Determination of the $\pi\Lambda\Sigma$ and $\pi\Sigma\Sigma$ Coupling Constants Utilizing Adler's Consistency Condition*

C. H. CHAN AND L. L. SMALLEY
Department of Physics, University of Alabama, Huntsville, Alabama 35807
(Received 6 July 1970)

An independent method to determine the $\pi\Sigma\Lambda$ and $\pi\Sigma\Sigma$ coupling constants is presented. It is accomplished by writing a once-subtracted dispersion relation for $\pi\Lambda$ and $\pi\Sigma$ scattering amplitudes slightly off the incident-pion mass shell and evaluating the subtraction constant at the point where Adler's consistency condition is valid. The results we obtained are $g_{\pi\Sigma\Lambda}^2/4\pi=20.9\pm6.7$ and $g_{\pi\Sigma\Sigma}^2/4\pi=11.4\pm5.5$, in good agreement with the earlier calculation of Chan and Meiere using a completely different method.

I. INTRODUCTION

THE determination of the meson-baryon coupling constants plays an important role in hadron physics. The πNN coupling constant has long been well known.[1] More recently, the $KN\Lambda$ and $KN\Sigma$ coupling constants have been determined[2] and tested[3] using the multichannel effective-range parameters of the $\bar{K}N$ system determined by Kim.[4] Finally, using the same analysis of the $\bar{K}N$ system, the $\pi\Sigma\Sigma$ and $\pi\Sigma\Lambda$ coupling constants were determined,[5] but whereas the coupling constants for the πN and KN systems satisfied both $SU(3)$ symmetry and the generalized Goldberger-Treiman relation,[6] those for the $\pi\Lambda$ and $\pi\Sigma$ systems did not.[5] In view of the importance of the question of whether the meson-baryon-baryon coupling constants satisfy $SU(3)$ symmetry or not, an independent determination of the $\pi\Sigma\Lambda$ and $\pi\Sigma\Sigma$ coupling constants is highly desirable.

We present here a determination of the $\pi\Sigma\Lambda$ and $\pi\Sigma\Sigma$ coupling constants using both Kim's analysis of the $\bar{K}N$ system[4] and Adler's consistency condition on the strong interactions.[7] This is accomplished by writing a once-subtracted dispersion relation for $\pi\Lambda$ and $\pi\Sigma$ scattering amplitudes slightly off the incident-pion mass shell and evaluating the subtraction constant at the point where Adler's consistency condition is valid. The consistency condition for πN scattering has been tested by Adler and is found to be experimentally satisfied.[7] We assume here that similar conditions also hold for $\pi\Lambda$ and $\pi\Sigma$ scatterings. Qualitatively, the consistency condition for $\pi\Lambda$ scattering has been tested by Martin[8] with very primitive experimental data.[9] Here, by using the better-known $\pi\Lambda$ and $\pi\Sigma$ scattering data and with the assumption of Adler's consistency conditions for $\pi\Lambda$ and $\pi\Sigma$ scatterings, we hope to determine the $\pi\Sigma\Lambda$ and $\pi\Sigma\Sigma$ coupling constants separately. The results we have obtained are

$$g_{\pi\Sigma\Lambda}^2/4\pi=20.9\pm6.7\,,\quad g_{\pi\Sigma\Sigma}^2/4\pi=11.4\pm5.5\,,\qquad(1)$$

in good agreement with the earlier calculation of Chan and Meiere using a completely different method.[5]

In Sec. II, a detailed calculation of the $\pi\Sigma\Lambda$ coupling constant is presented. We write a once-subtracted dispersion relation for $\pi\Lambda$ scattering and choose the subtraction point at which Adler's consistency condition is valid. In particular, it is shown explicitly how an extrapolation for the scattering amplitude is made from the physical $\pi\Lambda$ scattering where experimental data exist to the unphysical $\pi\Lambda$ scattering when one of the pion masses is equal to zero. In Sec. III, we summarize a similar calculation for the $\pi\Sigma\Sigma$ coupling constant and give our conclusion in Sec. IV.

II. DETERMINATION OF $\pi\Sigma\Lambda$ COUPLING CONSTANT

Let us first consider the matrix element M_Λ:

$$M_\Lambda=\tfrac{1}{2}(M_{\pi^-\Lambda}+M_{\pi^+\Lambda})\qquad(2)$$

for the $\pi\Lambda$ scattering process depicted in Fig. 1, where k and q are the initial and final four-momenta of the pion, and similarly p_i and p_f are those for the Λ particle. M_Λ

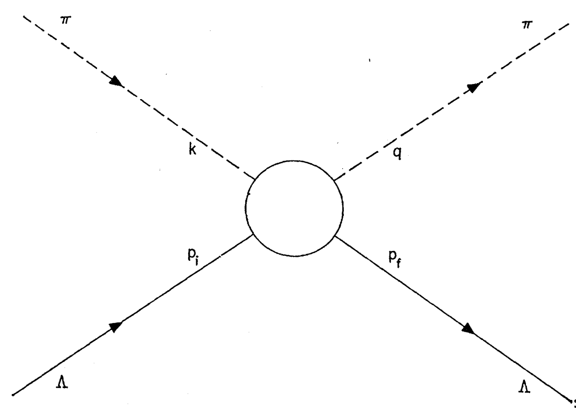

FIG. 1. π-Λ scattering.

---

* Work supported in part by the National Aeronautics and Space Administration under Grant No. NGL 01-002-001.
[1] M. H. MacGregor and R. A. Arndt, Phys. Rev. 139, B362 (1965).
[2] J. K. Kim, Phys. Rev. Letters 19, 1079 (1967).
[3] C. H. Chan and F. T. Meiere, Phys. Rev. Letters 20, 568 (1968).
[4] J. K. Kim, Phys. Rev. Letters 19, 1074 (1967).
[5] C. H. Chan and F. T. Meiere, Phys. Letters 28B, 125 (1968).
[6] S. L. Adler and R. F. Dashen, *Current Algebra* (Benjamin, New York, 1968).
[7] S. L. Adler, Phys. Rev. 137, B1022 (1965).
[8] B. R. Martin, Nuovo Cimento 43, 629 (1966).
[9] B. R. Martin, Phys. Rev. 138, B1136 (1965).

can be considered as a function of s, t, and k^2, where $s=-(k+p_i)^2$, $t=-(k-q)^2$, and k^2 is the four-momentum squared of the incident pion. We write k^2 explicitly since we shall consider processes in which the incident pion is off its mass shell. The matrix element M_Λ can be decomposed into the usual invariants,[10]

$$M_\Lambda(s,t,k^2)=\bar{u}(p_f)\times[-A_\Lambda(s,t,k^2)+\tfrac{1}{2}i\gamma\cdot(k+q)B_\Lambda(s,t,k^2)]u(p_i)\,. \quad (3)$$

The Adler condition is then obtained for the amplitude A_Λ in the limit $k_\mu\to 0$, in which case $s=\Lambda^2$, $t=\mu^2$, and $k^2=0$, where μ and Λ represent the mass of pion and the Λ, respectively. In this limit, Adler has shown[6]

$$A_\Lambda(s=\Lambda^2, t=\mu^2, k^2=0)=0\,. \quad (4)$$

We shall use the variable

$$\nu=(1/4\Lambda)(s-u)\,, \quad (5)$$

where

$$u=-(k-p_f)^2=2\Lambda^2+\mu^2-s-t-k^2\,.$$

Then Eq. (4) is identical to

$$A_\Lambda(\nu=0, t=\mu^2, k^2=0)=0\,. \quad (6)$$

In order to write a dispersion relation, we consider $A_\Lambda(\nu, t=\mu^2, k^2=0)$ as an analytic function of ν. Furthermore, A_Λ has poles when $s=\Sigma^2$ and $u=\Sigma^2$, i.e., in the ν plane when $\nu=\pm\nu_p$, where

$$\nu_p=(\Sigma^2-\Lambda^2)/2\Lambda\,, \quad (7)$$

and branch points at $s=(\mu+\Lambda)^2$ and $u=(\mu+\Lambda)^2$, i.e., cuts in the ν plane from $-\infty$ to $-\nu_C$ and ν_C to ∞, where

$$\nu_C=\mu+\mu^2/2\Lambda\,. \quad (8)$$

With the usually assumed asymptotic behavior, the function A_Λ satisfies the following once-subtracted dispersion relation:

$$\mathrm{Re}A_\Lambda(\nu, t=\mu^2, k^2=0)=\mathrm{Re}A_\Lambda(\nu_0, t=\mu^2, k^2=0) -\frac{g_{\pi\Sigma\Lambda}^2}{4\pi}\left[\frac{4\pi(\Sigma-\Lambda)}{\Lambda}\frac{\nu_p(\nu^2-\nu_0^2)}{(\nu_p^2-\nu^2)(\nu_p^2-\nu_0^2)}\right] +\frac{2(\nu^2-\nu_0^2)}{\pi}P\int_{\nu_C}^{\infty}\frac{\nu'\,\mathrm{Im}A_\Lambda(\nu', t=\mu^2, k^2=0)}{(\nu'^2-\nu^2)(\nu'^2-\nu_0^2)}d\nu'\,, \quad (9)$$

where the pole term has been explicitly separated out from the integral and $g_{\pi\Sigma\Lambda}$ is the rationalized, renormalized $\pi\Sigma\Lambda$ coupling constant (e.g., $g_{\pi N}^2/4\pi=14.5$).

The subtraction constant is evaluated at $\nu_0=0$, where the Adler condition, Eq. (6), is applicable. Hence Eq. (9) reduces to

$$\mathrm{Re}A_\Lambda(\nu, t=\mu^2, k^2=0)=-\frac{g_{\pi\Sigma\Lambda}^2}{4\pi}\frac{4\pi(\Sigma-\Lambda)}{\Lambda}\frac{\nu^2}{\nu_p(\nu_p^2-\nu^2)} +\frac{2\nu^2}{\pi}P\int_{\nu_C}^{\infty}\frac{\mathrm{Im}A_\Lambda(\nu', t=\mu^2, k^2=0)}{\nu'(\nu'^2-\nu^2)}d\nu'\,. \quad (10)$$

This equation is essentially what we shall use to determine the $\pi\Sigma\Lambda$ coupling constant. In order to carry out the numerical calculation, we shall replace the amplitude $A_\Lambda(\nu)$ in terms of $\pi\Lambda$ phase shifts. However, in this case, the amplitude A_Λ for the process shown in Fig. 1 is unphysical since the incident-pion mass is equal to zero, i.e., $k^2=0$, while all other particles are on their mass shells. We first write A_Λ in terms of the amplitudes f_1 and f_2 as[11]

$$A_\Lambda=4\pi\left(\frac{f_1(W+\Lambda)}{[(E_i+\Lambda)(E_f+\Lambda)]^{1/2}}-\frac{f_2(W-\Lambda)}{[(E_i-\Lambda)(E_f-\Lambda)]^{1/2}}\right), \quad (11)$$

where $W=\sqrt{s}$ is the total c.m. energy,

$$E_i=(W^2+\Lambda^2)/2W \quad (12)$$

is the incident Λ energy in the c.m. system, and

$$E_f=(W^2+\Lambda^2-\mu^2)/2W \quad (13)$$

is the outgoing Λ energy in the c.m. system. The amplitudes f_1 and f_2 are given by

$$f_1=\sum_{l=0}^{\infty}[f_{l+}P_{l+1}'(x)-f_{l-}P_{l-1}'(x)] \quad (14)$$

and

$$f_2=\sum_{l=1}^{\infty}(f_{l-}-f_{l+})P_l'(x)\,, \quad (15)$$

where $P_l(x)$ are Legendre polynomials, $x=\cos\theta$ is the cosine of the c.m. scattering angle which at $t=\mu^2$ (and $k^2=0$, $q^2=-\mu^2$) is

$$x=\cos\theta=\left(1+\frac{\mu^2}{E_f^2-\Lambda^2}\right)^{1/2}, \quad (16)$$

and $f_{l\pm}$ are the off-mass-shell $\pi\Lambda$ partial-wave scattering amplitudes with orbital angular momentum l and total angular momentum $l\pm\frac{1}{2}$. Since Kim's parameters and all the Y_1^* resonance data are for $\pi\Lambda\to\pi\Lambda$ with both pions on their mass shells, in order to take the off-mass-shells effect into account we multiply their partial-wave scattering amplitudes by a factor

$$(q_{\mathrm{off}}/q_{\mathrm{on}})^l\,, \quad (17)$$

where l is the orbital-angular-momentum quantum number for each partial wave, and

$$q_{\mathrm{off}}=(E_i^2-\Lambda^2)^{1/2}, \quad q_{\mathrm{on}}=(E_f^2-\Lambda^2)^{1/2}. \quad (18)$$

We then evaluate Eq. (10) at the $\bar{K}N$ threshold $\nu=\nu_{\bar{K}N}$, i.e.,

$$\nu_{\bar{K}N}=(1/2\Lambda)[(N+K)^2-\Lambda^2]\,, \quad (19)$$

where Kim's phase-shift analysis is most reliable.

[10] K. Nishizima, *Fundamental Particles* (Benjamin, New York, 1964), p. 148.

[11] W. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

The integral in Eq. (10) was evaluated in two parts: (a) the low-energy region from ν_C at threshold up to a cutoff energy corresponding to the total c.m. energy $W=1612$ MeV (in this region Kim's parameters for $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ waves were used); (b) the resonance-energy region (above $W=1612$ MeV) where we used all the known Y_1^* resonances from the Particle Properties Tables[12] and assumed the narrow-width approximation to evaluate their contributions. The narrow-width approximation consists of noting that near a resonance the partial-wave scattering amplitude f_l may be represented by

$$f_l=\frac{1}{q}\frac{\frac{1}{2}\Gamma_p}{(W_R-W)-\frac{1}{2}i\Gamma_T}, \qquad (20)$$

where Γ_p and Γ_T are the partial and total widths, and q is the momentum of the particle in the c.m. system. Then the imaginary part of f_l is given by

$$\mathrm{Im} f_l=(\Gamma_p/2q)\pi\delta(W-W_R). \qquad (21)$$

Table I lists the relevant Y^* resonances and their parameters which we use in our evaluation. The last two columns give their individual contributions to the resonance-energy-region integral.

The numerical results are[13]

$$\begin{aligned} \mathrm{Re}A_\Lambda(\nu_{\bar{K}N},\mu^2,0)&=-39.28\ \mathrm{F}^2, \\ \text{coefficient of pole term}&=+\ 2.25\ \mathrm{F}^2, \\ \text{integral (low-energy region)}&=-86.52\ \mathrm{F}^2, \\ \text{integral (resonance region)}&=-\ 0.03\ \mathrm{F}^2. \end{aligned} \qquad (22)$$

We hence obtain

$$g_{\pi\Sigma\Lambda}^2/4\pi=20.9\pm6.7. \qquad (23)$$

We note here that the claculation is sensitive only to the value of ReA and the low-energy integral. On the other hand, the contribution from the resonance-energy region is insignificant. Any change in the parameter and the shape of the resonances will not change our result more than 2 or 3%. We also estimated the contribution from the integration beyond the resonance energy by assuming a $\pi\Lambda$ total cross section of 30 mb or less. Due to the highly convergent behavior of the integrand, the contribution was less than 0.5% of that from the low-energy region and hence negligible in comparison to the uncertainty in the low-energy integral.

Because of the insignificance of the contributions from the resonance-energy and high-energy regions, the quoted error in Eq. (23) is solely due to the uncertainty of Kim's effective-range parameters. We varied each of Kim's 24 parameters used in this calculation within their quoted errors and assumed that they were not correlated. Our final error given in Eq. (23) is the square root of the sum of the squares of these errors.

[12] A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970).
[13] Units are in fermis.

TABLE I. The relevant Y^* resonances and their parameters.

Mass (MeV)	I	J^P	Γ (MeV)	Branching ratios (%) $\pi\Lambda$	$\pi\Sigma$	Contribution to resonance-energy integral (F^2) $\pi\Lambda$	$\pi\Sigma$
1520	0	$\frac{3}{2}^-$	16	···	41	···	+9.268
1670	0	$\frac{1}{2}^-$	30	···	50	···	−0.074
1690	0	$\frac{3}{2}^-$	45	···	55	···	+2.189
1815	0	$\frac{5}{2}^+$	75	···	11	···	+0.474
1830	0	$\frac{5}{2}^-$	80	···	30	···	−0.734
2100	0	$\frac{7}{2}^-$	140	···	1	···	+0.020
1670	1	$\frac{3}{2}^-$	50	32	50	−1.609	−5.609
1750	1	$\frac{1}{2}^-$	80	20	0	+0.046	−0.0
1765	1	$\frac{5}{2}^-$	100	15	1	+0.830	+0.107
1915	1	$\frac{5}{2}^+$	50	5	0.4	−0.087	−0.012
2030	1	$\frac{7}{2}^+$	120	35	5	+0.795	+0.188

It has also been suggested that Kim's parameters may not be meaningful enough to be used near the $\pi\Lambda$ threshold. In order to test the significance of this calculation, we examined the various contributions from the low-energy integral by subdividing it into several parts; our results are

low-energy integral

$$\begin{aligned} &(\text{from } W=1254\text{–}1329\ \mathrm{MeV})=-\ 6.95\ \mathrm{F}^2, \\ &(\text{from } W=1329\text{–}1404\ \mathrm{MeV})=-57.59\ \mathrm{F}^2, \\ &(\text{from } W=1404\text{–}1479\ \mathrm{MeV})=-20.61\ \mathrm{F}^2, \\ &(\text{from } W=1479\text{–}1554\ \mathrm{MeV})=-\ 0.49\ \mathrm{F}^2, \\ &(\text{from } W=1554\text{–}1612\ \mathrm{MeV})=-\ 0.88\ \mathrm{F}^2. \end{aligned}$$

This calculation shows that the first integral corresponding to the integration from the $\pi\Lambda$ threshold to the $\pi\Sigma$ threshold is not important and the last integral is negligible, which means that if we had chosen a different cutoff energy in the low-energy integral, it would hardly change our result. The dominant contribution is from the second integral where the $Y_1^*(1385)$ lies. This is in analogy to Adler's calculation for the π-N case,[7] in that the $N^*(1238)$ dominates the integral.

Lastly, we completed our calculation by using a set of K-matrix parameters for the S wave given by Martin and Sakitt.[14] This again did not affect our result. The coupling constant we obtained is $g_{\pi\Sigma\Lambda}^2/4\pi=18.7$. This is because the integral is dominated by the $Y_1^*(1385)$, so changing the set of S-wave parameters will not change our result.

III. DETERMINATION OF $\pi\Sigma\Sigma$ COUPLING CONSTANT

For the $\pi\Sigma$ system, the scattering amplitude, after crossing, often contains isospin 2. Since we have no knowledge of $I=2$ phase shifts, we must consider a combination of $I=0$ and 1 amplitudes which is crossing-even. Investigation of the $\pi\Sigma$ system shows that there is

[14] B. R. Martin and M. Sakitt, Phys. Rev. **183**, 1345 (1969); **183**, 1352 (1969).

only one crossing-even combination that does not involve $I=2$ amplitudes:

$$M_\Sigma(\nu)=2M_\Sigma(I=1)-M_\Sigma(I=0)$$
$$=7M_{\pi^0\Sigma^+}(\nu)-3[M_{\pi^-\Sigma^+}(\nu)+M_{\pi^+\Sigma^+}(\nu)], \quad (24)$$

where we have suppressed $t=\mu^2$ and $k^2=0$ arguments. For the combination $M_\Sigma(\nu)$, we decompose it into invariants $A_\Sigma(\nu)$ and $B_\Sigma(\nu)$:

$$M_\Sigma(\nu)=\bar{u}(p_f)[-A_\Sigma(\nu)+\tfrac{1}{2}i\gamma\cdot(k+q)B_\Sigma(\nu)]u(p_i). \quad (25)$$

In order to find Adler's condition for the above $\pi\Sigma$ combination, we make the following observations. For the $\pi\Lambda$ system the Adler condition is a null condition. This occurs because the Born term in $\pi\Lambda$ scattering is due to Σ exchange, and the $\Sigma\Lambda$ mass difference is nonzero. Now in the $\pi\Sigma$ scattering, there are both Σ and Λ exchanges; the Λ exchange does not contribute because of the $\Sigma\Lambda$ mass difference, but the Σ exchange does. Its contribution can be easily obtained using a method similar to that used by Adler to derive his consistency condition for πN scattering. However, they differ by a factor of 4 due to the different normalizations of the amplitudes. For the πN case, $M_N=\frac{1}{2}(M_{\pi^+p}+M_{\pi^-p})$; the s-channel nucleon pole occurs only in the π^-p scattering, and hence the Born term is proportional to $\frac{1}{2}(\sqrt{2}g_{\pi NN})^2$. For $\pi\Sigma$ scattering, $M_\Sigma=7M_{\pi^0\Sigma^+}-3(M_{\pi^-\Sigma^+}+M_{\pi^+\Sigma^+})$; the s-channel Σ pole occurs in both $\pi^0\Sigma^+$ and $\pi^-\Sigma^+$ scatterings, and the Born term is therefore proportional to $7(g_{\pi\Sigma\Sigma})^2-3(g_{\pi\Sigma\Sigma})^2$, i.e., a factor 4 in difference. Therefore, after a straightforward calculation, we obtain

$$\frac{A_\Sigma(\nu=0,t=\mu^2,k^2=0)}{K^{\Sigma\Sigma\pi}(0)}=\frac{16\pi}{\Sigma}\frac{g_{\pi\Sigma\Sigma}^2}{4\pi}, \quad (26)$$

where the pion form factor for the $\pi\Sigma\Sigma$ vertex, $K^{\Sigma\Sigma\pi}(k^2)$, is evaluated at $k^2=0$. In the derivation here, we have used a generalized form of the Goldberger-Treiman relation for Σ β decay.

The pion form factor $K^{\Sigma\Sigma\pi}(k^2)$ is normalized to unity at $k^2=-\mu^2$, i.e., when the incident pion is on its mass shell. Since we have no direct knowledge of $K^{\Sigma\Sigma\pi}$ at $k^2=0$, we expand it in a perturbation series about $k^2=-\mu^2$, at which point it is 1, and assume that it does not vary significantly from 1 at $k^2=0$. This is partially substantiated by the good agreement obtained by Adler in the πN case.[7]

We then write a dispersion relation for A_Σ by considering $A_\Sigma(\nu,t=\mu^2,k^2=0)$ as an analytic function of ν. Similar to the $\pi\Lambda$ case, $A_\Sigma(\nu)$ is an even function of ν. It has poles when $s=\Lambda^2$, Σ^2 and $u=\Lambda^2$, Σ^2, and branch points at $s=(\mu+\Lambda)^2$ and $u=(\mu+\Lambda)^2$. However, only the Λ pole contributes here. This is because the Σ pole does not contribute to the invariant amplitude A_Σ, because the residue is proportional to the mass difference between the scattered and exchanged baryons and is therefore zero. Thus in the ν plane $A_\Sigma(\nu)$ has poles only at $\nu=\pm\nu_\Lambda$, where

$$\nu_\Lambda=(\Lambda^2-\Sigma^2)/2\Sigma, \quad (27)$$

and cuts from $-\infty$ to $-\nu_C'$ and ν_C' to ∞, where

$$\nu_C'=[(\Lambda+\mu)^2-\Sigma^2]/2\Sigma. \quad (28)$$

Again, with the usually assumed asymptotic behavior, the function A_Σ satisfies the following once-subtracted dispersion relation:

$$\mathrm{Re}A_\Sigma(\nu,t=\mu^2,k^2=0)=\mathrm{Re}A_\Sigma(\nu_0,t=\mu^2,k^2=0)$$
$$+\frac{g_{\pi\Sigma\Lambda}^2}{4\pi}\left[\frac{12\pi(\Lambda-\Sigma)}{\Sigma}\frac{\nu_\Lambda(\nu^2-\nu_0^2)}{(\nu_\Lambda^2-\nu^2)(\nu_\Lambda^2-\nu_0^2)}\right]$$
$$+\frac{2(\nu^2-\nu_0^2)}{\pi}P\int_{\nu_C'}^{\infty}\frac{\nu'\,\mathrm{Im}A_\Sigma(\nu',t=\mu^2,k^2=0)}{(\nu'^2-\nu^2)(\nu'^2-\nu_0^2)}d\nu', \quad (29)$$

where the pole has been explicitly separated out from the integral. (The extra factor of -3 in the pole term occurs because only the $M_{\pi^-\Sigma^+}$ term which has a factor of -3 in the M_Σ amplitude can have a Λ intermediate state.) The subtraction constant is evaluated at $\nu_0=0$ where the Adler condition, Eq. (26), is applicable, and Eq. (29) then becomes

$$\mathrm{Re}A_\Sigma(\nu,t=\mu^2,k^2=0)=\frac{16\pi}{\Sigma}\frac{g_{\pi\Sigma\Sigma}^2}{4\pi}+\frac{g_{\pi\Sigma\Lambda}^2}{4\pi}\frac{12\pi(\Lambda-\Sigma)\nu^2}{\Sigma\nu_\Lambda(\nu_\Lambda^2-\nu^2)}$$
$$+\frac{2\nu^2}{\pi}P\int_{\nu_C'}^{\infty}\frac{\mathrm{Im}A_\Sigma(\nu',t=\mu^2,k^2=0)}{\nu'(\nu'^2-\nu^2)}d\nu'. \quad (30)$$

Using the value for $g_{\pi\Sigma\Lambda}^2/4\pi$ previously calculated, Eq. (30) becomes an equation for $g_{\pi\Sigma\Sigma}^2/4\pi$. Similarly to the $\pi\Lambda$ case, we evaluate Eq. (30) at the $\bar{K}N$ threshold. From the fact that A_Σ is a particular combination of $I=0,1$ amplitudes [Eq. (24)], we can use Kim's analysis of the $\bar{K}N$ system for the low-energy region but this time for the $I=0,1$ phase shifts for the $\pi\Sigma$ system. The contribution in the resonance region is obtained using both the $I=0,1$ Y^* resonances listed in Table I. We use a similar extrapolating procedure as that in the $\pi\Lambda$ scattering. The numerical results are[13]

$$\begin{aligned}\mathrm{Re}A_\Sigma(\nu_{\bar{K}N},t=\mu^2,k^2=0)&=-134.19\ \mathrm{F}^2,\\ \text{pole term}&=-146.29\ \mathrm{F}^2,\\ \text{integral (low-energy region)}&=-\ \ 89.10\ \mathrm{F}^2,\\ \text{integral (resonance region)}&=+\ \ \ 5.82\ \mathrm{F}^2.\end{aligned} \quad (31)$$

Hence we obtain[15]

$$g_{\pi\Sigma\Sigma}^2/4\pi=11.4\pm5.5. \quad (32)$$

IV. CONCLUSIONS

The results we have obtained here are in extremely good agreement with those in a previous calculation.[5]

[15] The estimated error is obtained by varying both the value of $g_{\pi\Sigma\Lambda}^2$ and Kim's parameters about their mean values.

However, we shall also point out here that in the previous calculation the resonance data used were from the 1968 Particle Properties Tables.[16] These data have been somewhat revised, especially the dubious 1690 Y_1^* resonance, which has not been seen in later experiments and has since been removed from the 1970 tables.[12] In order to compare more meaningfully the results of this calculation with that of the previous one, we recalculated the $\pi\Sigma\Lambda$ and $\pi\Sigma\Sigma$ coupling constants using the method of Chan and Meiere[5] with the newer resonance data listed in Table I. We found that the resonance contribution to the $\pi\Lambda$ scattering has changed from

$$5.16 \text{ F}^2 \quad \text{to} \quad 0.33 \text{ F}^2 \tag{33}$$

and for the $\pi\Sigma$ scattering the resonance contribution has changed from

$$-8.29 \text{ F}^2 \quad \text{to} \quad -3.57 \text{ F}^2, \tag{34}$$

so that the $\pi\Lambda\Sigma$ coupling constant changes from

$$21.5\pm7 \quad \text{to} \quad 17.8\pm7 \tag{35}$$

and the $\pi\Sigma\Sigma$ coupling constant changes from

$$11.4\pm5 \quad \text{to} \quad 9.0\pm5. \tag{36}$$

It should be noted that these values, though changed, are still consistent with the previous calculation and are in good agreement with the results of our calculations, Eqs. (23) and (32). None of these values agrees with the $SU(3)$ prediction. Table II compares these results with the $SU(3)$ limit. Although the values obtained for $g_{\pi\Sigma\Sigma}$ do agree with the $SU(3)$ prediction, it should be emphasized that this value follows directly from the value of $g_{\pi\Sigma\Lambda}$, which does not agree with exact $SU(3)$. If instead a value of 10 were used for $g_{\pi\Sigma\Lambda}{}^2/4\pi$, then Eq. (30) would yield 3.5 for $g_{\pi\Sigma\Sigma}{}^2/4\pi$, which definitely is not the $SU(3)$ result.

[16] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

TABLE II. Comparison of coupling constants calculated from Kim's analysis of the $\bar{K}N$ system and those obtained from exact $SU(3)$. The resonance data are from the 1970 Particle Properties Tables [Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970)].

Coupling constants	Dispersion relation: A amplitude plus Adler condition (this calculation)	Dispersion relation: B amplitude (Chan-Meiere method)	$SU(3)$ limit
$g_{\pi\Sigma\Lambda}{}^2/4\pi$	20.9±6.7	17.8±7	7
$g_{\pi\Sigma\Sigma}{}^2/4\pi$	11.4±5.5	9 ±5	9

We would also like to comment here that, though these two calculations utilize the same experimental data for input, there is no reason that they should give similar results since the methods used are completely different. In Chan and Meiere's calculation, they used an unsubtracted dispersion relation for the scattering amplitude B, while in this calculation we use a once-subtracted dispersion relation for the amplitude A. It is well known that the A and B amplitudes emphasize the various partial waves quite differently.

From the good agreement between these two calculations, we tend to be more confident in saying that the meson-baryon coupling constants may indeed violate $SU(3)$ symmetry as in the case of decuplet $\rightarrow$ baryon + meson.[17]

ACKNOWLEDGMENTS

Part of this work was completed while one of the authors (C.H.C.) was at Purdue University, Lafayette, Indiana. He would like to thank Professor F. T. Meiere for many very enlightening discussions. The authors would also like to express their appreciation to Professor J. E. Rush for his continued interest.

[17] G. Goldhaber, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).