requirement of *TCP* invariance can be stated in the form

$$\langle\alpha|T|\bar{K}^0\rangle=\lambda_\alpha A_\alpha{}^* e^{i\delta_\alpha} \quad (5)$$

if

$$\langle\alpha|T|K^0\rangle=A_\alpha e^{i\delta_\alpha}. \quad (6)$$

Here δ_α is the scattering eigenphase in the channel α. *CP* invariance requires $A_\alpha=\mathrm{Re}A_\alpha$. Combining Eqs. (5) and (6) with Eq. (1), the right-hand side of Eq. (3) can be expressed in terms of ϵ and the A_α's and, if $A_\alpha=\mathrm{Re}A_\alpha$ for all channels, assumes the simple form

$$2(1+|\epsilon|^2)^{-1}(\epsilon \sum_{\lambda_\alpha=+1} A_\alpha{}^2+\epsilon^* \sum_{\lambda_\alpha=-1} A_\alpha{}^2).$$

The imaginary part of Eq. (3) can then be written as

$$(m_2-m_1)\,\mathrm{Re}\epsilon=\mathrm{Im}\epsilon\,(\sum_\alpha \lambda_\alpha A_\alpha{}^2). \quad (7)$$

Beside Eq. (3), unitarity also requires

$$\gamma_{1,2}=\sum_k |\langle k|T|K_{1,2}{}^0\rangle|^2. \quad (8)$$

If we again choose the complete set k as the set of self-conjugate channels α, we obtain

$$\gamma_1-\gamma_2=2\left(\frac{1-|\epsilon|^2}{1+|\epsilon|^2}\right)(\sum_\alpha \lambda_\alpha A_\alpha{}^2). \quad (9)$$

Equations (7) and (9) yield

$$\tan\phi_\epsilon=\frac{\mathrm{Im}\epsilon}{\mathrm{Re}\epsilon}=2\left(\frac{1-|\epsilon|^2}{1+|\epsilon|^2}\right)\left(\frac{m_2-m_1}{\gamma_1-\gamma_2}\right). \quad (10)$$

Thus, while unitarity and *TCP* invariance require the sum on the right-hand side of Eq. (3) to have the phase ϕ_W, Eq. (4), the same conditions require ϵ to have the phase ϕ_ϵ given above,[8] which could also have been deduced directly from Eq. (65) of Ref. 2. Equation (10) admits two solutions for ϕ_ϵ differing by 180°; thus it might be thought that a sign factor, e.g., the sign of $\cos\phi_\epsilon$, is an additional independent parameter.[9] However, this can always be taken positive *by convention.* The redefinitions $|K^0\rangle\leftrightarrow|\bar{K}^0\rangle$, $\epsilon\to-\epsilon$, $|K_1{}^0\rangle\to|K_1{}^0\rangle$, and $|K_2{}^0\rangle\to-|K_2{}^0\rangle$ leave Eq. (1) unchanged; hence it is possible to regard $\mathrm{Re}\epsilon>0$ as the *convention* which defines the distinction between K^0 and $\bar{K}^0$ states.[10]

Consequently, in superweak theories (defined by $A_\alpha=\mathrm{Re}A_\alpha$ for all channels), all *CP*-noninvariant phenomena in neutral kaon decays are determined by the single real parameter $|\epsilon|$.

[8] If all channels α could be characterized by the same value of λ_α, we would have $\tan\phi_\epsilon=\lambda_\alpha\tan\phi_\omega$.

[9] In this connection, see the discussion of M. Nauenberg, T. D. Lee, C. N. Yang, and L. Wolfstein, in *Proceedings of the Thirteenth International Conference on High Energy Physics* (University of California Press, Berkeley, 1967), p. 81.

[10] By a happy coincidence, this convention agrees with the usua terrestrial definition of K^0 and $\bar{K}^0$ states.

PHYSICAL REVIEW D VOLUME 2, NUMBER 1 1 JULY 1970

Veneziano Model for the Reaction $\pi^-p\to\eta n$†

MAURICE L. BLACKMON AND KAMESHWAR C. WALI
Department of Physics, Syracuse University, Syracuse, New York 13210

(Received 29 January 1970)

A Veneziano model is proposed for the reaction $\pi^-p\to\eta n$. This reaction is interesting since only the A_2 trajectory can contribute in the t channel, and only $T=\frac{1}{2}$ trajectories appear in the s and u channels. We limit the terms in our model to those which have leading asymptotic behavior in two channels. We emphasize the intimate connection between resonance parameters and both forward and backward cross sections.

I. INTRODUCTION

In this paper, we consider a Veneziano-type model for the reaction $\pi^-p\to\eta n$. Our main purpose is to emphasize the intimate connection between the low-energy resonance parameters and the residue functions for *both* the forward (small t) and the backward (small u) differential cross sections for large s. With a similar purpose in mind, the Veneziano scheme has been investigated for πN elastic scattering by several authors.[1] It has been found that, in general, there is a satisfactory correlation between the low-energy resonance parameters and the forward differential cross sections. However, the predicted backward cross sections are, on the whole, unsatisfactory. In I, the predicted cross section is too big by several orders of magnitude. Berger and Fox[1] come within a factor of 2 for the Δ width, but they do not consider the fit to be

† Work supported by the U. S. Atomic Energy Commission.

[1] S. Fenster and K. C. Wali, Phys. Rev. D 1, 1409 (1970), hereafter called I; see also E. Berger and G. Fox, Phys. Rev. 188, 2120 (1969). References to their related papers are contained in these papers.

quantitatively acceptable. It has been suggested[2] that absorption effects are necessary. But prior to drastic modification of the simple Veneziano scheme, it would be interesting to see whether the discrepancy is confined to the πN system in which both isospin $T=\frac{1}{2}$ and $T=\frac{3}{2}$ baryonic trajectories, and both isospin $T=0$ and $T=1$ mesonic trajectories, contribute. In contrast, the reaction $\pi^- p \to \eta n$ has considerable simplifying features. It has only one trajectory in the t channel, the A_2, and only $T=\frac{1}{2}$ trajectories in the s and u channels. Thus we are at least free of the ambiguities associated with the Δ exchange.[3]

From the known mass spectrum in the low-energy region ($\sim$1.6 GeV), we introduce two $T=\frac{1}{2}$ baryonic trajectories N_α and N_γ, along with the A_2 trajectory in the t channel. By imposing suitable resonance conditions, we can ensure that only the observed particles appear in the energy region of interest. By fitting the principal features of forward scattering, we predict the backward scattering at several energies. At present, there is no experimental information in the reaction $\pi^- p \to \eta n$ for sufficiently high s and small u. Such information will be very useful in testing the model.

In Sec. II, we discuss the specific form of the scattering amplitude. In Sec. III, the resonance conditions and the forward differential cross sections are used to evaluate the constants in the model. Using these constants, we plot the backward differential cross sections. The concluding section is devoted to the discussion of the results.

II. FORM OF AMPLITUDE

As in I, we start with the invariant amplitudes $A(s,t,u)$ and $B(s,t,u)$ expressed as sums of Veneziano-type terms. Following Miyamura,[4] we consider only those terms which give rise to leading Regge behavior in all the channels. Thus,

$$B(s,t,u)=4\pi[\beta_1 B_{N_\alpha}^-(\tfrac{1}{2},1)+\beta_2 B_{N_\gamma}^-(\tfrac{1}{2},1)+\phi B_{N_\alpha N_\gamma}^-(\tfrac{1}{2},\tfrac{1}{2})], \quad (2.1)$$

$$A(s,t,u)=4\pi[\mu_1 C_{N_\alpha}^+(\tfrac{3}{2},1)+\mu_2 C_{N_\gamma}^+(\tfrac{3}{2},1)+\lambda_1 C_{N_\alpha}^+(\tfrac{3}{2},\tfrac{3}{2})+\lambda_2 C_{N_\gamma}^+(\tfrac{3}{2},\tfrac{3}{2})+\lambda_3 C_{N_\alpha N_\gamma}^+(\tfrac{3}{2},\tfrac{3}{2})], \quad (2.2)$$

where

$$B_x^\pm(\tfrac{1}{2}m,n)=\frac{\Gamma(\frac{1}{2}m-\alpha_x(s))\Gamma(n-\alpha(t))}{\Gamma(\frac{1}{2}m+n-\alpha_x(s)-\alpha(t))}\pm s\leftrightarrow u,$$

$$B_{xy}^\pm(\tfrac{1}{2}m,\tfrac{1}{2}n)=\frac{\Gamma(\frac{1}{2}m-\alpha_x(s))\Gamma(\frac{1}{2}n-\alpha_y(u))}{\Gamma(\frac{1}{2}m+\frac{1}{2}n-\alpha_x(s)-\alpha_y(u))}\pm s\leftrightarrow u, \quad (2.3)$$

$$B_x(\tfrac{1}{2}m,\tfrac{1}{2}n)=\frac{\Gamma(\frac{1}{2}m-\alpha_x(s))\Gamma(\frac{1}{2}n-\alpha_y(u))}{\Gamma(\frac{1}{2}m+\frac{1}{2}n-\alpha_x(s)-\alpha_y(u))}.$$

The functions $C_x^\pm$, $C_{xy}^\pm$, and C_x are analogously defined beginning with

$$C(a,b)=\Gamma(a)\Gamma(b)/\Gamma(a+b-1).$$

If we define the signature factors

$$\xi^+=(1+e^{-i\pi\alpha(t)}), \quad (2.4)$$

$$\xi_x^\pm=\tfrac{1}{2}(1\pm e^{-i\pi[\alpha_x(u)-1/2]}), \quad (2.5)$$

then for fixed t, $s\to\infty$,

$$B/4\pi \to (-\beta_1-\beta_2)\xi^+\Gamma(1-\alpha(t))(\alpha' s)^{\alpha(t)-1}, \quad (2.6)$$

$$A/4\pi \to (\mu_1+\mu_2)\xi^+\Gamma(1-\alpha(t))(\alpha' s)^{\alpha(t)}; \quad (2.7)$$

for fixed u, $s\to\infty$,

$$B/4\pi \to \{[(-\beta_1-\phi)\xi_{N_\alpha}^+ + (-\beta_1+\phi)\xi_{N_\alpha}^-]\times(\alpha' s)^{\alpha_{N\alpha}(u)-1/2}\Gamma(\tfrac{3}{2}-\alpha_{N_\alpha}(u)) + [(-\beta_2+\phi)\xi_{N_\gamma}^+ + (-\beta_2-\phi)\xi_{N_\gamma}^-]\times(\alpha' s)^{\alpha_{N\gamma}(u)-1/2}\Gamma(\tfrac{3}{2}-\alpha_{N_\gamma}(u))\}, \quad (2.8)$$

$$A/4\pi \to \{[(\mu_1+\lambda_1+\lambda_3)\xi_{N_\alpha}^+ + (\mu_1-\lambda_1-\lambda_3)\xi_{N_\alpha}^-]\times(\alpha' s)^{\alpha_{N\alpha}(u)-1/2}\Gamma(\tfrac{3}{2}-\alpha_{N_\alpha}(u)) + [(\mu_2+\lambda_2+\lambda_3)\xi_{N_\gamma}^+ + (\mu_2-\lambda_2-\lambda_3)\xi_{N_\gamma}^-]\times(\alpha' s)^{\alpha_{N\gamma}(u)-1/2}\Gamma(\tfrac{3}{2}-\alpha_{N_\gamma}(u))\}. \quad (2.9)$$

To ensure the correct signatures for the N_α and N_γ trajectories, we require

$$-\beta_1+\phi=-\beta_2+\phi=0, \quad (2.10)$$

$$\mu_1-\lambda_1-\lambda_3=\mu_2+\lambda_2+\lambda_3=0, \quad (2.11)$$

so that we are left with four constant parameters which we shall take to be μ_1, μ_2, β_1, and λ_1. Note that the high-energy differential cross sections computed from (2.6)–(2.9) depend only on μ_1, μ_2, and β_1.

III. NUMERICAL EVALUATION OF PARAMETERS

As has been customary in the Veneziano framework, we assume linear trajectories all with the same slope, i.e.,

$$\alpha(t)=\tfrac{1}{2}+\alpha' t, \quad (3.1)$$

$$\alpha_{N_\alpha}(s)=\tfrac{1}{2}+\alpha'(s-M^2), \quad (3.2)$$

$$\alpha_{N_\gamma}(s)=\tfrac{3}{2}+\alpha'(s-M_\gamma^2), \quad (3.3)$$

where M is the nucleon mass and M_γ is the mass of $N^*(1518)$. The universal slope α' is chosen to be 0.9 GeV^{-2}.

The main feature of the differential cross section for $\pi^- p \to \eta n$, as in the reaction $\pi^- p \to \pi^0 n$, is the turnover near $t\sim 0$. This requires a delicate cancellation between the invariant amplitudes $A(s,t,u)$ and $B(s,t,u)$. Thus, from our fit (see Fig. 1) to the forward differential cross-section data,[5] we find

$$\beta_1=1.3\ \mathrm{GeV}^{-2} \quad (3.4)$$

[2] C. Lovelace, Nucl. Phys. **B12**, 252 (1969).

[3] V. Barger and D. Cline, Phys. Rev. Letters **19**, 1504 (1967).

[4] O. Miyamura, Progr. Theoret. Phys. (Kyoto) **42**, 305 (1969). Miyamura has considered the same set of Veneziano terms used here. He has also discussed the resonance spectrum. Our analysis differs from his in only minor details. We are mainly concerned with correlating forward and backward cross sections with these resonance parameters.

[5] O. Guisan, Phys. Letters **18**, 200 (1965).

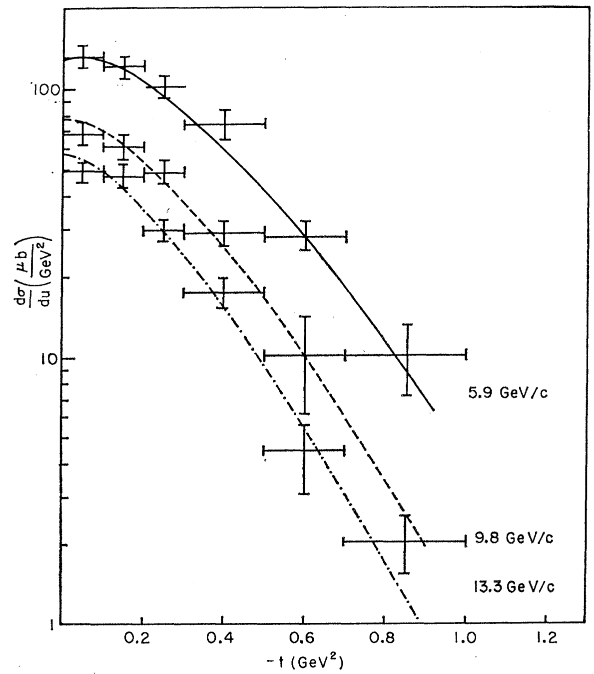

FIG. 1. Differential cross section for $\pi^- p \to \eta n$ in the small-t region.

and

$$\mu_1+\mu_2=\tfrac{5}{6}\beta_1/\alpha' M\,. \qquad (3.5)$$

This implies an s-channel helicity-flip amplitude with coupling five times as large as the helicity nonflip amplitude. In the language of Gell-Mann, Frautschi, and Zachariasen,[6] if one writes

$$f_+=\alpha(t)b_1(t)\xi^+(\alpha' s)^{\alpha(t)-1/2}, \qquad (3.6)$$

$$f_-=(-t/4M^2)^{1/2}\alpha(t)[b_1(t)-b_2(t)]\xi^+(\alpha' s)^{\alpha(t)-1/2}, \qquad (3.7)$$

then

$$b_2(0)=6b_1(0)\,. \qquad (3.8)$$

The particle at $\alpha_{N_\alpha}(M^2)=\frac{1}{2}$ is a pure $J^P=\frac{1}{2}^+$ state, since there is no pole in the invariant amplitude $A(s,t,u)$ at this point. It can, therefore, be identified with the nucleon pole, yielding the residue condition

$$\sqrt{2}g_{NN\pi}g_{NN\eta}/4\pi=2\beta_1/\alpha'\,. \qquad (3.9)$$

If we use exact $SU(3)$ to relate $g_{NN\eta}$ to $g_{NN\pi}$ (we use the notation of Ref. 7),

$$g_{NN\eta}=-(1/\sqrt{3})(1-4f)g_{NN\pi}\,, \qquad (3.10)$$

we find from (3.4) and

$$g_{NN\pi}{}^2/4\pi=15$$

that

$$f=0.31. \qquad (3.11)$$

[6] S. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).

[7] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

To determine μ_1 and μ_2 separately, we require that there be no parity partner at $\alpha_{N_\gamma}(M_\gamma{}^2)=\frac{3}{2}$. From Eqs. (A4) and (A5) in I, this condition implies that the ratio of the coefficients of the z_s term (z_s is cosine of the scattering angle) in the invariant amplitudes A and B must be $(M-M_\gamma)$. Evaluating this ratio in our model, we find

$$\mu_2/\beta_2=\mu_2/\beta_1=M_\gamma-M\,. \qquad (3.12)$$

Combining (3.12) with (3.4) and (3.5),

$$\mu_1=0.52\ \mathrm{GeV}^{-1},\quad \mu_2=0.75\ \mathrm{GeV}^{-1}. \qquad (3.13)$$

With this information, we can predict the backward cross section, the partial width for the $N^*(1518)\to N\eta$, and the f/d ratio for the coupling of N_γ octet to the octets of the pseudoscalar mesons and the baryons. The predicted backward cross sections for several energies are shown in Fig. 2. From the known total width of $N_\gamma(1518)$,

$$\Gamma_{N^*\to N\eta}/\Gamma_{\mathrm{total}}=(1.0\times10^{-2})\%\,, \qquad (3.14)$$

to be compared with an experimental number $\sim\frac{1}{2}\%$. The corresponding f/d ratio parameter f is given by

$$f=0.27\,. \qquad (3.15)$$

To determine the remaining constant λ_1, we examine the additional states predicted by the model. At $\alpha_{N_\alpha}(s^*)=\frac{3}{2}$, there are no $J^P=\frac{3}{2}^\pm$ particles, since we have im-

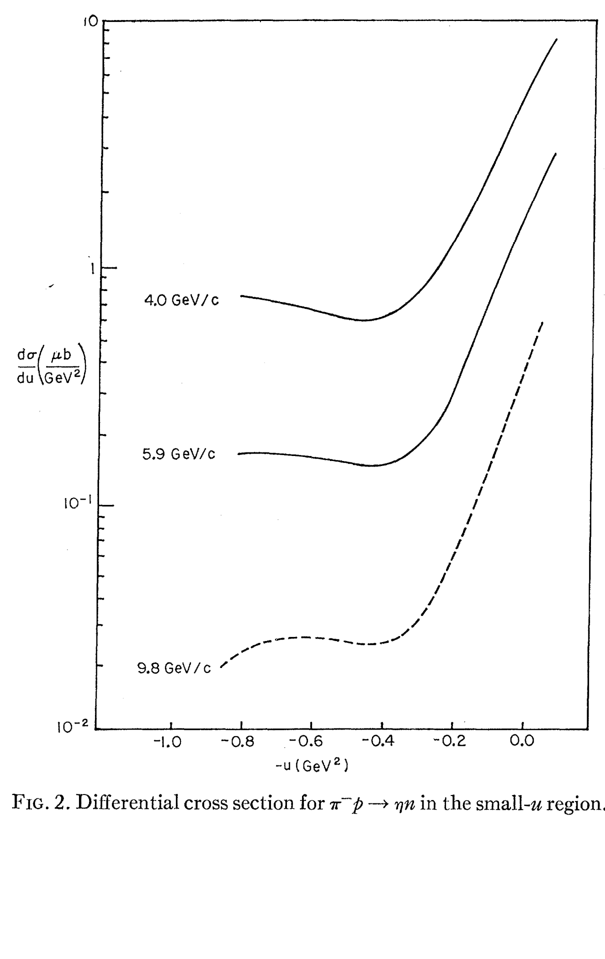

FIG. 2. Differential cross section for $\pi^- p \to \eta n$ in the small-u region.

posed the signature conditions (2.8) and (2.9), but these are additional states with $J^P = \frac{1}{2}^\pm$. Since the B amplitude does not vanish at this point, we cannot remove both the $J^P = \frac{1}{2}^\pm$ states. We choose to identify the Roper resonance N_R(1.47 GeV) as the daughter state of $\alpha_{N_\alpha}(s^*) = \frac{3}{2}$. This is not inconsistent with the presently known uncertainties in the mass and the width of the Roper resonance. If we make this identification, again appealing to Eqs. (A4) and (A5) of I, we find

$$\lambda_1 = 1.98 \text{ GeV}^{-1}. \quad (3.16)$$

This completes the determination of all the constants in the model. A Yukawa-type coupling constant $g_{N_R N\eta}$ of the Roper resonance to $N\eta$ can be determined by using the experimental width of the Roper resonance decaying into πN. If we take this width to be 0.26 GeV, we find $g_{N_R N\eta}/4\pi = 2.6$. If the Roper resonance belongs to an octet, the corresponding f/d ratio parameter f has a value of $f = 0.34$.

The particle spectrum in the region of interest is completed by a parity doublet $J^P = \frac{1}{2}^\pm$ at $\alpha_{N_\gamma} = \frac{3}{2}$. The $\frac{1}{2}^-$ state has a much larger coupling to ηN compared to the $\frac{3}{2}^-$ or the $\frac{1}{2}^+$ states at the same mass. We identify this $\frac{1}{2}^-$ state to be the $N_{11}^*(1550)$. We use the experimental numbers

$$\Gamma_{N_{11}^* \to \pi N}/\Gamma_T = 0.35, \quad \Gamma_{N_{11}^* \to \eta N}/\Gamma_T = 0.65$$

to calculate Γ_T for the $N^*(1550)$; we find

$$\Gamma_T = 160 \text{ MeV}.$$

For energies higher that 1600 MeV, the particle spectrum becomes parity-doubled on both the leading trajectories and their daughter trajectories.

IV. CONCLUSIONS

We have written a Veneziano model for $\pi^- p \to \eta n$ using only terms with leading Regge behavior in two channels. At the same time, we have obtained qualitative agreement with the experimental widths of the known resonances. Unlike the πN problem, where terms with nonleading Regge behavior must be added to obtain the correct resonance spectrum, the leading terms suffice. (The small width for the $J^P = \frac{1}{2}^+$ daughter at $N_\gamma = \frac{3}{2}$ would make the particle hard to find in this reaction.)

We have also predicted a backward differential cross section with our model. It should be pointed out that the backward cross section is sensitive to the universal slope[8] of the Regge trajectories. If α' is increased to 1.0 GeV^{-2}, and if the other parameters are changed slightly to maintain the correct particle spectrum and widths, the contribution of the N_γ trajectory tends to cancel. This cancellation lets the N_α trajectory dominate the backward cross section, and a sharp dip appears near $N_\alpha = -\frac{1}{2}$. For the slope $\alpha' = 0.9$ GeV^{-2}, there is less cancellation, and hence more effect on the small-u cross section from the N_γ trajectory. Experimental data for the reaction $\pi p \to \eta n$ at small u would therefore be quite important in testing the model in general, as well as determining more accurately the slope α'. A recent experiment by Coleman *et al.*[9] has shown the reaction $\pi^- p \to \eta n$ in the backward region. Although they were unable to measure a cross section, they deduce a value $g_\eta^2/g_\pi^2 \simeq 0.18 \pm 0.06$ which is far too large to agree with Eqs. (3.10) and (3.11). In a resonance fit to $\pi^- p \to \eta n$ and $\gamma p \to \eta n$, Deans and Holladay[10] find $g_\eta^2/g_\pi^2 \lesssim 10^{-3}$. Further study is needed to determine the coupling $g_{\eta NN}$.

---

[8] Some analyses of data involving A_2 exchange indicate that the effective A_2 trajectory does not have a slope near 0.9 GeV^{-2}. See R. D. Mathews, Nucl. Phys. **B11**, 339 (1969). One may explain the flatter effective trajectory as being due to effects of Regge cuts. See M. Blackmon, Phys. Rev. **178**, 2385 (1969). Since we find a reasonable fit to the forward data, and since universal slopes are essential to a Veneziano model, we have not considered changing the slope of the A_2 in our numerical work.

[9] S. Coleman *et al.*, Phys. Letters **30B**, 659 (1969).

[10] S. R. Deans and W. G. Holladay, Phys. Rev. **165**, 1886 (1968).