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Cheng-Wu Picture Applied to $\gamma p \to \rho^0 p$ and φp

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The Cheng-Wu physical picture, with some additional assumptions, is applied to ρ^0 (φ) photoproduction and comparison is made with available experimental data. The comparison is not favorable for the assumption that the intermediate particles are two pions (kaons). On the other hand, the asymptotic prediction for $\gamma p \to \rho^0 p$ is consistent with extrapolation of present data under an alternative assumption that the intermediate particles are a quark-antiquark pair. In addition, under the latter assumption there is a justification for the additivity hypothesis of the quark picture of high-energy scattering.

I. INTRODUCTION

THROUGH a systematic investigation, Cheng and Wu[1] recently obtained the $s \to \infty$, t fixed, limit of all two-body elastic scattering amplitudes in quantum electrodynamics to first nonvanishing order. After carrying out this tour de force, they proposed[2] a simple physical picture and a set of explicit rules yielding correctly these same results but with considerably less calculation. It is this latter aspect of their work which we wish to try to apply to hadron physics in this paper.

Their physical picture for high-energy scattering, say, $\gamma p \to \gamma p$, is briefly the following: The incident photon has a certain probability of virtually dissociating into a e^+e^- pair. So, if initially the photon is moving in the z direction with a very large momentum W, then the virtual e^+e^- pair must share this large longitudinal momentum, one particle having βW and the other $(1-\beta)W$. The invariant mass of this e^+e^- system is finite, provided $\beta \neq 0, 1$. For large W, due to time dilatation, this virtual state can be present for a period of order W. During this time, the separations of the constituent e^+e^- pair will be of order $1/W$ in the z direction, owing to Lorentz contraction, and of order 1 in the transverse directions. Thus, the electron and positron will interact simultaneously and independently with the target proton. After this interaction, the e^+e^- pair recombine to form the final photon. This entire process is nicely described by the upper "impact diagram" in Fig. 1. It is to be read from left to right.

To us the simplest analog to this process in hadronic physics would be $\gamma p \to \rho^0 p$, as indicated by the lower diagram in Fig. 1—the principal difference in the hadronic case being the $\pi^+\pi^-$ virtual state instead of the e^+e^- pair. Mainly for the sake of simplicity, we shall here ignore the contribution of diagrams representing the dissociation of the photon into four pions, six pions,.... Perhaps they are important.

Because of the presence of the strong interactions, as well as the electromagnetic, additional assumptions of impact, since the pions are near their energy shell for two distinct types are now necessary. First, before forward scattering, we assume that the photon coupling to the charged $\pi^+\pi^-$ pair is just the electric charge e. And furthermore, after impact, we treat the ρ meson as being the p wave final-state interaction of the $\pi^+\pi^-$ system as Kramer and Uretsky[3] have done in their treatment of this problem. Second, during impact, we shall assume that the amplitude for virtual πp scattering can be described as though it were dominated completely by diffractive scattering mediated by vector meson exchanges.

Before proceeding, it is instructive to recall the Glauber[4] theory of multiple scattering, since it is natural here to separate $\gamma p \to \rho^0 p$ via a $\pi^+\pi^-$ pair into a dynamical process in which only one pion scatters and another process in which both undergo scattering. Within the framework of Glauber theory, the interference contribution, due to double impact, to the

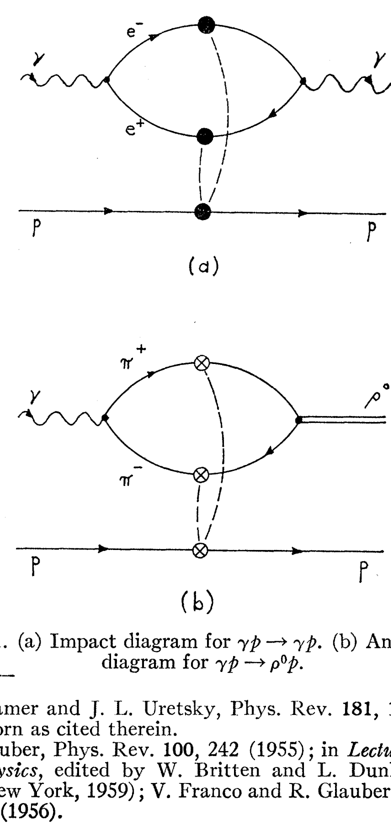

FIG. 1. (a) Impact diagram for $\gamma p \to \gamma p$. (b) Analogous diagram for $\gamma p \to \rho^0 p$.

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[1] H. Cheng and T. T. Wu, Phys. Rev. Letters **22**, 666 (1969); Phys. Rev. **182**, 1852 (1969); **182**, 1868 (1969); **182**, 1873 (1969); **182**, 1899 (1969).

[2] H. Cheng and T. T. Wu, Phys. Rev. Letters **23**, 670 (1969); Phys. Rev. D **1**, 1069 (1970); **1**, 1083 (1970).

[3] G. Kramer and J. L. Uretsky, Phys. Rev. **181**, 1918 (1969), and D. Horn as cited therein.

[4] R. Glauber, Phys. Rev. **100**, 242 (1955); in *Lectures in Theoretical Physics*, edited by W. Britten and L. Dunham (Interscience, New York, 1959); V. Franco and R. Glauber, Phys. Rev. **142**, 1195 (1956).

differential cross section in the forward direction for $\gamma p \to \rho^0 p$ would be expected to be about 35% of the single contribution, whereas in pion-deuteron scattering it is about 6%. The difference arises because the πp differential slope dominates here, and the larger target size dominates in the case of the deuteron.

The organization of this paper is the following. To introduce notation and for convenience, we give, in Sec. II, for the c.m. system, the kinematic analysis and list the relevant Cheng-Wu rules. The "diffractive assumption" which we make is represented in terms of the vector-meson propagator connecting the two parts of the impact diagram. In Sec. III, we apply these rules and calculate the differential cross section for $\gamma p \to \rho^0 p$. We then proceed in Sec. IV to compare our application of the Cheng-Wu picture with available experimental data. Obviously there is a second comparison since the analysis is also valid for $\gamma p \to \varphi p$ via a K^+K^- pair. This section then ends with consideration of an alternative assumption that the intermediate particles are a quark-antiquark pair. Section V contains the conclusions.

II. DIPION PHOTOPRODUCTION

We begin by considering the particular process $\gamma p \to \pi^+\pi^- p$ in which a photon and proton of four-momenta k and p, respectively, collide, producing a charged pion pair and proton with respective momenta q_+', q_-' and p'.

A. Kinematic Analysis

In the c.m. system, we choose coordinates so that the positive z axis is along $\hat{k}$, the direction of the incident photon, and $\hat{p}'$ for the final proton is in the xz plane. Then at high photon momentum W, the respective 4-momenta for the initial particles are

$$k=(\mathbf{0},\, W;\, W)\,,$$
$$p=(\mathbf{0},\, -W;\, W+M^2/2W)\,, \tag{2.1}$$

and for the final pion pair and proton

$$q_+'=\left[-\mathbf{\Delta}+\mathbf{q}',\, \beta W-\tfrac{1}{2}a-\frac{b}{2W};\; \beta W-\tfrac{1}{2}a-\frac{b}{2W}+\frac{(\mathbf{\Delta}-\mathbf{q}')^2+\mu^2}{2\beta W}\right],$$
$$q_-'=\left[-\mathbf{\Delta}-\mathbf{q}',\, (1-\beta)W-\tfrac{1}{2}a-\frac{b}{2W};\; (1-\beta)W-\tfrac{1}{2}a-\frac{b}{2W}+\frac{(\mathbf{\Delta}+\mathbf{q}')^2+\mu^2}{2(1-\beta)W}\right], \tag{2.2}$$
$$p'=\left(2\mathbf{\Delta},\, -W+a+\frac{b}{W};\; W-a-\frac{b}{W}+\frac{4\mathbf{\Delta}^2+M^2}{2W}\right),$$

where $\mathbf{\Delta}$, the transverse momentum transfer, is in the x direction, $\mathbf{q}'$ denotes a two-dimensional vector in the transverse plane, and $0<\beta<1$. Also μ and M are the pion and proton masses. By over-all energy-momentum conservation, a and b are determined to be

$$a=0\,,$$
$$b=\mathbf{\Delta}^2+\frac{(\mathbf{\Delta}-\mathbf{q}')^2+\mu^2}{4\beta}+\frac{(\mathbf{\Delta}+\mathbf{q}')^2+\mu^2}{4(1-\beta)}. \tag{2.3}$$

Note that the total invariant mass of the two-pion system is given by

$$m^2=(q_+'+q_-')^2\,,$$

which implies

$$4\mathbf{\Delta}^2+m^2=\beta^{-1}[(\mathbf{\Delta}-\mathbf{q}')^2+\mu^2]+(1-\beta)^{-1}[(\mathbf{\Delta}+\mathbf{q}')^2+\mu^2]. \tag{2.4}$$

Using this, we have

$$q_+'=(-\mathbf{\Delta}+\mathbf{q}',\, \beta W-(2W)^{-1}(2\mathbf{\Delta}^2+\tfrac{1}{4}m^2);\; \beta W-(2W)^{-1}(2\mathbf{\Delta}^2+\tfrac{1}{4}m^2)+(2\beta W)^{-1}[(\mathbf{\Delta}-\mathbf{q}')^2+\mu^2])\,,$$
$$q_-'=(-\mathbf{\Delta}-\mathbf{q}',\, (1-\beta)W-(2W)^{-1}(2\mathbf{\Delta}^2+\tfrac{1}{4}m^2);\; (1-\beta)W-(1/2W)(2\mathbf{\Delta}^2+\tfrac{1}{4}m^2)+[2(1-\beta)W]^{-1}[(\mathbf{\Delta}+\mathbf{q}')^2+\mu^2])\,, \tag{2.5}$$
$$p'=(2\mathbf{\Delta},\, -W+W^{-1}(2\mathbf{\Delta}^2+\tfrac{1}{4}m^2);\; W+(2W)^{-1}(M^2-\tfrac{1}{2}m^2))\,.$$

The p-wave final-state pion interaction we shall shortly identify as a ρ meson having 4-momentum

$$Q'=q_+'+q_-'=(-2\mathbf{\Delta},\, W-W^{-1}(2\mathbf{\Delta}^2+\tfrac{1}{4}m^2);\; W+m^2/4W)\,. \tag{2.6}$$

Its polarization vectors are explicitly

$$\varepsilon^*(\pm)=\mp(\sqrt{\tfrac{1}{2}})\left(1+\frac{4\Delta^2}{Wm}-\frac{4\Delta^2}{W^2},\, \mp i,\, -\frac{2\Delta}{m}+\frac{2\Delta}{W}-\frac{1}{W^2}\left(\Delta m-\frac{4\Delta^3}{m}\right);\, -\frac{2\Delta}{m}\right),$$
$$\varepsilon^*(0)=\left(-\frac{2\Delta}{m}+\frac{2\Delta}{W}-\frac{1}{W^2}\left(\Delta m-\frac{4\Delta^3}{m}\right),\, 0,\, \frac{W}{m}+\frac{1}{W}\left(\frac{1}{4}m-\frac{4\Delta^3}{m}\right)+\frac{4\Delta^2}{W^2};\; \frac{W}{m}-\frac{1}{W}\left(\frac{1}{4}m+\frac{2\Delta^2}{m}\right)\right), \tag{2.7}$$

which correspond to the standard ρ polarization vectors in the ρ rest frame

$$\varepsilon^*(\pm)=\mp(\sqrt{\tfrac{1}{2}})(1,\, \mp i,\, 0;\, 0)\,,$$
$$\varepsilon^*(0)=(0,0,1;\, 0)\,.$$

It is trivial to check that these polarization vectors in the c.m. frame are orthonormal and gauge invarient.

B. Cheng-Wu Rules and Diffractive Assumption

From the Cheng-Wu viewpoint, we discussed in the Introduction how $\gamma p \to \rho^0 p$ may be analogous to $\gamma p \to \gamma p$, the physical picture being that the incident photon virtually dissociates into two pions, instead of electrons, which simultaneously and independently interact with the target proton, and after this impact, the pions recombine to form the ρ. The additional input now to be added is the "diffractive assumption" that the amplitude for scattering can be described as though it were dominated completely by diffractive scattering mediated by vector meson exchange. Therefore, this total diffractive scattering is represented by an impact diagram having "crosslines" and "crossdots" which are to be interpreted in the c.m. frame with the following Cheng-Wu rules, which we list here for the sake of convenience:

(i) For each crossdot, write $(-i2\beta_i W)$ for positive pion lines and $(-i\gamma_0)$ for proton lines.

(ii) For each crossline write (in line with our assumption)

$$\mathcal{S}(\mathbf{\Delta}) = (2\pi)^2 \delta(\mathbf{\Delta}) - MAe^{-(a/2)\Delta^2},$$

where $A = \sigma_T/2M$, $\sigma_T = \frac{1}{2}[\sigma_T(\pi^+ p) + \sigma_T(\pi^- p)]$, and $\mathbf{\Delta}$ is the transverse momentum carried by the crossline.

(iii) There is an over-all factor of 2 (two longitudinal crossline polarizations).

For example, the covarient $\mathfrak{M}$ amplitude for πp complete diffractive scattering is then given by

$$\mathfrak{M} = -iAse^{at/2}.$$

The other Cheng-Wu rules we shall need are as follows [note that these are for our kinematics (z axis in photon direction) and Feynman rule conventions[5]]:

(iv) For each virtual particle a factor $1/2\beta_i W$.

(v) For each intermediate state to the left of crossdots, write

$$-\frac{1}{\sum_i (\mathbf{q}_i^2 + m_i^2)/2\beta_i - i\epsilon} W.$$

(vi) For each vertex write $\delta_{p,p'}(i\mathcal{L}_I)$ for 4-momentum conservation.

(vii) Integrate over all possible transverse momenta with

$$\prod_i \int \frac{d\mathbf{q}_i}{(2\pi)^2}.$$

III. ρ^0 PHOTOPRODUCTION

The impact diagram for the process $\gamma p \to \rho^0 p$ is given in Fig. 2, where the 4-momenta of the pions in the

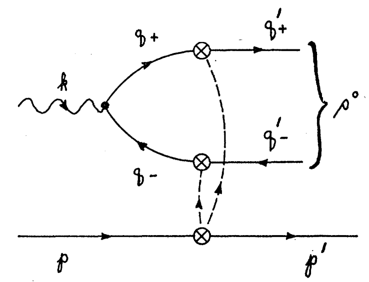

FIG. 2. Impact diagram for $\gamma p \to \rho^0 p$. The ρ^0 is seen as a final-state interaction of the particle-antiparticle pair.

intermediate state before impact are

$$q_+ = \left(\mathbf{q},\, \beta W;\, \beta W + \frac{\mathbf{q}^2 + \mu^2}{2\beta W}\right),$$
$$q_- = \left(-\mathbf{q},\, (1-\beta)W;\, (1-\beta)W + \frac{\mathbf{q}^2 + \mu^2}{2(1-\beta)W}\right). \quad (3.1)$$

Using the diffractive rules of Sec. II, we obtain the corresponding invariant amplitude

$$(-i\mathfrak{M}^{\pm}) = e\frac{4W^2\beta(1-\beta)}{(2\pi)^2 M} \times \int d\mathbf{q} \frac{\mathbf{q}\cdot\boldsymbol{\varepsilon}^{\pm}}{\mu^2 + \mathbf{q}^2} \mathcal{S}(\mathbf{q}_+' - \mathbf{q}_+)\mathcal{S}(\mathbf{q}_-' - \mathbf{q}_-), \quad (3.2)$$

where $\boldsymbol{\varepsilon}^{\pm}$ is the incident photon's transverse ($\pm$) polarization vector which we shall denote by a superscript on the invariant amplitude. The photon coupling to the pion line has been taken to be the charge e since the virtual pions are close to their energy shell for forward scattering. By analogy with the Glauber theory, it is useful both physically and mathematically to separate the single and double crossdot contributions to this amplitude. Thus,

$$\mathfrak{M} = \mathfrak{M}_S + \mathfrak{M}_D,$$
$$\mathfrak{M}_S = i8eAW^2\beta(1-\beta)e^{at/2}\left[\frac{\boldsymbol{\varepsilon}\cdot\mathbf{q}_-'}{\mu^2 + \mathbf{q}_-'^2} - \frac{\boldsymbol{\varepsilon}\cdot\mathbf{q}_-'}{\mu^2 + \mathbf{q}_+'^2}\right],$$
$$\mathfrak{M}_D = i\frac{8}{(2\pi)^2} eA^2MW^2\beta(1-\beta) \times e^{at/4} \int d\mathbf{q} \frac{\boldsymbol{\varepsilon}\cdot\mathbf{q}}{\mathbf{q}^2 + \mu^2 - i\epsilon} e^{a(\mathbf{q}'-\mathbf{q})^2}. \quad (3.3)$$

First, we shall obtain the p-wave projection of the single crossdot amplitude. This is

$$\mathfrak{M}_{S1}^{\pm} = i\left(\frac{16\pi}{3}\right)^{1/2} \frac{eA\eta}{m} s\mathfrak{A}_1^{\pm}, \quad (3.4)$$

where, in the ρ rest frame,

$$\mathfrak{A}_1^{\pm} = \mathfrak{A}_+^{\pm} Y_+^1 + \mathfrak{A}_0^{\pm} Y_0^1 + \mathfrak{A}_-^{\pm} Y_-^1 \quad (3.5)$$

reduces to an expansion in terms of the positive pion's spherical harmonics. The constant $\eta = (1 - 4\mu^2/m^2)^{1/2}$ is

[5] We follow the Feynman rule conventions as stated in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

the pion's velocity in this frame. The p-wave coefficients can be defined covariantly by

$$\mathfrak{A}_m{}^{\pm}=-\left(\frac{3}{4\pi}\right)^{1/2}\int d\Omega_+\,\varepsilon^*(m)\cdot r\,\mathfrak{A}^{\pm}\,, \tag{3.6}$$

where

$$r=\frac{q_+{}'-q_-{}'}{m\eta}\,,\quad \left(\mathbf{r}=\frac{2\mathbf{q}'}{m\eta}\right) \tag{3.7}$$

and

$$\mathfrak{A}^{\pm}=\left(\frac{3}{4\pi}\right)^{1/2}\frac{m\beta(1-\beta)}{\eta}e^{at/4}\left(\frac{\varepsilon\cdot\mathbf{q}_-{}'}{\mu^2+\mathbf{q}_-{}'^2}-\frac{\varepsilon\cdot\mathbf{q}_+{}'}{\mu^2+\mathbf{q}_+{}'^2}\right). \tag{3.8}$$

Explicitly, the coefficients are[6]

$$\begin{aligned}
&\mathfrak{A}_+{}^+=\mathfrak{A}_-{}^-=-(1+y)^{-2}(1-y+2yb)e^{at/2}\,,\\
&\mathfrak{A}_0{}^+=-\mathfrak{A}_0{}^-=-(1+y)^{-2}(2y)^{1/2}(1-b)e^{at/2}\,,\\
&\mathfrak{A}_-{}^+=\mathfrak{A}_+{}^-=0\,,
\end{aligned} \tag{3.9}$$

with

$$\begin{aligned}
&y=-t/m^2\,,\\
&b(\eta)=\frac{3}{2\eta^2}\left(1-\frac{1-\eta^2}{2\eta}\ln\frac{1+\eta}{1-\eta}\right).
\end{aligned} \tag{3.10}$$

Secondly, we find the corresponding p-wave coefficients for the double crossdot amplitude. This amplitude can be written in the following form, expected covariantly:

$$\mathfrak{M}_D{}^{\pm}=i\left(\frac{16\pi}{3}\right)^{1/2}\frac{eA\eta}{m}s\mathfrak{B}^{\pm}\,, \tag{3.11}$$

$$\mathfrak{B}^{\pm}=\left(\frac{3}{4\pi}\right)^{1/2}\frac{m^2MA\beta(1-\beta)}{8\pi}e^{at/4}\varepsilon\cdot\mathbf{r}\mathcal{P}(\mathbf{r}^2)\,, \tag{3.12}$$

where

$$\mathcal{P}(\mathbf{r}^2)=\int_0^\infty dz\left\{\left[\frac{2z}{(z^2+1)^2}\right]\cos\left(zd+\frac{cr^2z}{z^2+1}\right)+\left[\frac{1-z^2}{(z^2+1)^2}\right]\times\sin\left(zd+\frac{cr^2z}{z^2+1}\right)\right\}\exp\left(-\frac{cr^2z^2}{z^2+1}\right), \tag{3.13}$$

with

$$d=a\mu^2\quad\text{and}\quad c=\tfrac{1}{4}am^2\eta^2.$$

This form for $\mathcal{P}(\mathbf{r}^2)$ was obtained by exponentiating the denominater, completing the square of the exponent, and then doing the integration over the transverse momentum. By definition,

$$\mathfrak{B}_m{}^{\pm}=-\left(\frac{3}{4\pi}\right)^{1/2}\int d\Omega_+\,\varepsilon^*(m)\cdot r\mathfrak{B}^{\pm}\,. \tag{3.14}$$

This can be simplified further, using

$$\int d\Omega_+=\frac{2}{\eta}\int\frac{d^3q_+}{q_+{}^0}\frac{d^3q_-}{q_-{}^0}\delta(\tilde{q}_++\tilde{q}_-)\delta(Q^0-q_+{}^0-q_-{}^0)\,, \tag{3.15}$$

where the primes on the 4-momenta have been suppressed, and $q_i{}^0=(\mu^2+\tilde{q}_i{}^2)^{1/2}$ with $\tilde{q}_i$ denoting the 3-momenta:

$$\int d\Omega_+=-8\eta\int_0^1 rdr\int_0^{2\pi}d\varphi\frac{(1-\beta)}{\mathfrak{D}}\,, \tag{3.16}$$

which can be obtained by introducing the variables

$$\begin{aligned}
&K=q_++q_-\,,\\
&L=q_+-q_-=m\eta r\,.
\end{aligned} \tag{3.17}$$

Here,

$$\mathfrak{D}=4(1-\beta)^2(1+y)-y-2y^{1/2}\eta r\cos\varphi-\eta^2r^2+\eta^2-1 \tag{3.18}$$

and

$$\beta=[-f+(f^2-2ge)^{1/2}]/2e\,, \tag{3.19}$$

with

$$\begin{aligned}
&e=1+y\,,\\
&f=-1-y+y^{1/2}\eta r\cos\varphi\,,\\
&4g=y-2y^{1/2}\eta r\cos\varphi+\eta^2r^2+1-\eta^2.
\end{aligned}$$

Note that, because of symmetry between the two pions, the range of β now is $\frac{1}{2}\leq\beta<1$. Finally, then,

$$\beta_m{}^{\pm}=-\frac{3\sigma_Tm^2\eta}{8\pi^2}e^{at/4}\int_0^1 rdr\,\mathcal{P}(r^2)\times\int_0^{2\pi}d\varphi\,\varepsilon^*(m)\cdot r\varepsilon^{\pm}\cdot r\frac{\beta(1-\beta)}{\mathfrak{D}}\,, \tag{3.20}$$

with

$$\begin{aligned}
&\varepsilon^{\pm}\cdot r=\pm(\sqrt{\tfrac{1}{2}})re^{\pm i\varphi}\,,\\
&\varepsilon^*(\pm)\cdot r=\pm(\sqrt{\tfrac{1}{2}})re^{\mp i\varphi}\pm(\sqrt{\tfrac{1}{2}})y^{1/2}\left(\frac{2\beta-1}{\eta}\right),\\
&\varepsilon^*(0)\cdot r=y^{1/2}r\cos\varphi-\frac{1}{\eta}[\beta(1-y)-1]-\frac{1}{4\eta(1-\beta)}\times[y+2y^{1/2}r\eta\cos\varphi+\eta^2r^2+1-\eta^2].
\end{aligned} \tag{3.21}$$

Note that only the real part of $\varepsilon^*(m)\cdot r\varepsilon^{\pm}\cdot r$ contributes to $\mathfrak{B}_m{}^{\pm}$ since β and $\mathfrak{D}$ are even functions of φ, and that

$$\mathfrak{B}_+{}^+=\mathfrak{B}_-{}^-\,,\quad\mathfrak{B}_-{}^+=\mathfrak{B}_+{}^-\,,\quad\mathfrak{B}_0{}^+=-\mathfrak{B}_0{}^-. \tag{3.22}$$

Having determined the p-wave projections, we must now include the final-state interaction of the pion pair in order to obtain the differential cross section for ρ^0 photoproduction. To do this, we adopt the viewpoint that the final-state π-π interactions in $e^++e^-\to\rho^0\to\pi^++\pi^-$ and in ρ^0 photoproduction are the same, and

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[6] One way to obtain the single crossdot projection coefficients is to rewrite covariantly the amplitude

$$\mathfrak{M}_s=ieA\exp(\tfrac{1}{2}at)\left(s_-\frac{\epsilon\cdot q_+{}'}{k\cdot q_+{}'}-s_+\frac{\epsilon\cdot q_-{}'}{k\cdot q_-{}'}\right),$$

where $s_{\pm}=(p+k-q_{\pm}{}')^2$. Then evaluate it in the ρ frame where $\mathbf{k}$ is still in the z direction and $\mathbf{p}$ and $\mathbf{p}'$ are in the xz plane. Use spherical coordinates for the pions. That is, reexpress the above amplitude using

$$k=\tfrac{1}{2}m(1+y)\,(0,0,1;\,1),$$
$$q_{\pm}{}'=\tfrac{1}{2}m(\pm\eta\cos\varphi\sin\theta,\,\pm\sin\varphi\sin\theta,\,\pm\eta\cos\theta;\,1),$$

and

$$p=p'=\frac{s}{2m}\left(\frac{2y^{1/2}}{1+y},\,0,\,\frac{y-1}{y+1};\,1\right)$$

correct to leading order.

proceed to make use of the enhancement factor $F(m^2)$, obtained by Gounaris and Sakurai[7] to describe the former. Near the ρ mass, this pion form factor

$$F(m^2) \simeq \frac{m_\rho^2}{m^2 - m_\rho^2 - i m_\rho \Gamma_\rho (\eta/\eta_\rho)^3 (m/m_\rho)^2}. \quad (3.23)$$

Thus, whereas the differential cross section for pion production is

$$\frac{d\sigma^4}{dt dm^2 d\Omega_+} = \frac{\eta M^2}{(4\pi)^4 s^2} \left(\tfrac{1}{2} \sum_{\lambda=\pm} |\mathfrak{M}^\lambda|^2\right), \quad (3.24)$$

that for ρ photoproduction is

$$\frac{d\sigma}{dt} = \frac{\eta M^2}{(4\pi)^4 s^2} \int d\Omega_+ \int dm^2 |F(m^2)|^2 \left(\tfrac{1}{2} \sum_{\lambda=\pm} |\mathfrak{M}_1^\lambda|^2\right) \quad (3.25)$$

$$= \frac{\alpha \sigma_T^2}{g_{\rho\pi\pi}^2} \Pi \Big|_{m^2 = m_\rho^2} \quad (3.26)$$

in the narrow-width approximation, where

$$\Pi = \tfrac{1}{2} \sum_{\lambda=\pm} (\Pi_A{}^\lambda + \Pi_{AB}{}^\lambda + \Pi_B{}^\lambda), \quad (3.27)$$

with

$$\Pi_m{}^\pm|_A = |\mathfrak{A}_m{}^\pm|^2,$$
$$\Pi_m{}^\pm|_{AB} = 2\mathfrak{A}_m{}^\pm \mathfrak{B}_m{}^\pm, \quad (3.28)$$
$$\Pi_m{}^\pm|_B = |\mathfrak{B}_m{}^\pm|^2.$$

Our notation is $A \sim$ single crossdot, $B \sim$ double crossdot, and $AB \sim$ interference contributions, respectively. The coupling constant $g_{\rho\pi\pi}$ is related to the ρ width by

$$\Gamma_{\rho\to\pi\pi} = g_{\rho\pi\pi}{}^2 m_\rho \eta^3 / 48\pi. \quad (3.29)$$

IV. COMPARISON WITH AVAILABLE DATA

A. ρ^0 Photoproduction

Having with some additional assumptions applied the Cheng-Wu physical picture to ρ^0 photoproduction, we now wish to compare the result with the experimental data to test our ideas. While the result is expected to be valid only at asymptotically large energies, we would hope that it would be relevant at present accelerater energies even though the only energy dependence of the result is through the $\pi^\pm p$ total cross section and slope. Therefore, we take from the $\pi^\pm p$ scattering data the values $\sigma_T = 23$ mb ($\pi^+ p = \pi^- n$, 60 GeV, IHEP-CERN)[8] and $a = 7$ GeV^{-2} [average $\pi^+ p$ ($\pi^- p$), 16.7 (17) GeV, Foley *et al.*],[9] and $\Gamma_{\rho\to\pi\pi} = 125$ MeV.[10] For comparison with our expectations based on the Glauber theory, we have plotted in Fig. 3 the absolute magnitude

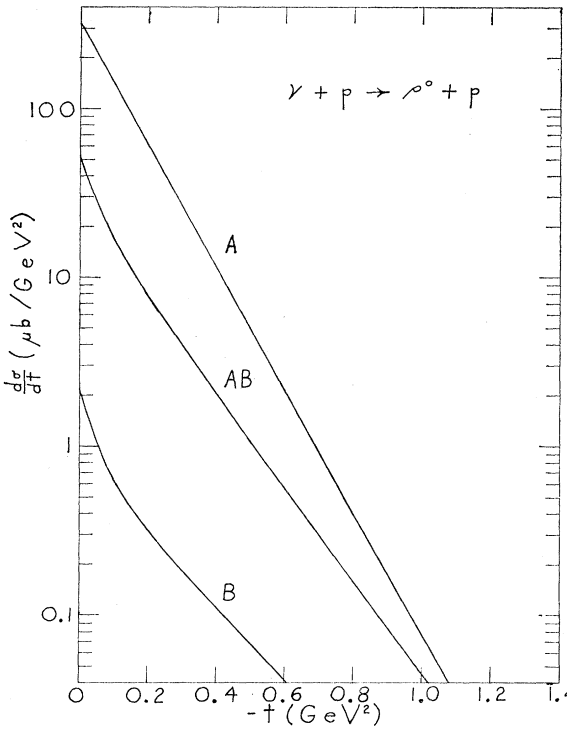

FIG. 3. The magnitude of the theoretical components of the asymptotic differential cross section for $\gamma p \to \rho^0 p$ as a function of t. Curve A represents the contribution of the single crossdot amplitude. Curve AB is the contribution of the interference of the single and double crossdot processes and is of opposite sign. Curve B represents the contribution of the double crossdot amplitude.

of the resulting contributions to $d\sigma/dt$ ($\gamma p \to \rho^0 p$) from the single crossdot, interference, and double crossdot terms. As anticipated, the double crossdot amplitude is 180° out of phase with the single amplitude. In the extreme forward direction, the interference term is only 16% of the single crossdot contribution and it remains significantly smaller at wider angles, so there will not be dip in the total differential cross section in the region of interest. The smallness is surprising.

The total, then, is plotted in Fig. 4 for comparison with the recent SLAC data[11] at 16 GeV. It is not favorable, the prediction being in magnitude 275 μb/GeV^2 at $t \simeq 0$, or about four times greater than the data, and the slope for small t values is somewhat steeper than the data. Note that in making this application we assumed that the intermediate particles are pions.

B. φ Photoproduction

Merely by reinterpretation of the relevant quantities, the result obtained in the preceding sections is applicable to $\gamma p \to \varphi p$ via a K^+K^- pair. Other than the obvious changes, $\mu_\pi \to \mu_K$, $m_\rho \to m_\varphi$, $\Gamma_{\rho\to\pi\pi} \to \Gamma_{\varphi\to K^+K^-}$, $\sigma_T(\pi p) \to \sigma_T(Kp)$, there is a modification of the final-state interaction factor $F(M^2)$. We use the form which would be correct in the limit of asymptotic $SU(3)$

[7] G. J. Gounaris and J. J. Sakurai, Phys. Rev. Letters **21**, 244 (1968).
[8] J. A. Allaby *et al.*, IHEP-CERN Collaboration, Phys. Letters **30B**, 500 (1969).
[9] K. J. Foley *et al.*, Phys. Rev. Letters **11**, 425 (1963); **15**, 45 (1965). For a compilation of elastic meson-nucleon scattering data, see Y. Sumi, Progr. Theoret. Phys. (Kyoto) Suppl. **41-42**, 3 (1967).
[10] Particle Data Group, Rev. Mod. Phys. **41**, 87 (1970).
[11] R. Anderson *et al.*, Phys. Rev. D **1**, 27 (1970).

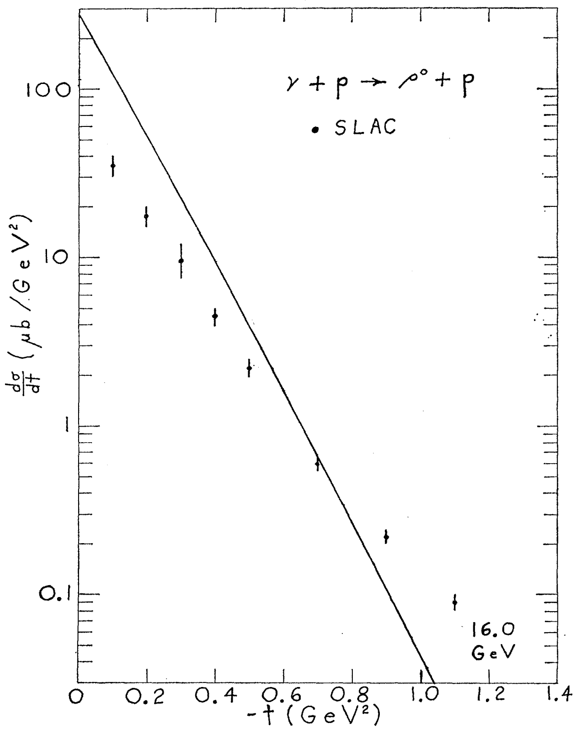

FIG. 4. The differential cross section for $\gamma p \to \rho^0 p$ versus t. The solid curve is the parameter-free, asymptotic prediction of the theory assuming a $\pi^+\pi^-$ intermediate pair. The experimental data are from Ref. 11.

symmetry

$$F(m^2)=\frac{1}{9}\frac{m_\varphi^2}{m_\varphi^2-m^2-im_\varphi\Gamma_\varphi(\eta/\eta_\varphi)^3(m/m_\varphi)^2}. \quad (4.1)$$

Therefore, in the narrow-width approximation the differential cross section for $\gamma p \to \varphi p$ is

$$\frac{d\sigma}{dt}=\frac{\alpha^2\sigma_T^2}{9g_{\varphi K\bar{K}}}\Pi\bigg|_{m^2=m_\varphi^2}. \quad (4.2)$$

From the $K^\pm p$ data, we take $\sigma_T=17$ mb (K^+p, 20 GeV, Galbraith *et al.*)[12] and $a=6.4$ GeV^{-2} [average K^+p (K^-p), 14.8 (15.9) GeV, Foley *et al.*],[13] and use a φ width[10] of $\Gamma_{\varphi\to K\bar{K}}=1.8$ MeV appropriate for a $\varphi \to K\bar{K}$ branching ratio of 45.5%. We find the relative contributions of the single and double crossdot amplitudes to be nearly the same as in the ρ case so we have not plotted them. The predicted total distribution is plotted in Fig. 5 along with the 16.0-GeV SLAC data.[11] Again it is not favorable, the predicted value in the forward direction being 30 μb/GeV^2.

C. Quark-Model Interpretation

The comparison with experimental data for $\gamma p \to \rho^0 p$ and φp is not favorable for the assumption that the initial photon dissociates into two pions (kaons) which impact with the target. Rather than leave matters as they stand, let us note what happens under an alternative assumption based on the quark picture for high-energy scattering.

Suppose that the incident photon, instead of dissociating into two pions or kaons, dissociates into a quark-antiquark pair which interacts with the proton and then recombines to form a ρ. Because the q-p and $\bar{q}$-p effective total cross sections are half that of $\pi^\pm p$, the single crossdot contribution to the total differential cross section is thereby reduced by a factor of 4 in agreement with experiment for $\gamma p \to \rho^0 p$. But our result for the differential cross section also depends on the mass of the intermediate particles which undergo scattering, so we must choose an *effective mass* for the bound quarks. Since small momentum transfers are involved here as in many other applications of the quark model, such an effective mass is the property of a bound quark averaged over lengths and times of the order $|M|^{-1}$ and is rather independent of the properties of a free quark. To be faithful to previous successful applications of the quark model to high-energy scattering, we could simply take an *effective* quark mass of one-third the mass of the proton. This, for instance, is the case in the approach of Van Hove and Kokkedee.[14] However, the relevant

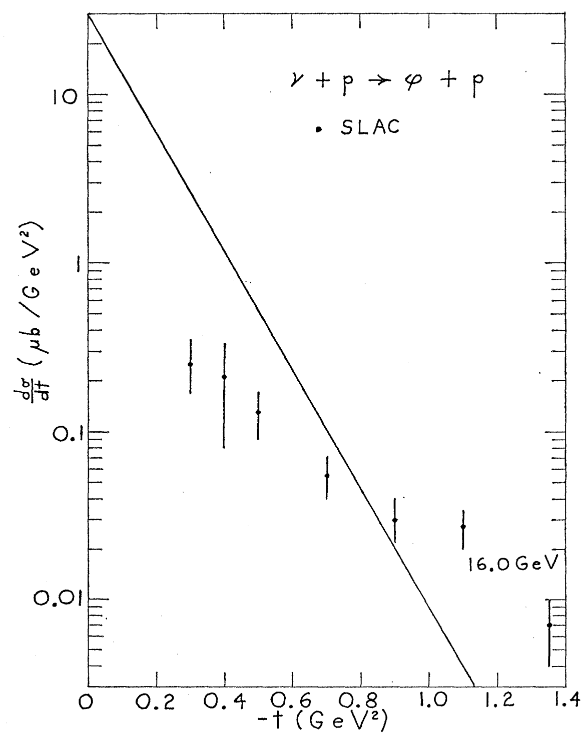

FIG. 5. The differential cross section for $\gamma p \to \varphi p$ versus t. The solid curve is the parameter-free, asymptotic prediction of the theory assuming a K^+K^- intermediate pair. The experimental data are from Ref. 11.

[12] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965).
[13] K. J. Foley *et al.*, Phys. Rev. Letters **11**, 503 (1963); **15**, 45 (1965).
[14] L. Van Hove, in *Particle Physics at High Energy*, edited by T. W. Priest and L. L. J. Vick (Oliver and Boyd, Edinburgh, 1967); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42**, 711 (1966).

assumptions of their procedure also follow from the Cheng-Wu picture if for small momentum transfer we average, in the above sense, the quark energy-momentum four-vectors and assume additivity is valid for the energy; i.e., the three quarks in the baryon and two in the meson carry all the energy. Then (1) additivity holds for both momentum and energy, and (2) the average energy-momentum four-vectors of the quarks are proportional to that of the hadron containing them with a factor of $\frac{1}{3}$ for a baryon and $\frac{1}{2}$ for a meson. This value for the effective mass of the quark also agrees with the magnitude of the baryon magnetic moments under the assumption of additivity.[15] Finally, if free quarks do indeed exist, there are model-dependent calculations indicating that scalar binding might give such a low quark effective mass.[16]

Plotted, then, in Fig. 6 is the resulting prediction for $\gamma p \to \rho^0 p$ via a $q\bar{q}$ pair, assuming a quark effective mass of one-third the mass of the proton. Plotted with the prediction are the DESY data[17] at 2.5–3.5 GeV and the SLAC data[11] at 6.5 and 16.0 GeV. This is a satisfactory asymptotic cross section, quite reminiscent of

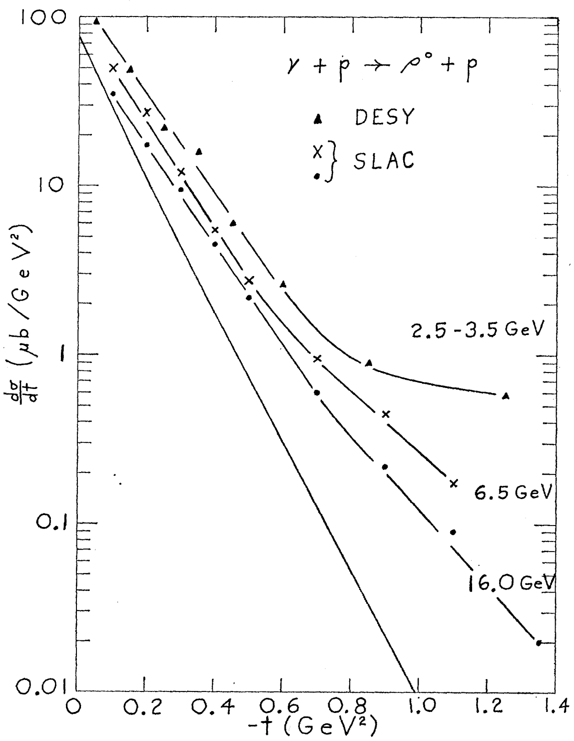

FIG. 6. The differential cross section for $\gamma p \to \rho^0 p$ versus t. The solid curve is the parameter-free, asymptotic prediction of the theory assuming a quark-antiquark intermediate pair. The experimental data are from Refs. 11 and 17.

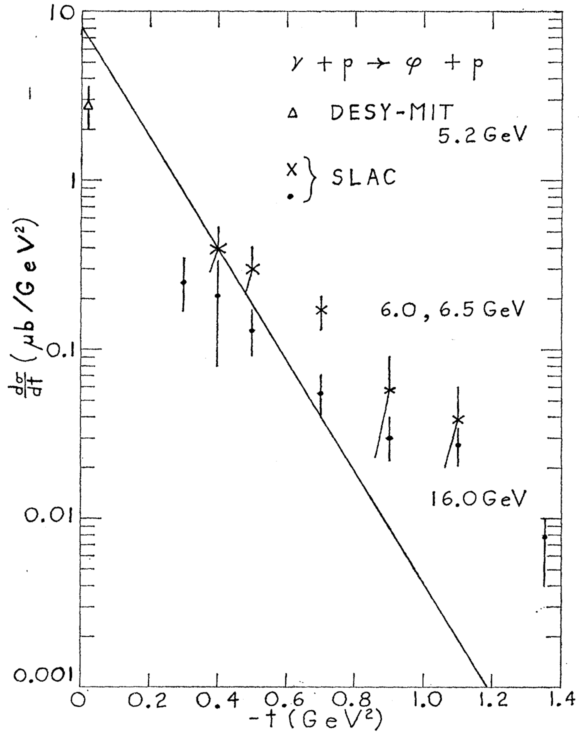

FIG. 7. The differential cross section for $\gamma p \to \varphi p$ versus t. The solid curve is the parameter-free, asymptotic prediction of the theory assuming a quark-antiquark intermediate pair. The experimental data are from Refs. 11 and 20.

the asymptotic pp differential cross section calculated by Durand and Lipes from the Chou-Yang model.[18]

In the extreme forward direction, the differential cross section is 76 μb/GeV^2 and the interference term is 7% of the single crossdot term. The fact that the double crossdot contribution is small, even via interference with the single crossdot amplitude, justifies the "additivity" hypothesis in the quark model. On the other hand, in the past, calculations of double scattering corrections to additivity for other processes have been carried out by the eikonal method introduced by Glauber. However, often the results obtained by this alternate method have tended to go in the wrong direction relative to experimental data.[19]

We have plotted in Fig. 7 the analogous prediction for $\gamma p \to \varphi p$ along with the SLAC data[11] at 6.0, 6.5, and 16.0 GeV as well as the DESY-MIT counter experiment's forward point[20] at 5.2 GeV. In the extreme forward direction, the height is 8 μb/GeV^2 and the interference term is 8% the single crossdot's. While the proper interpretation is not clear, it seems that the

[15] W. E. Thirring, Phys. Letters **16**, 335 (1965); Acta Phys. Austriaca, Suppl. **II**, 205 (1966); C. Becchi and G. Morpurgo, Phys. Letters **17**, 352 (1965); Phys. Rev. **140**, B687 (1965).

[16] N. N. Bogoliubov, B. V. Struminskij, and A. N. Tavkhelidze, JINR Report No. D-1968 (1965); H. J. Lipkin and A. N. Tavkhelidze, Phys. Letters **17**, 331 (1965); O. W. Greenberg, *ibid.* **19**, 423 (1965).

[17] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München-Collaboration, Phys. Rev. **175**, 1669 (1968).

[18] T. T. Chou and C. N. Yang, Phys. Rev. Letters **20**, 1213 (1968); L. Durand III and R. Lipes, *ibid.* **20**, 637 (1968).

[19] D. R. Harrington, Rutgers University report, 1967 (unpublished); V. Franco, Phys. Rev. Letters **18**, 1159 (1967); M. V. Barnhill, Phys. Rev. **163**, 1735 (1967); D. R. Harrington and A. Pagnamenta, *ibid.* **173**, 1599 (1968); N. W. Dean, Nucl. Phys. **B4**, 534 (1968); **B7**, 311 (1968).

[20] J. G. Ashburg *et al.*, as cited by S. C. C. Ting, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki (CERN, Geneva, 1968).

experimental forward height should increase if the asymptotic prediction is correct and the $K^{\pm}p$ parameters are representative of their "asymptotic" values.

V. CONCLUSION

In this paper, the Cheng-Wu physical picture, with some additional assumptions, has been applied to $\gamma p \rightarrow \rho^0 p$ and φp, and comparison has been made with available experimental data. The assumptions which we have made regarding the strong interactions are perhaps naive but we think simple. Moreover, the resulting amplitude is purely imaginary and qualitatively satisfies the properties one anticipates from the Glauber theory of multiple scattering.

The comparison is not favorable for the assumption that the intermediate particles are two pions (kaons). Of course, we have not investigated the contribution of diagrams representing the dissociation of the photon into four pions, six pions, . . . , the so called "tower diagrams" which may be important even in electrodynamics.[21]

Next, we considered an alternative assumption that the intermediate particles are quarks, since these objects have previously been a useful idea in analyzing high-energy scattering. Thereby, the asymptotic prediction for $\gamma p \rightarrow \rho^0 p$ is consistent with extrapolation of present data. In addition, under the latter assumption there is a justification for the additivity hypothesis of the quark picture of high-energy scattering.

There are several points which require further investigation: Our kinematic analysis here has assumed the intermediate virtual particles to be lighter than the final vector meson and to be spinless. To study the dependence of the application on these two items, we are now calculating the differential cross section for the so-called "fisheye" diagram for both spinless and spin-$\frac{1}{2}$ particles. This will also enable a different treatment of the final-state interaction, a general subject which will remain relevant at very high energies. Of direct interest is a similar application to photoproduction of vector mesons from nuclei. Finally, there are the possibilities of a more detailed approach and the inclusion of non-diffractive exchanges.

[21] H. Cheng and T. T. Wu, Phys. Rev. D 1, 467 (1970), and papers II and III of this series as cited therein.

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Spectral-Function Sum Rules for ρ-ω and ρ-ϕ Transitions*

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Assuming the field algebra of Lee, Weinberg, and Zumino, we point out that two spectral-function sum rules remain valid when the electromagnetic interaction is turned on. From these sum rules we derive formulas for calculating the electromagnetic ρ-ω and ρ-ϕ transitions. A tentative numerical estimate gives a value for the ρ-ω transition close to quark-model predictions, but an opposite sign for the ρ^+-ρ^0 mass splitting (-7.5 MeV). The branching ratio $\Gamma(\omega \rightarrow 2\pi)/\Gamma(\omega \rightarrow 3\pi)$ is estimated to be $\sim 1\%$, and the phase of the $\omega \rightarrow 2\pi$ amplitude relative to the $\rho \rightarrow 2\pi$ amplitude is between 80° and 130°.

I. INTRODUCTION

The decay mode $\omega \rightarrow 2\pi$ has recently been measured through the reaction $\pi^+ p \rightarrow \pi^+\pi^-\Delta^{++}$,[1,2] the e^+-e^- colliding-beam experiment,[3] and the $p\bar{p}$ annihilation process.[4] The e^+-e^- annihilation experiment, which measured the phase of the decay amplitude too, does not seem to be in good agreement with theoretical analysis[5] based upon an electromagnetic ρ-ω transition. Together they may imply a violation of charge independence which is nonelectromagnetic in origin.[6] To analyze this reaction, it is very important to have a reliable estimate of the ρ^0-ω mixing due to the electromagnetic interaction. The $SU(3)$ symmetry alone is not capable of determining the ρ-ω transition until one has

* Research supported in part by the U. S. Atomic Energy Commission.

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[1] G. Goldhaber, W. R. Butler, D. G. Coyne, B. H. Hall, J. N. MacNaughton, and G. H. Trilling, Phys. Rev. Letters 23, 1351 (1969).

[2] For an earlier experiment, see S. M. Flatté, D. C. Huwe, J. J. Murray, J. Button-Schaffer, F. T. Solmitz, M. L. Stevenson, and C. Wohl, Phys. Rev. 145, 1050 (1966). There is also a negative evidence for the $\omega \rightarrow 2\pi$ decay mode in the $\pi^- p$ reaction [W. Lee (private communication)].

[3] J. E. Augustin, J. C. Bizot, J. Buon, J. Haissinski, D. Lalanne, P. C. Marin, J. Perez-y-Jorba, F. Rumpf, E. Silva, and S. Tavernier, Phys. Rev. Letters 20, 126 (1968); Phys. Letters 28B, 517 (1969); and Nuovo Cimento Letters 2, 214 (1969).

[4] W. W. M. Allison, W. A. Cooper, F. Fields, and D. S. Rhines, Phys. Rev. Letters 24, 618 (1970).

[5] M. Gourdin, L. Stodolsky, and F. M. Renard, Phys. Letters 30B, 347 (1969).

[6] However, this is not conclusive according to a different analysis by R. G. Sachs and J. F. Willemson, Phys. Rev. D 2, 133 (1970).