e.g., looking at the perturbation-theory results. There is a marked difference though between the [1, 1] Padé and the SU_3 values for μ_{Σ^-} and μ_{Ξ^-}. Since these are exactly the magnetic moments which were found to be insensitive to small changes in α, these can be looked upon as definite predictions of our work.

Our results are inconclusive for two reasons. One is that too little is known experimentally about the magnetic moments we have been able to calculate with the [1, 1] Padé approximant. Also, the reliability of the lowest-order Padé approximant has first to be checked. For both of these reasons it would be very informative as to the usefulness of our approach to calculate the results of the [1, 2] and [2, 1] Padé approximation. This would, apart from testing the practical convergence of the method, also give the magnetic moments of the neutral-octet baryons. The magnetic moments of the neutron and Λ^0 being known experimentally, one would then have a severe check on the theory.

Though our results are not conclusive, it seems that they are sufficiently promising to call for further work along these lines. On the positive side, we have the excellent agreement between the value $\alpha=0.725$ required to reproduce the proton magnetic moment and the experimental value $\alpha=0.75$. On the other hand, the result for Σ^+, $\mu_{\Sigma^+}=1.5$, is too low compared to the present-day experimental value $\mu_{\Sigma^+}=2.5\pm0.5$.

Extension of this work to calculate the electromagnetic form factors of the proton is in progress.

The calculation of the magnetic moments in the next-order [1, 2] and [2, 1] Padé approximation might also be undertaken in the future.

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Linearly Rising Regge Trajectories and the Isovector Form Factor $F_1{}^V(t)$*

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A simple ansatz for the isovector Dirac form factor is examined. A fit to an approximate dipole form is achieved, and deviations from this form are also accounted for. A ρ' is no longer necessary; the next vector meson used in the saturation has a mass of approximately 1.7 GeV. The fit also gives a ρNN coupling constant which has the right value, as well as providing a good value for the necleon charge radius.

I. INTRODUCTION

ONE of the major problems in electromagnetic interactions in recent years has been to find a theoretical explanation for the experimental features of the pion and nucleon electromagnetic form factors. It is difficult to understand the t dependence of the form factors on the basis of the usual pole-dominance model, which works at low momentum transfers and has approximate validity in the timelike region. Far from the vector-meson pole in the spacelike region, nucleon form factors fall off faster than implied by the pole dominance, and have approximately the dipole form. The asymptotic behavior of the pion form factor is not yet known accurately. The dipole formula, unless accompanied by a pole, is very difficult to understand theoretically. One idea is that the dipole form of the nucleon form factors might be simulated by more complicated functions obtained by saturating the form factors by an infinite number of poles. Away from the pole in the spacelike region, we might try the narrow-resonance approximation for such a string of poles. Physically, this would imply that the virtual photon can change not only into a vector meson like the ρ meson, but also into other vector mesons with higher masses and the same quantum numbers as the ρ meson.

Such a model in which the meson spectrum includes an infinity of vector mesons with the same quantum numbers as the ρ is provided by the Veneziano[1] model, devised to satisfy duality, Regge behavior in all channels, and crossing symmetry. Several authors[2] have used the spectrum provided by the Veneziano model to saturate electromagnetic form factors. To this end they have used an ansatz for the form factors being approximated by the ratio of two gamma functions with a proper normalization constant. Making this ansatz for the Sachs form factors, they obtained the following results: (1) By choosing suitable parameters that appear in the gamma functions, an approximate dipole form can be obtained at high t, although the form factor is only saturated by single poles. (2) The deviation from

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[1] G. Veneziano, Nuvo Cimento **57A**, 190 (1968).
[2] P. Di Vecchia and F. Drago, Nuovo Cimento Letters **1**, 917 (1969); R. Lengo and E. Remiddi, *ibid.* **1**, 922 (1969); P. H. Frampton, Phys. Rev. **186**, 1419 (1969); Y. Oyanagi, Progr. Theoret. Phys. (Kyoto) **42**, 898 (1969); Fayyazuddin and Riazuddin, Phys. Letters **28B**, 8 (1969); M. Namiki and I. Ohba, Progr. Theoret. Phys. (Kyoto) **42**, 1166 (1969).

the dipole form can also be fitted. (3) The mean-square radius for the nucleon is obtained correctly from the previous fit.

There are, however, two drawbacks: First, a ρ' meson with mass around 1.3 GeV, a first daughter of the A_2, appears in the saturation of the form factor. There is no experimental evidence for such a meson. Second, once the form factor is fitted, the residue of the ρ meson is determined, giving the ρNN coupling constant. In the phenomenological theories under consideration, one finds a ρNN coupling constant too large by almost an order of magnitude.

To evade the first difficulty, Frampton[2] has tried a form for the Sachs form factors that does not include first daughters, and hence the ρ', but then the fit becomes worse.

In this paper we propose to use a similar ansatz (Veneziano form with the omission of the first daughters) for the Dirac form factors instead of the Sachs form factors. There is no *a priori* reason why the Sachs form factors should be saturated with poles rather than the Dirac form factors. From one point of view, the Sachs form factors appear more fundamental since they obey the simple scaling laws. On the other hand, the Sachs form factors are constrained by threshold conditions, whereas the Dirac form factors are unconstrained and seem more fundamental from this viewpoint. Anyway, any ansatz is purely phenomenological, pending a deeper understanding of the structure of the form factors. What we find is the following: (1) An approximate fit to the dipole form can still be achieved and deviations from this form can still be accounted for. (2) A ρ' is no longer necessary, the next vector meson used in the saturation having a mass of approximately 1.7 GeV. Such a meson may well exist in the R region. (3) The fit also gives a ρNN coupling constant which has the right value, as well as providing a good value for the nucleon charge radius.

II. BASIC EQUATIONS

First we give the definitions and the relations between form factors. The Dirac and Pauli form factors are defined from the matrix elements of the electromagnetic current as follows:

$$\langle p'|J_\mu(0)|p\rangle = \frac{e}{(2\pi)^3}\bar{U}(p') \times\left[\gamma_\mu F_1{}^p(t)+\frac{i\sigma_{\mu\nu}q_\nu}{2M}F_2{}^p(t)\right]U(p), \quad (1)$$

where $q=(p'-p)$, $t=q^2$, M=proton mass, e=proton charge, and for the normalization we have chosen $F_1{}^p(0)=1$, $F_2{}^p(0)=\mu_\alpha$, μ_α being the anomalous magnetic moment of the proton. Similarly, one defines the neutron form factors $F_1{}^n(t)$ and $F_2{}^n(t)$ with a proper normalization at $t=0$. The Sachs form factors are defined from the Dirac and Pauli form factors in the following way:

$$\begin{aligned} G_E(t)&=F_1(t)+(t/4M^2)F_2(t)\,,\\ G_M(t)&=F_1(t)+F_2(t)\,, \end{aligned} \quad (2)$$

and are known as charge and magnetic-moment form factors. Since recent experiments at high momentum transfer[3,4] indicate no significant deviations from the scaling laws, the relations

$$G_E{}^p(t)=G_M{}^p(t)/\mu_p=G_M{}^n(t)/\mu_n \quad (3)$$

will be assumed in the following analysis. We note that the electric form factor of the neutron $G_E{}^n(t)$ is not taken to be identically zero for all values of t. It is known that for small values of $|t|$, $G_E{}^n(t)$ is not zero and has a nonzero slope[5] at $t=0$. After the assumption of scaling laws, however, the magnetic-moment Sachs form factor can be expressed[12] in terms of $F_1{}^V(t)$, the isovector part of the Dirac form factor, and the neutron electric form factor $G_E{}^n(t)$ in the form

$$\frac{G_M{}^p(t)}{\mu_p}=\frac{1-\tau}{1-(\mu_p-\mu_n)\tau}F_1{}^V(t)+\frac{G_E{}^n(t)}{1-(\mu_p-\mu_n)\tau}, \quad (4)$$

where μ_p= the total proton magnetic moment, μ_n= the neutron magnetic moment, and $\tau=t/4M^2$. The normalization of $F_1{}^V(t)$ in Eq. (4) is $F_1{}^V(0)=1$.

III. RESULTS

We want the expression for $F_1{}^V(t)$ to satisfy the following conditions. It should (a) contain an infinite number of poles corresponding to even-daughter (1^-) vector poles, (b) behave like a power of $|t|$ for large $|t|$, and $\arg t\neq 0$, and (c) have the proper normalization. In what follows, we make an ansatz for $F_1{}^V(t)$ that has these properties, and omit the first daughters, and hence ρ'. We propose

$$F_1{}^V(t)=\frac{\Gamma(\frac{1}{2}\beta-\frac{1}{2}a)\Gamma(\frac{1}{2}-\frac{1}{2}\alpha_\rho(t))}{\Gamma(\frac{1}{2}-\frac{1}{2}a)\Gamma(\frac{1}{2}\beta-\frac{1}{2}\alpha_\rho(t))} \quad (5)$$

for the Dirac isovector form factor, where β is a parameter determined from the best fit of Eq. (5) to the data points, and a comes from the trajectory function $\alpha_\rho(t)$ which is assumed to have the form $\alpha_\rho(t)=a+bt$. The Dirac form factor of Eq. (5) is chosen to have the normalization $F_1{}^V(0)=1$. We will use the trajectory function derived by Lovelace[6] and used in similar analyses[2]

[3] J. Litt *et al.*, Phys. Letters **31B**, 40 (1970).

[4] C. Berger *et al.*, Phys. Letters **28B**, 276 (1968). The deviations are not considered to be significant for our purpose. The latest results (Ref. 4) are compatible with scaling laws.

[5] W. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva 1968), p. 23.

[6] C. Lovelace, Phys. Rev. Letters **28B**, 265 (1968).

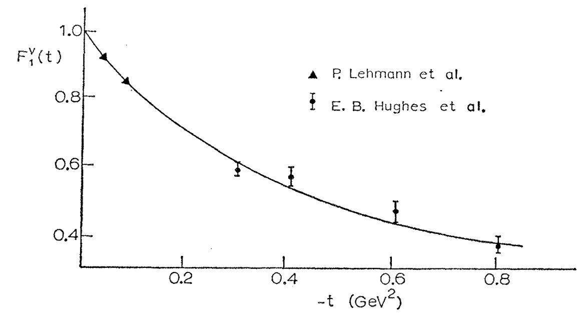

FIG. 1. The solid line is the plot of $F_1{}^V(t)$ corresponding to Eq. (8) with $\beta=4\frac{3}{4}$. The experimental points are from Refs. 7 and 13.

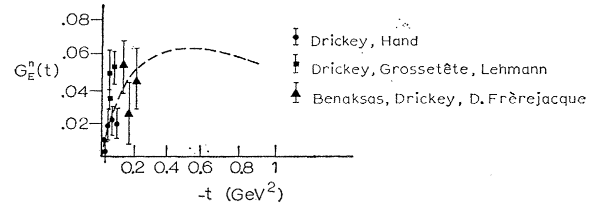

FIG. 3. Curve shown corresponds to our parametrization of $G_E{}^n(t)$, and $A=0.482$ GeV^{-2} is chosen to produce the slope of $G_E{}^n(t)$ at $t=0$. The experimental points are taken from Ref. 5.

before, namely,

$$\alpha_\rho(t)=\tfrac{1}{2}+t/2M_\rho{}^2. \tag{6}$$

For the best choice of the parameter β, a fit of Eq. (5) to data is used which also gives a correct fit for the Sachs form factors through Eq. (4). First let us look at the available data for the isovector Dirac form factor. Hughes *et al.*[7] have combined the neutron-form-factor measurements with the proton-form-factor data[8] to obtain the experimental data of the isotopic form factors (separated as isovector and isoscalar). We find that the best fit to the experimental points requires $\beta=4\frac{3}{4}$. The result of this choice is shown in Fig. 1 for the isovector Dirac form factor.

Now we can calculate the mean-square radius corresponding to Eq. (5). The Dirac isovector radius[9] is given by

$$\langle r_1{}^2\rangle^V=6[dF_1{}^V(t)/dt]_{t=0}. \tag{7}$$

Employing Eqs. (5) and (6), one obtains the formula

$$\langle r_1{}^2\rangle^V=\frac{6}{2.34}\left[\psi\left(\frac{2\beta-1}{4}\right)-\psi\left(\frac{1}{4}\right)\right]\ \mathrm{GeV}^{-2}, \tag{8}$$

where $\psi(\alpha)=\Gamma'(\alpha)/\Gamma(\alpha)$. When we replace the previously determined value of the parameter β, the numerical value reads, from Eq. (8),

$$\langle r^2\rangle_1{}^V=0.49\times10^{-23}\ \mathrm{cm}^2,$$

while the experimental value[9,10] is

$$\langle r_1{}^2\rangle_{\mathrm{expt}}{}^V=0.52\times10^{-23}\ \mathrm{cm}^2.$$

The coupling constants of the 1^- resonances to the nucleon can be discussed once we have fitted the form factors. The residue of the ρ pole will give us the ρNN coupling constant. The form factor containing a string of poles is

$$F_1{}^V(t)=g_{\rho NN}F_\rho/(M_\rho{}^2-t)+g_{\rho_1NN}F_{\rho_1}/(M_{\rho_1}{}^2-t)+\cdots, \tag{9}$$

where $g_{\rho NN}$, g_{ρ_1NN}, ..., are the coupling constants for the vector part of ρNN and ρ_1NN interactions (ρ_1 is the next vector meson), and the constants F_ρ, F_{ρ_1}, ..., may be taken from the measurements[11] of the coupling constant between the vector mesons and the photon. If we assume for F_ρ the value given by these analyses,[11] namely, $F_\rho=0.120$ GeV^2, then the residue of Eq. (5) at the ρ pole determines the ρNN coupling constant as

$$g_{\rho NN}{}^2/4\pi=2.84,$$

which can be compared to the value of Ref. 12,

$$g_{\rho NN}{}^2/4\pi=2.86.$$

The second residue can determine the nucleon coupling constant to the next vector meson if a value for F_{ρ_1} is given. As an estimate, we can take the value given in Ref. 12 for a second vector meson in the region of interest. We find

$$g_{\rho_1NN}/4\pi=2.57,$$

which is near the ρNN coupling constant.

We have expressed the Sachs form factors in terms of $F_1{}^V(t)$ by making use of the scaling laws. Therefore, we can now compare them with the recent data.[13] In Fig. 2

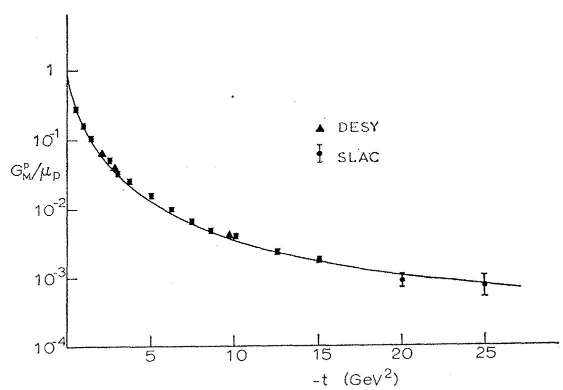

FIG. 2. Plot of the proton magnetic-moment form factor with a zero neutron electric form factor. A nonzero neutron electric form factor with a parametrization of Eq. (10) removes the small discrepancy. The experimental points are from Ref. 13.

[7] E. B. Hughes *et al.*, Phys. Rev. **139**, B458 (1965).
[8] T. Janssens *et al.*, Phys. Rev. **142**, 922 (1966).
[9] M. A. B. Bég *et al.*, Phys. Rev. **173**, 1523 (1968).
[10] R. Fukuda, Progr. Theoret. Phys. (Kyoto) **42**, 332 (1969).
[11] J. E. Augustin *et al.*, Phys. Letters **28B**, 503 (1969); G. McClellan *et al.*, Phys. Rev. Letters **22**, 374 (1969); F. Budos *et al.*, *ibid.* **22**, 490 (1969); S. C. C. Ting, in Ref. 5, p. 43.
[12] J. G. Cordes and P. J. O'Donnell, Phys. Rev. **185**, 1858 (1969); J. G. Cordes and P. J. O'Donnell, Phys. Rev. Letters **20**, 1462 (1968).
[13] P. Lehmann *et al.*, Nuovo Cimento **28**, 18 (1962); D. H. Coward *et al.*, Phys. Rev. Letters **20**, 292 (1968); W. Albrecht *et al.*, *ibid.* **18**, 1014 (1967); M. Goitein *et al.*, *ibid.* **18**, 1016 (1967); C. Berger *et al.*, Phys. Letters **28B**, 276 (1968).

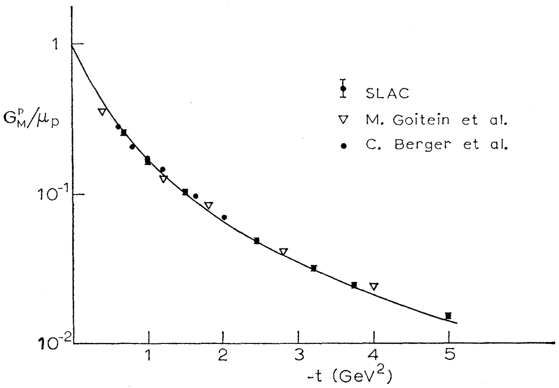

FIG. 4. Effect of nonzero $G_E^n(t)$.

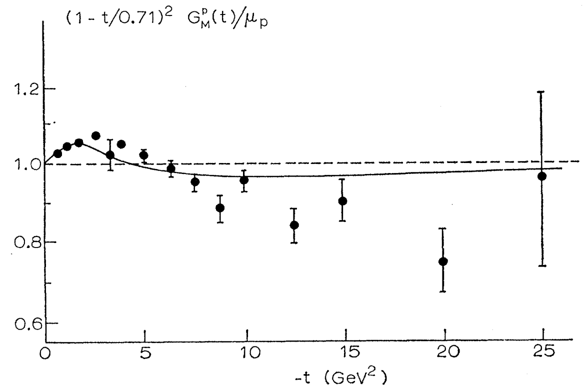

FIG. 5. $G_M^p(t)/\mu_p$ plotted relative to the empirical dipole fit. The experimental points are from Coward *et al.* (Ref. 13).

we plot $G_M^p(t)/\mu_p$ as a function of t, where $G_E^n(t)$ is taken to be zero. As expected, for small values of $|t|$ there is a small discrepancy between data points and the curve of Eq. (4). This is due to the fact that $G_E^n(t)$ is not zero for small $|t|$. To show the effect of a nonzero $G_E^n(t)$ in the calculation of $G_M^p(t)$, we will assume a functional form for the neutron electric form factor which is discussed in Ref. 12, namely,

$$G_E^n(t) = \frac{At}{1-t/4M^2} F_1^V(t)\,, \tag{10}$$

where the constant A is determined from the known slope of $G_E^n(t)$ at the point $t=0$. Figure 3 shows the corresponding neutron electric form factor plotted as a function of t. Once we have a functional form for the $G_E^n(t)$ term in formula (4), we can use it to remove the discrepancy in Fig. 2. In Fig. 4 we have shown $G_M^p(t)/\mu_p$ for small $|t|$ with a nonzero neutron electric form factor. Since a plot of $G_M^p(t)/\mu_p$ relative to the empirical dipole fit $G_M^p(t)/\mu_p = (1-t/0.71)^{-2}$ will clearly show deviation of the theoretical curve from the data, such a plot of $G_M^p(t)/\mu_p$ is given in Fig. 5, where the $G_E^n(t)$ term is also included. We also note that if we calculate a mean-square radius from the expression of $G_M^p(t)$ and take the slope of $G_E^n(t)$ from experiment, the measured value of $\langle r^2 \rangle$ is obtained. It may finally be remarked that the choice $\beta=5$ which would give exact dipole behavior asymptotically gives a slightly less good fit for low t, but is not yet excluded by experiment.

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Physical-Region Constraints on Low-Energy Partial-Wave Amplitudes*

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On the basis of analyticity, crossing, and positivity of the imaginary parts of the partial-wave amplitudes, we derive inequalities on integrals involving the low partial waves of elastic $\pi^0\pi^0$ scattering in the physical region. The integrals are sensitive only to the low-energy region, and can therefore be tested once a phase-shift analysis is given. The relations can be used to discriminate between various proposed $\pi^0\pi^0$ phase shifts.

I. INTRODUCTION

ANALYTICITY, crossing, and unitarity have long been considered essential ingredients of strong-interaction physics, and much effort has been devoted to elucidating their consequences. Apart from the implications of unitarity for individual partial-wave amplitudes, most tests of these general principles (such as the verification of dispersion relations, or the Froissart bound) have involved the full amplitudes, and not merely a few partial waves. Recently, however, many different results on the partial-wave amplitudes of $\pi\pi$ scattering below threshold have been obtained. In particular, Common[1] and Yndurain[2] have found the implications of the positivity of the absorptive parts for

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[1] A. K. Common, Nuovo Cimento **63A**, 863 (1969).
[2] F. J. Yndurain, Nuovo Cimento **64A**, 225 (1969).