Study of $\Delta^{++}(1236)$+Boson Production in π^+p Interactions at 18.5 GeV/c*

N. N. BISWAS, N. M. CASON, M. S. FARBER, J. A. HELLAND, M. J. HONES,† V. P. KENNEY, J. T. MCGAHAN,‡ J. A. POIRIER, J. T. POWERS, O. R. SANDER, AND W. D. SHEPHARD
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556
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The production of the ρ^0, f^0, g^0, η^0, and ω^0 bosons in the pseudo-two-body final state with the $\Delta^{++}(1236)$ baryon is studied. The cross sections and their t dependence are discussed. The s dependence of $\Delta^{++}\rho^0$ and $\Delta^{++}\omega^0$ final states is fitted with the expression $\sigma=k(p/p_0)^{-n}$ and yields values of n in the range 1.5–2.5. Using $SU(3)$ invariance, we determine the phase angle between ρ^0 and ω^0 production as a function of incident momentum. The t dependence for $\rho^0\Delta^{++}$ production shows that the Regge model of the conspiring pion is not correct. Correlations between the Δ^{++} and the ρ^0 decays are observed and are consistent with quark-model predictions.

I. INTRODUCTION

THE study of the reactions

$$\pi^+p\rightarrow p\pi^+\pi^+\pi^- \quad (1)$$

and

$$\pi^+p\rightarrow p\pi^+\pi^+\pi^-\pi^0 \quad (2)$$

has been undertaken in order to study general features of the production of pseudo-two-body final states at high energy. Reactions (1) and (2) were selected for study because of the possibility of observing such "simple" reactions as

$$\pi^+p\rightarrow \Delta^{++}(1236)\rho^0 \quad (3)$$

$$\rightarrow \Delta^{++}f^0 \quad (4)$$

$$\rightarrow \Delta^{++}g^0 \quad (5)$$

$$\rightarrow \Delta^{++}\eta^0 \quad (6)$$

$$\rightarrow \Delta^{++}\omega^0. \quad (7)$$

The term "simple" is used here in the context that the exchange of a single Regge trajectory would be expected to dominate in each case. The incident π^+ momentum, 18.5 GeV/c, is quite high in this experiment, and thus from a Regge-exchange point of view one expects the lowest-lying trajectory to dominate. Thus the pion trajectory should dominate in reactions (3)–(5), whereas ρ exchange should dominate in reaction (7). In reaction (6) the only known trajectory which can contribute is the A_2.

Similar studies of pseudo-two-body final states have been carried out at lower energies.[1–5] Therefore, we are able to study the energy dependence of such variables as the cross section σ, the differential cross section $d\sigma/dt$, and the density-matrix elements for some of the final states under consideration.

II. EXPERIMENTAL PROCEDURES

This experiment was done utilizing the facilities of the 80-in. hydrogen bubble chamber at the Alternating Gradient Synchrotron at Brookhaven National Laboratory. The exposure consisted of 152 000 photographs taken with a π^+ beam at an incident momentum of 18.5 GeV/c. The film was scanned for all three- and four-prong events which were then measured using conventional film-plane and image-plane measuring machines on-line to a DDP-124 computer. After geometrical reconstruction and kinematic fitting and after applying suitable cuts, we have separated a sample of 4292 events of reaction (1) and 3616 events of reaction (2). Details of the event selection techniques are discussed elsewhere.[6–8]

III. CROSS-SECTION DETERMINATION

Figure 1(a) shows the scatter plot of $M(p\pi_1^+)$ versus $M(\pi_2^+\pi^-)$ for reaction (1). The $\Delta^{++}(1236)$ band is clearly evident as is the ρ^0 band. The clustering of events in the overlap region of these two bands is more dense than expected from their simple overlap and is therefore indicative of the presence of reaction (3). To determine the cross section for this reaction, we have studied the scatter plot outside the overlap region and linearly extrapolated this background into the overlap region. The contribution due to "tails" of the Breit-Wigner resonances have been taken into account. The cross section for reaction (3) has in this way been determined to be 87 ± 22 μb.

Beside $\Delta^{++}\rho^0$ production, Fig. 1(a) shows an increased density of events in the Δ^{++} band at the f^0 and g^0 masses. This can be seen clearly in Fig. 2(a), which

* Supported in part by the National Science Foundation.
† Present address: Villanova University, Villanova, Pa.
‡ Present address: General Research Corporation, Arlington, Va.
1 Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Nuovo Cimento **31**, 729 (1964).
2 Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, B897 (1965).
3 Aachen-Berlin-CERN Collaboration, Phys. Letters **19**, 608 (1965).
4 Aachen-Berlin-CERN Collaboration, Phys. Letters **22**, 533 (1966).
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6 M. J. Hones, Ph.D. thesis, University of Notre Dame, 1969 (unpublished).
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8 M. J. Hones, N. M. Cason, N. N. Biswas, J. A. Helland, V. P. Kenney, J. T. McGahan, J. A. Poirier, O. R. Sander, and W. D. Shepard, Phys. Rev. D **2**, 827 (1970).

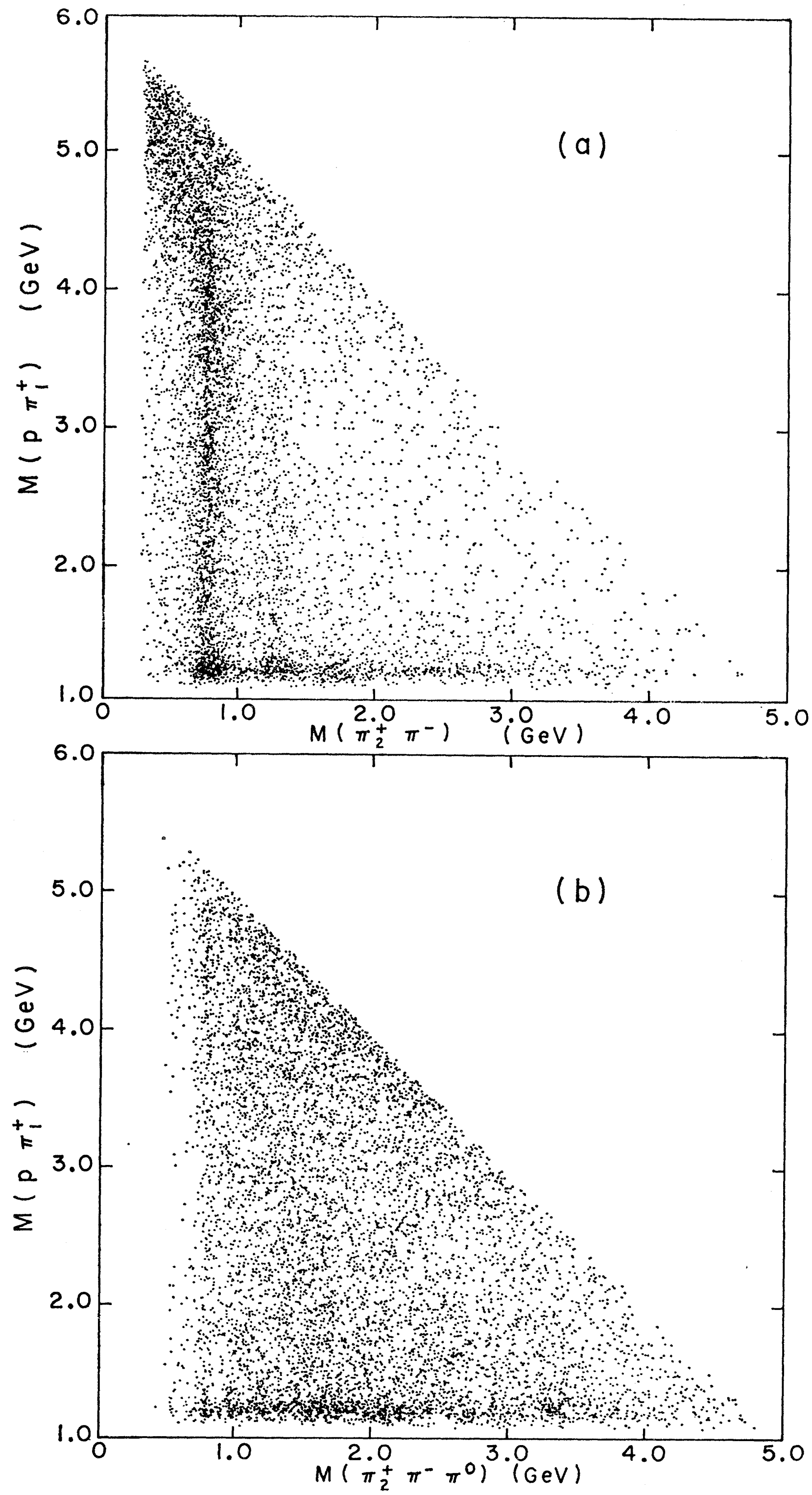

FIG. 1. (a) Scatter plot of $M(p\pi_1^+)$ versus $M(\pi_2^+\pi^-)$ for reaction (1). (b) Scatter plot of $M(p\pi_1^+)$ versus $M(\pi_2^+\pi^-\pi^0)$ for reaction (2).

is the plot of $M(\pi_2^+\pi^-)$ for events in the Δ^{++} band. The cross section for $\Delta^{++}f^0$ [reaction (4)] production has been determined in a manner analogous to that for $\Delta^{++}\rho^0$ production [reaction (3)] and is 41 ± 10 μb.

Both ρ^0 and f^0 mesons are produced outside the Δ^{++} band in addition to their pseudo-two-body production in reactions (3) and (4). This is not true for the g^0 meson. As far as one can tell from Fig. 1(a), there is

TABLE I. Cross sections and t dependence.

Final state	18.5 GeV/c[a] σ (mb)	18.5 GeV/c[a] α (GeV/c)$^{-2}$	8 GeV/c[b] σ (mb)	8 GeV/c[b] α (GeV/c^{-2}	4 GeV/c[c] σ (mb)	4 GeV/c[c] α (GeV/c)$^{-2}$
$\Delta^{++}\rho^0$	0.087 ± 0.022	20.0 ± 1.8	0.43 ± 0.04	13.9 ± 1.6	0.6	11.2[d]
$\Delta^{++}f^0$	0.041 ± 0.010	16.2 ± 2.4	0.18 ± 0.045	13.7 ± 2.9	0.1	e
$\Delta^{++}g^0$	0.034 ± 0.008	14.1 ± 3.0	e	e	e	e
$\Delta^{++}\eta^0$	0.020 ± 0.008	f	0.034 ± 0.011	f	0.03 ($\eta\to3\pi$)	e
$\Delta^{++}\omega^0$	0.012 ± 0.003	f	0.10 ± 0.012	f	0.35 ($\omega\to3\pi$)	f

[a] This experiment.
[b] Reference 3.
[c] Reference 2.
[d] Estimated from the data (not fitted by authors).
[e] Not reported.
[f] Distribution not exponential near $t=0$.

no g^0 production outside the Δ^{++} region, and we conclude that g^0 production is completely correlated with Δ^{++} production in our experiment. Thus the cross section for reaction (5) has been determined to be 34 ± 8 μb from the excess of events in the g^0 region in Fig. 2(a) using the hand-drawn background curve shown.

Figure 1(b) shows $M(p\pi_1^+)$ versus $M(\pi_2^+\pi^-\pi^0)$ for reaction (2). As in Fig. 1(a), the Δ^{++} band is quite clear. Close inspection of Fig. 1(b) reveals the presence of ω^0 and η^0 bands. Again, the $\Delta^{++}-\omega^0$ overlap region is more dense than expected from a simple extrapolation of the Δ^{++} band and ω^0 band into the overlap region. The cross section for reaction (7), determined the same way as the cross sections for reactions (3) and (4) and corrected for unseen decays, was found to be 12 ± 3 μb.

The η^0 mesons produced in association with Δ^{++} production have very little background as one can see from Fig. 1(b). The small cluster of events in the $\Delta^{++}\eta^0$ overlap region is surrounded by a region with very few events. Thus the cross section for reaction (6) has been determined from the $M(\pi^+\pi^-\pi^0)$ projection for events in the Δ^{++} region in Fig. 2(b). The value of the cross section, corrected for neutral decays, was found to be 20 ± 8 μb.

IV. ENERGY DEPENDENCE OF CROSS SECTIONS

The cross sections discussed in Sec. III are given in Table I along with cross sections determined in lower-energy experiments. As one can see from this table, the cross sections are all decreasing significantly with increasing energy. This is consistent with a Regge-pole-exchange picture, where one expects cross sections to decrease rapidly at high energy in the absence of Pomeranchukon exchange.

In Figs. 3(a) and 3(b), we show the cross sections[5,9] for reactions (3) and (7) as a function of incident laboratory momentum p. It has been suggested by Morrison[10] that the cross section be parametrized as

$$\sigma=k(p/p_0)^{-n}, \tag{8}$$

where p_0 is taken to be 1 GeV/c. Expression (8) has been used to fit a large number of two-body reactions and seems to work well empirically. We have, therefore, fitted the data for reactions (3) and (7) with this expression. The results are shown in Table II. For these two reactions, separate fits have been carried out using the data for $p\leq4.1$ GeV/c and for $p\geq7.0$ GeV/c. The solid lines in Figs. 3(a) and 3(b) are these fitted curves. The dashed lines in these figures correspond to the best fits using all the data. For both reactions, we find that the slope of the line for the high-momentum region is different from that for the low-momentum region. Morrison[10] has suggested that for two-body reactions involving meson exchanges, the values of n in Eq. (8) lie

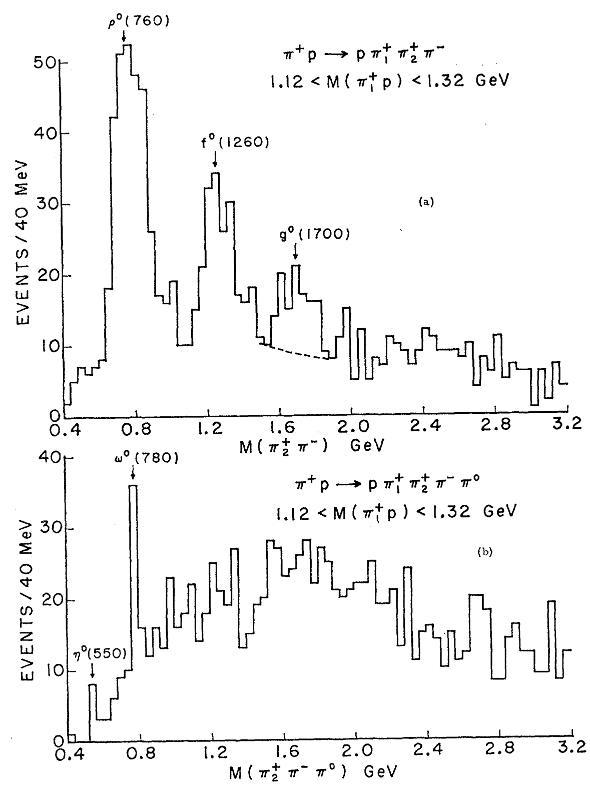

FIG. 2. (a) $M(\pi_2^+\pi^-)$ distribution for the events of reaction (1) in which $M(p\pi_1^+)$ lies in the $\Delta(1236)$ band: $1.12\leq M(p\pi_1^+)\leq1.32$ GeV. (b) $M(\pi_2^+\pi^-\pi^0)$ distribution for the events of reaction (2) in which $M(p\pi_1^+)$ lies in the $\Delta^{++}(1236)$ band.

[9] Low-energy data are summarized by D. Brown *et al.*, Phys. Rev. Letters 19, 664 (1967). Additional high-energy data on reactions (3) and (7) are reported by H. H. Kung, Columbia University Report No. Nevis-171, 1969 (unpublished), and by J. A. Gaidos *et al.*, Phys. Rev. D 1, 3190 (1970).

[10] D. R. O. Morrison, Phys. Letters 22, 528 (1966); see also D. Brown *et al.* (Ref. 9).

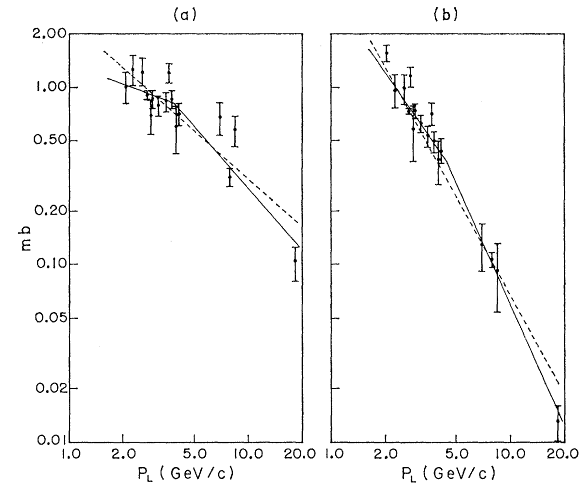

FIG. 3. Cross sections for reactions (a) $\pi^+p\rightarrow\Delta^{++}\rho^0$ and (b) $\pi^+p\rightarrow\Delta^{++}\omega^0$ as a function of incident momentum. The dashed lines are the best fits of all the data using expression (8). The solid lines show the same fits using two different regions of momenta, $p\leq4.1$ GeV/c and $p\geq7.0$ GeV/c.

in the range 1–2. Since at low energies, the value of n for reaction (3) is about 0.4 (see Table II), Brown *et al.*[9] have concluded that the rule suggested by Morrison is violated for this reaction. However, our study shows that at higher energies the value of n changes to about 1.7 for this reaction. Thus meson-exchange reactions *at high energy* do indeed seem to have a value of $n\approx2$.

V. $SU(3)$ INVARIANCE IN Δ^{++} VECTOR-MESON REACTIONS

It is of interest to test[11] $SU(3)$ invariance for the amplitudes of reactions (3) and (7) along with the amplitude for the reaction

$$K^+p\rightarrow\Delta^{++}K^{0*} \tag{9}$$

at high energy. From $SU(3)$ invariance, one obtains

TABLE II. Fitted values of the parameters in the expression $\sigma=k(p/p_0)^{-n}$.

Reaction	Incident momentum region (GeV/c)	n	k (mb)	C. L. (%)[a]
$\pi^+p\rightarrow\Delta^{++}\rho^0$	2.08– 4.10	0.36±0.22	1.32± 0.32	16.0
	7.00–18.5	1.68±0.22	13.0 ± 6.6	1.5
	2.08–18.5	1.00±0.06	2.72± 0.23	0.1
$\pi^+p\rightarrow\Delta^{++}\omega^0$	2.08– 4.10	1.50±0.19	3.47± 0.70	2.0
	7.00–18.5	2.50±0.29	19.2 ±12.2	50.0
	2.08–18.5	1.89±0.07	5.21± 0.40	0.5

[a] Confidence levels have been computed using the error estimates given by the authors.

[11] Using lower-energy data for the reactions studied here, $SU(3)$ invariance has been tested earlier; see, e.g., S. Meshkov *et al.*, Phys. Rev. Letters **12**, 87 (1964); V. Barger and M. Olsson, *ibid.* **15**, 930 (1965); S. Meshkov and G. B. Yodh, *ibid.* **19**, 603 (1967).

the vector sum rule

$$(4/\sqrt{3})[(1/\sqrt{2})\cos\lambda+\tfrac{1}{4}\sin\lambda]A_1+A_2 =(2/\sqrt{3})[(1/\sqrt{2})\sin\lambda+\tfrac{1}{2}\cos\lambda]A_3, \tag{10}$$

where the A_i's are the amplitudes for $\Delta^{++}\rho^0$, $\Delta^{++}\omega^0$, and $\Delta^{++}K^{*0}$ production, respectively, and λ is the $\omega-\phi$ mixing angle. Using the value[12] $\lambda=40°$, Eq. (10) becomes

$$1.62A_1+A_2=0.97A_3. \tag{11}$$

In order to estimate the cross section for reaction (9),

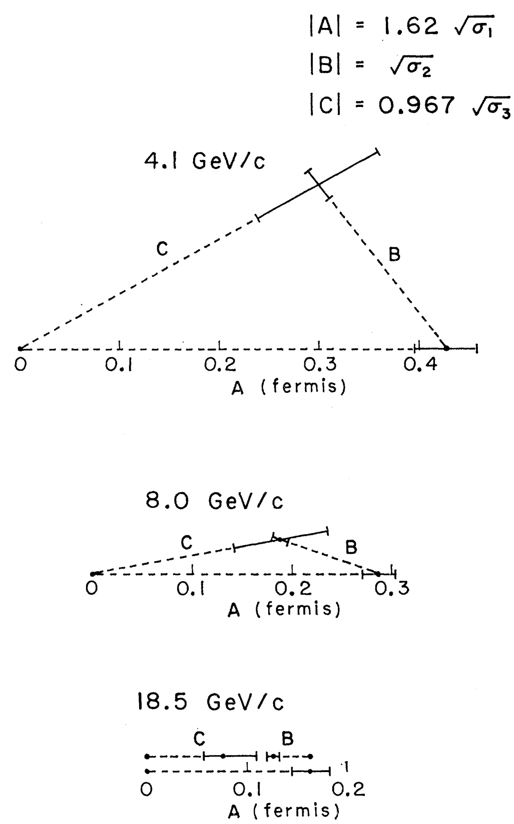

FIG. 4. Triangular relations [Eq. (11)] relating $A=1.62\sqrt{\sigma_1}$, $B=\sqrt{\sigma_2}$, and $C=0.97\sqrt{\sigma_3}$ (see text) at 4.1, 8.0, and 18.5 GeV/c. The values of β are $0.91_{-0.46}^{+0.34}$ rad, $0.66_{-0.66}^{+0.52}$ rad, and ≈0 rad, respectively.

we have fitted the data[13] above 3.0 GeV/c to Eq. (8). The fit yields the values $n=1.42\pm0.20$ and $k=9.30\pm2.82$ mb. (Since few experimental values are available for this reaction, separate fits for high and low momenta could not be made.)

In Fig. 4 we show the triangular relation (11) for

[12] A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970).

[13] J. C. Berlinghieri *et al.*, Nucl. Phys. **B8**, 333 (1968); and the complilation by L. R. Price *et al.*, LRL Report No. UCRL-20000 K^+N, 1969 (unpublished).

three different beam momenta,[14] 4.1, 8.0, and 18.5 GeV/c. The quantities $A=1.62\sqrt{\sigma_1}$, $B=\sqrt{\sigma_2}$, and $C=0.97\sqrt{\sigma_3}$ should form a closed triangle. [σ_1, σ_2, and σ_3 are the cross sections for reactions (3), (7), and (9).] The results are consistent with the $SU(3)$ vector sum rule.

VI. RELATIVE PHASE OF ρ^0-ω^0 PRODUCTION AMPLITUDES

The angle β between A and B in the triangular relation determines the phase (modulo π) between the amplitudes of reactions (3) and (7). We find that β equals $0.91_{-0.46}^{+0.34}$ rad at 4.1 GeV/c, $0.66_{-0.66}^{+0.52}$ rad at 8.0 GeV/c, and 0.00 ± 0.53 rad at 18.5 GeV/c.

G. Goldhaber *et al.*[15] have determined β using ρ^0-ω^0 interference in reactions (3) and (7) at an incident momentum of $\sim$4 GeV/c. These authors observed destructive interference in the 2π mass spectrum of the reaction

$$\pi^+p\to\Delta^{++}\pi^+\pi^- \tag{12}$$

in the ρ^0-ω^0 mass region. This effect depends on (a) the phase difference β between ρ^0 and ω^0 production amplitudes and (b) the phase difference β' between $\rho^0\to2\pi$ and $\omega^0\to2\pi$ decay amplitudes. Assuming $\beta'\approx\frac{1}{2}\pi$, G. Goldhaber *et al.*[15] determined β to be 1.5 ± 0.3 rad at $3.7-4.0$ GeV/c. Our value of $\beta=0.91_{-0.46}^{+0.34}$ rad at 4.1 GeV/c is consistent with their value and is free of any assumption about β'.

A. S. Goldhaber *et al.*[16] point out that the nature of the ρ^0-ω^0 interference pattern may vary with final state, since β depends on the production reaction. The decrease of β with energy which our data suggest would imply a variation with energy in the ρ^0-ω^0 interference pattern for reaction (12). At about 4 GeV/c, the total phase difference $(\beta+\beta')$ is approximately π rad and hence the amplitudes interfere destructively. If β approaches zero at high energy, as suggested by Fig. 4, the total phase difference would approach $\frac{1}{2}\pi$ rad. Thus one would expect the destructive interference effects to decrease with increasing energy.

VII. FOUR-MOMENTUM TRANSFER DEPENDENCE

Figures 5(a) and 5(b) show the Chew-Low plots of t, the square of the four-momentum transfer from the target proton to the Δ^{++}, versus $M(\pi^+\pi^-)$ and $M(\pi^+\pi^-\pi^0)$ for events in the Δ^{++} mass band for reactions (1) and (2), respectively. The data show that both reactions are characterized by low values of $|t|$. However, there is an interesting difference between the reactions—namely, reaction (1) is considerably more peripheral for Δ^{++} production than is reaction (2) for boson masses up to about 2 GeV. This is probably because pion exchange is allowed only in the former reaction.

The t' $(=t-t_{\min})$ distributions for reactions (3)–(7) are shown in Fig. 6. (Here $t_{\min}$ is the minimum allowed value of t for the $\pi^+\pi^-$ and the π^+p masses for the event.) The t' dependence for ρ^0, f^0, and g^0 production is clearly different from that for η^0 and ω^0 production. For η^0 and ω^0 production, the t' distributions are essentially flat at low $|t'|$. For ρ^0, f^0, and g^0 production [reactions (3)–(5)], the t' distributions are well fitted by the expression

$$d\sigma/dt=e^{-\alpha|t'|}. \tag{13}$$

The values of α for these reactions are given in Table I. These high values of α are consistent with the dominance of pion exchange.

It is interesting to compare these t distributions with those obtained at lower energies. The data[3] at 8 GeV/c are qualitatively quite similar to our data. In particular, the nonexponential t dependence for η^0 and ω^0 production is apparent at 8 GeV/c, whereas ρ^0 and f^0 production are well fitted by Eq. (13). However, there is a significant difference between the values of α obtained here and those obtained at lower energy. A shrinkage of the width of the forward peak with increasing incident energy is apparent. This shrinkage has been observed previously,[17] and is due to the variation of $t_{\min}$ with incident energy and over the broad ρ^0 and Δ^{++} widths. (The reaction $\pi^+p\to\Delta^{++}g^0$ was not studied at the lower energy, so no comparison can be made in that case.)

Of particular significance with respect to Regge-pole theory is the t dependence for $\Delta^{++}\rho^0$ production near $|t'|=0$. The "conspiring-pion" theory predicts a dip near $|t|=0$. Lower-energy experiments[18] have noted the absence of such a dip, and thus seem to rule out the conspiracy hypothesis. Our data, which should be an even better test of the model due to the higher incident energy, confirm this earlier work [Fig. 6(a)] and rule out the conspiracy hypothesis with a high degree of certainty.

VIII. $\pi\pi$ SCATTERING

The fact that reactions (3)–(5) are consistent with pion exchange suggests studying the $\pi\pi$ scattering angular distributions. Figure 7 shows the scattergram of $\cos\theta(\pi^+\pi^-)$ versus $M(\pi^+\pi^-)$ for events in the Δ^{++} band for which $|t|<15\ \mu^2$ (μ is the pion mass). Here $\theta(\pi\pi)$, the Jackson angle, is the angle between the

[14] The cross sections for reactions (3) and (7) are experimentally known at these beam momenta and in fact are determined by the same group. To obtain cross sections for reaction (9) at the same beam momenta we have used the fitted values obtained from Eq. (8), since no direct measurements are available. Data at lower energy have been compared at the same Q values of these reactions to account for kinematic effects due to mass differences in the initial and final states (see Ref. 11). At high energy such kinematic effects become unimportant and hence we have compared these cross sections at the same incident momentum.

[15] G. Goldhaber *et al.*, Phys. Rev. Letters **23**, 1351 (1969).

[16] A. S. Goldhaber *et al.*, Phys. Letters **30B**, 249 (1969).

[17] G. Wolf, Phys. Rev. Letters **19**, 925 (1967).

[18] See J. A. Gaidos *et al.* (Ref. 9).

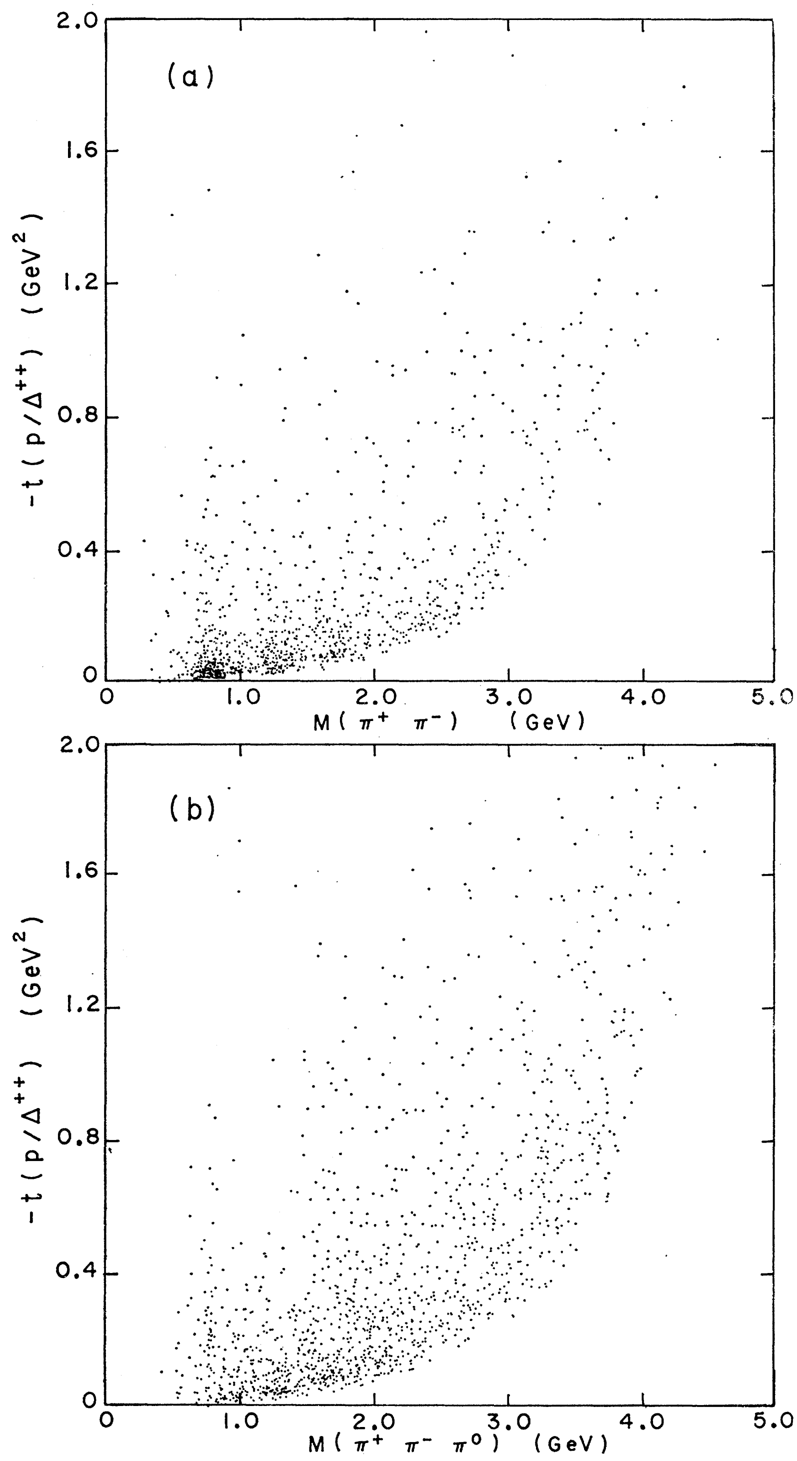

FIG. 5. Chew-Low plot of $-t(p\Delta)$ versus: (a) $M(\pi_2{}^+\pi^-)$ for events of reaction (1) in which $M(\pi_1{}^+p)$ lies in the $\Delta^{++}(1236)$ band; (b) $M(\pi_2{}^+\pi^-\pi^0)$ for events of reaction (2) in which $M(\pi_2{}^+p)$ lies in the $\Delta^{++}(1236)$ band.

incident π^+ and the outgoing π^+ from the boson decay in the boson reference frame. Projections for the ρ^0, f^0, and g^0 regions are shown in Fig. 8. The distributions for the ρ^0 and f^0 regions are similar to those which one observes in such reactions as $\pi p \rightarrow \pi\pi\eta$, where $\pi\pi$ scattering has been studied in detail. They tend to confirm that pion exchange is the dominant mechanism in ρ^0 and f^0 production in our experiment.

The strong asymmetry in the g^0 region indicates a more complicated situation. The data show a relatively

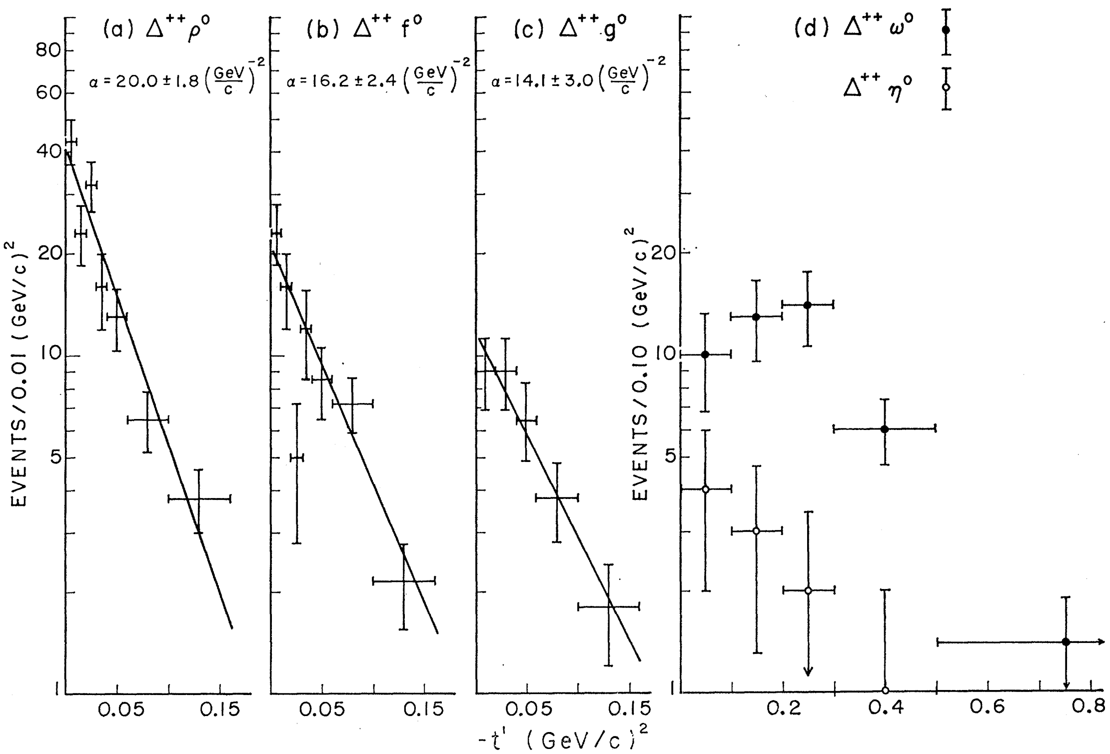

FIG. 6. Four-momentum transfer distributions for reactions (3)–(7).

isotropic distribution in $\cos\theta(\pi\pi)$ with a very strong forward peak. A natural question to ask is whether the g^0 is associated with the relatively isotropic part of the distribution, with the forward peak (which would suggest the presence of many angular momentum states), or with both.

The forward peak appears to be associated with background, as one can see from Fig. 7. The data from the g^0 region are apparently forward-peaked just as the data immediately outside the g^0 region. This forward peak can be variously interpreted as $\pi\pi$ diffraction scattering[19] or as diffraction dissociation[20] of the nucleon into an N^* which decays into $\Delta^{++}\pi$ (or $p\pi^+\pi^-$). Either explanation is consistent with the data, but neither mechanism predicts peaking in the g^0 mass region of the $M(\pi^+\pi^-)$ distribution. This seems to indicate that g^0 resonance production is associated with the isotropic part of the data.

TABLE III. Spin-density-matrix elements for the reaction $\pi^+p\rightarrow\Delta^{++}\rho^0$ at 18.5 GeV/c.

Matrix element	$0<\lvert t'\rvert<2\mu^2$	$2\mu^2<\lvert t'\rvert<15\mu^2$	$\lvert t'\rvert>15\mu^2$
$(A)\Delta^{++}$			
ρ_{33}	0.07 ± 0.05	0.21 ± 0.06	0.13 ± 0.04
$\mathrm{Re}\rho_{3-1}$	-0.03 ± 0.05	-0.04 ± 0.06	-0.03 ± 0.04
$\mathrm{Re}\rho_{31}$	-0.02 ± 0.06	0.03 ± 0.06	0.01 ± 0.04
$(B)\rho^0$			
ρ_{00}	0.91 ± 0.07	0.66 ± 0.08	0.80 ± 0.06
ρ_{1-1}	-0.06 ± 0.04	0.07 ± 0.06	0.00 ± 0.03
$\mathrm{Re}\rho_{10}$	0.06 ± 0.05	-0.01 ± 0.05	-0.04 ± 0.03
$\rho_{00}-\rho_{11}$	0.87 ± 0.10	0.49 ± 0.13	0.70 ± 0.08
$\mathrm{Re}\rho_{00}^{\mathrm{int}}$	0.28 ± 0.06	0.18 ± 0.06	0.24 ± 0.04
$\mathrm{Re}\rho_{10}^{\mathrm{int}}$	-0.02 ± 0.03	0.02 ± 0.03	0.00 ± 0.02

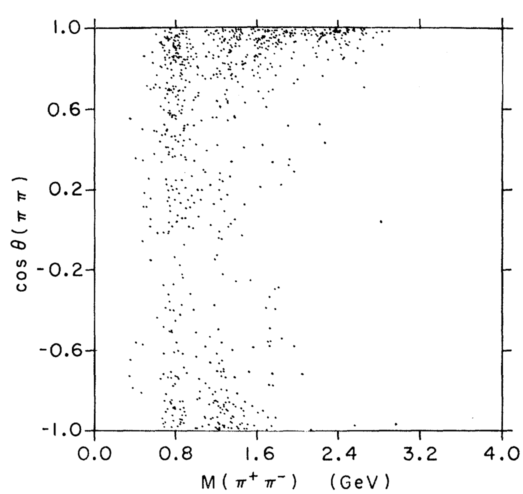

FIG. 7. Distribution of $\cos\theta(\pi^+\pi^-)$ versus $M(\pi^+\pi^-)$ for the events of reaction (1) which lie in the $\Delta^{++}(1236)$ band and for which $\lvert t'(p\Delta)\rvert<15\mu^2$.

[19] N. N. Biswas *et al.*, Phys. Rev. Letters **18**, 273 (1967).
[20] W. D. Walker *et al.*, Phys. Rev. Letters **20**, 133 (1968).

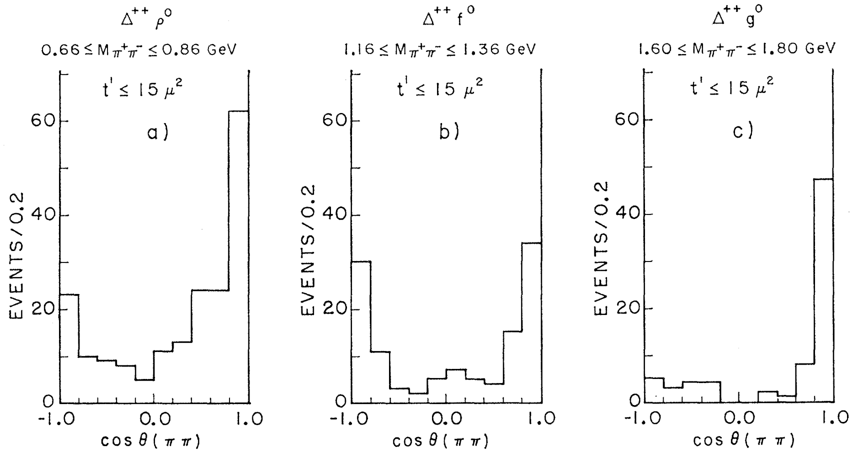

FIG. 8. Distribution of $\cos\theta(\pi^+\pi^-)$ for the final states: (a) $\Delta^{++}\rho^0$; (b) $\Delta^{++}f^0$; and (c) $\Delta^{++}g^0$.

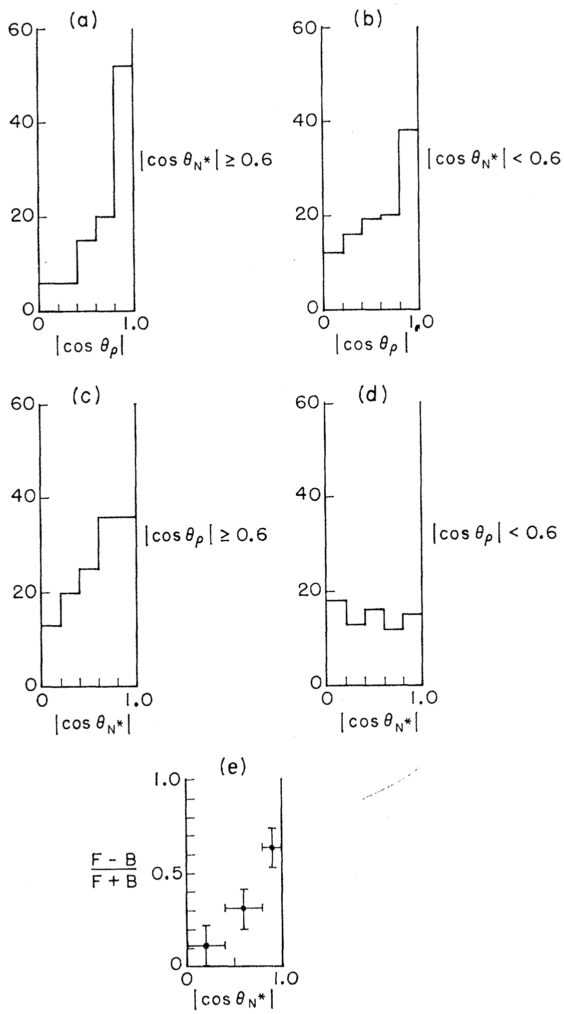

FIG. 9. Various distributions showing correlations between ρ^0 and Δ^{++} decays for events of reaction (1): (a) and (b) show the ρ^0 decay angular distributions for which $|\cos\theta_\Delta| \geq 0.6$ and for which $|\cos\theta_\Delta| < 0.6$, respectively; (c) and (d) show the $\Delta^{++}(1236)$ decay angular distribution for which $|\cos\theta_\rho| \geq 0.6$ and for which $|\cos\theta_\rho| < 0.6$, respectively; (e) shows the ρ^0 decay asymmetry as a function of the decay angle of the $\Delta^{++}(1236)$.

IX. DENSITY MATRICES AND ANGULAR CORRELATIONS

The density-matrix elements for reaction (3) are given in Table III. The values were determined using the method of moments[21] and were evaluated in the Jackson reference frame. The ρ^0-decay asymmetry requires the coherent admixture of some *s* wave (the ϵ^0) and the values shown correspond to the inclusion of these interference effects; we note the presence of nonzero values for $\mathrm{Re}\rho_{00}^{\mathrm{int}}$. The matrix elements are given for several regions of t'.

The ρ^0 and Δ^{++} are rather strongly correlated as shown in Fig. 9. Besides correlations between the ρ^0 and Δ^{++} polar decay angles, the figure shows a very strong correlation between the ρ^0-decay asymmetry and the Δ^{++} decay angle. The asymmetry is maximal for the fore-aft decay of the Δ^{++} and goes to zero for the transverse decay

We now compare the quark-model predictions of Białas and Zalewski[22] with the data. These predictions relate the ρ^0 and Δ^{++} density-matrix elements and thus are predictions regarding the correlations. For this analysis we have used the $\rho^0\Delta^{++}$ events for which $|t'| < 15\ \mu^2$ and we have tested the theory in both the Jackson and the helicity reference frames.

The quark-model predictions have been classified[22] into three categories which differ in the assumptions made. For all three classes, the additivity[23] of quark-quark scattering amplitudes is assumed. (Additivity implies that the amplitude for a reaction is simply the sum of all possible contributing two-body quark-quark or quark-antiquark scattering amplitudes.) The most reliable predictions [class (a)] were derived without further assumptions. Other relations [classes (b) and (c)] were obtained after making certain additional assumptions about the spin dependence of the quark-

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[22] A. Białas and K. Zalewski, Nucl. Phys. **B6**, 465 (1968).
[23] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 171 (1966).

quark scattering amplitudes, from which one obtains equality between certain amplitudes.

The relations in all three classes have been tested using our data for reaction (3). For the six relations in class (a), agreement is found in both frames, within about 2 standard deviations. Since these relations are covariant with respect to arbitrary rotations of the vector-meson, or baryon, reference frame about the production plane normal, they are expected to be valid in both frames.

The experimental results for class (b) also show agreement to within 2 standard deviations in both frames. The six class-(b) relations are covariant with respect to equal rotations of the vector-meson and baryon reference systems in the production plane. The angle to transform the Jackson frame to the helicity frame is not the same for vector meson and baryon. Thus, the class-(b) relations cannot hold in both frames. However, the difference in the transformation angles is small for high energy and low momentum transfer, and thus, since the agreement is about the same in both frames, the class-(b) relations do not provide useful information on the preferred frame.

The seven class-(c) relations have no covariance properties. Agreement is observed in the Jackson frame to within 2 standard deviations for all relations. However, in the helicity frame, one relation is violated by 5 standard deviations. This seems to indicate some preference for the Jackson frame.

The density-matrix elements for reactions (4) and (7) are given in Tables IV and V. In both cases we find that the matrix elements are essentially identical to those found at lower incident energy.[4] Thus, just as for the case of $\Delta^{++}\rho^0$ production, the production mechanism for $\Delta^{++}f^0$ and $\Delta^{++}\omega^0$ reactions does not seem to be dependent on incident momentum.

TABLE IV. Single-vertex spin-density-matrix elements for the reaction $\pi^+p \rightarrow \Delta^{++}f^0$ at 18.5 GeV/c.

Matrix element	$0<\|t'\|<5\mu^2$	$5\mu^2<\|t'\|<15\mu^2$	$\|t'\|<15\mu^2$
$(A)f^0$			
ρ_{00}	0.88 ± 0.11	0.92 ± 0.18	0.89 ± 0.09
ρ_{11}	0.26 ± 0.04	0.16 ± 0.08	0.24 ± 0.04
ρ_{22}	-0.20 ± 0.04	-0.12 ± 0.09	-0.19 ± 0.04
ρ_{1-1}	-0.10 ± 0.06	-0.20 ± 0.09	-0.12 ± 0.05
ρ_{2-2}	-0.06 ± 0.03	-0.01 ± 0.08	-0.05 ± 0.03
$\mathrm{Re}\rho_{10}$	-0.08 ± 0.06	-0.18 ± 0.12	-0.10 ± 0.06
$\mathrm{Re}\rho_{20}$	0.08 ± 0.02	0.11 ± 0.06	0.09 ± 0.02
$\mathrm{Re}\rho_{21}$	-0.04 ± 0.03	-0.07 ± 0.06	-0.04 ± 0.03
$\mathrm{Re}\rho_{2-1}$	0.03 ± 0.05	0.04 ± 0.07	0.03 ± 0.04
$(B)\Delta^{++}$			
ρ_{33}	-0.02 ± 0.06	0.10 ± 0.11	0.00 ± 0.05
$\mathrm{Re}\rho_{3-1}$	-0.02 ± 0.04	-0.14 ± 0.10	-0.05 ± 0.04
$\mathrm{Re}\rho_{31}$	-0.07 ± 0.06	-0.02 ± 0.11	-0.06 ± 0.05

TABLE V. Spin-density elements for the reaction $\pi^+p \rightarrow \Delta^{++}\omega^0$ at 18.5 GeV/c.

	$\|t'\|<0.2$ (GeV/c)^2	$\|t'\|<0.5$ (GeV/c)^2
$(A)\omega^0$		
ρ_{00}	0.32 ± 0.14	0.26 ± 0.10
ρ_{1-1}	0.20 ± 0.11	0.07 ± 0.09
$\mathrm{Re}\rho_{10}$	0.00 ± 0.10	0.05 ± 0.07
$(B)\Delta^{++}$		
ρ_{33}	0.21 ± 0.10	0.22 ± 0.08
$\mathrm{Re}\rho_{3-1}$	0.03 ± 0.09	-0.01 ± 0.08
$\mathrm{Re}\rho_{31}$	-0.11 ± 0.09	-0.04 ± 0.08

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