$V-A$ Elastic Scattering of Electrons by Fission Antineutrinos*

F. T. AVIGNONE, III
University of South Carolina, Columbia, South Carolina 29208
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The energy spectrum of electron antineutrinos from ^{235}U fission products in secular equilibrium has been recalculated. Assuming the validity of $V-A$ theory, cross sections for the elastic scattering reaction $\bar{\nu}_e+e^-\rightarrow\bar{\nu}_e+e^-$, integrated over recoil electron energy for this antineutrino spectrum, are given for several values of minimum electron energy from 1 to 5.5 MeV. Theoretical error bars, which reflect the uncertainties in the input data, are given for the antineutrino spectrum and the cross sections.

I. INTRODUCTION

In the recent theoretical treatment of weak interactions by Gell-Mann *et al.*,[1] the effective weak interaction is decomposed into a diagonal and a nondiagonal part. According to Ref. 1, only the nondiagonal part can be expected to have the usual property of universality of the weak interaction. As a result, there is no reason to expect universality to hold for the interaction governing the process

$$\bar{\nu}_e+e^-\rightarrow\bar{\nu}_e+e^-. \tag{1}$$

Recently, Reines and Gurr[2] have reported an upper limit for the cross section of reaction (1) which is four times larger than that predicted with the $V-A$ theory of Feynman and Gell-Mann.[3] These experiments are continuing and are being conducted with the antineutrino flux of a large nuclear reactor.

The main goals of the present investigation are to recalculate the energy spectrum of antineutrinos from a reactor, to determine the scattering cross section and recoil spectrum of electrons scattered by reactor antineutrinos, and finally to calculate theoretical errors on these quantities, which realistically reflect the uncertainties in the large body of experimental data used in such calculations. The results of the present work were those compared to the experimental cross section in Ref. 2.

There were several motivations for the present work.

(i) Earlier antineutrino spectrum calculations did not determine the theoretical errors which, in this case, might dictate the point of diminishing returns for the experimental efforts.

(ii) Since our earlier calculations,[4] charge distributions of primary products of fission have been experimentally determined by a mass-separator technique.[5]

(iii) More up-to-date tables of the most probable fission-product charges have been published.[6]

(iv) More recent calculations of β Q values have been published by Seeger and Perisho,[7] which take into account shell effects, nuclear deformation, and pairing energies. These results were used to assign β end-point energies and their uncertainties in cases involving unknown decay schemes.

(v) Finally, the serious disagreement at $\sim$10 MeV between the earlier spectra and that determined experimentally by Nezrick and Reines[8] is perplexing and should be clarified. (See Fig. 3 of Ref. 4.)

II. ANTINEUTRINO SPECTRUM

The energy spectrum of antineutrinos from a reactor is assumed to be that of ^{235}U fission products in secular equilibrium. The general methods of calculation, as well as references to earlier work, are given in Ref. 4.

The number of antineutrinos per fission, of energy E_ν, is given by

$$N(E_\nu)=\sum_j Y_j(Z,A)b_jP_j(E_\nu), \tag{2}$$

where $Y_j(Z,A)$ is the primary yield of the nuclide (Z,A) which decays via the jth branch, b_j is the branching ratio, and $P_j(E_\nu)$ is the theoretical, allowed Coulomb-corrected antineutrino spectrum for the jth β branch.

The sum in Eq. (2) involved a total of 548 β decays, 260 of which proceed through known decay schemes. The methods for considering those which proceed through unknown decay schemes is discussed later.

The yields in Eq. (2) were calculated assuming the usual Gaussian form as follows:

$$Y(ZA)=\frac{R(A)}{(c\pi)^{1/2}}\exp\left(-\frac{[Z-Z_p(A)]^2}{c}\right), \tag{3}$$

where $R(A)$ is a normalized mass yield for the primary fission product of mass number A taken from Zysin

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[1] M. Gell-Mann, M. L. Goldberger, N. M. Kroll, and F. E. Low, Phys. Rev. **179**, 1518 (1969).

[2] F. Reines and H. S. Gurr (private communication); Phys. Rev. Letters **24**, 1448 (1970).

[3] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[4] F. T. Avignone, III, S. M. Blankenship, and C. W. Darden, III, Phys. Rev. **170**, 931 (1968).

[5] K. Sistemich, P. Armbruster, J. Eidens, and E. Roeckl, Nucl. Phys. **A139**, 289 (1969).

[6] E. A. C. Crouch, Atomic Energy Research Establishment, Harwell, England, Report No. AERE-R 5488, 1967 (unpublished).

[7] P. A. Seeger and R. C. Perisho, Los Alamos Scientific Laboratory Report No. LA-3751, 1967 (unpublished).

[8] F. A. Nezrick and F. Reines, Phys. Rev. **142**, 852 (1966).

et al.,[9] Z is the charge of the primary product, $Z_p(A)$ is the most probable charge for a primary fission product of mass number A given in Ref. 6, and c was experimentally determined in Ref. 5 as $c=0.84\pm0.07$.

It should be mentioned here that, using the primary fission yields calculated with Eq. (3), we have been able to predict cummulative yields which are in excellent agreement with all of the experimental values given by Farrar *et al.*,[10] for which the yield is greater than 0.1%. These comparative results are preliminary and will be discussed in a future paper.

The branching ratios and end-point energies for β decays proceeding through known decay schemes were taken from the Table of Isotopes,[11] the Nuclear Data Tables,[12] and the current literature through December, 1969. In the cases where the decay schemes are not experimentally known, the decay schemes shown in Fig. 1 were assumed. The branching ratios to the ground states and to average excited states, as well as the excitation energies for even-even, odd-odd, odd-even, and even-odd nuclei, are the averages of all the values given in Refs. 11 and 12. The quoted error in each value of the single effective excited level of each daughter nucleus shown in Fig. 1 is the rms deviation taken from the computation of the average value appropriate to each category.

The main sources of error in the antineutrino spectrum were introduced by (a) the uncertainty in the yields $Y_j(ZA)$, (b) the uncertainties in the 288 theoretically calculated β end-point energies taken from

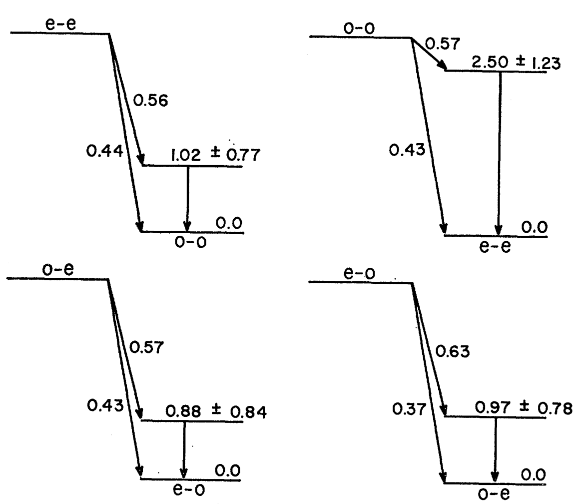

FIG. 1. Average properties of all known decay schemes. (Energies in MeV.)

Ref. 7, and (c) the large uncertainties in the average excited-state energies shown in Fig. 1.

The error in $N(E_\nu)$ due to (a) was found by directly calculating $N(E_\nu)$ with the central and extreme values in the primary yields. These are obtained by calculating Y_j using the $Z_p(A)$ curves of Ref. 6 and those used in our earlier work reported in Ref. 4 and also by varying the constant c in Eq. (3) over its experimental limits.

The errors in $N(E_\nu)$ at energy E_ν, introduced by (b) and (c) above, were calculated by varying each end-point energy over its range of uncertainty and calculating the resulting change $\delta_j(E_\nu)$ in each spectrum shape function $P_j(E_\nu)$. The error in $N(E_\nu)$ due to the uncertainties in the β end-point energies is then given by

$$\delta N(E_\nu)=\{\sum_i a_i[\delta_i(E_\nu)]^2\}^{1/2}, \qquad (4)$$

where a_i is a normalized weighting factor proportional to the product of the ith branching ratio and primary yield.

The total error in $N(E_\nu)$ at energy E_ν was then taken to be the square root of the sum of the squares of the errors introduced by the three main sources mentioned above.

The normalization of the antineutrino spectrum is easily accomplished by computing the number of β decays (or antineutrinos) per fission as follows:

$$N_{\bar{\nu}}=\sum_k (n_k+m_k)Y_k, \qquad (5)$$

where Y_k is the primary fission yield of the kth fission product, n_k is the number of β decays to stability via known decay schemes, and m_k is the number of decays to stability via unknown decay schemes. The results of this sum are 6.0 ± 0.1 antineutrinos per fission. Of these, 3.8 are found to come from β decays which proceed via known decay schemes.

The antineutrinos spectrum is then normalized by requiring

$$\int_0^\infty N(E_\nu)dE_\nu=6.0. \qquad (6)$$

The numerical results of the calculated spectrum are given in part in Table I. A plot of the spectrum resulting from the present work is compared to that of Ref. 4 in Fig. 2. By reference to Fig. 3 of Ref. 4, one readily sees that the high-energy tail of the experimental spectrum of Ref. 8 is not theoretically explained by the present calculations.

If we consider the error bars quoted in Table I and the relatively minor role played by the errors introduced through the primary fission yields, it is possible to conclude that if the spectrum of Ref. 8 is correct, the mass calculations of Ref. 7 must systematically predict β Q values significantly lower (by several MeV) than those which could give rise to such a high value of $N(E_\nu)$ in the 10-MeV region. Uncertainties in the

[9] Yu. A. Zysin, A. A. Lboz, and L. I. Selchenkov, *Fission Product Yields and Their Mass Distribution* (Consultants Bureau, New York, 1964).
[10] H. Farrar, H. R. Fickel, and R. H. Thomlinson, Can. J. Phys. 40, 1017 (1962).
[11] C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (Wiley, New York, 1967).
[12] *Nuclear Data Tables*, edited by K. Way *et al.* (Academic, New York).

$Z_p(A)$ curves of Ref. 6 could explain such a discrepancy only if they were much lower than presently accepted estimates, which would result in a displacement of the curve several full charge units further from stability.

As a final note, it is interesting to consider the average energy per fission carried off by β particles. The present work predicts 7.14±0.35 MeV/fission, which is in good agreement with the average experimental value of 7.0±0.4 MeV/fission given by James.[13] The average β energy calculated from the work of Ref. 4 is 8.2±0.4 MeV.

III. SCATTERING CROSS SECTION

The cross section for elastic antineutrino-electron scattering [Eq. (1)], in which an incident antineutrino of energy E_ν imparts an energy between E and $E+dE$ to an electron initially at rest, is given by

$$d\sigma=\frac{2G^2m^2}{\pi}\frac{(E_\nu-E+1)^2}{E_\nu^2}dE, \tag{7}$$

where $G^2m^2=1.4\times10^{-44}$ cm^2. (See the Appendix.)

TABLE I. Theoretical spectrum of antineutrinos from ^{235}U fission products in secular equilibrium. $N(E_\nu)$ is given in antineutrinos per MeV per fission. The power of 10 is given in the second parentheses.

E_ν (MeV)	$N(E_\nu)$	E_ν (MeV)	$N(E_\nu)$
0.5	(2.58±0.20)(0)	5.5	(7.31±0.37)(−2)
1.0	(2.18±0.19)(0)	6.0	(4.47±0.23)(−2)
1.5	(1.67±0.15)(0)	6.5	(2.74±0.10)(−2)
2.0	(1.35±0.14)(0)	7.0	(1.55±0.05)(−2)
2.5	(9.63±0.98)(−1)	7.5	(8.75±0.27)(−3)
3.0	(6.82±0.64)(−1)	8.0	(4.77±0.15)(−3)
3.5	(4.65±0.44)(−1)	8.5	(2.70±0.21)(−3)
4.0	(3.06±0.26)(−1)	9.0	(1.73±0.12)(−3)
4.5	(1.94±0.16)(−1)	9.5	(1.01±0.07)(−3)
5.0	(1.17±0.08)(−1)	10.0	(5.00±0.36)(−4)

For a given electron recoil energy E, the effective partial cross section for reactor antineutrinos is

$$S(E)=\int_{E_{\nu0}}^{\infty}\left(\frac{d\sigma}{dE}\right)N(E_\nu)dE_\nu\bigg/\int_0^\infty N(E_\nu)dE_\nu, \tag{8}$$

where the kinematics require that the minimum contributing antineutrino energy is

$$E_{\nu0}=\tfrac{1}{2}[E-1+(E^2-1)^{1/2}].$$

A plot of $S(E)$ in cm$^2/\bar{\nu}$ versus E is given in Fig. 3. The error bars reflect the uncertainty in the spectrum of antineutrinos.

The average scattering cross sections, integrated over ranges of experimentally observed electron recoil energies, are given in Table II. The threshold energies

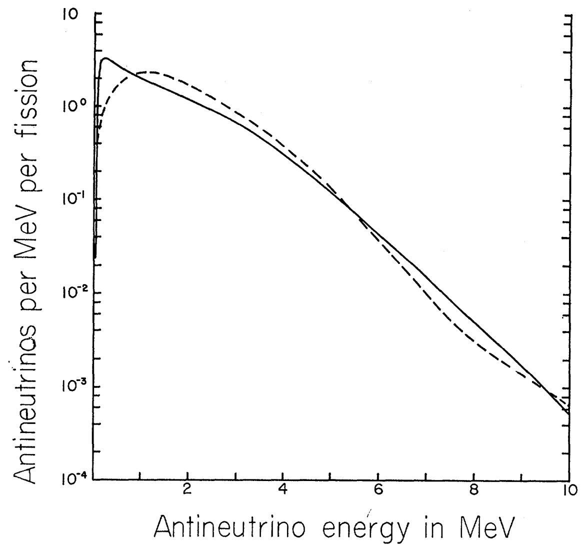

FIG. 2. Energy spectrum of antineutrinos from ^{235}U fission products in secular equilibrium. The solid curve is the present spectrum. The dashed curve is the spectrum of Ref. 4.

chosen were those which are important in the interpretation of the experiments of Reines and Gurr. The theoretical errors quoted would indicate that meaningful comparison with theory will be possible even for

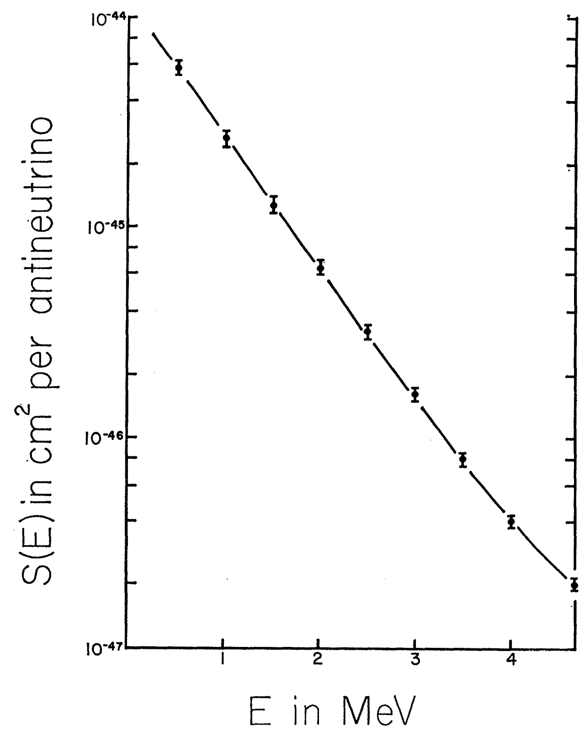

FIG. 3. Effective partial cross section $S(E)$ for reactor antineutrino scattering of electrons as a function of electron recoil energy E.

[13] M. F. James, J. Nucl. Energy **23**, 517 (1969).

TABLE II. Average scattering cross section for observed recoil electron energy from E(min) to infinity.

E(min) (MeV)	$\langle\sigma\rangle$ (10^{47} cm$^2/\bar{\nu}$)	E(min) (MeV)	$\langle\sigma\rangle$ (10^{47} cm$^2/\bar{\nu}$)
1	200 ±17	3.7	4.7 ±0.3
2	50.0±3.5	3.8	4.1 ±0.2
3	12.5±0.7	4.0	3.1 ±0.2
3.2	9.5±0.5	4.5	1.54±0.07
3.4	7.2±0.4	5.0	0.79±0.04
3.6	5.5±0.3	5.5	0.36±0.02

experimental accuracy greatly improved over that quoted in Ref. 2.

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APPENDIX: OUTLINE OF THEORETICAL CONSIDERATIONS

In accordance with the Fermi theory as modified by Feynman and Gell-Mann,[3] the weak-interaction Hamiltonian is written

$$H_\omega = \frac{G}{\sqrt{2}} \int d^3x \, J_\lambda(x) J^{\lambda\dagger}(x) . \tag{A1}$$

For the scattering process $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$,

$$J_\lambda(x) = \bar{\psi}_e \gamma_\lambda (1-\gamma_5) \psi_e . \tag{A2}$$

For reference see Marshak *et al.*[14] or Gasiorowicz.[15]

According to Ref. 14, the differential cross section for the above scattering process is given by

$$\frac{d\sigma}{d\Omega} = \frac{G^2}{(2\pi)^2} \frac{(S-m^2)^2}{4S^3} [(S-m^2)\cos\theta + S + m^2]^2 , \tag{A3}$$

where m is the electron rest mass, S is the invariant $-[\mathbf{P}_e(\text{initial}) - \mathbf{P}_{\bar{\nu}}(\text{initial})]^2$ and $\hbar = c = 1$. The quantity $t \equiv -[\mathbf{P}_e(\text{final}) - \mathbf{P}_e(\text{initial})]^2$ is also an invariant. Using these relations, one can derive the well-known expression

$$\cos\theta = 1 + 2tS/(S-m^2)^2 , \tag{A4}$$

and it is easily seen that

$$\frac{d\sigma}{dE} = -\frac{G^2}{(2\pi)^2} \frac{(S+t-m^2)^2}{(S-m^2)^2} . \tag{A5}$$

Transformation to laboratory coordinates gives the result

$$\frac{d\sigma}{dE} = \frac{2G^2m^2}{\pi} \frac{(E_\nu - E + 1)^2}{E_\nu^2} , \tag{A6}$$

where E is the energy of the recoil electron and E_ν is the energy of the incident antineutrino.

The dependence of $d\sigma/dE$ on recoil electron energy, as given in (A6), is in agreement with that given by Feynman and Gell-Mann,[2] but in disagreement with that given by King *et al.*[16] Furthermore, the denominator of that given in Ref. 16 contains E_ν^3 rather than E_ν^2.

We have independently calculated this energy dependence starting with the form of the leptonic current as given in (A2). The results are in agreement with (A6).

The coupling constant $G^2m^2 = 1.4 \times 10^{-44}$ cm^2, given in Ref. 4, was used in the numerical evaluation of the scattering cross section.

[14] R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Interscience, New York, 1969).

[15] S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966).

[16] R. W. King, D. C. Peaslee, and J. F. Perkins, Phys. Rev. **117**, 1614 (1960).