experimental forward height should increase if the asymptotic prediction is correct and the $K^{\pm}p$ parameters are representative of their "asymptotic" values.

V. CONCLUSION

In this paper, the Cheng-Wu physical picture, with some additional assumptions, has been applied to $\gamma p \to \rho^0 p$ and φp, and comparison has been made with available experimental data. The assumptions which we have made regarding the strong interactions are perhaps naive but we think simple. Moreover, the resulting amplitude is purely imaginary and qualitatively satisfies the properties one anticipates from the Glauber theory of multiple scattering.

The comparison is not favorable for the assumption that the intermediate particles are two pions (kaons). Of course, we have not investigated the contribution of diagrams representing the dissociation of the photon into four pions, six pions, ..., the so called "tower diagrams" which may be important even in electrodynamics.[21]

Next, we considered an alternative assumption that the intermediate particles are quarks, since these objects have previously been a useful idea in analyzing high-energy scattering. Thereby, the asymptotic prediction for $\gamma p \to \rho^0 p$ is consistent with extrapolation of present data. In addition, under the latter assumption there is a justification for the additivity hypothesis of the quark picture of high-energy scattering.

There are several points which require further investigation: Our kinematic analysis here has assumed the intermediate virtual particles to be lighter than the final vector meson and to be spinless. To study the dependence of the application on these two items, we are now calculating the differential cross section for the so-called "fisheye" diagram for both spinless and spin-$\frac{1}{2}$ particles. This will also enable a different treatment of the final-state interaction, a general subject which will remain relevant at very high energies. Of direct interest is a similar application to photoproduction of vector mesons from nuclei. Finally, there are the possibilities of a more detailed approach and the inclusion of nondiffractive exchanges.

[21] H. Cheng and T. T. Wu, Phys. Rev. D 1, 467 (1970), and papers II and III of this series as cited therein.

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Spectral-Function Sum Rules for ρ-ω and ρ-ϕ Transitions*

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Assuming the field algebra of Lee, Weinberg, and Zumino, we point out that two spectral-function sum rules remain valid when the electromagnetic interaction is turned on. From these sum rules we derive formulas for calculating the electromagnetic ρ-ω and ρ-ϕ transitions. A tentative numerical estimate gives a value for the ρ-ω transition close to quark-model predictions, but an opposite sign for the ρ^+-ρ^0 mass splitting (-7.5 MeV). The branching ratio $\Gamma(\omega \to 2\pi)/\Gamma(\omega \to 3\pi)$ is estimated to be $\sim 1\%$, and the phase of the $\omega \to 2\pi$ amplitude relative to the $\rho \to 2\pi$ amplitude is between 80° and 130°.

I. INTRODUCTION

THE decay mode $\omega \to 2\pi$ has recently been measured through the reaction $\pi^+ p \to \pi^+\pi^-\Delta^{++}$,[1,2] the e^+-e^- colliding-beam experiment,[3] and the $p\bar{p}$ annihilation process.[4] The e^+-e^- annihilation experiment, which measured the phase of the decay amplitude too, does not seem to be in good agreement with theoretical analysis[5] based upon an electromagnetic ρ-ω transition. Together they may imply a violation of charge independence which is nonelectromagnetic in origin.[6] To analyze this reaction, it is very important to have a reliable estimate of the ρ^0-ω mixing due to the electromagnetic interaction. The $SU(3)$ symmetry alone is not capable of determining the ρ-ω transition until one has

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[1] G. Goldhaber, W. R. Butler, D. G. Coyne, B. H. Hall, J. N. MacNaughton, and G. H. Trilling, Phys. Rev. Letters 23, 1351 (1969).
[2] For an earlier experiment, see S. M. Flatté, D. C. Huwe, J. J. Murray, J. Butten-Schaffer, F. T. Solmitz, M. L. Stevenson, and C. Wohl, Phys. Rev. 145, 1050 (1966). There is also a negative evidence for the $\omega \to 2\pi$ decay mode in the $\pi^- p$ reaction [W. Lee (private communication)].
[3] J. E. Augustin, J. C. Bizot, J. Buon, J. Haissinski, D. Lalanne, P. C. Marin, J. Perez-y-Jorba, F. Rumpf, E. Silva, and S. Tavernier, Phys. Rev. Letters 20, 126 (1968); Phys. Letters 28B, 517 (1969); and Nuovo Cimento Letters 2, 214 (1969).
[4] W. W. M. Allison, W. A. Cooper, F. Fields, and D. S. Rhines, Phys. Rev. Letters 24, 618 (1970).
[5] M. Gourdin, L. Stodolsky, and F. M. Renard, Phys. Letters 30B, 347 (1969).
[6] However, this is not conclusive according to a different analysis by R. G. Sachs and J. F. Willemson, Phys. Rev. D 2, 133 (1970).

experimental measurements of the ρ^+-ρ^0 and K^{*+}-K^{*0} mass splittings, as well as an accurate determination of the ω-ϕ mixing angle. Those mass splittings have not been measured accurately, mainly because of their large widths and of possible large systematic errors. Until now, therefore, one has relied on the ρ-ω transition strength predicted by the $SU(6)$ symmetry or some version of the quark model.[7]

In this paper, using the field algebra of Lee, Weinberg, and Zumino,[8] we give spectral-function sum rules in the presence of the electromagnetic interaction. This will enable us to derive a formula for calculating the ρ-ω transition. In principle, the formula can be evaluated using the results of e^+-e^- colliding-beam experiments and appropriate symmetry arguments. When we saturate our formula tentatively with lower continuum states, we find that the ρ-ω transition is close to the quark-model prediction. Higher intermediate states are likely to increase the absolute magnitude, but the sign will remain the same. The magnitude of the ρ-ϕ transition is sizable, in contrast to the quark-model predictions, so that the ρ^+-ρ^0 mass difference, through an $SU(3)$ relation, turns out to be negative in sign provided that the K^{*+}-K^{*0} mass difference is negative.[9] This may be soon tested by experiment.

II. SUM RULE

Let us start with a time-ordered product

$$T^{\alpha\beta}{}_{\mu\nu}=i\int d^4x\, e^{-iqx}\langle 0|\,T(J^\alpha{}_\mu(x)J^\beta{}_\nu(0))\,|0\rangle\,, \quad (2.1)$$

where $J^\alpha{}_\mu$ stands for a neutral component of unitary spin currents with α referring to unitary spin components including a singlet current. We recall that the neutral components of the unitary spin currents ($\alpha=0$, 3, and 8) are conserved even in the presence of the electromagnetic interaction. We write down the spectral representation as follows:

$$T^{\alpha\beta}{}_{\mu\nu}=\int dm^2\frac{\delta_{\mu\nu}+q_\mu q_\nu/m^2}{m^2+q^2}\rho^{\alpha\beta}(m^2) -\delta_{\mu4}\delta_{\nu4}\int dm^2\frac{\rho^{\alpha\beta}(m^2)}{m^2}\,. \quad (2.2)$$

In writing this representation we note that a one-photon intermediate state does not contribute as a pole at $q^2=0$, because of gauge invariance. We assume as usual that a matrix element of a T product falls off as $iq_0\to\infty$ to bar any additional entire function in q^2. This representation is valid to all orders of the electromagnetic interaction. It does not hold for nonconserved currents for which we need two independent spectral functions.

To see the relation between the sum rules we are going to give and those of Weinberg, we first outline the derivation by Lee, Weinberg, and Zumino. The starting point is the Lagrangian[8]

$$L=-\tfrac{1}{4}\tilde{F}^\alpha{}_{\mu\nu}\tilde{F}^{\alpha\mu\nu}-\tfrac{1}{2}m_0{}^2\tilde{\Phi}^\alpha{}_\mu\tilde{\Phi}^{\alpha\mu}+L_m(\tilde{\Psi},D_\mu\tilde{\Psi})\,, \quad (2.3)$$

where

$$F^\alpha{}_{\mu\nu}\equiv\partial_\mu\tilde{\Phi}^\alpha{}_\nu-\partial_\nu\tilde{\Phi}^\alpha{}_\mu+g_0f_{\alpha\beta\gamma}\tilde{\Phi}^\beta{}_\mu\tilde{\Phi}^\gamma{}_\nu\,,$$
$$D_\mu\tilde{\Psi}\equiv\partial_\mu\tilde{\Psi}+g_0T_\alpha\tilde{\Phi}^\alpha{}_\mu\tilde{\Psi}\,,$$

where the tilde denotes that the electromagnetic interaction is switched off. Except for $f_{\alpha\beta\gamma}$, the $SU(3)$ structure constants,[10] the same definitions and the notations as those in Ref. 8 are used. The unitary spin current is related to the field as $J^\alpha{}_\mu(x)=(m_0{}^2/g_0)\tilde{\Phi}^\alpha{}_\mu(x)$.

The first and second Weinberg sum rules can be derived by taking the vacuum expectation value of the commutators

$$[\tilde{J}^\alpha{}_0(\mathbf{x},t),\tilde{J}^\beta{}_i(\mathbf{y},t)] =if_{\alpha\beta\gamma}\tilde{J}^\gamma{}_i(\mathbf{x},t)\delta^3(\mathbf{x}-\mathbf{y}) +i(m_0{}^2/g_0{}^2)\delta_{\alpha\beta}\partial_i\delta^3(\mathbf{x}-\mathbf{y})\,, \quad (2.4)$$

$$[\partial_0\tilde{J}^\alpha{}_i(\mathbf{x},t)-\partial_i\tilde{J}^\alpha{}_0(\mathbf{x},t),\tilde{J}^\beta{}_j(\mathbf{y},t)] =-i(m_0{}^4/g_0{}^2)\delta_{\alpha\beta}\delta_{ij}\delta^3(\mathbf{x}-\mathbf{y}) +f_{\alpha\beta\gamma}\tilde{J}^\gamma{}_i(\mathbf{x},t)\partial_j\delta^3(\mathbf{x}-\mathbf{y}) -i(g_0{}^2/m_0{}^2)f_{\alpha\delta\gamma}f_{\beta\delta\epsilon}\tilde{J}^\gamma{}_i(\mathbf{x},t)\tilde{J}^\epsilon{}_j(\mathbf{x},t)\delta^3(\mathbf{x}-\mathbf{y})\,, \quad (2.5)$$

respectively. The vacuum expectation value of the last term in (2.5) is dropped according to the usual argument.[8]

It has to be realized, however, that the sum rules thus obtained are inconsistent with broken $SU(3)$ and the vector-dominance model.[11] Clearly, $SU(3)$ symmetry breaking must be introduced in the original Lagrangian. We introduce the symmetry breaking by inserting a factor $1+Dd_{8\alpha\alpha}$ in the kinetic-energy term.[12] Here a parameter D is a small real number characteristic of $SU(3)$ breaking. The once-integrated commutators of the algebra of currents will not be changed by the introduction of the symmetry-breaking term.

Then the sum rules become

$$\int dm^2\frac{\tilde{\rho}^{\alpha\beta}(m^2)}{m^2}=\left(\frac{m_0{}^2}{g_0{}^2}\right)\delta_{\alpha\beta}\,, \quad (2.6)$$

$$(\delta_{\alpha\gamma}+Dd_{8\alpha\gamma})\int dm^2\tilde{\rho}^{\gamma\beta}(m^2)=\left(\frac{m_0{}^4}{g_0{}^2}\right)\delta_{\alpha\beta}\,. \quad (2.7)$$

The electromagnetic interactions are introduced through a substitution law[13]

$$\Phi^3{}_\mu\to\Phi^3{}_\mu+(e/g_0)A_\mu\,,$$
$$\Phi^8{}_\mu\to\Phi^8{}_\mu+(\sqrt{\tfrac{1}{3}})(e/g_0)A_\mu\,. \quad (2.8)$$

[7] See, e.g., J. Yellin, Phys. Rev. **147**, 1080 (1966).
[8] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).
[9] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).
[10] Note that the extension to the singlet component of the unitary current can be made by setting $f_{\alpha\beta\gamma}=0$ for g, β, or $\gamma=0$.
[11] J. J. Sakurai, Phys. Rev. Letters **19**, 803 (1967).
[12] See, e.g., I. Kimel, Phys. Rev. Letters **21**, 177 (1968).
[13] T. D. Lee and B. Zumino, Phys. Rev. **163**, 1667 (1967).

It can be seen that for conserved components of α and β (0, 3, and 8), equal-time commutation relation (2.4) and (2.5) remain the same if all operators are understood as including electromagnetic interactions. Therefore, Weinberg's sum rules also remain valid even when the electromagnetic interaction is turned on. Thus, defining

$$\Delta\rho^{\alpha\beta}(m^2)=\rho^{\alpha\beta}(m^2)-\tilde{\rho}^{\alpha\beta}(m^2),$$

we have

$$\int dm^2\frac{\Delta\rho^{\alpha\beta}(m^2)}{m^2}=0, \tag{2.9}$$

$$\int dm^2\Delta\rho^{\alpha\beta}(m^2)=0, \tag{2.10}$$

for conserved neutral currents. For charged currents, (2.6) must be modified to include a contribution from the nonconserved parts of the spectral functions, but (2.7) will hold.

III. ρ^0-ω AND ρ^0-ϕ TRANSITIONS

Let us write the renormalized fields of the ρ^0, the ω, and the ϕ in a linear combination of the three neutral currents ($\alpha=0,3,8$). Most generally, they are written as[14]

$$\Phi^\rho{}_\mu=(f_\rho/m_\rho^2)J^I{}_\mu, \tag{3.1}$$

$$\Phi^\omega{}_\mu=(f_B/m_\omega^2)\alpha J^B{}_\mu+(f_Y/m_\omega^2)\beta J^Y{}_\mu, \tag{3.2}$$

$$\Phi^\phi{}_\mu=(f_B/m_\phi^2)\gamma J^B{}_\mu+(f_Y/m_\phi^2)\delta J^Y{}_\mu, \tag{3.3}$$

where the fields Φ_μ's are renormalized fields of the corresponding vector mesons, and

$$\begin{aligned}&J^I{}_\mu=J^3{}_\mu, \quad J^B{}_\mu=J^0{}_\mu\\&J^Y{}_\mu=\tfrac{2}{3}\sqrt{3}J^8{}_\mu, \quad f_\rho/f_Y>0,\end{aligned} \tag{3.4}$$

$$\begin{aligned}&\alpha=\cos\theta_Y/\cos(\theta_B-\theta_Y), \quad \beta=\sin\theta_B/\cos(\theta_B-\theta_Y),\\&\gamma=\sin\theta_Y/\cos(\theta_B-\theta_Y), \quad \delta=-\cos\theta_B/\cos(\theta_B-\theta_Y).\end{aligned} \tag{3.5}$$

The mixing angles and transition strengths are defined as usual in the current-mixing model.[14] Now we make use of the two kinds of sum rules (2.9) and (2.10). We consider the sum rules for off-diagonal propagators

$$\langle 0|T(\Phi^\rho{}_\mu(x)\Phi^\omega{}_\nu(0))|0\rangle \tag{3.6}$$

and

$$\langle 0|T(\Phi^\rho{}_\mu(x)\Phi^\phi{}_\nu(0))|0\rangle. \tag{3.7}$$

Each of these spectral representations contains two poles due to one-vector-meson states. Separating out those poles in the stable-particle approximation, one finds for the ρ-ω transition

$$\frac{\Delta m_{\rho\omega}{}^2(m_\omega{}^2)}{m_\omega{}^2-m_\rho{}^2}+\frac{\Delta m_{\rho\omega}{}^2(m_\rho{}^2)}{m_\rho{}^2-m_\omega{}^2}+\int_{\text{cont}}dm^2\Delta\rho^{\rho\omega}(m^2)=0, \tag{3.8}$$

$$\frac{\Delta m_{\rho\omega}{}^2(m_\omega{}^2)}{m_\omega{}^2(m_\omega{}^2-m_\rho{}^2)}+\frac{\Delta m_{\rho\omega}{}^2(m_\rho{}^2)}{m_\rho{}^2(m_\rho{}^2-m_\omega{}^2)}+\int_{\text{cont}}dm^2\frac{\Delta\rho^{\rho\omega}(m^2)}{m^2}=0, \tag{3.9}$$

where $\Delta m_{\rho\omega}{}^2(m_\rho{}^2)$ is the electromagnetic transition mass between the ρ and the ω, $\Delta\rho^{\rho\omega}(m^2)$ stands for the spectral function for (3.6), and the integral is over the continuum states. Combining (3.8) and (3.9), one derives

$$\Delta m_{\rho\omega}{}^2(m_\omega{}^2)=m_\omega{}^2\int_{\text{cont}}dm^2(m_\rho{}^2-m^2)\frac{\Delta\rho^{\rho\omega}(m^2)}{m^2}. \tag{3.10}$$

This is, together with (3.11), our final formula. When one separates the vector meson poles out of $\Delta\rho^{\rho\omega}(m^2)$, (3.10) will look similar to the dispersion representation of a Feynman diagram. However, one should realize that (3.10) has an extra factor of $m_\omega{}^2/m^2$ which may improve the convergence in some cases. For the ρ-ϕ transition, we have to define the transition mass at a certain value of q^2. Since the $\phi\to 2\pi$ decay is determined by the transition mass at $q^2=-m_\phi{}^2$, we define it at $q^2=-m_\phi{}^2$.

Replacing ω by ϕ in (3.8) and (3.9), we obtain

$$\Delta m_{\rho\phi}{}^2(m_\phi{}^2)=m_\phi{}^2\int_{\text{cont}}dm^2(m_\rho{}^2-m^2)\frac{\Delta\rho^{\rho\phi}(m^2)}{m^2}. \tag{3.11}$$

One immediate consequence of our formulas (3.10) and (3.11) is that, if one saturates the first and the second sum rules with one-vector-meson states alone to the order of $e^2/4\pi$, both the ρ-ω and the ρ-ϕ transitions turn out to be zero. This is in a sharp contrast to the results by Mathur and Okubo[15] from asymptotic-symmetry sum rules. In fact, their second sum rule is different from our Eq. (2.10).

For the purpose of numerical estimate, we shall therefore have to take account of the continuum contributions. The spectral function $\Delta\rho^{\rho\omega}(m^2)$ is written as

$$(\delta_{\mu\nu}-q_\mu q_\nu/q^2)\Delta\rho^{\rho\omega}(-q^2)=\sum_{n,\mathbf{p}_n=\mathbf{q}}\langle 0|\Phi^\rho{}_\mu(0)|n\rangle\langle n|\Phi^\omega{}_\nu(0)|0\rangle\delta(q_0-p_{n0}). \tag{3.12}$$

We shall classify the possible continuum intermediate

[14] N. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967); R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967). We follow the notations of the latter paper. According to Oakes and Sakurai, the current-mixing model is consistent with vector-meson dominance, but the mass-mixing models are not.

[15] V. S. Mathur and S. Okubo, Phys. Rev. **188**, 2435 (1969).

states into two classes, those consisting of hadrons alone and those containing a photon. The vector-meson dominance seems to work well for the spectral-function sum rules in the zeroth order of $e^2/4\pi$.[11] It implies

$$|\langle 0|\Phi^{\rho}{}_{\mu}(0)|\rho\rangle|\gg|\langle 0|\Phi^{\rho}{}_{\mu}(0)|n_1\rangle| \qquad (3.13)$$

and also

$$|\langle\omega|\Phi^{\omega}{}_{\mu}(0)|0\rangle|\gg|\langle n_1|\Phi^{\omega}{}_{\mu}(0)|0\rangle|, \qquad (3.14)$$

where $|n_1\rangle$ is a hadronic continuum state. Therefore, it is quite reasonable to discard such states $(2\pi,3\pi,\ldots)$ when we estimate the ρ^0-ω transition. Among the intermediate states belonging to the second class, the lowest intermediate state is $\pi\gamma$; then follow $\eta\gamma$, $(2\pi)_{s\text{ wave}}\gamma$, $A\gamma$, and so forth ($\rho\gamma$, $\omega\gamma$, and $\phi\gamma$ do not contribute because of C invariance). It will be a decent approximation to retain the $\pi\gamma$ and $\eta\gamma$ intermediate states when there is no particular evidence for unusual enhancement of higher continuum states.

Before ending this section we make a comment on the convergence of our formula. The convergence of Weinberg's second sum rule can be tested directly by measurements of the e^+-e^- total cross section. Even if it turns out to be divergent, we can still expect that our second sum rule (2.10) may be convergent, since (2.10) is the difference of two second sum rules of Weinberg type. The convergence of (2.10) may be tested by measuring e^+-$e^-\rightarrow$hadron$+\rho$ (or ω) which can be related to a matrix element $\langle 0|J_\mu(0)|\text{hadron}+\gamma\rangle$.

IV. NUMERICAL ESTIMATE

In this section we shall tentatively estimate the $\pi\gamma$ and $\eta\gamma$ contribution to the spectral function. The invariant coupling for $VP\gamma$ is defined as follows:

$$(4E_PE_\gamma)^{1/2}\langle P\gamma|\Phi^V{}_\mu(0)|0\rangle = G_{VP\gamma}(q^2)\epsilon_{\mu\nu\lambda\kappa}\epsilon_\nu{}^{(\gamma)}p^{(\gamma)}{}_\lambda p^{(P)}{}_\kappa, \qquad (4.1)$$

where

$$q=p^{(\gamma)}+p^{(P)}.$$

This relation defines "current"-$\pi\gamma$ couplings. By means of the field-current identity, which is fully consistent with our previous assumption on the Schwinger term, one can rewrite $G_{VP\gamma}(q^2)$ by factoring out the vector-meson poles as

$$G_{VP\gamma}(q^2)=(q^2+m_V{}^2)^{-1}\bar{G}_{VP\gamma}(q^2). \qquad (4.2)$$

The decay matrix element of $V\rightarrow P\gamma$ is given as

$$(8E_VE_PE_\gamma)^{1/2}\langle P\gamma|V\rangle=i(2\pi)^4\delta(p_P+p_\gamma-p_V) \times g_{VP\gamma}F(p_V{}^2)\epsilon_{\mu\nu\lambda\kappa}\epsilon^{(V)}{}_\mu\epsilon^{(\gamma)}{}_\nu p^{(\gamma)}{}_\lambda p^{(P)}{}_\kappa, \qquad (4.3)$$

where $g_{VP\gamma}$ is given by $\bar{G}_{VP\gamma}(q^2)$ as

$$\bar{G}_{VP\gamma}(q^2)=g_{VP\gamma}F(q^2). \qquad (4.4)$$

The $SU(3)$ symmetry relates various $g_{VP\gamma}$ couplings as[16]

$$g_{\rho\pi\gamma}=(m_\rho{}^2/f_\rho)g, \qquad (4.5)$$

$$g_{\omega\pi\gamma}=(\sqrt{3}m_\omega{}^2/2f_Y)[\sqrt{3}\cos(\theta_B-\theta_Y)/\sin\theta_B]g, \qquad (4.6)$$

$$g_{\phi\pi\gamma}=0, \qquad (4.7)$$

$$g_{\rho\eta\gamma}=\sqrt{3}(m_\rho{}^2/f_\rho)g, \qquad (4.8)$$

$$g_{\omega\eta\gamma}=(\sqrt{3}m_\omega{}^2/2f_Y)[\cos(\theta_B+\theta_Y)/\sin\theta_B]g, \qquad (4.9)$$

$$g_{\phi\eta\gamma}=(\sqrt{3}m_\phi{}^2/2f_Y)(2\cos\theta_Y)g, \qquad (4.10)$$

where a free parameter associated with the ratio of "singlet current"-$P\gamma$ coupling to "octet current"-$P\gamma$ coupling has been eliminated by imposing the restriction that $\bar{G}_{\phi\pi\gamma}(q^2)$ is practically zero[17] as compared with $\bar{G}_{\omega\pi\gamma}(q^2)$.

The universal form factor $F(q^2)$ must be determined from experiments such as $e^+e^-\rightarrow\rho\pi$, $e^+e^-\rightarrow\pi\omega$ for large timelike q^2. At present, we do not have any direct determination. For the purpose of numerical evaluation of our formula, we cut off our integrals at $2m_\phi{}^2$. We think this is reasonable since the vector-dominance model gives us, at least, a qualitative understanding of the region $q^2\lesssim 1$ BeV^2. When results of the above experiments are available, a quantitative test of our formula will be possible.

By using the decay rate of $\omega\rightarrow\pi\gamma$, and the values for θ_Y, θ_B, f_ρ, and f_Y determined in the "current-mixing" model,[14] we determine the scale of the coupling constant g,[18]

$$\frac{m_\rho{}^2}{f_\rho}\frac{g}{e}=0.75\text{ GeV}^{-1}. \qquad (4.11)$$

Substituting all of these into (3.10) and (3.11), we finally find

$$\text{Re}\Delta m_{\rho\omega}=-1.7\text{ MeV}, \qquad (4.12)$$

$$\text{Re}\Delta m_{\rho\phi}=-1.0\text{ MeV}. \qquad (4.13)$$

These are the contributions from the $\pi\gamma$ and $\eta\gamma$ states alone. What will happen if we consider higher intermediate states? Suppose we take account of an $M\gamma$ intermediate state, where M is a neutral octet meson (necessarily with $C=+$) or a mixture of a singlet and an octet obeying approximately the ordinary mixing pattern of quark type. From the $SU(3)$ Clebsch-Gordan coefficients and the fact that the invariant mass of such a state is largely above the vector-meson mass m_V, one can show that the contribution from such an intermediate state always adds constructively to the above results. In this sense, (4.12) and (4.13) are considered as the lower bounds on $|\text{Re}\Delta m_{\rho\omega}|$ and $|\text{Re}\Delta m_{\rho\phi}|$. Predictions of $\text{Re}\Delta m_{\rho\omega}$ from quark models, although the figures depend on the details of the assumptions, are all

[16] A phase convention adopted there is that $\rho^0=(T_1{}^1-T_2{}^2)/\sqrt{2}$, $\omega_8=(T_1{}^1+T_2{}^2-2T_3{}^3)/\sqrt{6}$, and $\omega_1=(T_1{}^1+T_2{}^2+T_3{}^3)/\sqrt{3}$.

[17] Experimentally the ratio of reduced decay rates is bounded by $\gamma(\phi\rightarrow\rho\pi)/\gamma(\omega\rightarrow\rho\pi)\lesssim 0.9\times 10^{-2}$ if the calculation for the $\omega\rightarrow 3\pi$ decay is taken from M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962). No event of the $\phi\rightarrow\pi\gamma$ process has been measured.

[18] We have chosen the lifetime from Ref. 9.

negative. For example,

$$\mathrm{Re}\Delta m_{\rho\omega} \simeq -2.0 \text{ MeV} \tag{4.14}$$

by Radicati, Picasso, Zanello, and Sakurai,[19] and

$$\mathrm{Re}\Delta m_{\rho\omega} \simeq -(3.6\pm 0.7) \text{ MeV} \tag{4.15}$$

by Yellin.[7] In the limit of $SU(3)$ symmetry or in simpler versions of the quark model, there is no transition between the ρ and the ϕ.

V. DISCUSSION

Let us first see the implications of our transition masses in the mass splittings of the octet vector mesons. For this purpose we shall use the Coleman-Glashow formula. Since $SU(3)$-breaking effects steal into our estimates (4.12) and (4.13) through the physical masses, it is not fully consistent to use the formula which neglects all medium-strong $SU(3)$ breakings. We hope, however, that the possible corrections are reasonably small as compared with $SU(3)$-symmetric terms. In the present phase convention, the ρ-ω and ρ-ϕ transition masses are related by $SU(3)$ to the ρ^+-ρ^0 and K^{*+}-K^{*0} mass differences as

$$\begin{aligned}[m_{\rho^+}{}^2 - m_{\rho^0}{}^2] - [m_{K^{*+}}{}^2 - m_{K^{*0}}{}^2] &= \sqrt{3}(\Delta m_{\rho\phi}{}^2 \cos\theta_Y - \Delta m_{\rho\omega}{}^2 \sin\theta_Y) \\ &\simeq -0.3\times 10^{-3} \text{ BeV}^2, \end{aligned}\tag{5.1}$$

where we have put the vector-meson masses and coupling strengths with currents equal to their $SU(3)$-symmetric values. We shall first notice that our result on the right-hand side of (5.1) is much smaller than the predictions from various quark models and $SU(6)$ symmetry. If we choose the most recent data for the electromagnetic mass difference of the K^*,[20] we find

$$m_{\rho^0} - m_{\rho^+} \simeq 7.5 \text{ MeV}. \tag{5.2}$$

This disagrees with the quark-model predictions[7,19] in which $m_{\rho^+} - m_{\rho^0}$ is of the same sign as $m_{\pi^+} - m_{\pi^0}$. The experimental data are not definite at this moment because there may be large systematic errors, but as far as a simple statistical average is concerned, the trend is in favor of our prediction (5.2), not of the quark-model predictions. Therefore, the experimental determination of the electromagnetic mass difference of the ρ meson will help to show whether our calculation based upon spectral-function sum rules is valid or not.

Let us turn to the $\omega\to 2\pi$ decay rate. If we simply calculate the $\omega\to 2\pi$ decay rate on the assumption that a direct $\omega\to 2\pi$ process should be negligible, the rate is obtained as

$$[\Gamma(\omega\to 2\pi)]^{1/2} \approx 0.35 \text{ MeV}^{1/2} \tag{5.3}$$

or

$$\Gamma(\omega\to 2\pi)/\Gamma(\omega\to 3\pi) \approx 1\%,$$

and the phase of the $\omega\to 2\pi$ amplitude in the e^+-e^- collision is given as

$$80^\circ \lesssim \arg M(\omega\to 2\pi) \lesssim 130^\circ, \tag{5.4}$$

where we have used $|(\mathrm{Im}\Delta m_{\rho\omega}{}^2)/m_\rho| \lesssim 1.6$ MeV,[5] which corresponds to

$$|\varphi_\lambda| \equiv |\arg\Delta m_{\rho\omega}{}^2| \leqq 25^\circ. \tag{5.5}$$

The phase of the ρ-ω transition coupling is not in serious disagreement with the colliding-beam experiment ($\varphi_\lambda = 54^\circ \pm 28^\circ$), and the decay rate is not far from the results of various measurements.[1,3,4] Quite recently, the ρ-ω interference has been measured through the e^+e^- and $\pi^+\pi^-$ pairs in photoproduction.[21,22] The branching ratio they quote is

$$\Gamma(\omega\to 2\pi)/\Gamma(\omega\to \text{total}) = (0.80_{-0.22}{}^{+0.28})\%, \tag{5.6}$$

which is very close to our estimate of $\sim 1\%$. The phase of the $\omega\to 2\pi$ decay amplitude, or the phase of the ρ-ω transition, is deduced by combining this measurement with the observed production phases in the electron-pair experiment. This yields

$$\varphi_\lambda = (-99_{-30}{}^{+38})^\circ, \tag{5.7}$$

which is in disagreement with our value. But, if one takes the production phases in the preliminary result at DESY,[23] one obtains

$$\varphi_\lambda = -(21\pm 25)^\circ. \tag{5.8}$$

This is consistent with our value as well as with other theoretical estimates.

The value of $\varphi_\lambda = (-99_{-30}{}^{+38})^\circ$ indicates a vanishingly small value for $\mathrm{Re}\Delta m_{\rho\omega}{}^2$. To reconcile this with the observed branching ratio of $\omega\to 2\pi$, one must have a larger value for $\mathrm{Im}\Delta m_{\rho\omega}{}^2$. An upper bound for the absolute magnitude of $\mathrm{Im}\Delta m_{\rho\omega}{}^2$ is furnished by the partial decay rates of the ρ and the ω. It is remarked here that the decay rates setting the bound on $\mathrm{Im}\Delta m_{\rho\omega}{}^2$ are "nonresonant" contributions or direct decays; they must not include a contribution such as $\rho\to\omega\to 3\pi$ or $\omega\to\rho\to 2\pi$.[24] Therefore, if the nonresonant contributions of $\omega\to 2\pi$ and/or $\rho\to 3\pi$ are huge by some un-

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[20] $M_{K^{*0}} - M_{K^{*\pm}} = 6.3\pm 6$ or 8 ± 3.5 MeV according to Ref. 9. However, we must be somewhat careful about possible systematic errors, which the Appendix of Ref. 9 discusses in detail. We have chosen 6.3 MeV for the numerical computation in (5.2).

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[24] See, e.g., Ref. 5. A similar formalism is given by D. Horn, Phys. Rev. D **1**, 1421 (1970).

known mechanism, either $\mathrm{Im}\Delta m_{\rho\omega}^2$ may be much larger than $\mathrm{Re}\Delta m_{\rho\omega}^2$, or the nonresonant direct $\omega\to 2\pi$ process may dominate over the $\omega\to\rho\to 2\pi$ decay. The former is compatible with the observed $\omega\to 2\pi$ rate only if $\mathrm{Re}\Delta m_{\rho\omega}^2$ turns out to be much smaller than a few MeV^2. In the latter case the direct decay simulates the resonant $\omega\to 2\pi$ decay through an imaginary coupling. Unfortunately, we have no theoretical explanation for such a enormous enhancement in the direct $\omega\to 2\pi$ decay or the $\rho\to 3\pi$ decay.

PHYSICAL REVIEW D VOLUME 2, NUMBER 11 1 DECEMBER 1970

Eikonal Regge Model for Elastic Scattering Processes*

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The Frautschi-Margolis version of the Regge eikonal model is extended to include secondary Regge trajectories. Physical properties of the model are discussed. In particular, the "shrinkage" of $d\sigma/dt$ observed at present energies (rapid shrinkage for pp and K^+p, little or no shrinkage for $\pi^\pm p$ and K^-p, antishrinkage for $\bar{p}p$) is related to the energy dependence of σ_{tot} (pp and K^+p nearly flat, $\pi^\pm p$ and K^-p falling slowly, $\bar{p}p$ falling rapidly).

I. INTRODUCTION

In recent years it has become increasingly apparent that in order to understand hadron scattering one must consider cuts in the complex angular momentum plane as well as Regge poles.[1] A popular conjecture is to identify the cuts with multiple-scattering terms in the eikonal model.[2] Frautschi and Margolis[3] have aplied this idea to the diffraction peak in elastic scattering, noting the special features which arise when exchange of a Pomeranchuk pole (P) with appreciable slope is supplemented by cuts representing multiple scattering (PP, PPP, ...).

In the present paper we extend the Frautschi-Margolis model, which originally dealt only with the Pomeranchon, to include other Regge poles (R) and their associated cuts (RP, etc.). The formalism of the extension is trivial and follows closely work by Chiu and Finkelstein,[4] but we present it here as a basis for discussion of physical features of the model, and for the detailed fit of the following paper[5] to several elastic reactions at small t.

While the physical nature of the model was already evident in the original paper by Frautschi and Margolis, explicit addition of secondary trajectories clarifies some points, in addition to putting the discussion on a more quantitative basis. The main point we wish to give fresh emphasis has to do with the shrinking (or nonshrinking) of elastic diffraction peaks. Empirically the pp and K^+p forward peaks shrink noticeably, the $\pi^\pm p$ and K^-p peaks exhibit little energy dependence, and the $\bar{p}p$ peak actually broadens with increasing energy. This variable behavior was a well-known embarrassment for the early Regge-pole model. Further variable behavior is found in the total cross section, which is almost flat for pp and K^+p, falls slowly with energy in $\pi^\pm p$ and K^-p scattering, and falls rapidly for $\bar{p}p$. Thus empirically the faster $\sigma_{\mathrm{tot}}(E)$ falls, the less the forward peak shrinks.

We have a natural explanation for this correlation. Multiple-scattering corrections in the eikonal model sharpen the forward peak, by an amount roughly proportional to σ_{tot} [since $\mathrm{Im}A(0)=\sigma_{\mathrm{tot}}/4\pi$ and the double-scattering correction involves the square of the amplitude]. Factors associated with secondary trajectories also act in the same direction, as discussed in Sec. III. This is in accord with the geometrical picture, wherein the width of a diffraction peak is $\Delta\theta\sim h/pR\sim h/p(\sigma_{\mathrm{tot}}/\pi)^{\frac{1}{2}}$. Thus if σ_{tot} *falls* with increasing energy, multiple scattering is *reduced*, the contribution of secondary trajectories is also *reduced*, and the peak tends to *broaden*. The broadening is in competition with the usual Regge shrinkage factor, and the result at present energies is that shrinkage wins when σ_{tot} is flat (pp and K^+p), shrinkage and broadening almost balance when σ_{tot} falls slowly ($\pi^\pm p$ and K^-p), and broadening wins when σ_{tot} falls rapidly ($\bar{p}p$).

Our explanation for the correlation between shrinkage and the energy dependence of σ_{tot} is closely related to the familiar "crossover effect," where if $\sigma_{\mathrm{tot}}(\bar{A}B)$

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