Meson-Baryon Consistency Conditions from Duality*

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A duality requirement and a simply proportionality hypothesis involving the residues of baryonic Regge trajectories are used to derive a self-consistency condition for the scattering of pseudoscalar mesons from spin-$\frac{1}{2}$ baryons. Essentially, the condition states that the differences between the residues of even- and odd-signature baryon trajectories are the components of an eigenvector of the $s \rightleftarrows u$ channel crossing matrix. Several different forms of the consistency condition are discussed, one of which involves residues of trajectories in the physical region, at the energies of the lowest resonances on the trajectories. Experimental values of partial widths of resonances are used to check this latter form for πN, $\pi\Lambda$, and $\pi\Sigma$ scattering. The agreement with experiment is good.

I. INTRODUCTION

SEVERAL years ago, Chew used the static model of pion-nucleon scattering to derive a simple bootstrap condition.[1] The condition is that the residues of the possible poles in the P-wave channels (of $2I$ and $2j$ equal to 11, 13, 31, and 33) form a four-component eigenvector of the static-model crossing matrix, with eigenvalue one.[2] The relation between the Δ width and the πNN coupling constant implied by this condition is satisfied fairly well.

On the other hand, the corresponding condition for K-meson–nucleon scattering is violated badly. Since KN states are exotic (contain no resonances or bound-state poles) the condition implies that the residues of the poles in any P-wave $\bar{K}N$ channel vanish, in contradiction with experiment. Recently, however, a different self-consistency condition obtained from a duality assumption has been successful for the KN and $\bar{K}N$ systems. This is the condition that the baryonic Regge trajectories that contribute to KN scattering occur in exchange-degenerate pairs, and that the residues of the even- and odd-signature members of each pair are equal.[3]

In this paper we derive a more general consistency condition for the residues of baryonic Regge trajectories, a condition applicable to amplitudes that are not exotic in either the s or u channel. If one of the channels is exotic, as in the case with the KN amplitudes, the condition reduces to that mentioned above. If the odd-signature couplings are small (as they are in pion-nucleon scattering) the condition is a simple modification of the Chew bootstrap condition.

This type of consistency condition has been derived previously, from an idealized Veneziano model in which the external particles are spinless.[4] The present derivation is an improvement in two important ways: First, we use a duality assumption that is weaker than that of the Veneziano model; second, we include the baryon spin. Including the spin allows us to derive realistic conditions that may be compared to experiment.

The consistency conditions are derived in Sec. II and are applied to πN, $\pi\Lambda$, and $\pi\Sigma$ scattering in Sec. III. Possible extensions of the model are discussed in Sec. IV.

II. DERIVATION OF CONSISTENCY CONDITIONS

A. Spinless External Particles

We consider a meson-baryon scattering amplitude T_i, where i denotes the internal quantum numbers of the initial and final particles. Temporarily, the baryon spins are neglected. We consider the s-channel amplitude at an intermediate energy, near the backward (small-u) direction, and assume that the contributions of t-channel Regge poles are negligible. The duality assumption may be stated[5]

$$\langle \mathrm{Im} T_{ui}{}^{\mathrm{Regge}} \rangle = \langle \mathrm{Im} T_{si}{}^{\mathrm{res}} \rangle , \qquad (1)$$

where $T_{ui}{}^{\mathrm{Regge}}$ is the contribution of u-channel Regge poles, $T_{si}{}^{\mathrm{res}}$ is the contribution of s-channel resonances, and $\langle\ \rangle$ denotes some semilocal average over s and u.

It is assumed that the even- and odd-signature trajectories that contribute to $T_{ui}{}^{\mathrm{Regge}}$ are exchange-degenerate (though the multiplicity and quantum numbers of the two different sets need not be the same). Each trajectory may be associated with a set of daughters, i.e., trajectories of spin lower by one or more units. We assume that the contribution of such daughters in the Regge region may be neglected, in which case Eq. (1) leads to

$$\alpha_i (X_{ui}{}^{(+)} - X_{ui}{}^{(-)}) = Z_{si}{}^{(+)} - Z_{si}{}^{(-)} , \qquad (2)$$

where α_i is a constant, $X_{ui}{}^{(\pm)}$ are the sums of the residues of the u-channel trajectories of signatures $\pm$ [averaged over the region of u involved in Eq. (1)], and $Z_{si}{}^{(\pm)}$ are the contributions of resonances of parities $\pm$ to the right-hand side of Eq. (1). The minus sign in the combi-

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[1] G. F. Chew, Phys. Rev. Letters 9, 233 (1962).
[2] The crossing-matrix form of the bootstrap condition is discussed in detail by R. H. Capps, Nuovo Cimento 34, 932 (1964).
[3] See, e.g., V. Barger, Phys. Rev. 179, 1371 (1969); R. H. Capps, Phys. Rev. Letters 22, 215 (1969).
[4] R. H. Capps, Phys. Rev. D 1, 2395 (1970).
[5] A lucid discussion of the duality assumption is given by Haim Harari, Phys. Rev. Letters 22, 562 (1969). This paper contains references to other papers on duality.

nation $X_{ui}^{(+)}-X_{ui}^{(-)}$ results from the different signature factors of the exchange-degenerate trajectories. The minus sign in the right-hand side of Eq. (2) is convenient because the imaginary parts of elastic amplitudes of opposite parities have opposite signs in the backward direction.

The s-channel resonance contributions themselves lie on Regge trajectories, so the Z are proportional to some average over s of the residues of these trajectories. We define proportionality constants $\beta_i^{(\pm)}$ by the relation $Z_{si}^{(\pm)}=\beta_i^{(\pm)}X_{si}^{(\pm)}$, where the X_s are the residues of the s-channel trajectories in the small-s region appropriate for backscattering in the u channel. Since the Z_i may involve contributions from daughters of the leading trajectories, the constants β_i depend on three things: the phase-space factors involved in the Z, the relative proportion of daughter contributions in the Z, and the variation of the residue between the resonance and Regge regions. If the above relation is substituted into Eq. (2), the result is

$$\alpha_i(X_{ui}^{(+)}-X_{ui}^{(-)})=\beta_i^{(+)}X_{si}^{(+)}-\beta_i^{(-)}X_{si}^{(-)}. \quad (3)$$

We must keep in mind the fact that the u- and s-channel processes corresponding to an amplitude i are different, in general.

Because of the unknown constants α_i and $\beta_i^{(\pm)}$, there is not much content in Eq. (3). One way of gaining content would be to adopt a specific model in which the α and β could be evaluated, such as some models of the Veneziano type. We wish to avoid specific models. We make the following alternate hypothesis, which we call the index-independence hypothesis: The ratios $\beta_i^{(+)}/\alpha_i$ and $\beta_i^{(-)}/\alpha_i$ are independent of the index i. This is a fairly strong hypothesis, so we shall discuss the conditions under which it is approximately true. It is appropriate in a model in which the external baryons are degenerate, the external mesons are degenerate, and the baryon trajectories are degenerate. The hypothesis is equivalent to the assumption that these degeneracies are not accidental.

If the external baryons, external mesons, even-signature trajectories, and odd-signature trajectories correspond to irreducible representations of a symmetry group, the index-independence hypothesis follows from group symmetry and duality. Duality, applied to exotic states, implies that the even- and odd-signature trajectories are exchange degenerate, with proportional residues, and group symmetry then leads directly to index independence. Thus, the physical motivation for the hypothesis is that the dominant even- and odd-signature trajectories seem to correspond to the 56- and 70-fold irreducible representations of $SU(6)$.[6] Of course, the trajectory-degeneracy assumption must be modified when baryon spins are included correctly. This is done in Sec. II B.

[6] This classification has been made by many authors; see, e.g., R. H. Capps, Phys. Rev. **158**, 1433 (1967).

We now complete the derivation of the consistency conditions, using the index-independence hypothesis. Application of Eq. (3) to a process i that is exotic in the s channel leads to the condition $X_{ui}^{(+)}=X_{ui}^{(-)}$. Application to the crossed amplitude (obtained by reversing the roles of the s and u channels) then implies $\beta^{(+)}=\beta^{(-)}$. We next assume temporarily that for some amplitude i, $X_{ui}^{(+)}-X_{ui}^{(-)}$ does not vanish. Application of Eq. (3) to this amplitude and its crossed amplitude and use of the relation $\beta^{(+)}=\beta^{(-)}$ lead to the condition $[\beta^{(+)}/\alpha]^2=1$. This implies that Eq. (3) may be written

$$X_{ui}^{(+)}-X_{ui}^{(-)}=\pm(X_{si}^{(+)}-X_{si}^{(-)}). \quad (4)$$

The choice of signs must be the same for all i. The proper choice depends on whether or not the residues change sign between the resonance and the scattering regions.

If $X_{ui}^{(+)}-X_{ui}^{(-)}$ vanishes for all i, the above argument breaks down, but Eq. (4) is valid, obviously.[7]

The u-channel residues are related to s-channel residues by the equation $X_{ui}^{(\pm)}=\sum_j C_{ij}X_{sj}^{(\pm)}$, where C is the $s\rightleftarrows u$ crossing matrix. Thus, Eq. (4) is the requirement that the $X_{si}^{(+)}-X_{si}^{(-)}$ are components of an eigenvector of the crossing matrix. If sufficient backscattering data are available, this type of condition can be tested. For some reactions, however, resonance data but no scattering data are available, so an alternative form of the condition is useful. In order to obtain this form, we eliminate the $X_{ui}^{(\pm)}$ terms in Eq. (2) by using the relation $Z_{ui}^{(\pm)}=\beta_{i^*}^{(\pm)}X_{ui}^{(\pm)}$, where the amplitude i^* is obtained from i by crossing the two mesons. One then modifies the index-independence hypothesis slightly by assuming that $\beta_{i^*}^{(\pm)}/\alpha_i$ is independent of i. One then uses an argument similar to that used above to obtain the result

$$Z_{ui}^{(+)}-Z_{ui}^{(-)}=\pm(Z_{si}^{(+)}-Z_{si}^{(-)}). \quad (5)$$

The Z are the integrals of the contributions of resonances, including daughters, at an intermediate energy.[8] Presumably, this energy should not be close to threshold.

It is convenient to extend the index-independence hypothesis in order to obtain still a third form of the consistency condition. We define $Y_{si}^{(\pm)}$ to be the sum of the residues of the s-channel trajectories at the s value of the lowest resonance or bound state of parity $\pm$ on the trajectory. Proportionality constants $\gamma_i^{(\pm)}$ are defined by the relation $Z_{si}^{(\pm)}=\gamma_i^{(\pm)}Y_{si}^{(\pm)}$. We assume that the $\gamma_i^{(\pm)}$ are independent of i, so that Eq. (5) leads to the condition

$$R_{ui}=\pm R_{si}, \quad (6)$$

[7] An abbreviated form of the proof of Eq. (4) has been given previously by R. H. Capps, Phys. Rev. D **2**, 780 (1970). The sign ambiguity in the consistency condition is not mentioned in this reference.

[8] When one of the channels is exotic, this condition becomes a form of the simple duality principle. The predictions of this kind of condition for meson-decuplet scattering have been compared to experiment by M. J. King and K. C. Wali, Phys. Rev. Letters **24**, 1460 (1970).

where

$$R_{\nu i}=Y_{\nu i}{}^{(+)}-\gamma Y_{\nu i}{}^{(-)}\,, \qquad (7)$$

and γ is the ratio $\gamma^{(-)}/\gamma^{(+)}$. This "low-energy" form of the condition is compared with data in Sec. III.

B. Scattering of Mesons from Spinning Baryons

We now consider the scattering of pseudoscalar mesons from $j^P=\frac{1}{2}^+$ baryons, treating the baryon spins correctly. The assumption concerning baryon trajectories must be modified slightly. There are two types of even-signature trajectories, N type and Δ type; the lightest physical states on these are of spin parity $\frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively. These two types of trajectories are not degenerate with each other but are assumed exchange degenerate, respectively, with odd-signature trajectories whose lightest states are of $j^P=\frac{3}{2}^-$ and $\frac{5}{2}^-$. The multiplicity and internal quantum numbers of the even-signature trajectories of either of the types need not be the same as those of the exchange-degenerate odd-signature trajectories.

We will apply the conditions to the A' and B amplitudes. We list the partial-wave expansions for the A, A', and B amplitudes[9]:

$$q^2A(s,u)=4\pi[(W+m)(E-m)f_1-(W-m)(E+m)f_2]\,, \qquad (8)$$

$$q^2B(s,u)=4\pi[(E-m)f_1+(E+m)f_2]\,, \qquad (9)$$

$$A'(s,u)=A+(s-u)B/(4m-t/m)\,, \qquad (10)$$

$$f_1=\sum_j(a_{j-}P_{j+1/2}'-a_{j+}P_{j-1/2}')\,, \qquad (11a)$$

$$f_2=\sum_j(a_{j+}P_{j+1/2}'-a_{j-}P_{j-1/2}')\,, \qquad (11b)$$

where m and E are the nucleon mass and energy in the c.m. system, $W=s^{1/2}$, $a_{j\pm}$ is the partial-wave amplitude for angular momentum j and orbital angular momentum $j\pm\frac{1}{2}$, and P_k' is the derivative with respect to $\cos\theta$ of the Legendre polynomial P_k. For convenience we list the relation between E and W:

$$E=(W^2+m^2-\mu^2)/2W\,, \qquad (12)$$

where μ is the meson mass. In the backward direction, the relation between s and u is

$$su=(m^2-\mu^2)^2\,. \qquad (13)$$

The differential cross section in the back direction is equal to $(E/4\pi W)^2|A'|^2$. It can be shown from the above formulas and the relation $s+t+u=2(m^2+\mu^2)$ that, in the back direction,

$$A'=(4\pi W/E)(f_1-f_2)\,. \qquad (14)$$

[9] These formulas are given by Virendra Singh, Phys. Rev. 129, 1889 (1963).

The amplitudes A or A' and B are convenient because, unlike f_1 and f_2, they are functions of s and u only, i.e., they are even in $s^{1/2}$ and $u^{1/2}$. The crossing properties are

$$A_i'(s,u)=A_{i^*}'(u,s)\,, \qquad (15a)$$

$$B_i(s,u)=-B_{i^*}(u,s)\,, \qquad (15b)$$

where i^* is the amplitude obtained from i by crossing the two mesons. The crossing behavior of A is the same as that of A'. The amplitudes that satisfy the u-channel partial-wave expansions obtained by replacing s by u, W by $u^{1/2}$, and $E(s)$ by $E(u)$ in the right-hand sides of Eqs. (8) and (9), and $\cos\theta_s$ by $\cos\theta_u$ in Eqs. (11a) and (11b), are $A_i(s,u)$ and $-B_i(s,u)$.

We will use Eq. (6), the low-energy form of the consistency condition. The $Y^{(+)}$ are the residues at the P-wave resonance energy, and the $Y^{(-)}$ are the residues at the D-wave resonance energy. The residues Y are defined for both the A' and B amplitudes, i.e.,

$$Y_{siA'}{}^{(\pm)}=\mathrm{Res}A_i'(s,u)\,, \qquad (16a)$$

$$Y_{siB}{}^{(\pm)}=\mathrm{Res}[q^2B(s,u)]\,, \qquad (16b)$$

where Res denotes the residue of the s-channel trajectory of the appropriate signature at the appropriate resonance energy. The q^2 factor is included in the residue definition for B only for convenience; this does not change the crossing property of the residue since q^2 is the same in the s and u channels along the back-direction curve of Eq. (13).

If one considers the A' and B amplitudes and repeats the argument that leads to Eq. (6), the following equations result:

$$R_{uiA'}=\pm R_{siA'}\,, \qquad (17a)$$

$$R_{uiB}=\mp R_{siB}\,, \qquad (17b)$$

where $R=Y^{(+)}-\gamma Y^{(-)}$. One should take either the upper or lower signs in both equations. These are the basic consistency equations for meson-baryon scattering.

We now show why the choices of signs in these two conditions are correlated. The proper choices depend on whether or not the residues change sign between the resonance (intermediate-s) and Regge (small-s) regions. Since s and u are both positive along the backscattering curve, we see from Eq. (12) that $E+m$ is larger than $E-m$ throughout the range between the resonance and Regge regions.[10] Hence, we expect the $(E+m)f_2$ term of Eq. (9) to dominate the B amplitude. The index-independence assumption for A', together with the fact that the relative magnitude of f_1 and f_2 is not independent of i, implies that the (Regge region)/(resonance region) ratios of f_1 and f_2 should be the same. Since E and W do not change signs between the resonance and Regge regions, it follows that $A'(s,u)$ and $q^2B(s,u)$

[10] Some of the quantities discussed here possess branch points at $s=0$ or $u=0$. These branch points are not relevant here, as we are concerned only with regions of positive s and u.

either both change sign or both do not change sign. However, if the s-channel residues are defined by Eqs. (16), the u-channel residues depend on $A'(s,u)$ and $-q^2B(s,u)$. This leads to an extra minus sign in the condition for B [Eq. (17b)] so that the upper sign in this equation goes with the upper sign in Eq. (17a).

A similar argument provides another reason that one should apply the index-independence hypothesis to A' rather than A. (The first reason is that the back-scattering amplitude is proportional to A'.) Since both terms of A [Eq. (8)] may be of comparable importance, and since the s dependences of the coefficients of these terms are different, a common s dependence of the residues of f_1 and f_2 would not lead necessarily to a common s dependence of all A_i.[11] The index-independence hypothesis would be more artificial if applied to the A_i.

Since the $Y^{(+)}$ are defined at the energy of the P-wave resonances, it is convenient to identify an exchange-degenerate trajectory by the spin ($\frac{1}{2}$ or $\frac{3}{2}$) of the P-wave state on the trajectory. The quantity $Y_{sij}{}^{(+)}$ is then the sum of the residues of the P_j poles in the amplitude i, and $Y_{sij}{}^{(-)}$ is the sum of the residues of the D-wave poles that correspond to angular momentum $j+1$.

One may use this notation to write the A' and B residues in terms of the $R_{si\frac{1}{2}}$ and $R_{si\frac{3}{2}}$ by using the P-wave part of the partial-wave expansions of A' and B in the back direction. These may be computed from Eqs. (9), (11a), (11b), and (14). If we neglect the $(E-m)$ term of Eq. (9), the result is

$$R_{siA'}=c_1(R_{si\frac{1}{2}}+2R_{si\frac{3}{2}}), \tag{18a}$$

$$R_{siB}=c_2(R_{si\frac{1}{2}}-R_{si\frac{3}{2}}), \tag{18b}$$

where c_1 and c_2 are s-dependent factors of proportionality.

If these equations are substituted into Eqs. (17) and the upper signs are taken in the latter equations, the resulting consistency conditions are algebraically the same as those of the static, reciprocal bootstrap model.[1,2] If the final $\frac{1}{2}$ or $\frac{3}{2}$ index of the R's is identified with the total angular momentum of the static model, the conditions are that the R's are components of an eigenvector of the static P-wave $s \rightleftarrows u$ crossing matrix, with eigenvalue one. In our case, however, the R's involve the couplings of baryons of both parities [See Eq. (7)]. The R's for some amplitudes are evaluated in Sec. III.

III. COMPARISON OF PION-BARYON CONDITIONS WITH DATA

We turn to the problem of checking the predictions for πN, $\pi\Lambda$, and $\pi\Sigma$ amplitudes, using partial-width data. We use Eqs. (17) with the upper signs, and write all u-channel residues in terms of s-channel residues by using the equation $R_{ui}=\sum_j C_{ij}R_{sj}$, where C is the crossing matrix.[12] All amplitudes then refer to the s channel, so the s may be suppressed. The result of the procedure is that the following six amplitudes are zero:

$$\begin{aligned}
&[BN0] && (\tfrac{2}{3})^{1/2}(R_{N\frac{1}{2}B}+2R_{N\frac{3}{2}B}),\\
&[A'N1] && (\tfrac{4}{3})^{1/2}(R_{N\frac{1}{2}A'}-R_{N\frac{3}{2}A'}),\\
&[B\Lambda 0] && (3)^{1/2}R_{\Lambda 1B},\\
&[B\Sigma 0] && \tfrac{1}{3}(R_{\Sigma 0B}+3R_{\Sigma 1B}+5R_{\Sigma 2B}),\\
&[A'\Sigma 1] && (\tfrac{1}{12})^{1/2}(2R_{\Sigma 0A'}+3R_{\Sigma 1A'}-5R_{\Sigma 2A'}),\\
&[B\Sigma 2] && (5/36)^{1/2}(2R_{\Sigma 0B}-3R_{\Sigma 1B}+R_{\Sigma 2B}),
\end{aligned} \tag{19}$$

where the $R_{\mu\nu A'}$ and $R_{\mu\nu B}$ are to be written as in Eqs. (18). The first subscript on the R denotes the external baryons, and the second the s-channel isotopic spin. The symbols in square brackets identify the amplitudes; here the third symbol is the t-channel isotopic spin.[12] These are the six amplitudes that are odd under spin-isospin, $s \rightleftarrows u$ crossing, i.e., the A' and B are odd and even, respectively, under crossing of the internal indices.

The different trajectories contributing to a particular $Y_i{}^{(+)}$ or $Y_i{}^{(-)}$ are not all degenerate, in practice. We ignore the possible effects of mass splitting on the coupling constants (residues at the resonance energies).

We now turn to the problem of evaluating the $R=Y^{(+)}-\gamma Y^{(-)}$. The $Y^{(+)}$ are the residues of the P-wave poles. We normalize all the $Y^{(\pm)}$ by setting the contribution to $Y_i{}^{(+)}$ of any P-wave resonance on a trajectory equal to Γ_i/k^3, where Γ_i is the partial width for the decay into the state i, and k is the decay momentum. If the P-wave pole is a bound-state pole, the corresponding quantity is $CG^2/[(M+m)^2-\mu^2]$, where G is a coupling constant, M, m, and μ are the masses of the bound state, baryon, and meson, respectively, and C is a numerical constant. For the case of the residue $Y_{N\frac{1}{2}\frac{1}{2}}{}^{(+)}$, the C corresponding to the nucleon-pole contribution is 6. The πNN coupling constant is taken as $G^2=14.5$. The other baryon-baryon-meson constants are calculated from the assumption of $SU(3)$ symmetry, with the F/D ratio taken as the $SU(6)$ value of $\frac{2}{3}$.

We take $a_j\Gamma_i/k^5$ for the contribution of an odd-signature trajectory to $\gamma Y_{si}{}^{(-)}$, where Γ_i is the appropriate partial width of the lowest state (D state) on the trajectory, and a_j is a constant, assumed to depend only on the spin. We determine the a_j by assuming that the duality principle is valid for $\bar{K}N$ scattering. This leads to the simple prediction that the contributions of Λ poles of opposite parities should cancel in the R for $\bar{K}N$ amplitudes, and that the contributions of Σ poles of opposite parities should cancel. This follows because KN states of both isospins are exotic. We obtain the value $a_{3/2}=1.51$ BeV^2 by assuming that the $\Lambda_{3/2^-}(1520)$ and $\Lambda_{1/2^+}(1116)$ contributions cancel in $\bar{K}N$ amplitudes,

[11] If one uses the Veneziano representation of the amplitudes, the bootstrap condition is a condition on the form of satellite terms. In the Veneziano model for meson-baryon scattering, it is more convenient to use the A then the A' amplitude because of the pole at $t=4m^2$ in A'. Because of this, the Veneziano form of the bootstrap condition used here is simple for spinless particles (see Ref. 4), but is not simple for meson-baryon scattering.

[12] The various s-u and s-t channel isotopic-spin crossing matrices are listed by C. Rebbi and R. Slansky, Rev. Mod. Phys. **42**, 68 (1970).

TABLE I. Contributions of different P- and D-wave resonances to the $R_i = Y_i^{(+)} - \gamma Y_i^{(-)}$. We take $\hbar = c = 1$ and the unit of mass to be 1 BeV.

State	Resonance	Contribution to R
$N\,\frac{1}{2}\,\frac{1}{2}$	$N^+(938)$	24.7
$N\,\frac{1}{2}\,\frac{1}{2}$	$N^-(1520)$	−4.6
$N\,\frac{1}{2}\,\frac{3}{2}$	$N^-(1670)$	−1.7
$N\,\frac{3}{2}\,\frac{1}{2}$	$\Delta^-(1670)$	−0.9
$N\,\frac{3}{2}\,\frac{3}{2}$	$\Delta^+(1236)$	9.7
$\Lambda\,1\,\frac{1}{2}$	$\Sigma^+(1193)$	2.6
$\Lambda\,1\,\frac{1}{2}$	$\Sigma^-(1670)$	−1.4
$\Lambda\,1\,\frac{3}{2}$	$\Sigma^+(1385)$	3.6
$\Lambda\,1\,\frac{3}{2}$	$\Sigma^-(1765)$	−0.6
$\Sigma\,0\,\frac{1}{2}$	$\Lambda^+(1116)$	7.8
$\Sigma\,0\,\frac{1}{2}$	$\Lambda^-(1520)$	−8.3
$\Sigma\,0\,\frac{1}{2}$	$\Lambda^-(1690)$	−3.4
$\Sigma\,0\,\frac{3}{2}$	$\Lambda^-(1830)$	−1.2
$\Sigma\,1\,\frac{1}{2}$	$\Sigma^+(1193)$	6.5
$\Sigma\,1\,\frac{1}{2}$	$\Sigma^-(1670)$	−4.4
$\Sigma\,1\,\frac{3}{2}$	$\Sigma^+(1385)$	2.2
$\Sigma\,1\,\frac{3}{2}$	$\Sigma^-(1765)$	~0

and obtain $a_{5/2} = 1.53$ BeV^2 by assuming that the $\Sigma_{5/2^-}(1765)$ and $\Sigma_{3/2^+}(1385)$ contributions cancel. All data are taken from the recent compilation of Barbaro-Galtieri *et al.*[13]

The index-independence hypothesis requires that certain ratios involving the residues are the same for N-type and Δ-type trajectories. In view of this, it is reassuring that the values of $a_{3/2}$ and $a_{5/2}$ computed above are nearly equal.

The contributions of individual P- and D-wave resonances to $Y_{si}^{(+)}$ and $\gamma Y_{si}^{(-)}$ are given in Table I. The three symbols in the state label are the external baryon, the s-channel isotopic spin, and the spin label of the trajectory. The angular momenta of the odd-parity resonances are one greater than this spin label. The sign of a contribution to R in the third column is the parity of the resonance. The R are those of the right-hand side of Eqs. (18). The particles contributing to the $Y_{i\frac{1}{2}}^{(+)}$, $Y_{i\frac{3}{2}}^{(+)}$, $\gamma Y_{i\frac{1}{2}}^{(-)}$, and $\gamma Y_{i\frac{3}{2}}^{(-)}$ are those of the N octet, Δ decuplet, $j^P = \frac{3}{2}^-$ nonet, and $j^P = \frac{5}{2}^-$ octet, respectively.

It would have been just as logical to calculate $a_{3/2}$ by assuming that the $\Lambda(1115)$ contribution to the R for $\bar{K}N$ scattering is cancelled by the sum of the $\Lambda^*(1520)$ and $\Lambda^*(1690)$ contributions, rather than by the $\Lambda^*(1520)$ contribution alone. A calculation shows that this would have decreased $a_{3/2}$ by only 6%, and all the contributions of $\frac{3}{2}^-$ resonances in Table I by the same percentage. The effect on the final results would be small.

In order to test the accuracy of the predictions that certain R combinations are small, we adopt a uniform normalization, based on the fact that $(W_\alpha/W_\beta)^{1/2}K_{\alpha\beta}$ are the elements of a unitary matrix, where K is the $t \rightleftarrows s$ crossing matrix, i.e., $T(\alpha_t) = \sum_\beta K_{\alpha\beta}T(\beta_s)$.[14] Here W_α is the number of states in the spin-isospin multiplet α. We use the amplitudes of Eqs. (18) with the c_1 and c_2 replaced by $(\frac{2}{3})^{1/2}$ and $(\frac{4}{3})^{1/2}$, respectively. With this convention, the amplitudes of Eq. (19) are normalized so that $\sum_{IJ} x_{IJ}^2/[(2I+1)(2J+1)] = 1$, where x_{IJ} is the coefficient of each R_{IJ}, and I and J are the s-channel isospin and spin indices.

The $Y^{(+)}$, $\gamma Y^{(-)}$, and R are listed in Table II. The fundamental unit of mass is 1 BeV. The first six rows correspond to the six crossing-odd amplitudes of Eq. (19) for which R is predicted to be zero. The next six rows correspond to the crossing-even amplitudes obtained by making the interchange $A' \rightleftarrows B$ in Eq. (19). The consistency conditions are satisfied fairly well; the crossing-odd R values are small compared to the larger of the crossing-even values and to the larger of the $Y^{(+)}$ and $\gamma Y^{(-)}$ values of the table.

It is important to note that if $SU(3)$ symmetry of the residues were satisfied exactly, the ratios of Y's would depend on seven independent parameters. Hence, $SU(3)$ symmetry and the satisfaction of two or three of the consistency conditions does not imply satisfaction of the others.

If the lower signs of Eqs. (17) had been taken, the prediction would be that the vector R_i is an eigenvector of the static P-wave crossing matrix with eigenvalue (-1). This would imply that the quantities R for both N- and Δ-type trajectories would vanish for $\pi^0 p$ scattering. This is in strong contradiction with experiment, for the odd-parity contributions $\gamma Y^{(-)}$ are relatively small for pion-nucleon scattering, as can be seen from Table I.

TABLE II. Values of linear combinations of the residue functions $R_{A'}$ and R_B.

Combination	$Y^{(+)}$	$Y^{(-)}$	R
$[BN0]$	5.0	4.3	0.7
$[A'N1]$	4.9	6.7	−1.8
$[B\Lambda 0]$	−2.0	1.4	−3.4
$[B\Sigma 0]$	7.9	9.1	−1.2
$[A'\Sigma 1]$	11.4	9.7	1.7
$[B\Sigma 2]$	1.3	3.3	−2.0
$[A'N0]$	42.2	6.5	35.7
$[BN1]$	45.9	2.7	43.2
$[A'\Lambda 0]$	13.9	3.8	10.1
$[A'\Sigma 0]$	11.1	7.4	3.7
$[B\Sigma 1]$	9.5	11.4	−1.9
$[A'\Sigma 2]$	−5.3	4.6	−9.9

IV. CONCLUDING REMARKS

The duality condition, as used here, relates s-channel resonances to the residues of u-channel Regge poles. If

[13] A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970). If a definite width is not given in the resonance table, and if a weighted average width is not given in the data-card listings, we have used as a width the simple numerical average of all measurements listed in the listings.

[14] This general property of crossing matrices is derived by R. H. Capps, Ann. Phys. (N. Y.) **43**, 428 (1967), Appendix.

one knows the behavior of the residues of these poles between the small-u region and the physical region for u-channel scattering, one can obtain consistency conditions of the bootstrap type.

In this paper we assume that the $P_{1/2}$ and $P_{3/2}$ resonances are dynamically similar and that the residues of the trajectories through them are dynamically similar. This leads to generalizations of the Chew reciprocal-bootstrap conditions. These agree with the present crude partial-width data reasonably well (i.e., the first six R values in Table II are small).

Actually, the N- and Δ-type trajectories are not alike. The P-wave states are not degenerate. It is interesting that the $P_{3/2}$ states occur at energies intermediate between those corresponding to P-state degeneracy and those required by trajectory degeneracy.[15] A useful way to improve the bootstrap condition would be to find a simple method of accounting for the difference between N-type and Δ-type trajectories.

We illustrate this problem by considering the Regge form [Eq. (4)] of the consistency condition for πN scattering. The condition states that $|\mathrm{Im}A'|$ in the back direction should be the same for π^+p and π^-p scattering. The experimental data in the range of lab momentum 4–10 BeV/c shows that at 180°, $|A'_{\pi^+p}/A'_{\pi^-p}|\sim 2$.[16] It appears from the Regge analyses that the ratio of imaginary parts is also about 2.[16] The prediction is off by a factor of 2.

On the other hand, if the semilocal average in Eq. (1) is taken to involve an average over angle as well as energy, it is likely that the prediction is satisfied better, because the π^+p cross section is peaked particularly sharply in the back direction. (Off the back direction, the data interpretation is complicated by the presence of B as well as A' terms in the differential cross section, however.) This example illustrates that the question of correcting for the difference in the N- and Δ-type trajectories is not disconnected from the question of what type of semilocal average one should take in applying the Regge or intermediate-energy form of the consistency condition.

Another useful way of extending the method introduced here would be to derive consistency conditions relating s- and t-channel residues. The t-channel residues would be proportional to products of the couplings of meson trajectories with meson-meson and baryon-antibaryon states. One could not test such a condition on a single amplitude without knowing something about the relation between the residues of baryon and meson trajectories. The condition could be tested for amplitude ratios, however; i.e., one could determine an over-all proportionality constant by applying the condition to one amplitude, and then use this constant to test the condition for other amplitudes. It is hoped that such a procedure will be carried out in the future.

In a previous reference, an idealized Veneziano model was used to derive conditions similar to those given here, for each of the three pairs of Mandelstam channels.[4,7] It was pointed out that if the even- and odd-signature trajectories correspond respectively to the $SU(6)$ representations **56** and **70**, the s-t and s-u channel conditions could not be satisfied simultaneously. This raises the question of whether or not one of these conditions is satisfied at the expense of the other. If only the $\mathbf{56}^+$ and $\mathbf{70}^-$ baryon trajectory multiplets contribute to meson-baryon scattering, the predictions of the s-t and s-u conditions differ only by a factor of 2 in the ratio of odd to even parity coupling, i.e., in the $\gamma Y^{(-)}/Y^{(+)}$ ratio.[7] The s-u condition is treated in this paper. However, since the $\gamma Y^{(-)}/Y^{(+)}$ ratio is determined by assuming that the condition is valid for $\bar{K}N$ scattering, our procedure is not suitable for studying the relative merits of the s-u and s-t conditions. However, it may be possible to distinguish between these conditions with the intermediate-energy or Regge form of the consistency equation [Eq. (5) or (4)] if sufficiently accurate data are available.

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[15] This may not be an accident; see M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters **22**, 83 (1969).

[16] For a review of backscattering data and Regge analyses, see V. D. Barger and D. B. Cline, *Phenomenological Theories of High Energy Scattering* (Benjamin, New York, 1969), Chap. 7.