions about implied physical consequences are beyond the scope of this paper.

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πN Charge-Exchange Polarization in the Veneziano Model

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Pion-nucleon charge-exchange polarization is calculated in various Veneziano representations for πN scattering. Our results are in qualitative agreement with the expermiental data for some sets of parameters. It is not possible, however, to resolve the nonuniqueness of the Veneziano representation from this information alone.

I. INTRODUCTION

It is well known that in a process like $\pi^- p \to \pi^0 n$, in the t channel only one Regge pole can be exchanged —in this case the pole corresponding to the ρ trajectory —and consequently the phases of the spin-nonflip and the spin-flip amplitudes should be equal. This results in a vanishing polarization at high energies where the Regge-pole model becomes applicable. However, there is experimental evidence of nonvanishing polarizations at pion laboratory energies of 5.9 and 11.2 GeV/c, although single-Regge-pole fits are good for differential cross sections at these energies.

Several models have been proposed to explain both the polarization and the differential cross sections for the charge-exchange process. All of them involve adding a background term to the ρ-trajectory contribution, and justify such an addition. The addition[1] of a secondary trajectory ρ' with the same quantum number as that of the ρ trajectory, but with a different intercept, can introduce the required phase difference between the spin-flip and spin-nonflip amplitudes without changing the cross section significantly. An alternative model[2] requires a Regge cut—which presumably is connected with the Gribov-Pomeranchuk phenomenon—to provide the necessary background. A fairly successful model[3] was to treat the polarization as arising from the interference between the Regge-trajectory contribution and the direct-channel resonances occurring on indefinitely rising baryon trajectories. It may be recalled that in this model the significant contributions were derived from the resonances in the neighborhood of the energy value, and that the Breit-Wigner tails of the resonances were not important.

The Veneziano model for the scattering amplitude explicitly contains both the resonance poles corresponding to a rising trajectory and the Regge asymptotic behavior. Naturally we should expect that it automatically contains the necessary interference terms to give the appropriate polarization. Indeed, polarization could form one of the stringent tests for the Veneziano model. However, it is not possible to write a unique Veneziano representation.

We make a comparative study of various Veneziano representations for πN charge-exchange scattering with respect to their prediction for polarization.

II. IGI'S MODEL

We first consider Igi's model[4] for the πN invariant amplitudes A and B. These are obtained by requiring (a) crossing symmetry, (b) Regge asymptotic behavior at high energies, and (c) constraints implied by isospin. We use ρ and f trajectories in the t channel and N_α, Δ_δ, and N_γ trajectories in the s and u channel; we identify amplitudes with t-channel isospin $I_t=0$ (1) by means of

[1] R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters **18**, 259 (1967).
[2] C. B. Chiu and J. Finkelstein, Nuovo Cimento **48A**, 820 (1967).
[3] B. R. Desai, D. T. Gregorich, and R. Ramachandran, Phys. Rev. Letters **18**, 565 (1967).
[4] K. Igi, Phys. Letters **28B**, 330 (1968).

the subscript f (ρ):

$$A_f(s,t,u)=(\beta_{f,N_\alpha}/\pi)[C(1-\alpha_f(t),\tfrac{3}{2}-\alpha_{N_\alpha}(s))+C(1-\alpha_f(t),\tfrac{3}{2}-\alpha_{N_\alpha}(u))+C(\tfrac{3}{2}-\alpha_{N_\alpha}(s),\tfrac{3}{2}-\alpha_{N_\alpha}(u))]$$
$$+(\beta_{f,\Delta_\delta}/\pi)[C(1-\alpha_f(t),\tfrac{3}{2}-\alpha_{\Delta_\delta}(s))+C(1-\alpha_f(t),\tfrac{3}{2}-\alpha_{\Delta_\delta}(u))-C(\tfrac{3}{2}-\alpha_{\Delta_\delta}(s),\tfrac{3}{2}-\alpha_{\Delta_\delta}(u))]$$
$$+(\beta_{f,N_\gamma}/\pi)[C(1-\alpha_f(t),\tfrac{3}{2}-\alpha_{N_\gamma}(s))+C(1-\alpha_f(t),\tfrac{3}{2}-\alpha_{N_\gamma}(u))-C(\tfrac{3}{2}-\alpha_{N_\gamma}(s),\tfrac{3}{2}-\alpha_{N_\gamma}(u))]\,, \quad (1)$$

$$B_f(s,t,u)=(\beta_f/\pi)\{B(1-\alpha_f(t),\tfrac{1}{2}-\alpha_{N_\alpha}(s))-B(1-\alpha_f(t),\tfrac{1}{2}-\alpha_{N_\alpha}(u))$$
$$+p[B(1-\alpha_f(t),\tfrac{1}{2}-\alpha_{\Delta_\delta}(s))-B(1-\alpha_f(t),\tfrac{1}{2}-\alpha_{\Delta_\delta}(u))]$$
$$+q[B(1-\alpha_f(t),\tfrac{1}{2}-\alpha_{N_\gamma}(s))-B(1-\alpha_f(t),\tfrac{1}{2}-\alpha_{N_\gamma}(u))]$$
$$+p[B(\tfrac{1}{2}-\alpha_{N_\alpha}(s),\tfrac{1}{2}-\alpha_{\Delta_\delta}(u))-B(\tfrac{1}{2}-\alpha_{\Delta_\delta}(s),\tfrac{1}{2}-\alpha_{N_\alpha}(u))]$$
$$+q[B(\tfrac{1}{2}-\alpha_{N_\alpha}(s),\tfrac{1}{2}-\alpha_{N_\gamma}u))-B(\tfrac{1}{2}-\alpha_{N_\gamma}(s),\tfrac{1}{2}-\alpha_{N_\alpha}(u))]\}\,, \quad (2)$$

with

$$p+q=1\,, \quad (3)$$

$$A_\rho(s,t,u)=(\beta_\rho/\pi)\{C(1-\alpha_\rho(t),\tfrac{3}{2}-\alpha_{N_\alpha}(s))-C(1-\alpha_\rho(t),\tfrac{3}{2}-\alpha_{N_\alpha}(u))$$
$$+p'[C(1-\alpha_\rho(t),\tfrac{3}{2}-\alpha_{\Delta_\delta}(s))-C(1-\alpha_\rho(t),\tfrac{3}{2}-\alpha_{\Delta_\delta}(u))]$$
$$+q'[C(1-\alpha_\rho(t),\tfrac{3}{2}-\alpha_{N_\gamma}(s))-C(1-\alpha_\rho(t),\tfrac{3}{2}-\alpha_{N_\gamma}(u))]$$
$$+p'[C(\tfrac{3}{2}-\alpha_{N_\alpha}(s),\tfrac{3}{2}-\alpha_{\Delta_\delta}(u))-C(\tfrac{3}{2}-\alpha_{u_\delta}(s),\tfrac{3}{2}-\alpha_{N_\alpha}(u))]$$
$$+q'[C(\tfrac{3}{2}-\alpha_{N_\alpha}(s),\tfrac{3}{2}-\alpha_{N_\gamma}(u))-C(\tfrac{3}{2}-\alpha_{N_\gamma}(s),\tfrac{3}{2}-\alpha_{N_\alpha}(u))]\}\,, \quad (4)$$

with

$$p'+q'=1\,, \quad (5)$$

$$B_\rho(s,t,u)=(\beta_{\rho,N_\alpha}/\pi)[B(1-\alpha_\rho(t),\tfrac{1}{2}-\alpha_{N_\alpha}(s))+B(1-\alpha_\rho(t),\tfrac{1}{2}-\alpha_{N_\alpha}(u))+B(\tfrac{1}{2}-\alpha_{N_\alpha}(s),\tfrac{1}{2}-\alpha_{N_\alpha}(u))]$$
$$+(\beta_{\rho,\Delta_\delta}/\pi)[B(1-\alpha_\rho(t),\tfrac{1}{2}-\alpha_{\Delta_\delta}(s))+B(1-\alpha_\rho(t),\tfrac{1}{2}-\alpha_{\Delta_\delta}(u))-B(\tfrac{1}{2}-\alpha_{\Delta_\delta}(s),\tfrac{1}{2}-\alpha_{\Delta_\delta}(u))]$$
$$+(\beta_{\rho,N_\gamma}/\pi)[B(1-\alpha_\rho(t),\tfrac{1}{2}-\alpha_{N_\gamma}(s))+B(1-\alpha_\rho(t),\tfrac{1}{2}-\alpha_{N_\gamma}(u))-B(\tfrac{1}{2}-\alpha_{N_\gamma}(s),\tfrac{1}{2}-\alpha_{N_\gamma}(u))]\,, \quad (6)$$

$$B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)\,,$$
$$C(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y-1)\,.$$

The invariant amplitudes at this stage satisfy crossing symmetry explicitly. It may be noticed that each of these amplitude has the right (t,s) and (t,u) terms to give the appropriate Regge behavior together with the right signature factors. Thus as $s\to\infty$

$$A_{\rho,f}\sim s^{\alpha_{\rho,f}(t)}(1\mp e^{-i\pi\alpha_{\rho,f}(t)})\,,$$
$$B_{\rho,f}\sim s^{\alpha_{\rho,f}(t)}(1\mp e^{-i\pi\alpha_{\rho,f}(t)})\,.$$

We have implicitly assumed that as $s\to\infty$, $\tan\pi\alpha(s)\to i$, which is easily incorporated in evaluating our gamma functions through Stirling's approximation. By this trick, which has by now become a standard practice in the Veneziano model, we have introduced a phase to the apparently real scattering amplitude. The nonleading terms in the above amplitude will carry the requisite phase difference between the spin-flip and the non-spin-flip amplitudes; together with this we have real background terms coming from the (s,u) terms. All this together would produce a nonzero polarization.

We may improve now the constraints due to isospin. From the t-channel amplitudes A_f and A_ρ, we may obtain amplitudes with s- or u-channel isospin $I_{s,u}=\frac{1}{2}$ or $\frac{3}{2}$ through crossing. They are, for example,

$$A_{1/2}{}^s(s,t,u)=-(A_f+2A_\rho)\,, \quad (7a)$$

$$A_{3/2}{}^s(s,t,u)=A_f-A_\rho\,. \quad (7b)$$

We must now ensure that, say, in the $I_s=\frac{1}{2}$ amplitude there will be no pole term corresponding to the $I=\frac{3}{2}$, Δ_δ trajectory and that there will be poles corresponding to N_α and N_γ only.

The absence of pole terms corresponding to the Δ_δ trajectory in (7a) requires

$$\alpha_f(t)=\alpha_\rho(t)\,,\quad \beta_{f,\Delta_\delta}=-2p'\beta_\rho\,,\quad \alpha_{N_\alpha}(s)=\alpha_{\Delta_\delta}(s)\,.$$

Similar conditions can be obtained from the other invariant amplitude B and the $I_s=\frac{3}{2}$ amplitudes. We may collect all the conditions implied by isospin constraints as follows:

$$\alpha_f(t)=\alpha_\rho(t)\,, \quad (8)$$

$$\alpha_{N_\alpha}(s)=\alpha_{\Delta_\delta}(s)=\alpha_{N_\gamma}(s)\,, \quad (9)$$

$$\beta_{f,\Delta_\delta}=-2p'\beta_\rho\,,\quad \beta_{\rho,\Delta_\delta}=-\tfrac{1}{2}p\beta_f\,,$$
$$\beta_{f,N_\alpha}=\beta_\rho\,,\quad \beta_{\rho,N_\alpha}=\beta_f \quad (10)$$
$$\beta_{f,N_\gamma}=(1-p')\beta_\rho\,,\quad \beta_{\rho,N_\gamma}=(1-p)\beta_f\,.$$

Equation (8) is the familiar exchange degeneracy and the Chew-Frautschi plot for mesons seems to support it. The relations in Eq. (10) are the same conditions used by Igi. The degeneracy implied by Eq. (9) is a consequence of the isospin crossing relation. It is, however, true that the baryon trajectories do not appear degenerate and this condition is only approximately satisfied. However, even when the trajectories are not degenerate, the fact that their *slopes are equal* together with Eqs. (8) and (10) would imply the absence of the unwanted poles from the leading trajectory. It is the daughters of the trajectories that will cause trouble and will have their poles appearing simultaneously in the $I=\frac{1}{2}$ as well as the $I=\frac{3}{2}$ amplitudes. With a view to examining how far the unadorned Veneziano model explains the polarization phenomenon, we shall consider two alternatives: Firstly, we shall keep Eqs. (8)–(10) intact and assume an average trajectory for the baryons. We then hope, at least, to derive some qualitative features implied by Igi's version of the Veneziano model. Next we shall consider a more realistic set of baryon trajectories (however, with the same slope), ignoring the presence of unwanted poles in the baryon channels. Again emphasis will be on the qualitative features. In both the alternatives we will assume, after Igi, a universal slope for the trajectories. Thus

$$\text{slope}=0.86\ \text{GeV}^{-2}, \tag{11}$$

$$\alpha_\rho(0)=\alpha_f(0)=0.5. \tag{12}$$

This leaves us with four parameters, β_ρ, β_f, p, and p', in terms of which the amplitudes will be completely specified.

Using Singh's[5] notation, we may define

$$A'(s,t)=A(s,t)+\left(\frac{\omega+t/4M_N}{1-t/4M_N^2}\right)B(s,t), \tag{13}$$

where the pion laboratory energy ω is given by

$$\omega=(s-M_N^2-M_u^2)/2M_N.$$

The charge-exchange (CEX) amplitude can be written in terms of the corresponding t-channel isospin amplitudes:

$$A_{\text{CEX}}'=-\sqrt{2}A_\rho',$$
$$B_{\text{CEX}}=-\sqrt{2}B_\rho.$$

The imaginary parts of A_ρ' and B_ρ in the forward direction ($t=0$) are related to the difference between π^-p and π^+p elastic scattering. Following Igi, we use them as inputs:

$$\text{Im}A_\rho'(s,\,t=0)=2.14s^{1/2}\ \text{mb},$$
$$\text{Im}B_\rho(s,\,t=0)=43.7s^{-1/2}\ \text{mb GeV}.$$

These yield

$$\beta_\rho=20.16\ \text{mb GeV} \tag{14}$$

and

$$(2-\tfrac{3}{2}p)\beta_f=\gamma_\rho=71.80\ \text{mb}. \tag{15}$$

Case 1. When the baryon trajectories are degenerate, no further parameters need be determined. For baryons we use $\alpha_{\text{eff}}(0)=-0.256$.

Case 2a. We need all the four parameters. The remaining parameters can be determined by using information on backward scattering. The set of parameters as determined by Igi are

$$\beta_\rho=20.16\ \text{mb GeV},\quad \beta_f=25.7\ \text{mb},\quad p=-0.526,$$
$$p'=0.852,\quad \alpha_{N_\alpha}(0)=-0.256,\quad \alpha_{\Delta_\delta}(0)=0.18,$$
$$\alpha_{N_\gamma}(0)=-0.55.$$

Case 2b. An alternative procedure to determine the parameters p, p', and β_f is to use the information on the residue of the f-Regge pole derived from finite energy sum rules (FESR). We use the following results for this purpose:

$$\text{Im}A_f'(s,\,t=0)=14.6s^{1/2}\ \text{mb}, \tag{16}$$

$$\text{Im}B_f(s,\,t=0)=54.1s^{-1/2}\ \text{mb GeV}. \tag{17}$$

Using Eqs. (16) and (17) together with Eq. (15), we get the parameters as

$$\begin{aligned}\beta_f&=44.5\ \text{mb},\\ p&=0.266,\\ p'&=0.22.\end{aligned} \tag{18}$$

III. FENSTER-WALI MODEL

In all the three cases of Igi's model that we considered in Sec. II, we are plagued with unwanted poles in the baryon spectrum. Since the trajectories appear as linear functions of the variable s rather than of W $(=\sqrt{s})$, parity doublets are unavoidable. However, by using more subsidiary Veneziano terms, it is possible (by a judicious choice of coefficients) to eliminate the first few poles of wrong parity in each trajectory. Fenster and Wali[6] have used this to write a Veneziano amplitude for πN scattering. The amplitude represents the low-energy region well up to 1.6 GeV. All parity doublets in this region are eliminated and the resonance parameters for $\Delta_\delta(1236)$ and $N_\gamma(1518)$ are reproduced. The various parameters that occur in the scattering amplitude are determined by using in addition the normalization at $t=0$ of the amplitudes and the shape for very small negative t for the charge-exchange-scattering data. We treat this amplitude as *Case 3.*

IV. RESULT AND DISCUSSION

With the explicit form for the amplitudes A' and B, so determined, it is straightforward to obtain the polarization, using

$$P(s,t)=\frac{2}{4k^2s}\left(\frac{\sqrt{s}}{8\pi}\right)(-4k^2t-t^2)^{1/2}\,\text{Im}(A^*B)\bigg/\frac{d\sigma}{dt}. \tag{19}$$

In Fig. 1, the calculated values of the polarization at pion laboratory energies of 5.9, 11.2, and 18.0 GeV are

[5] V. Singh, Phys. Rev. **129**, 1889 (1963).

[6] S. Fenster and K. C. Wali, Phys. Rev. D **1**, 1409 (1970).

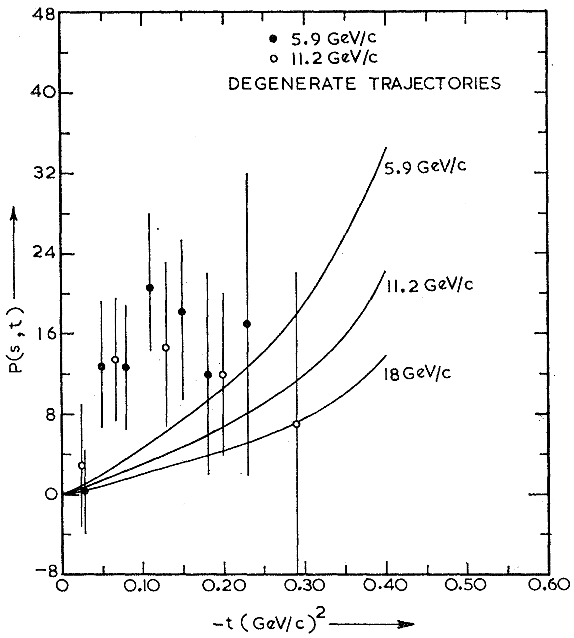

FIG. 1. Polarization results for πN charge-exchange scattering using a Veneziano model with case-1 parametrization. Baryon trajectories are treated as degenerate.

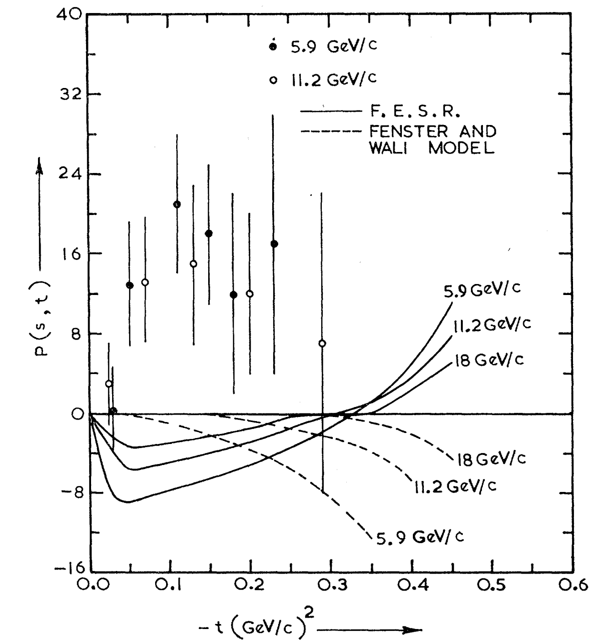

FIG. 3. Smooth curves correspond to the results with case-2b parametrization, which makes use of FESR results together with forward scattering data. Dashed curves show the polarization for case 3, the Fenster-Wali model.

plotted as a function of momentum transfer. Since the aim was merely to find the qualitative features, no attempt was made to obtain detailed parameter fits. In cases 1 and 2a, which use both forward and backward πN scattering data to fix the parameters, the calculated

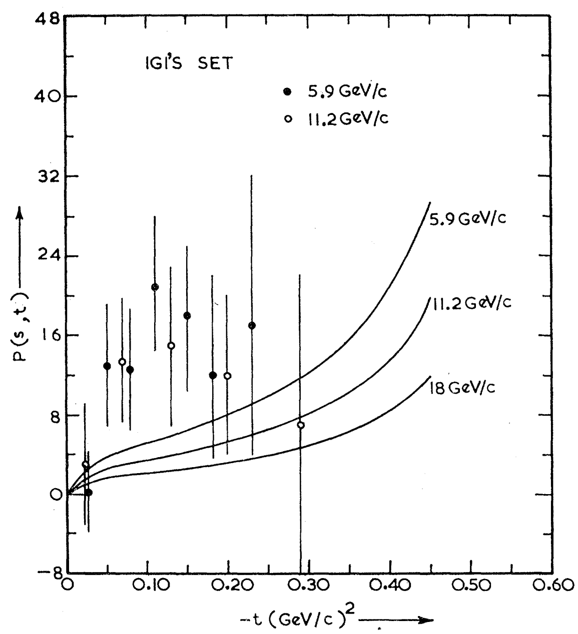

FIG. 2. Polarization results for πN charge-exchange scattering using a Veneziano model with case-2a parametrization. Baryon trajectories are nondegenerate and the parameters make use of forward and backward πN scattering data.

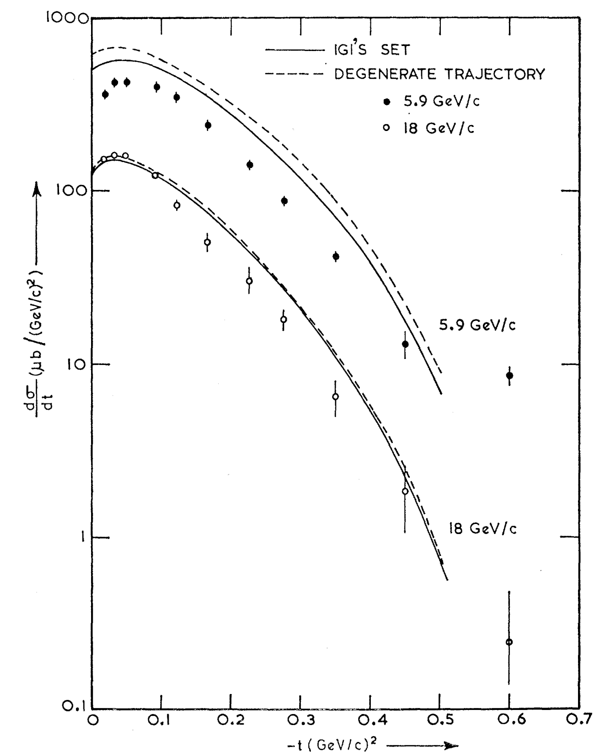

FIG. 4. Fits to πN charge-exchange differential cross section using a Veneziano model with case-1 and -2a parametrization for energies 5.9 and 18 GeV/c. Dashed curves correspond to case 1. Smooth curves show the differential cross section for case 2a.

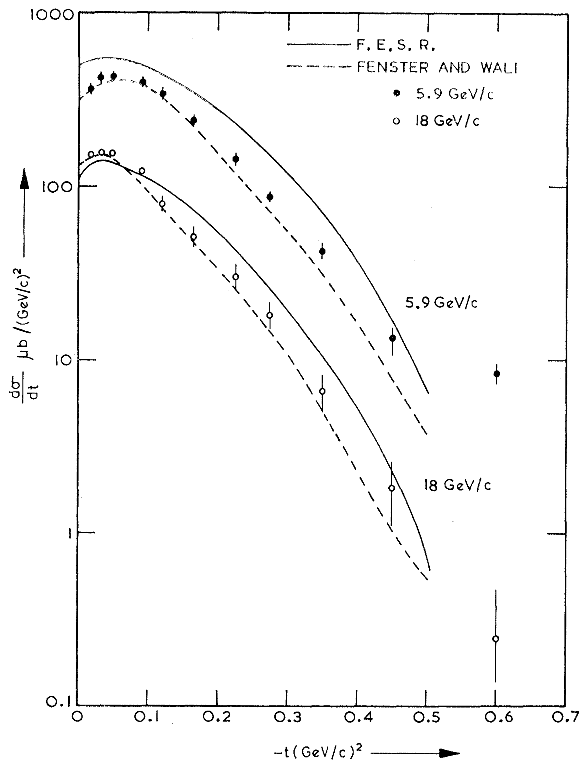

FIG. 5. Fits to πN charge-exchange differential cross section using a Veneziano model. Smooth curves correspond to case-2b parametrization, which makes use of FESR results together with forward scattering data. Dashed curves show the fits to the differential cross section for case 3.

values of the polarization have the same sign and approximately the same order of magnitude up to $t=-0.3$ $(\mathrm{GeV}/c)^2$ as the experimental numbers. Beyond this momentum transfer, the calculated values show a tendency to keep increasing, whereas the experimental indications, though scanty, have no such trend. (See Fig. 2.) However, it is seen that the polarization depends sensitively upon the details of parametrization. In case 2b, where the parameters for the amplitude are fixed by using FESR's rather than the backward scattering data, the polarization, though of the same order of magnitude, bears opposite sign. (See Fig. 3.) Further, in case 3, using the Fenster-Wali amplitude, we find a polarization that is even smaller in magnitude and again of opposite sign.

We have also plotted the $\pi^- p \rightarrow \pi^0 n$ differential cross sections for cases 1, 2, 2b, and 3 in Figs. 4 and 5, for energies 5.9 and 18.0 GeV. The predicted curves for the various cases follow the general trend of the experimental data, although they do not reproduce it precisely. This is only to be expected in view of the very limited number of parameters that the model is required to depend on. We may mention that a better fit is possible if more satellite terms are included.

In conclusion, we have considered a model with very few parameters to explain the qualitative features of the charge-exchange scattering data. The various versions of the Veneziano model for πN scattering yield a polarization that agrees in order of magnitude with the experimental value for small momentum transfers, but depends for its sign on the details of parametrization. It is significant that the correct order of magnitude for the polarization has been obtained without any new parameters as required in interference models. However, it is doubtful whether this information could be used to settle the problem of nonuniqueness[7] of the Veneziano representation.

ACKNOWLEDGMENTS

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[7] After completion of this work we received a preprint of A. W. Hendry, S. T. Jones, and H. W. Wyld, Nucl. Phys. **B15**, 389 (1970), who have done calculations of a similar type.