Note that since $\delta_{00}(m_K^2)\simeq\pi/4$ in our model, the value of δ_S which results is *twice* as large as that calculated by Arnowitt *et al.*[13] Hence in this model the calculated $K_L^0-K_S^0$ mass difference $\delta_L-\delta_S$ can no longer be said to be "in excellent agreement with experiment." [13]

ACKNOWLEDGMENTS

We would like to thank the Aspen Center for Physics for hospitality during our stay in Aspen, where most of this work was done. We wish to thank Professor S. Gasiorowicz for some discussion.

PHYSICAL REVIEW D VOLUME 2, NUMBER 11 1 DECEMBER 1970

Collinear Dispersion Relations and the K_{e4} Decay Rate

N. F. Nasrallah*
Faculty of Education, Lebanese University, Beirut, Republic of Lebanon

AND

K. Schilcher
Physics Department, American University of Beirut, Beirut, Republic of Lebanon

(Received 1 July 1970)

We study the K_{e4} decay form factors using a method introduced by Fubini and Furlan. The amplitudes which we extrapolate from the soft-pion limits to the physical point differ slightly from the previously used ones. Our choice is motivated by the collinear parametrization. These amplitudes are simply related to the K_{e4} form factors and to the K-π scattering amplitudes, which appear on equal footing. The calculated decay rate $\Gamma=(2.3\pm0.3)\times10^3$ sec^{-1} lies within the experimental error.

I. INTRODUCTION

The form factors for the decay K_{e4} were first calculated by Callan and Treiman[1] from current algebra. These authors contract over one of the pions of the final state at a time and obtain values for the form factors in the two soft-pion limits. Their results for the form factor F_3, however, differ considerably, depending on which of the momenta of the two pions is put equal to zero. Weinberg[2] later explained the rapid variation of F_3 by taking a nearby K pole explicitly into account.

In all these calculations the form factors F_1 and F_2, on which the decay rate $\Gamma_{K_{e4}^+}$ only depends, were taken to be constant. The results of Refs. 1 and 2 give for the K_{e4}^+ decay rate

$$\Gamma_{K_{e4}^+}=(1.6\pm0.2)\times10^3\ \text{sec}^{-1},$$

whereas experimentally,[3]

$$\Gamma_{K_{e4}^+}=(2.9\pm0.6)\times10^3\ \text{sec}^{-1}.$$

It seems to us, however, that the discrepancy between theory and experiment could be accounted for by the variation of the form factors between the soft-pion limit and the physical point.

In this paper we apply an extrapolation method of Fubini and Furlan[4] which makes use of the collinear parametrization in the rest frame of the particles to study the appropriate matrix elements. The method of Ref. 4 has several advantages:

(a) The ambiguity that different choices of the amplitudes to be extrapolated may lead to different results on the mass shell is resolved, the physical amplitudes at threshold and the ones related to them by crossing being directly related to the soft-pion limits through dispersion relations.

(b) The Low representation of the amplitudes determines their asymptotic behavior, thus giving information about the possibility of writing dispersion relations and the number of subtractions needed.

(c) Anomalous thresholds are absent in the physical sheet, where the dispersion relations are written.

(d) Since we are working in the rest frame, we can make use of strong parity and angular momentum selection rules to calculate the corrections to the soft-pion limits.

The form factors F_1, F_2, and F_3, and the K-π scattering amplitudes at the threshold will appear naturally on the same footing in sum rules.

We obtain for the K_{e4}^+ decay rate

$$\Gamma_{K_{e4}^+}=(2.3\pm0.3)\times10^3\ \text{sec}^{-1}.$$

II. COLLINEAR PARAMETRIZATION AND K_{e4} FORM FACTORS

The K_{e4}^+ form factors are defined in the following way:

* Supported by the National Council for Scientific Research in Lebanon.

[1] C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966).

[2] S. Weinberg, Phys. Rev. Letters 17, 336 (1966).

[3] R. P. Ely *et al.*, LRL Report No. UCRL 18626, 1968 (unpublished).

[4] S. Fubini and G. Furlan, Ann. Phys. (N. Y.) 48, 322 (1968).

$$\frac{i}{f_\pi m^2}\int dy\, e^{iq_b\cdot y}(m^2-q_b^2)\times\langle\pi^a(q_a)|TD_b(y)A_\lambda^{K^-}(0)|K^+(p)\rangle$$
$$=\frac{i}{\sqrt{2}M}[F_1{}^b(q_++q_-)_\lambda+F_2{}^b(q_+-q_-)_\lambda+F_3{}^b(p-q_+-q_-)_\lambda],\quad(1)$$

where m and M denote the masses of the π and K mesons, respectively, $\pi^a(q_a)$ $[\pi^b(q_b)]$ refers to either the $\pi^+(q_+)$ $[\pi^-(q_-)]$ or the $\pi^-(q_-)$ $[\pi^+(q_+)]$, $D\equiv\partial_\mu A_\mu$ is the divergence of the axial-vector current with the quantum numbers of the π meson, $\langle 0|D_{a,b}|\pi^{a,b}\rangle=f_\pi m^2$, $A_\lambda{}^{K^-}$ denotes the axial-vector current with the quantum numbers of the K^- meson, and the F_i's are functions of the invariants.

The amplitude defined in Eq. (1) contains a K pole close to the physical region and cannot be chosen as the smooth function used for extrapolation. We shall choose instead to study the following amplitude:

$$M_\lambda{}^b=\frac{i}{f_\pi m^2}\frac{(M^2-q_l^2)}{M^2}\int dy\, e^{iq_b\cdot y}\times\langle\pi^a|TD_b(y)A_\lambda{}^{K^-}(0)|K^+\rangle(m^2-q_b^2),\quad(2)$$

which is free of singularities, with

$$q_l=(p-q_+-q_-).$$

We consider the special configuration where all particles are at rest, $\mathbf{p}=\mathbf{q}_+=\mathbf{q}_-=0$, and use the collinear parametrization

$$q_b=xq_a=x(m/M)p.\quad(3)$$

With this parametrization, $M(x)$ is a function of the parameter x only. $M(x=1)$ is given in terms of the form factors F_i evaluated at the configuration where all particles are at rest. $M(x=-1)$ is simply related to the K-π scattering amplitudes at threshold. $M(x=0)$ corresponds to the soft-pion limits.

To simplify the notation, we define

$$\tilde{F}_i(x)=[(M^2-q_l^2)/M^2]F_i(x).\quad(4)$$

In this notation we have from Eqs. (1) and (2)

$$M_0(x)=(i/\sqrt{2}M)[\tilde{F}_1(x)(q_++q_-)_0+\tilde{F}_2(x)(q_+-q_-)_0+\tilde{F}_3(x)(p-q_+-q_-)_0].\quad(5)$$

We recall that, as we are working in the rest frame, only the time components of the currents contribute. Now standard soft-pion techniques give the following:

For the π^+ meson soft, i.e., $q_+=xq_-$,

$$\tilde{F}_1{}^+(x=0)-\tilde{F}_2{}^+(x=0)=0,\quad(6a)$$
$$\tilde{F}_3{}^+(x=0)=0;\quad(6b)$$

whereas for $q_-=xq_+$,

$$\tilde{F}_1{}^-(x=0)+\tilde{F}_2{}^-(x=0)=\frac{M^2-(M-m)^2}{M^2}[F_1{}^-(0)+F_2{}^-(0)]=\frac{M^2-(M-m)^2}{M^2}\left(\frac{-2Mf_+}{f_\pi}\right),\quad(7a)$$

$$\tilde{F}_3{}^-(x=0)=\frac{M^2-(M-m)^2}{M^2}\left(-\frac{M(f_+-f_-)}{f_\pi}\right),\quad(7b)$$

where f_+, f_- are the usual K_{l3} form factors

$$\langle\pi^0(q)|V_\lambda{}^{K^-}|K^+(p)\rangle=-(1/\sqrt{2})[f_+(p+q)_\lambda+f_-(p-q)_\lambda]$$

evaluated at the physical point with $\mathbf{p}=\mathbf{q}=0$.

$\tilde{F}_i(x=1)=\{[M^2-(M-2m)^2]/M^2\}F_i(x=1)$ are form factors evaluated at the physical point where all particles are at rest. We have additional information on $M(x)$ at $x=-1$, since we can see from Eqs. (2) and (5) that this point corresponds to the crossed amplitude:

$$M_0{}^b(x=-1)=(i/M\sqrt{2})[\tilde{F}_1{}^b(x=-1)(q_++q_-)_0+\tilde{F}_2{}^b(x=-1)(q_+-q_-)_0+\tilde{F}_3{}^b(x=-1)(p-q_1-q_-)_0]$$
$$=[(M^2-q_l^2)/M^2]\langle\pi^a|A_0{}^{K^-}|\pi^{-b},K^+\rangle,\quad(8)$$

because our particles are at rest ($q_l^2=M^2$) in the equation above and K-pole dominance of $A_0{}^{K^-}$ becomes exact at this point.

The K pole, on the other hand, contributes only to $\tilde{F}_3$, so that

$$\tilde{F}_1{}^b(x=-1)=\tilde{F}_2{}^b(x=-1)=0\quad(9)$$

and

$$\tilde{F}_3{}^b(x=-1)=(\sqrt{2}f_K/M)T_{\text{th}}(K^++\pi^a\to K^++\pi^a),\quad(10)$$

where f_K is defined similarly to f_π and where $T_{\text{th}}(K^++\pi^a\to K^++\pi^a)$ denotes the scattering matrix element for the corresponding reaction at threshold.

III. ASYMPTOTIC BEHAVIOR OF AMPLITUDES

In order to write down dispersion relations for the form factors, we start by examining the asymptotic behavior of $M(x)$.

The Low representation reads

$$M_\lambda{}^b(x)=-\frac{(2\pi)^3}{f_\pi}\frac{(m^2-q_b^2)}{m^2}\frac{(M-q_l^2)}{M^2}\times\left[\sum_n\frac{\langle\pi^a|D_b|n\rangle\langle n|A_\lambda{}^{K^-}|K^+\rangle}{(q_a+q_b-p_n)_0}\delta(\mathbf{p}_n-\mathbf{q}_+-\mathbf{q}_-)+\sum_n\frac{\langle\pi^a|A_\lambda{}^{K^-}|m\rangle\langle m|D_b|K^+\rangle}{(p-q_b-p_m)_0}\delta(\mathbf{p}-\mathbf{p}_m-\mathbf{q}_b)\right],\quad(11)$$

so that, for large x,

$$M_\lambda{}^b(x) \to \alpha x^3 + \beta x^2 + \cdots , \tag{12}$$

where α and β are linear combinations of the following constants:

$$c = \frac{m}{f_\pi M^2} \int d^3y\, e^{i\mathbf{q}_b\cdot\mathbf{y}} \langle \pi^a | [A_0{}^{K^-}(0), D_b(\mathbf{y},0)] | K^+ \rangle ,$$
$$c' = \frac{1}{f_\pi M^2} \int d^3y\, e^{i\mathbf{q}_b\cdot\mathbf{y}} \langle \pi^a | [A_0{}^{K^-}(0), \dot{D}_b(\mathbf{y},0)] | K^+ \rangle . \tag{13}$$

We assume these quantities exist.

To see what this result implies for the form factors, we use Eqs. (5) and (12); e.g., for $M^-(x)$,

$$q_{-0} M_0{}^-(x) = (i/M\sqrt{2}) (\tilde{F}_1{}^- + \tilde{F}_2{}^- - \tilde{F}_3{}^-)(q_+ \cdot q_-) + \tilde{F}_3{}^-(p\cdot q_-) + (\tilde{F}_1{}^- - \tilde{F}_2{}^- - \tilde{F}_3{}^-)(q_-)^2 \}$$
$$\underset{q_{-0}\to\infty}{\sim} \alpha q_{-0}{}^4 + \beta q_{-0}{}^3 + \cdots . \tag{14}$$

$(q_1\cdot q_-)$, $(p\cdot q_-)$, and $(q_-)^2$ can be considered as independent invariants; this means that every factor multiplying them in Eq. (14) has to have an asymptotic behavior similar to the one appearing on the right-hand side of this equation. In particular,

$$\tilde{F}_1{}^-(x) + \tilde{F}_2{}^-(x) \underset{x\to\infty}{\sim} \alpha_1 x^3 \times \beta_1 x^2 + \cdots , \tag{15a}$$

$$\tilde{F}_3{}^-(x) \underset{x\to\infty}{\sim} \alpha_2 x^3 + \beta_2 x^2 + \cdots , \tag{15b}$$

$$x[\tilde{F}_1{}^-(x) - \tilde{F}_2{}^-(x) - \tilde{F}_3{}^-(x)] \underset{x\to\infty}{\sim} \alpha_3 x^3 + \beta_2 x^2 + \cdots , \tag{15c}$$

where α_i and β_i do not depend on x.

The polynomials appearing on the right-hand side of Eqs. (15) do not contribute to the absorptive parts of the dispersion relations. Similar results hold for $\tilde{F}_i{}^+(x)$.

The constants α_i and β_i cannot be determined; they correspond to the usual polynomial ambiguity in dispersion relations, and we shall neglect them.

IV. DISPERSION RELATIONS AND SUM RULES

Using the results of Sec. II, we can write the following dispersion relations[5]:

$$\tfrac{1}{2}[\tilde{F}_1{}^-(x=1) + \tilde{F}_2{}^-(x=1) + \tilde{F}_1{}^-(x=-1) + \tilde{F}_1{}^-(x=-1) + \tilde{F}_2{}^-(x=-1)] = \tilde{F}_1{}^-(x=0) + \tilde{F}_2{}^-(x=0) + \frac{1}{\pi}\int \frac{dx}{x(x^2-1)} \mathrm{Abs}[\tilde{F}_1{}^-(x) + \tilde{F}_2{}^-(x)] , \tag{16a}$$

$$\tfrac{1}{2}[\tilde{F}_3{}^-(x=1) + \tilde{F}_3{}^-(x=-1)] = \tilde{F}_3{}^-(x=0) + \frac{1}{\pi}\int \frac{dx}{x(x^2-1)} \mathrm{Abs}\tilde{F}_3{}^-(x) . \tag{16b}$$

The dispersion relation for $x(\tilde{F}_1{}^- - \tilde{F}_2{}^- - \tilde{F}_3{}^-)$ is less reliable due to the enhancement of the intermediate region in the integral over the continuum; we shall therefore not use it.

From the quantity M^+, we obtain in a similar fashion

$$\tfrac{1}{2}[\tilde{F}_1{}^+(x=1) - \tilde{F}_2{}^+(x=1) + \tilde{F}_1{}^+(x=-1) - \tilde{F}_2{}^+(x=-1)] = +\tilde{F}_1{}^+(x=0) - \tilde{F}_2{}^+(x=0) - \frac{1}{\pi}\int \frac{dx}{x(x^2-1)} \times \mathrm{Abs}[\tilde{F}_1{}^+(x) - \tilde{F}_2{}^+(x)] , \tag{17a}$$

$$\tfrac{1}{2}[\tilde{F}_3{}^+(x=1) + \tilde{F}_3{}^+(x=-1)] = \tilde{F}_3{}^+(x=0) + \frac{1}{\pi}\int \frac{dx}{x(x^2-1)} \mathrm{Abs}\tilde{F}_3{}^+(x) . \tag{17b}$$

The next step is to examine the structure of the absorptive part of $M(x)$. The reduction technique can be applied to decompose Abs$M$ as follows:

$$\begin{aligned}\mathrm{Abs}M_0{}^b(x) = \tfrac{1}{2}(2\pi)^4 [& \sum_n \langle 0| j_b | n, K^+\rangle \langle n, \pi^a | j_{K^-} | 0\rangle \delta(p_n + p - q_b) + \sum_m \langle 0 | j_b | m\rangle \langle m, \pi^a | j_{K^-} | K^+\rangle_c \delta(p_m - q_b) \\ & + \sum_l \langle \pi^a | j_b | l, K^+\rangle_c \langle l | j_{K^-} | 0\rangle \delta(p_l + p - q_a - q_b) + \sum_k \langle \pi^a | j_b | k\rangle_c \langle k | j_{K^-} | K^+\rangle_c \delta(p_K - q_a - q_b) \\ & - \sum_{k'} \langle 0 | j_{K^-} | k', K^+\rangle \langle k', \pi^a | j_b | 0\rangle \delta(p_{k'} + q_a + q_b) - \sum_{l'} \langle 0 | j_{K^-} | l'\rangle \langle l', \pi^a | j_b | K^+\rangle_c \delta(p_{l'} + q_a + q_b - p) \\ & - \sum_{m'} \langle 0 | j_{K^-} | m', K^+\rangle_c \langle m' | j_b | 0\rangle \delta(p_{m'} - q_b) - \sum_{n'} \langle \pi^a | j_{K^-} | n'\rangle_c \langle n' | j_b | K^+\rangle_c \delta(p_n + q_b - p)] ,\end{aligned} \tag{18}$$

where the subscript c denotes the connected part of a matrix element and where

$$j_b = (1/f_\pi m^2)(\Box + m^2) D_b ,$$
$$j_{K^-} = (1/M^2)(\Box + M^2) A_0{}^{K^-} .$$

[5] The integrals may start at $x=1$, where the integrands are infinite. The integral converges nevertheless since $\mathrm{Abs}\tilde{F} \sim_{x\to 1} (x^2-1)^{1/2}$; see, e.g., Ref. 4.

For details of the derivation of Eq. (18) we refer the reader to the Appendix.

Figure 1 is a diagrammatic representation of the different terms in the decomposition of Eq. (18). Since we are working in the rest frame, the states n, n', k, k' can only be 0^+ states; the states m, m', l, l' can only be 0^- states. The contribution of m, $m' = \pi^b$ and l, $l' = K^+$

vanish identically owing to the presence of the factor $(m^2-q_b^2)(M^2-q_l^2)$ in our choice of a smooth function.

In the matrix elements $\langle\pi^a|j_{K^-}|n'\rangle_c$, $\langle n,\pi^a|j_{K^-}|0\rangle$, $\langle k|j_{K^-}|K^+\rangle_c$, and $\langle 0|j_{K^-}|k',K^+\rangle$, j_{K^-} is dominated by the K pole which contributes only to $\tilde{F}_3$; we expect therefore the contribution of the continuum to $F_1\pm F_2$ to be small compared to that to F_3. If we neglect the former as a first approximation, we obtain from (6a), (7a), (9), (16a), and (17a)

$$\tfrac{1}{2}(\tilde{F}_1(x=1)-\tilde{F}_2(x=1))=0\,,$$

$$\tfrac{1}{2}(\tilde{F}_1(x=1)+\tilde{F}_2(x=1))=\frac{M^2-(M-m)^2}{M^2}\left(\frac{-2Mf_+}{f_\pi}\right), \tag{19}$$

$$F_1(x=1)=F_2(x=1)=2x\frac{M^2-(M-m)^2}{M^2-(M-2m)^2}\times\left(-\frac{Mf_+}{f_\pi}\right). \tag{20}$$

Our result differs from that of Callan and Treiman[1] by the factor $2[M^2-(M-m)^2]/[M^2-(M-2m)^2]=1.2$ because of our different choice of a smooth function.

From Eq. (20) we can now calculate the $K_{e4}{}^+$ decay rate:

$$\Gamma_{K_{e4}{}^+}=(2.3\pm0.3)\times10^3\ \mathrm{sec}^{-1}\,, \tag{21}$$

whereas experimentally,

$$\Gamma_{K_{e4}{}^+}=(2.9\pm0.6)\times10^3\ \mathrm{sec}^{-1}. \tag{22}$$

We have used the result of Cabibbo and Maksymowicz[6] with the π-π s-wave, $I=0$ scattering length $a_0=0$.[7,8]

It might now be asked how our approximation differs from that of Callan and Treiman.[1] For comparison we rewrite Eqs. 16 and 17 as

$$\frac{1}{2}\frac{M^2-(M-2m)^2}{M^2}F(1)=\frac{M^2-(M-m)^2}{M^2}F(0)+\frac{1}{\pi}\int\frac{dx}{x(x^2-1)}\frac{M^2-(M-m-mx)^2}{M^2}\mathrm{Abs}F(x)\,, \tag{23}$$

where $F(x)=F_1(x)\pm F_2(x)$, or, approximately (taking $M=3.5\ m$),

$$F(1)=1.2F(0)+\frac{1}{\pi}\int\frac{dx}{x(x-1)}\,\mathrm{Abs}F(x)\frac{(6-x)}{5}. \tag{24}$$

Had we written once-subtracted dispersion relations for the quantities $F(x)$ themselves, we would have obtained

$$F(1)=F(0)+\frac{1}{\pi}\int\frac{dx}{x(x-1)}\,\mathrm{Abs}F(x)\,, \tag{25}$$

which yields the Callan-Treiman[1] result upon neglect of the continuum.

Both dispersion relations, Eqs. (24) and (25), are in our approach, of course, equivalent; however, the neglect of the continuum constitutes two different approximations. In our case the integrand is damped for $1<x<11$, i.e., in the low and particularly strongly in the intermediate π-π regions. Indeed the factor $(6-x)$ changes sign at $x=6$, thus leading to cancellations. If, as we expect, the variation of $F(x)$ is mainly due to the final-state π-π interaction, it becomes clear why our approximation is the better one.

What can we say about $\tilde{F}_3$? We have two sum rules at our disposal, (16b) and (17b). In this case the contribution of the dispersion integrals may be sizeable and one could as mentioned above estimate it by considering the intermediate states n, $n'=\kappa$, k, $k'=\sigma$ and using K-pole dominance of j_{K^-}. If we nevertheless neglect the contributions of the dispersion integrals, we have from

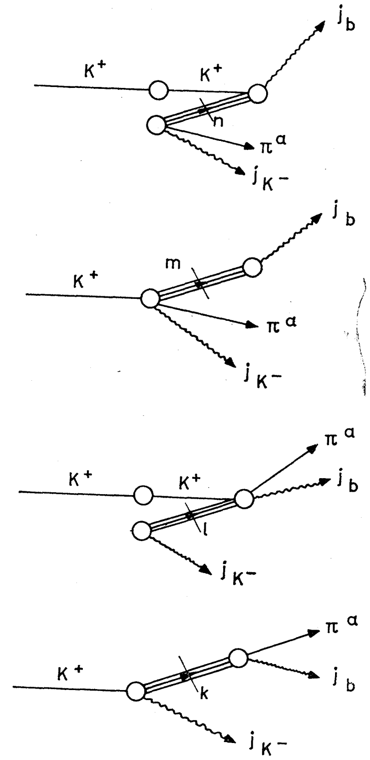

FIG. 1. Diagrammatic representation of the contributions to the absorptive part of $M(x)$ in Eq. (18). The crossed diagrams may be obtained by interchanging j_{K^-} and j_b.

[6] N. Cabibbo and A. Maksymowicz, Phys. Rev. **137**, B438 (1965); **168**, 1926(E) (1968).

[7] A current-algebra estimate of a_0 by one of us (see Ref. 8) gives $a_0=0.25$ or 0.6; the latter value would slightly decrease the value of $\Gamma_{K_{e4}{}^+}$. This of course also applies to all other calculations.

[8] N. F. Nasrallah, Nucl. Phys. **B11**, 240 (1969).

(16b), (7b) and (10)

$$\tfrac{1}{2}\left[\tilde{F}_3(1)+\frac{\sqrt{2}f_K}{M}T_{\rm th}(K^+\pi^+\to K^+\pi^+)\right]=\frac{M^2-(M-m)^2}{M^2}\left(-\frac{M(f_++f_-)}{f_\pi}\right),\quad (26)$$

and from (17b), (6b), and (10)

$$\tfrac{1}{2}[\tilde{F}_3(1)+(\sqrt{2}f_K/M)T_{\rm th}(K^+\pi^-\to K^+\pi^-)]=0.\quad (27)$$

In order to check to what extent the neglect of the continuum is justified, one would need more accurate information on the K-π scattering lengths and on the parameter $\xi=f_-/f_+$.

ACKNOWLEDGMENTS

One of us (N. F. N.) wishes to thank Professor P. Mourad and the Physics Department of the American University of Beirut for use of their facilities.

APPENDIX

In this appendix we present a simple derivation of Eq. (18) which shows the separation of Abs$M$ into the parts appearing in the figure. This will help to illustrate the notation.

$$\mathrm{Abs}M_0{}^b(x)=\tfrac{1}{2}\int dy\, e^{iq_b\cdot y}\langle\pi^a(q_a)|[j_b(y),j_{K^-}(0)]|K^+(p)\rangle=\tfrac{1}{2}\int dy\, e^{iq_b\cdot y}\langle\pi^a|j_b(y)j_{K^-}(0)-j_{K^-}(0)j_b(y)|K^+\rangle.\quad (\mathrm{A1})$$

This can be rewritten using the creation operators of the "in" states:

$$\begin{aligned}\mathrm{Abs}M_0{}^b(x)&=\tfrac{1}{2}\int dy\, e^{iq_b\cdot y}\langle 0|a_a(q_a)j_b(y)j_{K^-}(0)a_{K^+}{}^+(p)|0\rangle-\text{c.t.}\\&=\tfrac{1}{2}\int dy\, e^{iq_b\cdot y}\langle 0|(j_b(y)a_a(q_a)+[a_a(q_a),j_b(y)])(a_{K^+}{}^+(p)j_{K^-}(0)+[j_{K^-}(0),a_{K^+}{}^+(p)])|0\rangle-\text{c.t.}\\&=\tfrac{1}{2}\int dy\, e^{iq_b\cdot y}\{\sum_n\langle 0|j_b(y)a_{K^+}{}^+(p)|n\rangle\langle n|a_a(q_a)j_{K^-}(0)|0\rangle+\sum_m\langle 0|j_b(y)|m\rangle\\&\quad\times\langle m|a_a(q_a)[j_{K^-}(0),a_{K^+}{}^+(p)]|0\rangle+\sum_l\langle 0|[a_a(q_a),j_b(y)]a_{K^+}{}^+(p)|l\rangle\langle l|j_{K^-}(0)|0\rangle\\&\quad+\sum_k\langle 0|[a_a(q_a),j_b(y)]|k\rangle\langle k|[j_{K^-}(0),a_{K^+}{}^+(p)]|0\rangle-\text{c.t.}\},\quad (\mathrm{A2})\end{aligned}$$

where c.t. means crossed terms and where we used the fact that $[a_a(q_a),a_{K^+}{}^+(p)]=0$. Integrating over dy, we obtain finally

$$\begin{aligned}M_0{}^b(x)=\tfrac{1}{2}(2\pi)^4[&\sum_n\langle 0|j_b|n,K^+\rangle\langle n,\pi^a|j_{K^-}|0\rangle\delta(p_n+p-q_b)+\sum_m\langle 0|j_b|m\rangle\langle m,\pi^a|j_{K^-}|K^+\rangle_c\delta(p_m-q_b)\\&+\sum_l\langle\pi^a|j_b|l,K^+\rangle_c\langle l|j_{K^-}|0\rangle\delta(p_l+p-q_a-q_b)+\sum_k\langle\pi^a|j_b|k\rangle_c\langle k|j_{K^-}|K^+\rangle_c\delta(p_K-q_a-q_b)-\text{c.t.}],\quad (\mathrm{A3})\end{aligned}$$

where, for instance, $\langle k|j_{K^-}|K^+\rangle_c=\langle k|[j_{K^-}(0),a_{K^+}{}^+(p)]|0\rangle$ denotes the connected part of $\langle k|j_{K^-}|K^+\rangle$.