Theoretical Estimates of Quark Production Cross Sections at Ultrahigh Energies*

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We discuss various theoretical models for the production of quarks at ultrahigh energy. In particular, a well-defined temperature model (suggested by Yang sometime back) is used to illustrate the type of production cross section expected in this picture for a given quark mass and c.m. energy.

I. INTRODUCTION

RECENTLY McCusker and Cairns and others[1] have revived the intriguing possibility of the experimental existence of heavy-particle quarks with fractional charges. While these findings remain tentative,[2] it is perhaps useful at this stage to both update and extend the (equally) tentative theoretical situation concerning quark production cross-section estimates at high energies. Hopefully they can serve the purpose of stimulating the field of study.

The primary *raison d'être* for postulating heavy-particle triplets is of course rooted in the success of $SU(3)$ symmetry. It is perhaps fair to say that $SU(3)$ symmetry is well established as an approximate internal symmetry of strong interactions. It is correct to about 10%, which is as much as we can generally hope for. The evidence that the strong interactions are approximately invariant under an $SU(3)$ transformation is based in a *very* important part on the remarkably accurate Gell-Mann–Okubo mass formulas. Predictions of the existence of the η^0 meson and the Ω^- hyperon before their experimental discovery, on the basis of these formulas, play a critical role in increasing our confidence in the existence of a broken unitary symmetry higher than isospin symmetry.

In the derivation of $SU(3)$ mass formulas, one presumes, among other assumptions, that the symmetry-violating interactions are small. On the other hand, since the mass differences among, say, K, η, and π, which are members of the same octet multiplet, are *not* small compared to their actual masses, the violation of $SU(3)$ symmetry is apparently *not* weak. From this viewpoint, an important and relevant question to ask is whether basic heavy particles exist with a higher mass scale and strength of interaction [invariant under $SU(3)$], such that the mass formulas involving the known particles can be usefully described in terms of a *small perturbation* on the fundamental entities; in other words, to restore soundness to the derivation of the mass formulas, the success of which is after all the real reason why one believes $SU(3)$ in the first place. Because of the rather qualitative nature of our ability to do dynamical calculations in strong interactions, it may not be easy to construct a completely satisfactory dynamical model such that the well-established mass formulas involving the known hadrons arise naturally as compounds of the basic heavy particles. However, the conceptual picture appears quite clear and solid. In addition (as indicated below), the accuracy of the $SU(3)$ mass formulas themselves can be turned around to give an estimate of the mass scale of the heavy particles as something in the neighborhood of 10 BeV. We might add here that if we become accustomed to think in terms of a higher mass scale for the basic triplets, one order of magnitude more massive than the presently known hadrons, in order to establish the pattern for $SU(3)$ symmetry, then we should not be surprised if a similar mass scale exists relative to weak interactions. Hence a W meson (especially one with pairwise strong interactions with normal hadrons[3]) of higher mass scale (say, $\sim$10 BeV) need not be an unfamiliar feature of high-energy interactions.

Mass of triplets. In the absence of detailed calculation procedures, the higher mass scale for triplets can only be inferred heuristically from plausibility-type arguments and thus they are good only as an order-of-magnitude estimate. For instance, it has been suggested[4] that the accuracy of the mass formulas themselves gives us some semiquantitative indications about the higher mass scale M of the basic heavy triplet. It seems reasonable for, say, the *baryon octet*, that the mass formulas are linear in the perturbation parameter $\lambda = m/M \ll 1$, where m can be of the order of the nucleon mass. The accuracy of the octet baryon formulas is of the order of 1%. Hence the correction term (ignored in first-order mass formulas) is $(m/M)^2 \sim 1\%$, or $m/M \sim \frac{1}{10}$; hence $M = 10$ BeV.

There is another argument: Gürsey, Lee, and Nauenberg[5] have pointed out that the ratio of the Gamow-

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[1] C. B. A. McCusker and I. Cairns, Phys. Rev. Letters **23**, 658 (1969); W. T. Chu, Y. S. Kim, W. J. Beam, and N. Kwak, *ibid.* **24**, 917 (1970); I. Cairns *et al.*, Phys. Rev. **186**, 1394 (1969).

[2] R. K. Adair and H. Kasha, Phys. Rev. Letters **23**, 1355 (1969); H. Frauenfelder, U. E. Kruse, and R. D. Sard, *ibid.* **24**, 33 (1970); D. C. Rahm and R. I. Louttit, *ibid.* **24**, 279 (1970); A. W. Wolfendale (private communication).

[3] S. Pepper, C. Ryan, S. Okubo, and R. E. Marshak, Phys. Rev. **137**, B1259 (1965); C. Callan, Phys. Rev. Letters **20**, 809 (1968); **20**, 1134(E) (1968); S. Pakvasa, S. F. Tuan, and T. T. Wu, *ibid.* **20**, 1546 (1968); S. F. Tuan, Ann. Phys. (N. Y.) **59**, 80 (1970).

[4] T. D. Lee (private communication).

[5] F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964).

Teller coupling constant G_A to the Fermi coupling constant G_V can be of the form

$$G_A/G_V=-1+O(m/M)=-1.15\pm0.02\,,$$

which places M in the range 7–10 BeV. The form of the above equation is suggested by the zero-nucleon-mass limit, when we are back to a "neutrino-type nucleon" with $G_A/G_V=-1$; $O(m/M)$ then represents the effect of strong-interaction renormalization when the nucleon mass is switched on. Note, as pointed out by Treiman,[6] that the same consideration does not necessarily apply to $O(m_\pi/M)$ where, in the limit of zero pion mass, G_A retains a possibly small renormalization effect and $G_A\neq -G_V$.

The quark model is the simplest of such triplet schemes and is based on a single triplet, the quark q_α ($\alpha=1, 2, 3$), first proposed by Gell-Mann and Zweig.[7] The properties of the quark are listed for convenience in Table I, and require fractional charges for its members. It offers the most direct interpretation of the gross features of level classification for the known meson and baryon states as compounds of $(q\bar{q})$ and (qqq) (in a para-Fermi statistics picture) with L and radial excitations.[8]

Despite its simplicity, the quark picture introduces some conceptual questions, as first emphasized by Lee,[9] to wit, in the simplest quark structure, the charges are $2e_q$, $-e_q$, $-e_q$ ($e_q=\frac{1}{3}e$), where e_q is related to the observed charge of the proton e_p ($=e$) by

$$e_p=3e_q.$$

Similarly for the baryon number $N_p=3N_q$, where N_p and N_q are the baryon numbers of proton and quark, respectively. These equations present no difficulty, by themselves, since certainly if we regard He^4 as having "unit" baryon number then $N_p=\frac{1}{4}$ is no great surprise. However, the leptons which have no strong interactions are not composites of the quarks. Both the leptons and the quarks are regarded as fundamental particles. It appears strange that the basic unit of charge e_l for the leptons should be three times that of the quarks,

$$e_l=3e_q.$$

This is difficult to understand if the electron or muon is believed to be a fundamental particle. Of course, one might point out that the realistic quark model was motivated largely by $SU(3)$-symmetry considerations for strong interactions, and has essentially nothing to say about these leptons with no strong interactions—other than the trivial statement that they are singlets under $SU(3)$. In fact the symmetry of the leptons is an $U_2\times U_2$ symmetry.[10] The advantage of the quark model is that it does take advantage of maximal simplicity, especially for classification of states. The complexities involved in maintaining integral charge values for the subunits require two or more triplets[9,11] or a triplet together with a singlet.[9] They are likely in general to introduce a far richer pattern of hadron states than appears to be warranted.

TABLE I. Principal quantum numbers of the quark triplet q_α ($\alpha=1, 2, 3$). Here Q is the charge, B is the baryon number, S is the strangeness, J is the spin, and I is the isospin.

	Q/e	B	S	J	I
q_1	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{1}{2}$	$\frac{1}{2}$
q_2	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{2}$	$\frac{1}{2}$
q_3	$-\frac{1}{3}$	$\frac{1}{3}$	-1	$\frac{1}{2}$	0

Another problem which is inherent in any nonrelativistic molecular picture of the low-lying hadrons as compounds of heavy triplets or quarks is the *tight-binding* problem. For instance, in terms of the quark model, where the pion is a bound state of $q\bar{q}$ (with $m_q\sim 10$ BeV), is the motion of the constituent quarks a relativistic one (as in positronium) or a nonrelativistic one (as in the nucleus)?

It is common to assume that the range of force between m_q and $m_{\bar{q}}$ should be typically of Compton wavelength $1/m_{\bar{q}}$ in natural units. If this were so, the motion of m_q-$m_{\bar{q}}$ is necessarily relativistic, because the uncertainty in momentum Δp is of order m_q, so $(\Delta p/m_q)^2\sim 1$—a fully relativistic situation. A second circumstance is that the virial theorem tells us that $\langle T\rangle\sim -\frac{1}{2}\langle V\rangle$. Since the potential energy is necessarily of the order of the rest energy of particles which are bound (typically $\sim 2m_q$), then the kinetic energy must also be of this same order of magnitude, again fully relativistic. This is, of course, less than satisfactory for a realistic picture of heavy quarks bound in the fashion of molecular physics, whose orbital L excitations give the pattern of mesonic levels $^1S_0$, $^3S_1$ $(L=0)$; $^3P_2$, $^3P_1$, $^3P_0$, $^1P_1$ $(L=1)$, etc.

A conceptual solution,[12] which maintains the nonrelativistic motion for heavy real quarks, would be to propose that the range of force between m_q-$m_{\bar{q}}$ is characterized *not* by the Compton wavelength $1/m_q$, but by some mediating strength associated with the exchange of some intermediary of range $\sim 1/m_V$, where $m_V\ll m_q$ and may be typically of order 1 BeV. Then $(\Delta p/m_q)^2\approx(m_V/m_q)^2\ll 1$ and hence is *nonrelativistic*. This is not an impossible situation even though the exact nature of the intermediary m_V is not yet clear. Dalitz[13] has illustrated, with the Blankenbecler-Sugar equation, examples with flat-bottom potentials with roughly these properties, though the constructions obtained so far are not yet ones of direct physical interest.

[6] S. B. Treiman (private communication).
[7] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN report, 1964 (unpublished).
[8] R. H. Dalitz, in *Proceedings of the International Conference on Symmetries and Quark Models* (Gordon and Breach, New York, 1970).
[9] T. D. Lee, Nuovo Cimento **35**, 933 (1965).
[10] T. D. Lee, Nuovo Cimento **35**, 945 (1965).
[11] M. Y. Han and Y. Nambu, Phys. Rev. **139**, 1006 (1965).
[12] G. Morpurgo, Physics **2**, 95 (1965).
[13] R. H. Dalitz, in *Proceedings of the Second Hawaii Topical Conference in Particle Physics* (University of Hawaii Press, Honolulu, 1968), p. 348.

An ancillary question is whether possible two-body attractive systems (qq) and $(\bar{q}\bar{q})$ (which can, for instance, have charges $\pm 4/3$, $\pm 2/3$, etc.) may form tightly bound quark states of relatively low mass (say, ~ 2 BeV each) and hence become amenable to detection by present-day accelerators in reactions such as

$$p+p \to p+p+(qq)+(\bar{q}\bar{q})+\cdots.$$

While this may be an open possibility, the simplest bond picture[14] (with bond energy B) would suggest, to the contrary, that the low-lying baryons have energy $3m_q-3B$ and hence $B\approx m_q$. The (qq) [and $(\bar{q}\bar{q})$] system has then mass $2m_q-B\approx m_q$ and thus is comparable to the mass of the single heavy quark.

With the above background information on heavy triplets or quarks, we discuss in Sec. II various production mechanisms for producing these heavy objects. In Sec. III, a specific temperature model,[15] which essentially carries the Fermi thermodynamics theory[16] to its ultimate consequences, is used to illustrate the type of production cross section expected in this picture for a given quark mass and c.m. energy. The study here is undertaken not in the spirit of presenting something which we believe to be necessarily relevant to the physical facts, but rather one which provides a convenient basis for discussion in terms of a *well-defined* calculation. As emphasized by Wolfendale,[17] the usefulness of these theoretical production estimates is that they can stimulate the field. Calculations on the model are given in Sec. IV.

Finally, we wish to emphasize that the production mechanisms and estimates discussed here are in fact uniformly applicable to a wide class of postulated heavy objects (basic triplets, quarks, W meson,[3] a particle,[18] and the X object[19]) provided they have *pairwise strong interactions with the normal hadrons*. For definiteness we shall concentrate on the quark, which has features like fractional charge (and hence anomalous ionization) plus a stable member (also true of integrally charged triplets[9,11]) and thus can be especially amenable to experimental detection.

II. VARIOUS PRODUCTION MECHANISMS

There are no reliable estimates of production cross sections in strong interactions. The theoretical estimates available for q-pair production span many orders of magnitude. We can characterize these estimates by two classes: optimistic and pessimistic.

Optimistic estimates. These include naive order-of-magnitude estimates. For instance, it is noted that the cross section for $\bar{p}$ production in p-p interactions rises to a value near 1 mb at laboratory proton energies near 25 BeV, approximately four times the threshold energy.[20] This cross section is approximately equal to πa^2, where $a=1/m_p$ in natural units. Relative production of π, K, and $\bar{p}$ is also found to be roughly proportional to $m_\pi^{-2}:m_K^{-2}:m_p^{-2}$. If we take this simplest of arguments for the production of the quark q as well, then $\sigma(q\bar{q})$ from

$$p+p \to p+p+q+\bar{q}+\cdots \tag{2.1}$$

is of order $\pi(1/m_q)^2\approx 4\ \mu$b for m_q of order 10 BeV.[21] This saturation cross section is expected to be reached at four times the kinematic threshold lab energy $E_{\rm th}$ for the incident proton, i.e., $E\approx 1$ TeV (10^{12} eV). If we accept as a reasonable representation of the flux intensity I of incoming particles[22] up to very high energies ($\lesssim 10^{16}$ eV)

$$I=\frac{1.5\times 10^4\ \text{m}^{-2}\ \text{sr}^{-1}\ \text{sec}^{-1}}{E^{5/3}} \tag{2.2}$$

(E measured in BeV units),

then

$$\text{counting rate of quarks}=I\times\frac{\sigma(q\bar{q})}{\sigma_{\rm abs}(pp)}, \tag{2.3}$$

where $\sigma(q\bar{q})/\sigma_{\rm abs}(pp)\approx\sigma(q\bar{q})/30\ \text{mb}=$No. of quarks/incoming particle. The counting rate at $E_{\rm lab}=1$ TeV is

$$\frac{1.5\times 10^4}{10^5}\times\tfrac{4}{3}\times 10^{-4}=2\times 10^{-5}/\text{sec m}^2\ \text{sr}$$
$$\approx 600\ \text{events/year m}^2\ \text{sr}. \tag{2.4}$$

This type of counting rate and production cross section appears to be too high for consistency with some cosmic-ray data—at least in the case of the quark. A recent compilation[13] places rather stringent limits on the production cross section for such stable particles, which is, for quarks of mass $\lesssim 10$ BeV, less than $\sim 10^{-32}$ cm^2 and, for stable heavy triplets of integer charge, $\sim 10^{-30}$ cm^2 for masses in the range 3–10 BeV. However, it is not ruled out[22] that some cosmic-ray work may be overinterpreted with regard to the certainty of conclusions.

Some other favorable production mechanisms include those associated with diffraction dissociation. Like the situation with the postulated X object,[19] we choose to visualize the production at energies much greater than threshold. In this region, diffraction dissociation with Pomeranchuk trajectory exchange would seem to be the most dominant process.[23] Diagrams such as Fig. 1 are likely in which the group of intermediate states q^* is supposed to carry a major fraction of the incident

[14] I am indebted to Professor R. H. Dalitz and Dr. G. Kalbfleisch for discussions on this point.

[15] I wish to thank Professor C. N. Yang for aid in formulating this approach several years back.

[16] E. Fermi, Progr. Theoret. Phys. (Kyoto) **5**, 570 (1950).

[17] A. W. Wolfendale (private communication).

[18] T. D. Lee, Phys. Rev. **140**, B959 (1965).

[19] J. D. Bjorken, S. Pakvasa, W. A. Simmons, and S. F. Tuan, Phys. Rev. **184**, 1345 (1969).

[20] R. K. Adair and N. Price, Phys. Rev. **142**, 844 (1966).

[21] It is perhaps of passing interest to note that this estimate actually agrees with calculations based on single-pion-exchange peripheral diagrams for quark production performed by F. Chilton *et al.* [Phys. Letters **22**, 91 (1966)].

[22] R. K. Adair (private communication).

[23] R. K. Adair, Phys. Rev. **172**, 1370 (1968).

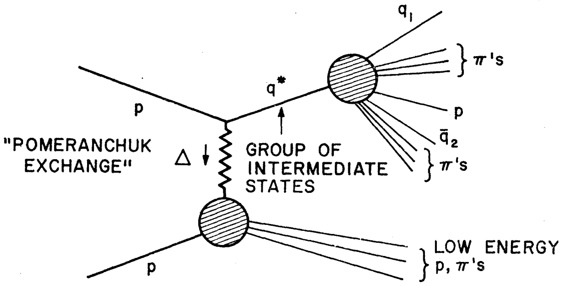

FIG. 1. Diffraction-dissociation model for quark production.

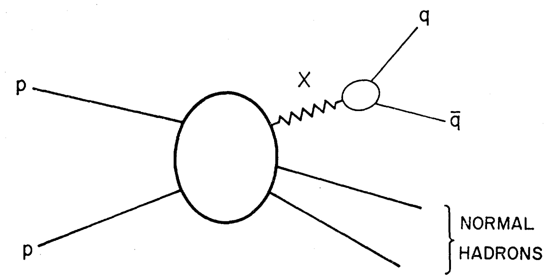

FIG. 3. Pair production of quarks in pp collision through a mesonic X intermediary. X may be a pion, ρ, ω, or anything else.

energy and then decay into quark pairs q_1 and $\bar{q}_2$ (plus, very likely, some associated π's). Such a mechanism, when summed over all channels containing $q_1\bar{q}_2$ pairs, is expected to lead to a cross section roughly constant with energy. If q_1, $\bar{q}_2$ are produced in the forward direction in the c.m. system, then the minimum momentum transfer to the target nucleon is $\Delta_{\min}\approx m_p E_{\rm th}/E$, where $E_{\rm th}$ is the kinematic threshold for the production of q. As the energy is decreased toward threshold, the minimum momentum transfer Δ^2 increases; at energies $\sim$3–4 times the threshold, $\Delta^2>0.1$ BeV^2, so that suppression of the diffractive process can be expected. We cut off the cross section at this point. Although the magnitude of the cross section for the diffractive processes is very uncertain, it seems reasonable that q_1 and $\bar{q}_2$ should emerge with a sizable finite fraction of the incident energy with good probability.

Another possible production mechanism is the exchange of q-pairs via q exchange in the peripheral model[19] (Fig. 2). A cross section of the order of a fraction of a mb can be expected only if it is assumed that all $\Delta^2\lesssim m_q^2$ are effective, without significant damping by form factors at the vertices. Again, if we interpret this diagram in terms of groups of states being exchanged, this may not be totally unreasonable.

Pessimistic estimates. In increasing order of pessimism we may categorize the following types of mechanisms. It has been suggested by Dorfan *et al.*[24] that pair production will come from the graph, Fig. 3, where X may be a pion, ρ, or anything else, and $(q\bar{q})$ could just as well be replaced by $(p\bar{p})$. The propagator for X is $1/(p_\mu^2+m_X^2)$, where $p_\mu^2=m_{q\bar{q}}^2$ and $m_{q\bar{q}}$ denotes the total rest energy of the $q\bar{q}$ pair. Hence p_μ^2 is typically of order m_q^2 (or m_p^2 for the $p\bar{p}$ case) and this is larger than the most likely m_X (m_π or m_ρ or m_ω). Hence the conclusion is that the graph is of order $1/m_q^2$. The cross section expected for the production of the heavy particle q [assuming this to be the process of $(q+\bar{q})$ pair production] is estimated from the observed $\bar{p}$ production cross section by assuming the plausible mass dependence $(m_p/m_q)^4$. For $m_q\sim 10$ BeV, we expect $\sigma(q\bar{q})\approx 10^{-31}$ cm^2. These arguments are very rough, of course—and on the pessimistic side since they assume that there are no large contributions from high-mass X.

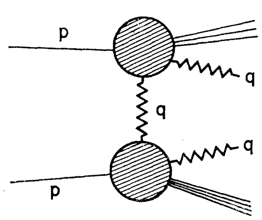

FIG. 2. Peripheral production of quark via quark exchange.

Again, Van Hove[25] suggests that one can try and guess what might be favorable kinematical conditions to create free quarks in high-energy proton-proton collisions (assuming that pion-proton collisions are less favorable *a priori* because of the lower intensity of pion beams). It seems likely, according to this speculation, that large momentum transfer collisions should be preferred, of the type

$$p+p\rightarrow (NN)+\cdots, \tag{2.5}$$

where (NN) is a two-nucleon system of low mass and low c.m. momentum (it could be a deuteron), and where "···" would contain a free quark-antiquark pair. Collisions of this type, involving baryon exchange, will of course have very small cross sections.

Finally, the statistical model[26] calculations give very low production rates, typically about 10^{-36} cm^2 for $m_q\sim 10$ BeV, falling very rapidly with increasing mass m_q. Such estimates are difficult to reconcile with the recent tentative evidence for heavy quarks,[1] since a quark production rate in the mb range is indicated.[27]

III. TEMPERATURE MODEL

In the statistical thermodynamics model of Hagedorn,[26] the minuscule production cross sections found for quarks are due to the assumption that the temperature T of the "hot-pot" interaction region is roughly equal to the pion mass (divided by the Boltzmann con-

[24] D. Dorfan, J. Eades, L. Lederman, W. Lee, and C. Ting, Phys. Rev. Letters **14**, 999 (1965).

[25] L. Van Hove (private communication).

[26] D. Chernavsky, E. Feinberg, V. Maximinko, and I. Sisakhan, Zh. Eksperim. i Teor. Fiz. Pis'na v Redaktsiya **3**, 340 (1966) [Soviet Phys. JETP Letters **3**, 219 (1966)]; G. Domokos and T. Fulton, Phys. Letters **20**, 546 (1966); R. Hagedorn, Nuovo Cimento Suppl. **6**, 311 (1968).

[27] J. Dooher, Phys. Rev. Letters **23**, 1471 (1969).

stant), while the quark mass is believed to be $\gtrsim 5$ BeV. Since the production cross section is generally proportional to $e^{-2m_q/T}$ in the thermodynamics picture, there is drastic suppression of production of heavy particles $m_q \gg T$. The limitation of T to the order of 158 MeV appears to give a good description of data at the accelerator energy, as particularly emphasized by Hagedorn.[26] However, at energies of the order of 10^{12}–10^{15} eV, we are a long extrapolation in concept from the relatively low-energy accelerator domain. The nature of ultrahigh-energy interactions can certainly be different.

It is the purpose of this section to raise the following question: Within the context of the Fermi statistical model,[16] how large can the quark production cross section be? In other words, we make an exploratory study of the production of heavy particles in strong interactions by choosing the assumptions in such a way that the temperature comes out as high as reasonably possible for given incident energies. More precisely, the assumptions are as follows:

(i) The statistical thermodynamics model of Fermi[16] is meaningful for a discussion of problems of strong interactions; (ii) all the available energy goes into the hot pot; and (iii) following a high-energy p-p collision, only hadrons stable in strong interactions participate in the statistical equilibrium.

Assumption (iii) differs markedly from the basis of calculation of Hagedorn.[26] Our picture is as follows. We assume that the strongly produced particles in high-energy p-p collisions are boiled off according to equilibrium statistical mechanics in an interaction volume Ω with radius of the order of a fermi. The particles stable in strong interaction have a lifetime much longer than the reaction time scale of strong interactions and hence can escape from the hot-pot interaction region. On the other hand, strong-interaction resonances like ρ and N^* have decay lifetimes comparable to the reaction time and thus may not have traversed far enough out of the interaction region (before decaying) to participate in the equilibrium considerations. We can replace (iii) by the more drastic assumption (iii′) in which only *nonstrange* hadrons stable in strong interactions participate. The exclusion of strange particles is perhaps not totally unreasonable since the evidence from photo-K production seems to indicate that the effective K couplings g_{KYN}^2 $(Y=\Lambda, \Sigma)$ can be smaller[28] than the corresponding pion-nucleon interaction. There is also corrobative evidence[29] from $p+\bar{p}\to Y+\bar{Y}$ that hyperon pair production is severely curtailed as the strangeness increases.[30] If the strange-particle interactions are weaker, they, like the weak and electromagnetic interactions, may not participate in equilibrium statistics according to (i)—because of the weaker coupling.

Assumption (ii) is equivalent to the statement that the c.m. energy W is equal to the energy of the hot pot. Physically, there is reason to doubt the validity of this assumption, as we shall discuss later. The effect of using (ii), as well as (iii′), is to overestimate the production cross sections for quarks.

Formulation.[15] We consider the process

$$p+p\to p+p+q+\bar{q}+\cdots, \tag{3.1}$$

where $(q,\bar{q})$ is the quark pair and "$\cdots$" denote the (possibly) many mesons and baryons produced in association with the $(q,\bar{q})$ pair.

If W is the c.m. energy for (3.1), then the energy density of the initial $p+p$ system in a volume Ω_0 is

$$\frac{W}{\Omega_0(2M/W)}, \tag{3.2}$$

where M is the nucleon mass and $2M/W$ is the Lorentz contraction factor. For convenience, we shall define the Lorentz-contracted volume Ω as

$$\Omega=\Omega_0(2M/W). \tag{3.3}$$

The energy density for the right-hand side of (3.1) includes a sum over densities calculated according to conventional statistical mechanics; $\rho_m^{(i)}$, where m denotes the meson octet $(\pi,K,\bar{K},\eta)$; $\rho_{(B,\bar{B})}^{(i)}$, where B encompasses the baryon octet (N,Λ,Σ,Ξ), the Ω^-, the deuteron d,[31] and the quark q, and $\bar{B}$ is the corresponding set of antiparticles. We have for the equality of initial- and final-state energy density of process (3.1)

$$\frac{W^2}{2M\Omega_0}=\sum_i \rho_m^{(i)}+\sum_i \rho_{(B,\bar{B})}^{(i)}, \tag{3.4}$$

where

$$\rho_m^{(i)}=\frac{2^{\nu_i}(2J_i+1)(2T_i+1)}{8\pi^3}\int\frac{(m_i^2+k^2)^{1/2}d^3k}{\exp[(m_i^2+k^2)^{1/2}/T]-1}$$
$$=\frac{2^{\nu_i}(2J_i+1)(2T_i+1)}{2\pi^2}I_{(m)}^{(i)} \tag{3.5}$$

and

$$\rho_{(B,\bar{B})}^{(i)}=\frac{2^{\nu_i}(2J_i+1)(2T_i+1)}{8\pi^3}\int\frac{(m_i^2+k^2)^{1/2}d^3k}{\exp[(m_i^2+k^2)^{1/2}/T]+1}$$
$$=\frac{2^{\nu_i}(2J_i+1)(2T_i+1)}{2\pi^2}I_{(B,\bar{B})}^{(i)}\quad (B\neq d). \tag{3.6}$$

[28] M. Gell-Mann, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (Benjamin, New York, 1964), p. 11.

[29] C. Y. Chien, *et al.*, Phys. Rev. **152**, 1171 (1966).

[30] The empirical Morrison rule which assumes $\sigma=\sigma_0 P_L^{-n}$ with $n\sim 2.5$ for strangeness exchange, as is involved in $p+\bar{p}\to Y+\bar{Y}$, already suggests large suppression of these processes as p_L (lab momentum of incident proton) increases.

[31] For the temperature range under consideration (see Sec. IV) it is a reasonable approximation to keep just the deuteron while dropping terms like the heavy nuclei, He^3, He^4, etc., since they, like the quarks q, actually contribute relatively little to the density. We keep, however, the quark contributions to ρ for completeness of discussion.

Here J_i and T_i are the spin and isospin of the ith particle, $\nu_i=0$ if particle=antiparticle, and $\nu_i=1$ if particle $\neq$ antiparticle; note that the deuteron satisfies Bose statistics (3.5) rather than Fermi statistics (3.6). We have taken the Boltzmann constant to be equal to unity, and the temperature is T.

The pion multiplicity calculated on this basis is

$$\langle n_\pi\rangle=\frac{3\Omega}{8\pi^3}\int\frac{d^3k}{\exp[(m_\pi^2+k^2)^{1/2}/T]-1}. \quad (3.7)$$

We remark here that Eq. (3.4) offers *an unambiguous determination of the c.m. energy W for a given temperature T.*

The problem of quark production (and in fact of baryons in general) raises the question of a *quark conservation law,* namely, quarks are pair-produced in (3.1). We can understand the problem as follows: The grand partition function (G.P.F.) is

$$\prod_\epsilon (1+\lambda e^{-\epsilon/T}+\lambda^2e^{-2\epsilon/T}+\cdots)\prod_{\epsilon'}\left(1+\frac{1}{\lambda}e^{-\epsilon'/T}+\frac{1}{\lambda^2}e^{-2\epsilon'/T}+\cdots\right)=\frac{1}{1-\lambda e^{-\epsilon/T}}\frac{1}{1-\lambda^{-1}e^{-\epsilon'/T}}, \quad (3.8)$$

where $\lambda e^{-\epsilon/T}$ $(\lambda^{-1}e^{-\epsilon'/T})$, $\lambda^2e^{-2\epsilon/T}$ $(\lambda^{-2}e^{-2\epsilon'/T})$, ..., etc., represent one quark (antiquark), two quarks (antiquarks), ...; λ is related to the chemical potential μ by $\lambda=e^{-\mu/T}$. The Bose statistics form of (3.8) is used for simplicity of discussion; the conclusions will, of course, be valid for the Fermi case as well.

The average number of quarks n_q and antiquarks $n_{\bar q}$ are then

$$\begin{aligned} n_q&=\frac{-\partial\ln(\prod_\epsilon\cdots\prod_{\epsilon'}\cdots)}{\partial(\epsilon/T)}=\frac{1}{\lambda^{-1}e^{\epsilon/T}-1},\\ n_{\bar q}&=\frac{-\partial\ln(\prod_{\epsilon'}\cdots\prod_\epsilon\cdots)}{\partial(\epsilon'/T)}=\frac{1}{-1+\lambda e^{\epsilon'/T}}. \end{aligned} \quad (3.9)$$

[Note that for the Fermi-statistics case, we would have (+) instead of (−) in the denominator of (3.9).] Quark conservation will then require $\lambda=1$. This in turn implies that the equilibrium assumption for quarks (and of course for baryons as well) together with their antiparticles is an *additive* contribution to the energy density equation [Eq. (3.4)], thus justifying Eqs. (3.5) and (3.6).

Actually the question of quark production in the temperature model must be considered in two separate limits:

(I) The multiplicity of quarks $\langle N_q\rangle$ in each spin-charge state per collision is large, i.e., $\langle N_q\rangle\gg1$.

(II) $\langle N_q\rangle\ll1$.

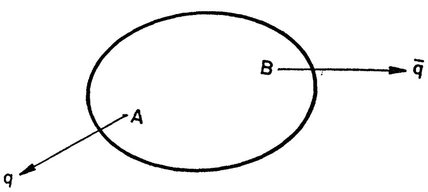

FIG. 4. The case of a large volume of interaction in which a quark produced at point A is *statistically independent* of the corresponding antiquark produced at point B if in equilibrium.

In case (I) it is meaningful to talk about quarks participating in the statistical equilibrium, and will generally correspond to a temperature T comparable to the quark mass. The meaningful experimental parameters are then the multiplicity $\langle N_q\rangle$ and the *total* cross section for process (3.1), where

$$\langle N_q\rangle=\Omega\frac{1}{8\pi^3}\int\frac{d^3k}{\exp[(m_q^2+k^2)^{1/2}/T]+1}. \quad (3.10)$$

For case (II), some additional features are present. Physically the equilibrium case corresponds to a "large" volume of interaction region (see Fig. 4), with a large number of quarks (and antiquarks), say, 25, being boiled off. A quark produced at point A is *statistically independent* of the corresponding antiquark produced at point B, if in equilibrium. On the other hand, for a small interaction region in which, say, only $\frac{1}{4}$ of a quark is produced, the selection rules and correlations become very important. The meaningful physical parameter is evidently the production cross section for quark pairs in this case where $\langle N_q\rangle\ll1$, and a different formulation is needed.

In practice, as will be evident from the calculations of Sec. IV, the range of temperature and energy of interest do correspond to case (II). Namely, the volume of interaction is probably "large" with respect to π, η, K, and the stable low-mass baryons, but "small" with respect to the quarks $(q,\bar q)$. In particular, we shall find that the temperature calculated from (3.4) is small compared with the quark rest mass m_q; thus the "particle" gas is cold as far as quarks are concerned.

We make the following assertion:

Statement. For $\langle N_q\rangle\ll1$, the probability per collision for creation of a pair $q\bar q$ per spin-charge state, irrespective of how many pions, K mesons, low-mass baryon pairs, etc., are produced with it, is

$$[P(q,\bar q)]^2=\left\{\frac{\Omega}{8\pi^3}\int_{-\infty}^{+\infty}\exp[-(m_q^2+k^2)^{1/2}/T]d^3k\right\}^2. \quad (3.11)$$

Note that (3.11) is entirely reasonable as the product of probabilities for producing a quark and an antiquark; it fits well with the intuitive expectation that pair production of quarks should have an exponential dependence $e^{-2m_q/T}$.

Justification. We divide the partition function $\sum_{\text{all states}} e^{-E_n/T}$ as follows:

$$\sum_{\text{all states}} e^{-E_n/T} = \sum_{n0} e^{-E_n/T} + \sum_{q,\bar{q}+\cdots} e^{-E_n/T} + \cdots = A + A[P(q,\bar{q})]^2 + \cdots . \quad (3.12)$$

Here we have used the fact that the number of particles produced without quarks is so large that the expression for pair production of quarks in association with other low-mass hadrons $\sum_{q,\bar{q}+\cdots} e^{-E_n/T}$ can be factorized as follows:

$$\sum_{q,\bar{q}+\cdots} e^{-E_n/T} = A[P(q,\bar{q})]^2 . \quad (3.13)$$

The "$\cdots$" in (3.12) contains terms with more than one pair of quarks and is therefore negligible. From (3.12), we have for the probability of pair emission of quarks

$$\frac{A[P(q,\bar{q})]^2}{A + A[P(q,\bar{q})]^2} \approx [P(q,\bar{q})]^2 , \quad (3.14)$$

and the Statement is established.

The production cross section for quark pairs is then

$$\sigma(q\bar{q}) = [P(q,\bar{q})]^2 \sigma_{\text{tot}}(pp) , \quad (3.15)$$

where $\sigma_{\text{tot}}(pp)$ is the asymptotic value of the total p-p cross section ($\sim$38 mb), and the corresponding c.m. energy W is determined from the temperature-energy relations (3.4)–(3.6). Counting rates can be readily obtained from a knowledge of the cosmic-ray flux, as discussed, for instance, in terms of Eqs. (2.2) and (2.3).

Remarks (1). Conservation laws, such as charge conservation, isospin conservation, baryon-number conservation, and strangeness conservation, are not taken into account in the determination of T from W. The effect of charge conservation is expected to be small, for the case of *small particle masses* (e.g., the pion), because of their copious production in high-energy collisions. We illustrate this with

$$\begin{aligned} &pp \text{ (net charge 2)} \to \cdots + \text{many pions},\\ &np \text{ (net charge 1)} \to \cdots + \text{many pions}. \end{aligned} \quad (3.16)$$

The final-state pions *lose memory* of the effect of a small initial charge and hence can be adjusted to give the required net charge. Such appears to be the case experimentally.

Remarks (2). The nucleon-pair-production effects are indeed *dependent* on taking care of the baryon conservation (in particular, nucleon conservation) law. By nucleon pair effect, we mean, for instance, the process

$$p + p \to p + p + p + \bar{p} + \cdots . \quad (3.17)$$

Initially we have nucleon number $N=2$, and finally we have $N=3$, $\bar{N}=1$; this imbalance is not taken into account in the statistical calculation as formulated by Eqs. (3.4)–(3.6). The whole nucleon-pair effect is likely to be small in the two extreme temperature limits $T \ll M$ and $T \gg M$, where M is the nucleon mass. In the former limit, the nucleon pair is not sufficiently excited and hence the effect is small. Indeed, a calculation of nucleon multiplicity will show that $\langle N \rangle \ll 1$, and hence it is more meaningful to talk about a production cross section for nucleon pairs in exact analogy to the formulation of quark pair production [Eqs. (3.11)–(3.15)]—rather than about statistical equilibrium. For the limit $T \gg M$, then, nucleon-pair effect is again unimportant. Here a large number of nucleon pairs is produced [similar to the pion case of Eq. (3.16)] following the statistical concept. The argument is then similar to the charge-conservation case of remark (1), namely, the great multiplicity of pairs produced causes it to lose memory of the initial production process (with the additional two protons). We have then approximate nucleon conservation: $N_n \approx N_{\bar{n}}$.

Remark (3). Nucleon-pair effects are *important* in the intermediate range of temperature $T \sim M$, where nucleon multiplicity is of the order of unity. This is readily evident from Eq. (3.17), say, where $N=2$ initially and $N=3$, $\bar{N}=1$ finally.

Conservation of the baryon number $\langle B \rangle$ can be formally introduced by the λ effect. We adjust λ for the baryon-antibaryon system such that

$$\begin{aligned} \langle B \rangle = 2 &= \frac{\Omega}{2\pi^2} \sum_{i(B\neq d)} (2J_i+1)(2T_i+1) \left[\int_\infty \frac{k^2 dk}{\lambda \exp[(m_i^2+k^2)^{1/2}/T]+1} - \int_\infty \frac{k^2 dk}{\lambda^{-1} \exp[(m_i^2+k^2)^{1/2}/T]+1} \right] \\ &\quad + \frac{\Omega}{2\pi^2} (2J_d+1)(2T_d+1) \left[\int_\infty \frac{k^2 dk}{\lambda^2 \exp[(m_d^2+k^2)^{1/2}/T]-1} - \int_\infty \frac{k^2 dk}{\lambda^{-2} \exp[(m_d^2+k^2)^{1/2}/T]-1} \right], \end{aligned} \quad (3.18)$$

subject to the constraint that Eq. (3.4) holds with $\rho_m^{(i)}$ given by (3.5) but that (3.6) is replaced by

$$\begin{aligned} \rho_{(B,\bar{B})}^{(i)} &= \frac{(2J_i+1)(2T_i+1)}{2\pi^2} 2I_{(B,\bar{B})}^{i}(\lambda) \\ &= \frac{(2J_i+1)(2T_i+1)}{2\pi^2} \left[\int \frac{k^2 dk (m_i^2+k^2)^{1/2}}{\lambda \exp[(m_i^2+k^2)^{1/2}/T]+1} + \int \frac{k^2 dk (m_i^2+k^2)^{1/2}}{\lambda^{-1} \exp[(m_i^2+k^2)^{1/2}/T]+1} \right] \quad (B\neq d) \\ &= \frac{(2J_d+1)(2T_d+1)}{2\pi^2} \left[\int \frac{k^2 dk (m_d^2+k^2)^{1/2}}{\lambda^2 \exp[(m_d^2+k^2)/T]-1} + \int \frac{k^2 dk (m_d^2+k^2)^{1/2}}{\lambda^{-2} \exp[(m_d^2+k^2)/T]-1} \right] \quad (B=d). \end{aligned} \quad (3.19)$$

Note that $\lambda=1$ in (3.18) corresponds to $\langle B\rangle=0$, and we are back to the original Eqs. (3.4)–(3.6).

The λ effect is not a complete cure for nucleon-pair effects at intermediate temperature, however. This can be readily illustrated by the following example:

Process	Nos. of nucleons	Nos. of antinucleon
$p+p\to p+p$	2	0
$p+p\to p+p+N+\bar{N}$	3	1

The λ effect cures the horizontal combinations (2,0) and (3,1), but does not cure the *individual cases* not following the horizontal pattern, namely, (2,1)- and (3,0)-type correlations. We hope to take up this more complex question in a future study.

IV. CALCULATIONS

The program of calculations is greatly simplified by taking advantage of the following summation form of the integrals[32] appearing in (3.5)–(3.7), (3.11), (3.18), and (3.19), to wit,

$$\frac{8I}{T^4}=\frac{8}{T^4}\int\frac{k^2(m^2+k^2)^{1/2}dk}{(\lambda\exp[(m^2+k^2)^{1/2}/T]\pm1)}=\left(\frac{m}{T}\right)^4\sum_{n=1}^{\infty}\lambda^{-n}\begin{bmatrix}(-1)^{n-1}\\1\end{bmatrix}\left[K_4\left(\frac{nm}{T}\right)-K_0\left(\frac{nm}{T}\right)\right]$$

$[(-1)^{n-1}$ for Fermi statistics, 1 for Bose statistics$]$ (4.1)

and

$$\int\frac{k^2dk}{(\lambda\exp[(m^2+k^2)^{1/2}/T]\pm1)}=\tfrac{1}{4}m^3\sum_{n=1}^{\infty}\lambda^{-n}\begin{bmatrix}(-1)^{n-1}\\1\end{bmatrix}\times\left[K_3\left(\frac{nm}{T}\right)-K_1\left(\frac{nm}{T}\right)\right],\quad(4.2)$$

$$\int k^2dk\exp[-(m^2+k^2)^{1/2}/T]=\tfrac{1}{4}m^3\left[K_3\left(\frac{m}{T}\right)-K_1\left(\frac{m}{T}\right)\right],\quad(4.3)$$

where $K_n(x)$ are the modified Hankel functions and the convergence of the series in (4.1) and (4.2) is very rapid for λ in the range of interest $0.5\leq\lambda\leq1$. Since the I and density ρ are both functions of mass m and temperature T, it has been found appropriate to plot the function $8I/T^4$ (which depends on m/T only) against m/T and read off the various contributions to $\rho^{(i)}$ for different masses of baryons and mesons at a fixed temperature; vary the temperature and repeat accordingly. The plot of $8I/T^4$ is, of course, somewhat different depending on the statistics (Bose-Einstein or Fermi-Dirac); the differentiation between them is, however, marked only for small values of m/T and $\lambda\sim1$.

Calculations of the temperature-energy relation from Eqs. (3.4)–(3.6) on the basis of propositions (i)–(iii) are completely straightforward. We take first a radius of interaction $R\sim0.8$ F, where

$$\Omega_0=\tfrac{4}{3}\pi R^3.\quad(4.4)$$

This is well within the range $0.6\leq R\leq0.9$ F preferred by Byers and Yang[33] as the limiting radius of interaction for high-energy collisions. We can proceed to tabulate some typical values obtained from the temperature-energy relation, corresponding to a quark mass $m_q=10$ BeV (with $\lambda=1$). The respective pion multiplicities can be calculated from Eq. (3.7). Comparison with an empirical multiplicity formula for pions, suggested by Adair,[22,34]

$$\langle n_\pi\rangle_{\text{emp}}\sim2+E^{1/4}(E\text{ in BeV units})\quad(4.5)$$

can also be made. As an illustration, these give for T (in pion units), W and E in BeV, $\langle n_\pi\rangle_{\text{calc}}$, and $\langle n_\pi\rangle_{\text{emp}}$ the following values:

$$\begin{aligned}&T=4,\quad W=23.2,\quad E=268,\quad \langle n_\pi\rangle_{\text{calc}}=1.4,\ \langle n_\pi\rangle_{\text{emp}}=6.05;\\&T=8,\quad W=107.7,\quad E=5799,\quad \langle n_\pi\rangle_{\text{calc}}=2.44,\ \langle n_\pi\rangle_{\text{emp}}=10.7.\end{aligned}\quad(4.6)$$

Note that the threshold for pair production of a 10-BeV quark, from Eq. (3.1), would correspond to a reaction temperature of about four pion masses. The pion multiplicities calculated are a factor 4 too small when compared with those of the empirical multiplicity formula (4.5). This can be readily understood when we recognize that propositions (i)–(iii) lead to the result that $\rho^{(\pi)}$ is only a small fraction of the total ρ which determines the temperature-energy, Eq. (3.4). Indeed, even at the lowest temperature relevant, $T=4m_\pi$,

$$\rho^{(\pi)}/\rho\sim10\%.$$

Introduction of the λ effect to take into account nucleon-pair effects [cf. Eqs. (3.18) and (3.19)] is a pertinent consideration since we are in the temperature range $T\sim M$, where nucleon-pair effects *are important.* We find that $\lambda=0.76$ corresponds to $\langle B\rangle=2$ at $T=4m_\pi$ ($m_q=10$ BeV). The corresponding pion multiplicity is, however, *decreased* from 1.4 [cf. Eq. (4.6)] to 1.38. This can perhaps be understood physically in that $\langle B\rangle=2$ constraint forces a slight increase in the baryon contribution to density function ρ when compared with the $\langle B\rangle=0$, $\lambda=1$ case. It is not clear whether the individual correlations to nucleon-pair effects discussed

[32] I wish to thank Professor T. T. Wu for aid in simplifying the mathematics of this model, and for collaboration during the early stages of this work.

[33] N. Byers and C. N. Yang, Phys. Rev. **142**, 976 (1966).

[34] This empirical multiplicity rule (4.5) cannot be used much above 10^{12} eV with confidence. The data at very high energies do not seem to be inconsistent with such a statement but there are strong biases. See also J. F. de Beer *et al.*, Proc. Phys. Soc. (London) **89**, 567 (1966).

[35] T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965).

TABLE II. Temperature-energy and pion multiplicities calculated on the basis of propositions (i), (ii), and (iii′) for a radius $R=1.4135$ F and $m_q=10$ BeV in Eqs. (3.4)–(3.6) (with $\lambda=1$). The empirical pion mulitplicities $\langle n_\pi\rangle_{\text{emp}}$ are computed from the Adair formula (4.5).

T (pion units)	W (BeV)	E (BeV)	$\langle n_\pi\rangle_{\text{calc}}$	$\langle n_\pi\rangle_{\text{emp}}$
4	32.52	527.7	5.5	6.8
7	111.40	6201.75	8.73	10.87
8	148.0	10 955.4	9.82	12.23
9	190.8	18 195.6	10.85	13.6
10	238.9	28 542	11.89	15
12	355.15	63 065	13.85	17.85

at the end of Sec. III and introduction of strangeness conservation will contribute adequately to cure the multiplicity difficulty.

It has long been recognized[16] that *increasing* the radius of interaction volume R has the desired effect of boosting pion multiplicity. Taking $R=1.4135$ F (Compton wavelength of the pion), we have for $T=4m_\pi$ ($W=54.64$ BeV) a calculated $\langle n_\pi\rangle=3.27$ compared with a corresponding empirical estimate of 6.2 at the same energy—still too low by itself as a major correction though undoubtedly one in the right direction.

We now consider the alternative propositions (i), (ii), and (iii′) and *drop* the $\Delta S\neq 0$ contributions to ρ from the known hadrons stable in strong interaction, retaining just the pion, η, nucleon pair, d, and $\bar{d}$ in Eqs. (3.4)–(3.6). Table II gives the corresponding tabulation for temperature-energy and pion multiplicity, for a typical quark mass of 10 BeV and $R=1.4135$ F.

The results are in fair agreement between empirical and calculated multiplicities—especially if the multiplicity should prove to be much smaller than (4.5) at very high energies.[22] Again the λ effect [Eqs. (3.18) and (3.19)] tends to *depress* the pion multiplicity slightly. For instance, at $T=4m_\pi$ and $\lambda=0.8$, corresponding to a $\langle B\rangle\approx 2.4$, we have $\langle n_\pi\rangle=5.44$ as opposed to $\langle n_\pi\rangle=5.5$ for $\lambda=1$ (Table II).

In Table III we have illustrated typical quark-pair-production cross sections in mb for the model based on propositions (i), (ii), and (iii′), with $R=1.4135$ F. Some representative quark masses from 7 to 20 BeV are used to indicate the general trend.

The following comments on the model are worthy of note.

(a) There is no angular dependence implicit in our relations (3.4)–(3.6). Hence angular distribution would tend to be isotropic. This is in sharp contradistinction with the spectacular angular distribution noted in $p+p$ elastic scattering at large angles,[35] at least in the accelerator range of energy. One possible interpretation would be to regard the thermodynamical model as only applicable to large-angle phenomena where the particles coming off more nearly resemble a statistical picture.[36] This is then superimposed over a mechanism more appropriate to the longitudinal case (where one might conceivably have the coherent droplet model of Byers and Yang[33]). What is less clear is how to share the c.m. energy W of the initial system between the two mechanisms. In other words, assumption (ii) is brought into question. There appears also to be no obvious connection between our theory and popular concepts like the fireballs model[23] for ultrahigh-energy phenomena.

(b) On the basis of the formulation proposed, we still get too many nucleon pairs for the temperature-energy under consideration (above threshold for pair quark production). The reason why so many nucleons of all kinds are formed compared to the pions is their statistical weight [8 for the nucleons, 3 for the pion; cf. Eqs. (3.5) and (3.6)]. There is very little direct evidence about baryon pairs from cosmic rays,[17] though the commonly accepted explanation is that baryon pairs do not contribute more than 10–20%, at least below 10^{15} eV. In our model, $\langle N_n\rangle>\langle n_n\rangle$ throughout the energy-temperature range discussed. However, to the extent that it remains unclear whether even one antiproton event has been unambiguously identified in cosmic rays,[22,37] we are perhaps justified in keeping an open mind on this question—especially in the energy range >1 TeV. Fermi,[16] in his original temperature model, had in fact speculated that the nucleons achieve relativistic conditions only when $W>100$ BeV (E_{lab}

TABLE III. Quark-pair-production cross sections (in mb). We assume typical quark masses $m_q=7$, 10, 13, 15, and 20 BeV. The corresponding temperature T (in pion units) and c.m. energy W (in BeV) are also listed.

T	W ($m_q=7$)	$\sigma(q\bar{q})$	W ($m_q=10$)	$\sigma(q\bar{q})$	W ($m_q=13$)	$\sigma(q\bar{q})$	W ($m_q=15$)	$\sigma(q\bar{q})$	W ($m_q=20$)	$\sigma(q\bar{q})$
4	32.526	10^{-6}	32.52	4.56×10^{-11}	32.52	21.3×10^{-16}	32.52	$\ll 10^{-16}$	32.52	$\langle\langle\langle 10^{-16}$
7	112.36	2.7×10^{-2}	111.38	1.37×10^{-4}	111.20	5.5×10^{-7}	111.20	1.67×10^{-8}	111.20	9.73×10^{-13}
8	151.15	0.15	148.03	1.67×10^{-3}	147.45	1.44×10^{-5}	147.38	5.47×10^{-6}	147.38	1.52×10^{-10}
9	197.40	0.524	190.77	1.15×10^{-2}	189.32	1.94×10^{-4}	189.07	1.06×10^{-5}	189.02	8×10^{-9}
10	250.01	1.49	238.93	0.56×10^{-1}	236.09	1.71×10^{-3}	235.56	1.06×10^{-4}	235.32	1.75×10^{-7}
12	382.00	6.93	355.15	0.486	346.23	2.7×10^{-2}	344.22	3.57×10^{-3}	342.88	1.81×10^{-5}
15	620.56	32.27	581.2	6.05	560.64	0.87	553.73	0.084	550.22	1.7×10^{-3}

[36] K. Huang, Phys. Rev. **146**, 1075 (1966); **156**, 1555 (1967).
[37] M. F. Kaplon (private communication).

$\gtrsim 5\times10^{12}$ eV); at somewhat lower energies the number of nucleon-antinucleon pairs formed will *decrease* very rapidly.

(c) On the positive side, our version of the thermodynamics model, in which the temperature is allowed to assume values as high as reasonably possible for given incident energies, *does* allow an understanding of the large production cross section for quarks as inferred from the experiment of McCusker and Cairns.[1,27] For instance, Table III shows that for $T\sim 15m_\pi$ and $m_q=10$ BeV, $\sigma(q\bar{q})\sim 6$ mb; this corresponds to an incident proton energy $E\sim 1.7\times10^{14}$ eV.

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Gribov-Pomeranchuk Pole in the Regge-Pole Residues*

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We consider a mechanism for generating a nonsense wrong-signature fixed pole in the scattering amplitude based on a certain class of Feynman diagrams. We show that the presence of a fixed pole in the Regge-pole residue is expected when we include an effect of the third double-spectral function in an appropriate way, and discuss its phenomenological implications in connection with dip-bump structures in differential cross sections. In particular, a certain channel-independent feature of a fixed-pole effect is pointed out and a comparison with the data is made.

I. INTRODUCTION

In nonrelativistic potential scattering theory, when a trajectory passes through a nonsense–wrong-signature point, its contribution to the scattering amplitude is expected to vanish; while in relativistic theory this contribution could be finite. The latter is due to the possible existence of a fixed pole in the Regge-pole residue at this point. This possibility was recognized some time ago.[1,2] By applying unitarity corrections to Regge-pole amplitudes, Mandelstam and Wang argued[2] that in general the presence of such a pole is expected. Their argument is based on a perturbative approximation to effects of the third double-spectral function. Recently, the Cambridge group,[3] Olive and Polkinghorne, and also Landsoff and Polkinghorne, have studied a closely related problem which has raised some doubts on the general validity for such an approximation. These authors investigated how a Gribov-Pomeranchuk pole[4] is reconciled with the unitarity relation in a perturbation theory. They demonstrated that in order to arrive at this reconciliation, effects to all orders in the third double-spectral function need to be considered simultaneously. In their particular example, keeping only to a given order in the third double-spectral function, one would be led to undesirable results. In view of the possible role of fixed poles in high-energy phenomenology, one is naturally prompted to ask whether it is possible to carry out arguments for the existence of fixed poles without restricting oneself to first order in the effect of the third double-spectral function. We found that it is in fact possible to recast the argument in Feynman-diagram language which is consistent with the result of the Cambridge group[3] and is also independent of the order of the third double-spectral function. In this paper we reexamine the argument and the assumption for the presence of this fixed pole and consider the effect of fixed poles in high-energy phenomenology.[5]

The plan of this paper is as follows. In Sec. II we first review the work of Olive and Polkinghorne and the argument of Mandelstam and Wang. Then we demonstrate how a similar argument can be formulated in a Feynman-diagram model. In Sec. III we discuss the behavior of Regge residues near $\alpha=0$ and investigate how the usual sense- and nonsense-choosing mechanisms deduced from constraints of analyticity and factorization are modified in the presence of fixed poles.

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[2] S. Mandelstam and L. L. Wang, Phys. Rev. 160, 1490 (1967).
[3] D. I. Olive and J. C. Polkinghorne, Phys. Rev. 171, 1475 (1968); see also P. V. Landshoff and J. C. Polkinghorne, *ibid.* 181, 1989 (1969).
[4] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters 2, 239 (1962).
[5] A summary of a portion of this work has been given elsewhere; see C. B. Chiu and S. Matsuda, Phys. Letters 31B, 455 (1970).