where β is the second "free" parameter and the function $h(t)$ is given by

$$h(t)=\frac{2Q(t)}{\sqrt{t}}\ln\left(\frac{\sqrt{t}+2Q}{2m_\pi}\right)-\frac{i\pi Q(t)}{\sqrt{t}}\quad (t\geq 4m_\pi^2);\quad (18)$$

consequently,

$$\frac{Q}{\sqrt{t}}\cot\delta_{00}=\left[-\frac{3}{16\pi}f(t)\right]^{-1}\mathrm{Re}\left[\frac{1}{F(t)}\right].\quad (19)$$

To fit the $I=0$ s-wave phase shift δ_{00} [which derives from expression (19)] to the data, we require that $\delta(m_R)=\frac{1}{2}\pi$ for $m_R=900$ and 750 MeV, and *either* one of the two low-energy conditions: (I) $a_{00}=0.2\ m_\pi^{-1}$ (Weinberg's soft-pion result[12]), or (II) $f(\frac{1}{2}m_\pi^2)=0$ (the location of the soft-pion s-wave amplitude zero[13]). [The location of the soft-pion s-wave amplitude zero[12] at $t=\frac{1}{2}m_\pi^2$, for which $B=4/(3F_\pi m_\pi^2)$, is found to be essentially unchanged in the hard-pion theory.[13]] The possible plots of δ_{00} shown in Fig. 1 would appear to lend support to the up-down phase-shift solution and, hence, a broad s-wave $\pi\pi$ resonance. Finally, we remark that for an independent determination of the parameters B and β, from *low-energy $\pi\pi$ data only*, one might fit our s-wave $I=0$ enhancement function, $\phi(t)=[F(t)/F(4m_\pi^2)]$, to the threshold $\pi\pi$ data recently obtained[14] in the reaction $p+d\rightarrow He^3+X^0$.

We thank Professor B. Maglić for stimulating our interest in this problem; we are grateful to members of the high-energy group for some discussions.

[12] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[13] R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. Letters **20**, 475 (1968).

[14] H. Brody *et al.*, Phys. Rev. Letters **24**, 948 (1970); B. Maglić (private communication).

PHYSICAL REVIEW D VOLUME 2, NUMBER 11 1 DECEMBER 1970

Axial-Vector-Meson Dominance and a New Estimate for the Decay $K^+\rightarrow\pi^+\gamma\gamma$

Gerald W. Intemann

Department of Physics, State University of New York, Binghamton, New York 13901

(Received 28 July 1970)

The radiative decay mode $K^+\rightarrow\pi^+\gamma\gamma$ is studied within the framework of an axial-vector dominance model (AVDM) with the exchange of the $A_1^+(1070)$, $B^+(1235)$, and $K_A^+(1320)$ dominating the decay. We obtain a new estimate for the branching ratio $R=\Gamma(K^+\rightarrow\pi^+\gamma\gamma)/\Gamma(K^+\rightarrow \text{all})=1.5\times10^{-5}$–$2.1\times10^{-4}$ depending on the d-wave–to–s-wave ratio in axial-vector-meson decays. For a modest amount of d wave, the AVDM result is comparable to the value for R based on the π^0-pole model and is also in good agreement with experiment. The π^+ energy spectrum is also calculated using AVDM and compared with the spectrum found in the π^0-pole model. The significance of these results for the testing of a proposed mechanism for $K^+\rightarrow\pi^+\pi^0$ is also discussed.

I. INTRODUCTION

The purpose of this paper is to present a new estimate for the rate of the decay process

$$K^+\rightarrow\pi^+\gamma\gamma.\quad (1)$$

There has been one reported experimental search[1] for this decay which places an upper limit on the branching ratio: $R=\Gamma(K^+\rightarrow\pi^+\gamma\gamma)/\Gamma(K^+\rightarrow \text{all})<1.1\times10^{-4}$.

A number of models have been proposed to describe process (1). Among these are (a) the π^0-pole model,[2] (b) the η-pole model,[3] (c) the σ-pole model,[2] and (d) the vector-dominance model.[4]

Among these various theoretical descriptions, the one which in many ways is the most interesting is (a). In this model, the decay (1) proceeds solely through a virtual π^0 state which then decays into two photons: $K^+\rightarrow\pi^+\pi^0\rightarrow\pi^+\gamma\gamma$. If this description were correct, it would be interesting, since it would enable one to test a proposed mechanism for the decay $K^+\rightarrow\pi^+\pi^0$. It has been suggested[4–7] that this latter decay, which appears to violate the $|\Delta I|=\frac{1}{2}$ rule, can occur because the two-pion final state may not be a pure isospin state on account of the π^+-π^0 mass difference. Thus, even if weak interactions exactly obey the $|\Delta I|=\frac{1}{2}$ rule, the effect of the pion mass difference will permit $K^+\rightarrow\pi^+\pi^0$ to take place. It then follows that, if the decay (1) proceeds via a π^0 intermediate state, the π^+ and virtual π^0 mass difference will be large enough to enhance the decay. Using current algebra to determine the off-the-mass-shell behavior of the $K^+\rightarrow\pi^+\pi^0$ amplitude, Fujii[4] predicts the branching ratio $R\simeq2.7\times10^{-5}$, which is

[1] M. Chen *et al.*, Phys. Rev. Letters **20**, 73 (1968).

[2] I. R. Lapidus, Nuovo Cimento **46A**, 668 (1966).

[3] G. Faldt, B. Petersson, and H. Pilkuhn, Nucl. Phys. **B3**, 234 (1967).

[4] Y. Fujii, Phys. Rev. Letters **17**, 613 (1966).

[5] N. Cabibbo and R. Gatto, Phys. Rev. Letters **5**, 382 (1960).

[6] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966).

[7] S. Okubo, R. E. Marshak, and V. S. Mathur, Phys. Rev. Letters **19**, 407 (1967).

about one order of magnitude smaller than the present experimental upper limit.

The η-pole model has also been suggested as a description of $K^+ \to \pi^+\gamma\gamma$ decay. Especially for π^+ kinetic energies $T_\pi < 70$ MeV, one might expect the η-pole contribution to dominate over the π^0-pole contribution. This model yields a branching ratio $R = 1.47 \times 10^{-6}$, which is considerably lower than the experimental upper limit.

One other pole model has been proposed, in which the two photons result from the decay of the $I=0$, scalar σ-meson resonance. This model yields a much larger branching ratio of $R = 4 \times 10^{-3}$, in disagreement with experiment.

One other promising model is the vector-meson-dominance model (VDM). As a result of the many successes of this model in describing other radiative processes,[8] one might hope that the VDM would yield a reasonably good estimate for process (1). In applying the VDM to this decay, one has contributions coming from both the ρ and K^* mesons: $K^+ \to \pi^+ \to \rho^+ + \gamma$ followed by $\rho^+ \to \pi^+\gamma$, and $K^+ \to K^{*+} + \gamma \to K^+ + \gamma + \gamma$ followed by $K^+ \to \pi^+$. By determining the weak coupling constant for the $K^+ \to \pi^+$ transition from current algebra[6] and using the estimates[9] $\Gamma(\rho^+ \to \pi^+\gamma) \approx \Gamma(K^{*+} \to K^+\gamma) \simeq 0.13$ MeV, one finds that the branching ratio $R = 4.6 \times 10^{-10}$, which is extremely small.

Among the various models that we have thus far discussed, the one which yields the largest decay rate and is still in agreement with experiment is the π^0-pole model. Thus, it would appear that, unless future experiments reduce the present upper limit on $K^+ \to \pi^+\gamma\gamma$ by roughly two orders of magnitude, the π^0-pole model offers a fairly accurate description of this decay and furnishes a test of the proposed mechanism for $K^+ \to \pi^+\pi^0$.

In this paper we present a new estimate for $K^+ \to \pi^+\gamma\gamma$. This calculation will be based on a new model which we shall call the axial-vector-dominance model (AVDM). This model will yield a range of predictions, $1.5 \times 10^{-5} \le R \le 2.1 \times 10^{-4}$, depending primarily on the d-wave–to–s-wave ratio in axial-vector-meson decays. Thus, we will find that for a modest amount of d wave, the AVDM yields values for R in agreement with experiment and also of the same order of magnitude as the value found from using the π^0-pole model. The results of this calculation would therefore suggest that there may be more than one dominant mechanism for $K^+ \to \pi^+\gamma\gamma$ and it may not be possible to give a test of the $K^+ \to \pi^+\pi^0$ decay mechanism solely from the decay rate of $K^+ \to \pi^+\gamma\gamma$. As we shall see later in the paper, the π^0-pole model and the AVDM do, however, yield a significantly different energy spectrum for the π^+. Thus, one can indeed differentiate, in principle, between these two models.

[8] For a review see, e.g., J. J. Sakurai, in *Lectures in Theoretical Physics* (Gordon and Breach, New York, 1969), Vol. XI.

[9] These estimates are based on SU_3, ω-ϕ mixing, and the experimental width $\Gamma(\omega \to \pi^0\gamma) = 1.19 \pm 0.04$ MeV.

II. AXIAL-VECTOR-DOMINANCE MODEL

A. Experimental Status of Axial-Vector Mesons

Before discussing the details of our model, let us attempt to briefly review the experimental status of axial-vector mesons. The experimental situation is not very clear.[10] The latest compilation[11] of data lists the $A_1(1070)$ with $I=1$ and $J^{PG} = 1^{+-}$; the $B(1235)$ with $I=1$ and $J^{PG} = 1^{++}$; the $D(1285)$ with $I=0$ and $J^{PG} = 1^{++}$; and perhaps two strange $I=\frac{1}{2}$ 1^+ mesons $K_C(1240)$ and $K_A(1280–1360)$. However, whereas the existence of the B meson seems to be definitely confirmed, the existence of the A_1 is still in considerable doubt. Furthermore, assuming that the A_1 does exist, the decay widths of the A_1 and B are still uncertain with the present combined data giving $\Gamma(A_1) = 95 \pm 35$ MeV and $\Gamma(B) = 102 \pm 20$ MeV. The dominant decay schemes for these mesons are believed to be $A_1 \to \rho\pi$ or 3π and $B \to \omega\pi$.

The situation is also very uncertain with regard to the D meson. Although there is some evidence for the D, its spin-parity assignment has not been firmly established although it appears that 1^+ is favored.

In the cases of the K_C and K_A mesons, the situation is not much better. In the $K\pi\pi$ mass region 1200–1350 MeV, the so-called Q region, some experiments show only a broad enhancement whereas others report definite structure with at least two resonances. The dominant decay channels for these resonances are believed to be $K^*\pi$, $K\rho$, and possibly $K\omega$. The total decay width for K_C is estimated to be[12] $\Gamma(K_C) = 90 \pm 40$ MeV while no reliable decay width for the K_A has been determined.

B. Matrix Element and Phase-Space Integration

Despite the present experimental confusion, we shall assume the existence of 1^+ mesons. For the sake of our calculation we shall only assume the existence of one $K\pi\pi$ resonance, the $K_A(1320)$. In describing $K^+ \to \pi^+\gamma\gamma$ we propose that this process is dominated by the exchange of the A_1^+, B^+, and K_A^+ mesons. Their contributions are shown by the Feynman diagrams in Fig. 1.

In evaluating these diagrams, we will take as the invariants for the axial-vector–pseudoscalar-meson–photon ($AP\gamma$) vertex and the K^+-π^+ vertex

$$f_{AP\gamma}[(\epsilon \cdot e_A) + 2(e_A \cdot q)(q \cdot \epsilon)/(m_A^2 - m_P^2)], \quad \sqrt{2} m_K^2 f_{K^+\pi^+},$$

where q is the pseudoscalar-meson momentum, ϵ and e_A are the polarization vectors of the photon and axial-vector meson, respectively, and m_A and m_P are the masses of the axial-vector and pseudoscalar mesons.

Evaluating each diagram, one obtains for the matrix element, choosing the gauge $p \cdot \epsilon = 0$, $p \cdot \epsilon' = 0$, where p

[10] N. Yeh (private communication).

[11] A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970).

[12] Actually, this estimate is only an educated guess (see Ref. 11).

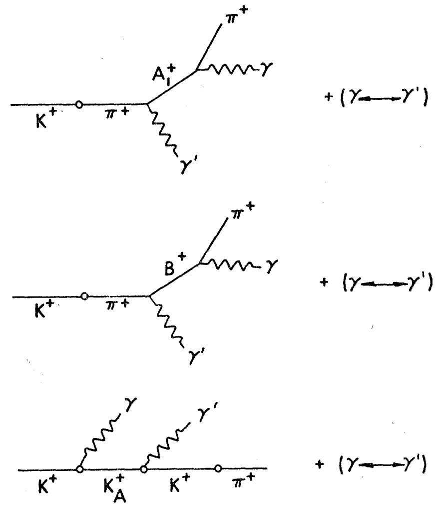

FIG. 1. Feynman diagrams for the decay $K^+\to\pi^+\gamma\gamma$ in the AVDM.

is the K^+ four-momentum,

$$\mathfrak{M}=\frac{\sqrt{2}f_{K^+\pi^+}m_K^2}{m_K^2-m_\pi^2}\sum_A \delta_A\frac{f_{AP\gamma}^2}{m_A^4}(P_A+P_A')$$
$$\times[m_A^4(\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}')+2m_A^2(\mathbf{k}'\cdot\boldsymbol{\varepsilon})(\mathbf{k}\cdot\boldsymbol{\varepsilon}')+2(\mathbf{k}\cdot\boldsymbol{\varepsilon}')(\mathbf{k}'\cdot\boldsymbol{\varepsilon})(k\cdot k')] \quad (2)$$

$(A=A_1^+,\ B^+,\ P=\pi^+$ and $A=K_A^+,\ P=K^+)$,

where

$$\delta_A=\begin{cases}+1 & \text{for } A=A_1^+,\ B^+\\ -1 & \text{for } A=K_A^+,\end{cases}$$
$$P_A=(m_A^2-m_K^2+2m_K^2k)^{-1},$$
$$P_A'=(m_A^2-m_K^2+2m_Kk')^{-1}, \quad (3)$$

and k and k' are the four-momenta of the two photons. We note that the K_A contribution to the matrix element tends to interfere destructively with the A_1 and B contributions.

Squaring $\mathfrak{M}$, summing over photon polarizations, and integrating over the three-particle phase space yields for the decay rate

$$\Gamma(K^+\to\pi^+\gamma\gamma)=\frac{m_K^3f_{K^+\pi^+}^2}{2(2\pi)^3(m_K^2-m_\pi^2)^2}\int_{m_\pi}^{E_\pi\,\max}dE_\pi\int_{\omega_{\min}}^{\omega_{\max}}M\,d\omega, \quad (4)$$

$$\omega_{\min}=\tfrac{1}{2}[m_K-E_\pi-(E_\pi^2-m_\pi^2)^{1/2}], \quad (5)$$

$$\omega_{\max}=\tfrac{1}{2}[m_K-E_\pi+(E_\pi^2-m_\pi^2)^{1/2}], \quad (6)$$

$$E_{\pi\,\max}=(m_K^2+m\pi^2)/2m_K, \quad (7)$$

and

$$M=[\sum_A \delta_A f_{AP\gamma}^2(P_A+P_A')]^2+4\left[\sum_A \delta_A\frac{f_{AP\gamma}^2}{m_A^2}(P_A+P_A')\right]^2\omega^2(m_K-E_\pi-\omega)^2$$
$$+\left[\sum_A \delta_A\frac{f_{AP\gamma}^2}{m_A^4}(P_A+P_A')\right]^2\omega^2(m_K-E_\pi-\omega)^2[(m_K^2+m_\pi^2)^2-4m_K(4\omega^2E_\pi+4\omega^3+m_K^2E_\pi+m_\pi^2E_\pi-m_KE_\pi^2)]$$
$$+4\left[\sum_A \delta_A\frac{f_{AP\gamma}^2}{m_A^2}(P_A+P_A')\right]\left[\sum_A \delta_A\frac{f_{AP\gamma}^2}{m_A^4}(P_A+P_A')\right]\omega^2(m_K-E_\pi-\omega)^2(m_K^2+m_\pi^2-2m_KE_\pi). \quad (8)$$

C. Evaluation of Coupling Constants

In order to obtain a numerical result for the decay rate we must evaluate the coupling constants $f_{K^+\pi^+}$, $f_{A_1\pi\gamma}$, $f_{B\pi\gamma}$, and $f_{K_AK\gamma}$.

Using current algebra and the Hara-Nambu prescription[6] for the extrapolated amplitude $A(K_S^0\to\pi^+\pi^-)$, one can obtain,[6] for the amplitude $A(K^+\to\pi^+)$,

$$A(K^+\to\pi^+)=F_\pi A(K_S^0\to\pi^+\pi^-)\times\tfrac{1}{16}(3m\pi^2+4m_{K^0}{}^2+m_\eta^2)/(m_{K^0}{}^2-m_\pi^2), \quad (9)$$

where F_π is the pion decay constant. Equation (9) gives $A(K^+\to\pi^+)=1.91\times10^{-2}$ MeV^2 and

$$f_{K^+\pi^+}=A(K^+\to\pi^+)/\sqrt{2}m_K^2=5.54\times10^{-8}. \quad (10)$$

In order to determine $f_{A_1\pi\gamma}$, $f_{B\pi\gamma}$, and $f_{K_AK\gamma}$, we shall make use of vector-meson dominance. In this scheme the decay $A_1^+\to\pi^+\gamma$ is related to $A_1^+\to\rho^0\pi^+$ and $\rho^0\to\gamma$; the decay $B\to\pi\gamma$ is related[13] to $B\to\omega\pi$ and $\omega\to\gamma$; the decay $K_A\to K\gamma$ is related[13] to $K_A\to K\rho(K\omega)$ and $\rho(\omega)\to\gamma$. One finds the relations

$$f_{A_1\pi\gamma}{}^2=\frac{\alpha}{2(f_{\rho\pi\pi}{}^2/4\pi)}\times\left[3g_{A\rho\pi}{}^2+\frac{1}{16}\frac{(m_{A_1}{}^2-m_\pi^2)^4}{m_{A_1}{}^4}h_{A\rho\pi}{}^2\right], \quad (11)$$

$$f_{B\pi\gamma}{}^2=\frac{\alpha}{18(f_{\rho\pi\pi}{}^2/4\pi)}\times\left[3g_{B\omega\pi}{}^2+\frac{1}{16}\frac{(m_B^2-m_\pi^2)^4}{m_B^4}h_{B\omega\pi}{}^2\right], \quad (12)$$

[13] We shall neglect the contribution from the ϕ meson.

TABLE I. Values of radiative decay widths of axial-vector mesons and $K^+ \to \pi^+\gamma\gamma$ based on the AVDM and using ρ data and δ as input.

$\Gamma(\rho)$ (MeV)	$-\delta$	$\Gamma(A_1^+ \to \pi^+\gamma)$ (MeV)	$\Gamma(B^+ \to \pi^+\gamma)$ (MeV)	$\Gamma(K_A^+ \to K^+\gamma)$ (MeV)	$\Gamma(K^+ - \pi^+\gamma\gamma)$ (10^4 sec^{-1})	R ($\times 10^{-4}$)
140	0.00	0.238	0.078	0.049	1.12	1.42
	0.20	0.193	0.064	0.040	0.74	0.94
	0.35	0.164	0.054	0.034	0.53	0.67
	0.50	0.138	0.045	0.028	0.38	0.48
	1.00	0.078	0.022	0.012	0.12	0.15
130	0.00	0.256	0.084	0.053	1.29	1.64
	0.21	0.206	0.068	0.043	0.84	1.07
	0.33	0.181	0.059	0.037	0.64	0.82
	0.50	0.149	0.048	0.030	0.44	0.56
	1.00	0.084	0.023	0.013	0.14	0.18
115	0.00	0.289	0.095	0.060	1.65	2.10
	0.17	0.243	0.080	0.051	1.16	1.48
	0.35	0.200	0.065	0.041	0.79	1.10
	0.50	0.169	0.054	0.034	0.56	0.71
	1.00	0.095	0.026	0.015	0.18	0.23

$$f_{K_AK\gamma}^2 = \frac{3\alpha}{2(f_{\rho\pi\pi}^2/4\pi)}(g_{K_AK\rho}+\tfrac{1}{3}g_{K_AK\omega})^2, \qquad (13)$$

where α is the fine-structure constant; $g_{A\rho\pi}$ and $h_{A\rho\pi}$ are the s-wave and d-wave coupling constants for $A_1 \to \rho\pi$ with similar definitions for $g_{B\omega\pi}$, $h_{B\omega\pi}$, $g_{K_AK\rho}$, and $g_{K_AK\omega}$, where we have neglected any d-wave contribution in $K_A \to K\rho$ $(K\omega)$. Finally, $f_{\rho\pi\pi}$ is the $\rho \to \pi\pi$ coupling constant and is related to the ρ width by

$$\Gamma(\rho) = \frac{1}{12}\frac{f_{\rho\pi\pi}^2}{4\pi}\frac{(m_\rho^2-4m_\pi^2)^{3/2}}{m_\rho^2}. \qquad (14)$$

In deriving Eqs. (11)–(13), we have used an ω-ϕ mixing angle of 35°18′. Owing to the experimental uncertainties in the decay widths of A_1, B, and K_A, we shall use the values of their s-wave and d-wave couplings which have been found from current algebra and SU_3 symmetry. For the $A_1\rho\pi$ system, using current algebra and the hard-pion techniques developed by Schnitzer and Weinberg[14] along with the KSRF relation[15] and the Weinberg second sum rule,[16] one can express $g_{A\rho\pi}$ and $h_{A\rho\pi}$ in terms of one dimensionless parameter δ,

$$g_{A\rho\pi} = m_\rho^2(2+\delta)/2F_\pi, \quad h_{A\rho\pi} = \delta/F_\pi, \qquad (15)$$

where δ is a measure of the d-wave–to–s-wave ratio.

The $SU_3 \otimes SU_3$ chiral algebra and the method of asymptotic SU_3 symmetry[17] lead to the relations[18]

$$g_{B\omega\pi} = 1.73 g_{A\rho\pi}, \quad h_{B\omega\pi} = 0.63 h_{A\rho\pi}. \qquad (16)$$

Finally, the techniques of current algebra and dispersion relations yield the sum rules[19]

$$g_{K_AK\rho} = -\frac{1}{F_K}\frac{m_\rho}{m_{K_A}}(m_{K_A}^2 - m_\rho^2)\left(1-\frac{F_K^2}{2F_\pi^2}\right)^{1/2}, \qquad (17)$$

$$g_{K_AK\omega} = -\frac{\sqrt{3}}{F_K}(0.43)^{1/2}\frac{m_\phi^2}{m_\rho m_{K_A}}\left(1-\frac{F_K^2}{2F_\pi^2}\right)^{1/2} \times\left(\frac{1.03}{(m_{K_A}^2-m_\phi^2)}+\frac{0.43m_\phi^2}{m^2_\omega(m_{K_A}^2-m_\omega^2)}\right)^{-1}, \qquad (18)$$

where F_K is the K_{l2} decay constant. Using the broken-SU_3 value $F_K/F_\pi = 1.22$,[20] Eqs. (17) and (18) yield

$$g_{K_AK\rho} = 0.48 g_{A\rho\pi}, \quad g_{K_AK\omega} = 0.15 g_{A\rho\pi}. \qquad (19)$$

Combining Eqs. (10)–(16) and (19) and inserting the numerical values into Eq. (8), we find the decay rate for $K^+ \to \pi^+\gamma\gamma$. We have calculated the rate using the ρ width $\Gamma(\rho)$ and the parameter δ as input. Our results appear in Table I. The π^+ energy spectrum has also been calculated, and it is shown in Fig. 2.

III. DISCUSSION OF RESULTS

We have calculated the decay rate and pion energy spectrum for the decay $K^+ \to \pi^+\gamma\gamma$ within the framework of an AVDM. This model yields a range of predictions (depending on the d-wave–to–s-wave ratio in $A_1 \to \rho\pi$ decay) for the branching ratio $R = (K^+ \to \pi^+\gamma\gamma)/(K^+ \to \text{all}) = (2.1–0.15)\times 10^{-4}$. For a modest amount of d wave, $\delta \approx -0.20$, the prediction is in good agreement with the present experimental upper limit of 1.1×10^{-4}. From Table I we see that R as well as the radiative decay widths of A_1, B, and K_A are quite sensitive to the value of the parameter δ but considerably less sensitive to the ρ width.

We would also like to compare our results with those based on other models or theories. We have exhibited

[14] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[15] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[16] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[17] S. Matsuda and S. Oneda, Phys. Rev. **158**, 1594 (1967); **174**, 1992 (1968).

[18] G. Fourez, Phys. Rev. **178**, 2454 (1969). We have assumed a mixing angle between K_C and K_A of 12°.

[19] C. S. Lai, Phys. Rev. **170**, 1443 (1968).

[20] S. Matsuda and S. Oneda, Phys. Rev. **185**, 1887 (1969).

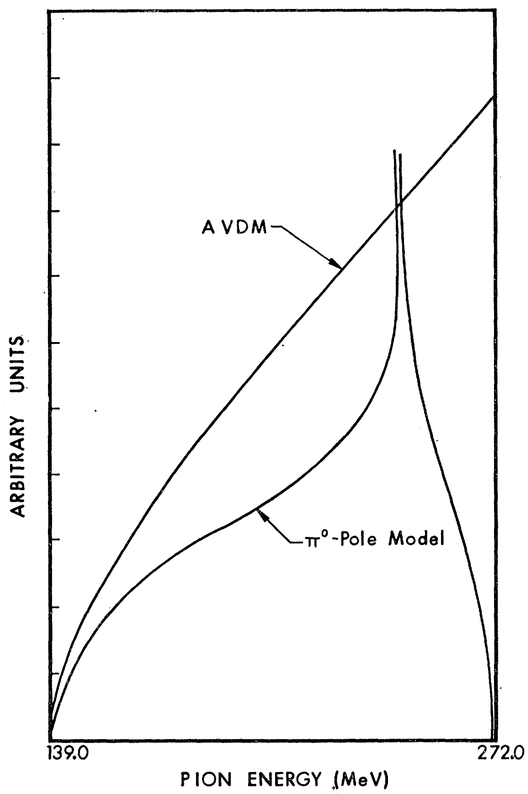

FIG. 2. Comparison of the pion energy spectrum for the decay $K^+ \to \pi^+\gamma\gamma$ based on the π^0-pole model and the AVDM.

this comparison in Table II. We observe that the AVDM yields a much larger decay rate for $K^+ \to \pi^+\gamma\gamma$ than either the η-pole model or the VDM. On the other hand, the AVDM results are comparable for the most part with the result of the π^0-pole model.

The fact that the AVDM and the π^0-pole model give the same order-of-magnitude estimate for $K^+ \to \pi^+\gamma\gamma$ has some definite consequences with regard to the understanding of the mechanism for the decay $K^+ \to \pi^+\pi^0$. If future experiments can further substantiate a value for $R \sim 10^{-4}$–10^{-5}, this information will not by itself be sufficient for determining whether or not the same mechanism proposed for $K^+ \to \pi^+\pi^0$ is responsible for the enhancement of $K^+ \to \pi^+\gamma\gamma$. In order to better understand the exact mechanism for $K^+ \to \pi^+\gamma\gamma$ it will be necessary to also determine the π^+ energy spectrum. From Fig. 2 we observe that this spectrum is noticeably different for the AVDM and the π^0-pole model. The AVDM spectrum increases monotonically and is nearly linear over most of the energy range of the π^+. On the other hand, the same spectrum based on the π^0-pole model has a dramatic peak around 110 MeV arising from the propagator for the π^0 and in fact the overwhelming portion of the spectrum is contained in the immediate neighborhood of the peak.[2] Thus, an experimental study of the π^+ energy spectrum should reveal a great deal of information as to the exact mechanism for the decay and should determine the feasiblity of testing the proposed mechanism for $K^+ \to \pi^+\pi^0$.

TABLE II. Comparison of theoretical and experimental predictions for $K^+ \to \pi^+\gamma\gamma$.

Source	$\Gamma(K^+ \to \pi^+\gamma\gamma)$ (sec^{-1})	R
π^0-pole model	2.19×10^3	2.70×10^{-5}
η-pole model	1.19×10^2	1.47×10^{-6}
σ-pole model	3.24×10^5	4.0×10^{-3}
VDM	3.73×10^{-2}	4.6×10^{-10}
AVDM	$(16.5\text{–}1.2)\times10^3$	$(2.1\text{–}0.15)\times10^{-4}$
Experiment	$<8.9\times10^3$	$<1.1\times10^{-4}$

The validity of using the values of the s- and d-wave couplings of the axial-vector-meson decays based on current algebra and asymptotic SU_3 symmetry cannot be tested at this time due to the present experimental confusion regarding axial-vector mesons. In fact it would be desirable to narrow the range of predictions of the AVDM for $K^+ \to \pi^+\gamma\gamma$. However, such a refinement would require some knowledge of the d-wave–to–s-wave ratio in axial-vector-meson decays. When we know more about the properties of 1^+ mesons, more specific predictions based on the AVDM will be possible.

Note added in proof. Since the time this calculation was performed, there has been another measurement of the branching ratio for $K^+ \to \pi^+\gamma\gamma$ by J. H. Klems *et al.*, Phys. Rev. Letters **25**, 473 (1970). They have found an upper limit on R of 4×10^{-5}, assuming a phase-space pion energy spectrum. This measurement is still consistent with the predictions of the AVDM which have a moderate d-wave contribution.