Magnetic Moments of the Charged Octet Baryons from Lagrangian Field Theory Using the Padé Approximation

L. B. RÉDEI
Institute of Physics, Department of Theoretical Physics, Umeå University, Umeå, Sweden

(Received 2 December 1969)

The magnetic moments of the charged octet baryons are calculated in the lowest-order nonvanishing Padé approximation assuming pseudoscalar Yukawa coupling between the octet baryons and pseudoscalar octet mesons. The experimental value is used for the pion-nucleon coupling constant. For the other coupling constants, the SU_3 values are assumed. It is found that $\alpha=0.725$ (α being the F/D mixing parameter, $\alpha_{\text{expt}}=0.75\pm0.03$) reproduces the experimental value of the proton magnetic moment. Using this value of α, one obtains $\mu_{\Sigma^+}=1.48$, $\mu_{\Sigma^-}=-0.31$, and $\mu_{\Xi^-}=-0.22$ for the total magnetic moments in proton magnetons.

I. INTRODUCTION

THE purpose of this paper is to report the results obtained from Lagrangian field theory, using the lowest-order Padé approximation, for the magnetic moment of the proton and the other charged octet baryons Σ^+, Σ^-, and Ξ^-.

The extensive existing literature on the magnetic moment of the baryons can be divided into two classes. Using symmetry arguments, one can obtain approximate relations between the octet baryon magnetic moments. Assuming, e.g., perfect SU_3 symmetry, one can relate all baryon magnetic moments to those of the proton and the neutron.[1] In the quark model, it is possible to relate the octet baryon magnetic moments to those of the proton if one assumes that the baryon magnetic moments are built up additively out of the quark moments.[2] On the dynamical side, theoretical work, to the author's knowledge, has been confined to the proton and the neutron. Lagrangian Yukawa coupling combined with straightforward perturbation expansion fails very badly.[3] The static model gives good results for the isovector part of the magnetic moment, but it goes wrong for the isoscalar component, apart from containing an uv-cutoff parameter.[4] More recent attempts to calculate the nucleon form factors and magnetic moments have been based on the dispersion-theoretical methods initiated by Chew *et al.*[5] in conjunction with the idea of vector-meson dominance.[6] For a critical survey and further references, see, e.g., Refs. 7 and 8. In spite of the successes of the dispersion-theoretical approach—in particular, the prediction of the existence of the vector-meson resonances—at its present stage of development it does not seem sufficiently quantitative, and it is therefore desirable to test the Padé-approximation method in Lagrangian field theory as a possible alternative approach.

The Padé approximation[9] has in the last two years been applied to calculating the S matrix in field theory to obtain phase shifts, Regge trajectories, resonances, etc., in strong-interaction physics.[10] In particular, the pion-pion,[11] pion-nucleon,[12] and nucleon-nucleon[13] systems have been studied.

For a recent review on the results obtained by the Padé approximation in strong interactions see Ref. 14.

In this paper, as a first step towards calculating the nucleon electromagnetic form factors, we shall discuss the magnetic moment of the proton and the other charged octet baryons in the lowest-order Padé approximation. The following assumptions are made. The strong interactions are given by the Yukawa-coupling Lagrangian between the octet baryons and the pseudoscalar octet mesons, the coupling constants being given by their SU_3 values.[15] We use the physical masses throughout, and SU_3 invariance is therefore broken. In addition, one could have a direct meson-meson coupling term. However, to the lowest-order Padé approximation we are considering here, such a term would not contribute. For the interaction of the baryons with an external electromagnetic field, we assume mini-

[1] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).
[2] W. Thirring, Acta Phys. Austriaca, No. 12 (1966). Reprinted in J.J.J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969). The same result is obtained from assuming perfect SU_6 symmetry. See M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).
[3] B. Fried, Phys. Rev. **88**, 1142 (1952).
[4] H. Miyazawa, Phys. Rev. **101**, 1564 (1955).
[5] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. **110**, 1265 (1958); also, P. Federbush, M. L. Goldberger, and S. B. Treiman, *ibid.* **112**, 642 (1958).
[6] Y. Nambu, Phys. Rev. **106**, 1366 (1957); W. R. Frazer and J. R. Fulco, *ibid.* **117**, 1609 (1960); M. Gell-Mann and F. Zachariasen, *ibid.* **124**, 953 (1964).
[7] See, e.g., S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966).
[8] E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. **139**, B458 (1965)
[9] For a general review of the use of the Padé approximation in physics see, e.g., G. A. Baker, in *Advances in Theoretical Physics* (Academic, New York, 1965), Vol. 1.
[10] For a survey of the mathematical properties of the Padé approximation in strong-interaction physics see the Appendix in J. L. Basdevant, D. Bessis, and J. Zinn Justin, Nuovo Cimento **60A**, 185 (1969).
[11] D. Bessis and M. Pusterla, Nuovo Cimento **54A**, 243 (1968); J. L. Basdevant, D. Bessis, and J. Zinn Justin, *ibid.* **60A**, 185 (1969); J. L. Basdevant and B. W. Lee, Saclay report, 1969 (unpublished).
[12] J. A. Mignaco, M. Pusterla, and E. Remiddi, Nuovo Cimento **64A**, 733 (1969).
[13] D. Bessis, S. Graffi, V. Grecchi, and G. Turchetti, Phys. Letters **28B**, 567 (1969).
[14] D. Bessis, Review Talk presented at the Seminar on the Analytic Properties of Scattering Amplitudes, Serpukhow, 1969 (unpublished). M. Jacob, Review Talk presented at the Lund Conference, Sweden, 1969 (unpublished).
[15] See, e.g., S. G. Gasiorowicz and S. L. Glashow, in *Advances in Theoretical Physics* (Academic, New York, 1968), Vol II.

mal coupling between the baryon and meson currents and the external field. We use the experimental value $g^2/4\pi = 14.6$ for the pion-nucleon coupling constant. Because of the assumed SU_3 constraints on the other coupling constants, there is only one parameter in the Lagrangian, the F/D mixing parameter[15] α. The latest experimental value obtained from the phenomenological analyses of the strong decays of the $\frac{3}{2}$ baryon resonances is reported to be[16] $\alpha=0.75\pm0.03$. The range of α from other determinations[15] seems to lie between[16a] $0.67<\alpha<0.75$. Using the Lagrangian described above, we have calculated the lowest-order nonvanishing $S[1, 1]$ Padé approximant to the S matrix for the scattering of a charged baryon in a static magnetic field. This required computing all Feynman diagrams to lowest order in the electromagnetic interaction and to order g^2 in the pion-nucleon coupling constant. The calculation of the magnetic moment of the neutral baryons would require going to order g^4 and this was not attempted here. [With one exception, the magnetic moment of Σ^0, because of the isospin properties of the electromagnetic current, can be expressed[17] as $\mu_{\Sigma^0}=\frac{1}{2}(\mu_{\Sigma^+}+\mu_{\Sigma^-})$.]

The following results are obtained. The total magnetic moment of the proton μ_p is sensitive to small variations in the value of α. For the range $0.67\leq\alpha\leq 0.75$, μ_p varies from 4.01 to 2.32 (in proton magnetons). The experimental value $\mu_p=2.79$ is reproduced by choosing $\alpha=0.725$, in excellent agreement with the experimental value $\alpha=0.75\pm0.03$.[16] Using this value, $\alpha=0.725$, we obtained $\mu_{\Sigma^+}=1.5(e/2M_p)$, in qualitative agreement with the reported experimental value[18] $\mu_{\Sigma^+}=2.5\pm0.5$. The calculated μ_{Σ^+} is somewhat less sensitive to α; it ranges between $1.5\leq\mu_{\Sigma^+}\leq2.15$ for $0.725\geq\alpha\geq0.67$. As for the other magnetic moments, we obtained $\mu_{\Sigma^-}=-0.3(e/2M_p)$ and $\mu_{\Xi^-}=-0.2(e/2M_p)$, both of these being insensitive to small changes in α. There is as yet no experimental information on these.

The paper is organized in the following way. We shall first very briefly sum up the relevant properties of the Padé approximation. Afterwards we discuss the Lagrangian. This is followed by an account of the calculation of the proton magnetic moment and a summary of the results for the other charged baryons. The last section contains a discussion of the results and an outline of possible extensions of this work.

II. PADÉ APPROXIMATION IN FIELD THEORY

We consider the scattering amplitude for an initial state $|a\rangle$ and final state $|b\rangle$. Let the S-matrix element be given by the formal renormalized perturbation series

$$\langle b | S | a\rangle = \langle b | T \exp\left(i\int \mathfrak{L}_{\text{int}}(x)\, d^4x\right) | a\rangle = \langle b | a\rangle + \sum_{n=1}^{\infty} g^n \langle b | S_n | a\rangle, \tag{1}$$

in terms of the renormalized coupling constant g. (The matrix elements $\langle b | S_n | a\rangle$ are given by renormalized Feynman diagrams.) The matrix elements of the $[N, M]$ left Padé approximant $S[N, M]_L$ to the scattering operator S are defined by

$$\langle b | S[N, M]_L | a\rangle = \langle b | Q_L^{(N)} (P_L^{(M)})^{-1} | a\rangle,$$

where $Q_L^{(N)}$ and $P_L^{(M)}$ are polynomials of degree N and M in g, to be determined in terms of the operator coefficients S_n by the conditions that

$$S[N, M]_L - S = g^{N+M+1}\Delta,$$

where Δ is a polynomial in g. Since, in general, the S_n are noncommuting operators, one can in an analogous fashion define the corresponding right Padé approximant $S[N, M]_R$. In this paper we use only diagonal $(N=M)$ Padé approximants and for these there are two important theorems.[10] The left diagonal Padé equals the right diagonal Padé, i.e.,

$$S[N, N]_L = S[N, N]_R = S[N, N].$$

For unitary S, the diagonal Padé approximant is also unitary, i.e.,

$$(S[N, N])^{-1} = (S[N, N])^{\dagger}.$$

III. INTERACTION LAGRANGIAN

We write the interaction Lagrangian density for the scattering of an octet baryon in an external electromagnetic field as

$$\mathfrak{L}_{\text{int}} = \mathfrak{L}_{\text{strong}} + \mathfrak{L}_{\text{E.M.}}, \tag{2}$$

where $\mathfrak{L}_{\text{strong}}$ describes the strong interaction between the baryons and mesons, and where $\mathfrak{L}_{\text{E.M.}}$ describes the interaction with the external field. (We are calculating to lowest order in the fine-structure constant and therefore can neglect the influence of the radiation field). For $\mathfrak{L}_{\text{strong}}$, we assume pseudoscalar Yukawa coupling between the octet baryons and the pseudoscalar octet mesons, i.e.,

$$\begin{aligned}\mathfrak{L}_{\text{strong}} = {}& g\bar{N}\gamma_5\boldsymbol{\tau}N\cdot\boldsymbol{\pi} + g_{\Xi\Xi\pi}\bar{\Xi}\gamma_5\boldsymbol{\tau}\Xi\cdot\boldsymbol{\pi} \\ &+ g_{N\Sigma K}(\bar{N}\gamma_5\boldsymbol{\tau}K\cdot\boldsymbol{\Sigma}+\text{H.c.}) + g_{\Xi\Sigma K}(\bar{\Xi}\gamma_5\boldsymbol{\tau}K_c\cdot\boldsymbol{\Sigma}+\text{H.c.}) \\ &+ g_{NN\eta}\bar{N}\gamma_5N\eta^0 + g_{\Xi\Xi\eta}\bar{\Xi}\gamma_5\Xi\eta^0 \\ &+ g_{N\Lambda K}(\bar{N}\gamma_5K\Lambda+\text{H.c.}) + g_{\Xi\Lambda K}(\bar{\Xi}\gamma_5K_c\Lambda+\text{H.c.}) \\ &+ g_{\Sigma\Sigma\pi}(\bar{\boldsymbol{\Sigma}}\gamma_5\times\boldsymbol{\Sigma}\cdot\boldsymbol{\pi}+\text{H.c.}) + g_{\Sigma\Lambda\pi}(\bar{\boldsymbol{\Sigma}}\gamma_5\Lambda\cdot\boldsymbol{\pi}+\text{H.c.}) \\ &+ g_{\Sigma\Sigma\eta}\bar{\boldsymbol{\Sigma}}\gamma_5\cdot\boldsymbol{\Sigma}\eta^0 + g_{\Lambda\Lambda\eta}\Lambda\gamma_5\Lambda\eta^0,\end{aligned} \tag{3}$$

[16] H. Pilkuhn and A. Swoboda, Lund Conference, Sweden, (unpublished).

[16a] The perfect-SU_6 prediction $\alpha=0.60$ is somewhat outside this range. See F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964).

[17] R. Marshak, S. Okubo, and G. Sudarshan, Phys. Rev. **106**, 599 (1957).

[18] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

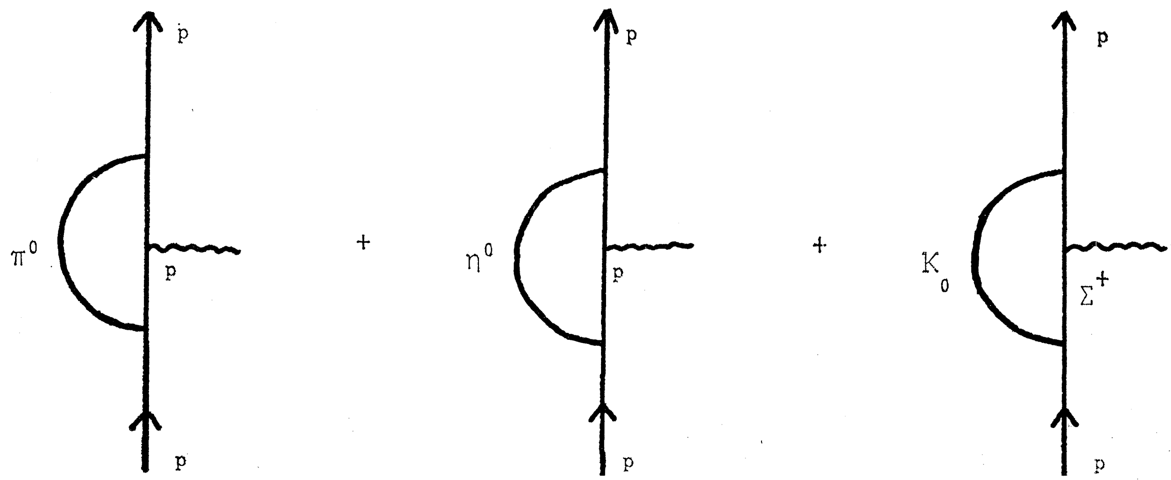

FIG. 1. Feynman diagrams contributing to $\langle \mathbf{p}' \mid S_2{}^{\mathrm{I}} \mid \mathbf{p} \rangle$.

in obvious notation.[15] In addition, there could be a direct meson-meson interaction term. As mentioned in the Introduction, however, this would not contribute to the lowest-order Padé approximation, and therefore we need not worry about it.

The phenomenological analyses of the baryon resonance decays indicate[16] that the coupling constants in Eq. (3) are given by their SU_3 values and, accordingly, we assume that

$$\begin{aligned} &g_{\Xi\Xi\pi} = -g_{N\Sigma K} = (2\alpha-1)g, \\ &g_{\Xi\Sigma K} = g, \\ &g_{NN\eta} = g_{\Xi\Lambda K} = (4\alpha-3)g/\sqrt{3}, \\ &g_{\Xi\Xi\eta} = g_{N\Lambda K} = (3-2\alpha)g/\sqrt{3}, \\ &g_{\Lambda\Lambda\eta} = -g_{\Sigma\Lambda\pi} = -g_{\Sigma\Sigma\eta} = 2\alpha g/\sqrt{3}, \\ &g_{\Sigma\Sigma\pi} = 2i(1-\alpha)g, \end{aligned} \tag{4}$$

where g is the pion-nucleon coupling constant, $g^2/4\pi = 14.6$, and α is the F/D mixing parameter.[15] We use the physical values for the particle masses and SU_3 invariance is therefore broken.

For the electromagnetic interaction, we assume minimal coupling[7] between the external electromagnetic field $eA_{\mathrm{ext}}{}^{\mu}$ and the baryon and meson fields, i.e.,

$$\mathcal{L}_{\mathrm{E.M.}} = eJ_{\mu}{}^{B}A_{\mathrm{ext}}{}^{\mu} + eJ_{\mu}{}^{M}A_{\mathrm{ext}}{}^{\mu}, \tag{5}$$

where J^B and J^M are the baryon and meson currents.

IV. MAGNETIC MOMENT OF PROTON

We consider elastic low-energy proton scattering in a static external magnetic field given by the vector potential $e\mathbf{A}^{\mathrm{ext}}(\mathbf{x})$. To lowest order in the fine-structure

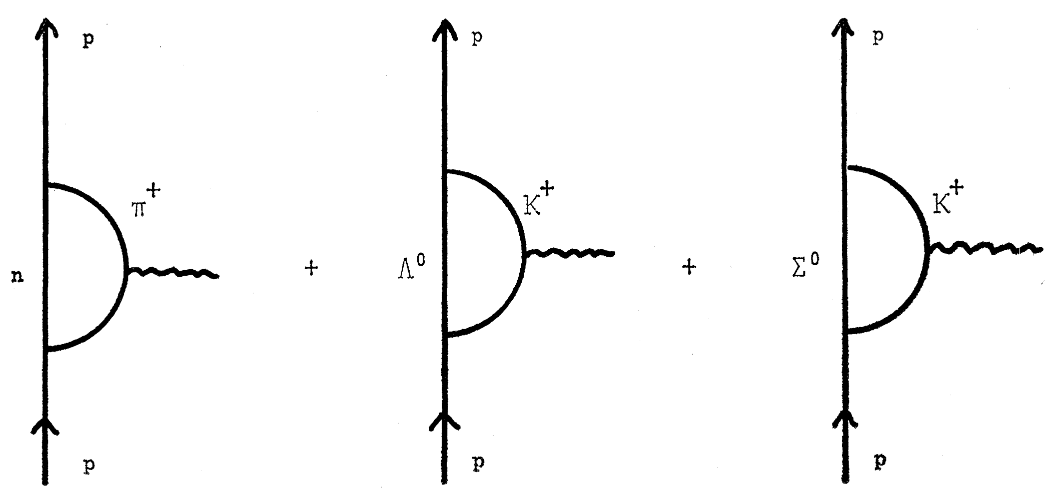

FIG. 2. Feynman diagrams contributing to $\langle p' \mid S_2{}^{\mathrm{II}} \mid \mathbf{p} \rangle$.

TABLE I. The proton magnetic moment (in proton magnetons) as a function of α.

α	μ_p [1, 1] Padé	μ_p perturbation theory
0.67	4.02	1.75
0.70	3.28	1.69
0.71	3.06	1.67
0.725	2.76	1.64
0.75	2.32	1.57

constant e^2, the scattering amplitude $\langle \mathbf{P}' \mid S \mid \mathbf{P} \rangle$, where $\mathbf{P}$ is the incoming and $\mathbf{P}'$ the outgoing proton momentum, is given by an infinite power series in the squared pion-nucleon coupling constant g^2:

$$\langle \mathbf{P}' \mid S \mid \mathbf{P} \rangle = \langle \mathbf{P}' \mid \mathbf{P} \rangle + e^2 \times (\langle \mathbf{P}' \mid S_B \mid P \rangle + g^2 \langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle + \cdots), \quad (6)$$

where $\langle \mathbf{P}' \mid S_B \mid \mathbf{P} \rangle$ is the Born term. Formally putting $e^2 = g^2\gamma$, $\gamma = e^2/g^2$, we can rewrite this as

$$\langle \mathbf{P}' \mid S \mid \mathbf{P} \rangle = \langle \mathbf{P}' \mid \mathbf{P} \rangle + \langle \mathbf{P}' \mid (g^2\gamma S_B + g^4\gamma S_2 + \cdots) \mid \mathbf{P} \rangle. \quad (7)$$

The lowest-order nonvanishing Padé approximant in g^2 is the [1,1] diagonal Padé, and it is given by (Sec. II)

$$\langle \mathbf{P}' \mid S[1, 1] \mid \mathbf{P} \rangle = \langle \mathbf{P}' \mid \mathbf{P} \rangle + e^2 \langle \mathbf{P}' \mid S_B (1 + g^2 S_B^{-1} S_2)^{-1} \mid \mathbf{P} \rangle. \quad (8)$$

The evaluation of the lowest Padé approximant requires, apart from the Born term, the lowest-order strong-interaction correction $\langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle$. In addition, one has to calculate the operator inverse $(1 + g^2 S_B^{-1} S_2)^{-1}$. We shall first discuss the evaluation of the matrix elements $\langle \mathbf{P}' \mid S_B \mid \mathbf{P} \rangle$ and $\langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle$. Since we are concerned here with the magnetic moment, i.e., the magnetic form factor at zero momentum transfer, it is sufficient to calculate these matrix elements to lowest nonvanishing order in the proton momentum. To this order, the Born term is given by[7]

$$\langle \mathbf{P}' \mid S_B \mid \mathbf{P} \rangle = i2\pi\delta(E' - E)[(1/2M_p)(\mathbf{P}' + \mathbf{P}) \cdot \mathbf{A}(0) + (i/2M_p)\xi' \boldsymbol{\sigma} \cdot \mathbf{q} \times \mathbf{A}(0)\xi], \quad (9)$$

where M_p is the proton mass, E the energy, ξ the spin, $\mathbf{q} = \mathbf{P}' - \mathbf{P}$ the momentum transfer, $\boldsymbol{\sigma}$ the Pauli spin matrices, and $\mathbf{A}(\mathbf{q})$ the Fourier transform of $\mathbf{A}^{\text{ext}}(\mathbf{x})$. Turning now to $\langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle$, there are two types of Feynman diagrams contributing to it:

$$\langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle = \langle \mathbf{P}' \mid S_2^{\text{I}} \mid \mathbf{P} \rangle + \langle \mathbf{P}' \mid S_2^{\text{II}} \mid \mathbf{P} \rangle, \quad (10)$$

where $\langle \mathbf{P}' \mid S_2^{\text{I}} \mid \mathbf{P} \rangle$ and $\langle \mathbf{P}' \mid S_2^{\text{II}} \mid \mathbf{P} \rangle$ are each given by the sum of three Feynman diagrams. These are illustrated in Figs. 1 and 2.

Using the Lagrangian given by Eqs. (2), (3), and (5), and the standard methods for computing Feynman diagrams,[7] it is straightforward to compute $\langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle$. To lowest order in the proton momentum, one obtains the following result:

$$\langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle = 2\pi i\delta(E' - E)(\Gamma_p^{\text{I}} + \Gamma_p^{\text{II}}) \times (i/2M_p)\xi' \boldsymbol{\sigma} \cdot \mathbf{q} \times \mathbf{A}(0)\xi, \quad (11)$$

where

$$\Gamma_p^{\text{I}} = -(1/8\pi^2)[L^{\text{I}}(M_p, M_p, m_{\pi^0}) + L^{\text{I}}(M_p, M_p, m_{\eta^0}) + 2L^{\text{I}}(M_p, M_{\Sigma^+}, m_{K^0})],$$

$$\Gamma_p^{\text{II}} = (1/8\pi^2)[2L^{\text{II}}(M_p, M_n, m_{\pi^+}) + L^{\text{II}}(M_p, M_{\Lambda^0}, m_{K^+}) + L^{\text{II}}(M_p, M_{\Sigma^0}, m_{K^+})]. \quad (12)$$

The expressions L^{I} and L^{II} are given by an elementary integral multiplied by the coupling constant squared:

$$L^{\text{I}}(M_A, M_B, m_C) = (g_{ABC})^2 \int_0^1 dy\, y^2 \times \frac{y + (M_B - M_A)/M_A}{y^2 + y[(M_B/M_A)^2 - 1] + (1-y)(m_C/m_A)^2},$$

$$L^{\text{II}}(M_A, M_B, m_C) = (g_{ABC})^2 \int_0^1 dy\, y(1-y) \times \frac{y + (M_B - M_A)/M_A}{y^2 + y[(M_B/M_A)^2 - 1] + (1-y)(m_C/M_A)^2} \quad (13)$$

This settles $\langle \mathbf{P}' \mid S_2 \mid \mathbf{P} \rangle$. It remains to calculate the inverse $(1 - g^2 S_B^{-1} S_2)^{-1}$ on the proton subspace. Since both S_B and S_2 are diagonal in the energy, this is straightforward in the representation where energy and angular momentum are diagonal. In fact, it is only the $l=0$ and $l=1$ waves that scatter, and the inversion reduces to diagonalizing an 8×8 matrix. This can be

TABLE II. The calculated magnetic moment of Σ and Ξ^- in units of proton magnetons.

	$\alpha = 0.67$	$\alpha = 0.70$	$\alpha = 0.725$
μ_{Σ^+}	2.15	1.75	1.48
μ_{Σ^0}	0.93	0.72	0.59
μ_{Σ^-}	−0.29	−0.30	−0.31
μ_{Ξ^-}	−0.22	−0.22	−0.22

done in closed form and one gets the simple result

$$\langle \mathbf{P}' | S[1,1] | \mathbf{P} \rangle = \langle \mathbf{P}' | \mathbf{P} \rangle + 2\pi i e^2 \delta(E'-E) \times ((1/2M_p)(\mathbf{P}'+\mathbf{P})\mathbf{A}(0) + \{1/[1-(\Gamma_p^{\mathrm{I}}+\Gamma_p^{\mathrm{II}})]\} \times (i/2M_p)\xi' \boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{A}(0)\xi). \quad (14)$$

Comparing this with the Born term in Eq. (9), one obtains the final result in units $c=\hbar=1$ for the proton magnetic moment

$$\mu_p = \{1/[1-(\Gamma_p^{\mathrm{I}}+\Gamma_p^{\mathrm{II}})]\}(e/2M_p), \quad (15)$$

where Γ_p^{I} and Γ_p^{II} are given by Eqs. (12) and (13). Using the experimental value $g^2/4\pi = 14.6$ and the SU_3 values as given by Eq. (4) for the other coupling constants, we have thus obtained μ_p as function of the F/D mixing parameter α. We determined α by requiring Eq. (15) to reproduce the experimental value $\mu_p = 2.79(e/2M_p)$ and we obtained $\alpha = 0.725$, in excellent agreement with the value one obtains from the phenomenological analyses of the baryon resonance decays,[16] $\alpha = 0.75 \pm 0.3$. The calculated value of μ_p is, however, sensitive to small changes in the value of α and we have therefore calculated μ_p for a few points in the interval $0.67 \leq \alpha \leq 0.75$, which seems to be the generally accepted range[15] for α. The result is given in Table I, where for comparison we have also listed the perturbation-theoretical values.

Thus, even if the calculated μ_p is sensitive to small changes in α, for the range $0.70 \leq \alpha \leq 0.75$, the lowest-order Padé-approximant result is in fair agreement with the experimental proton magnetic moment, the discrepancy being less than 20%, a very substantial improvement upon the perturbation-theoretical result.

We mention briefly that though the pion contribution to the magnetic moment is the most important one, according to our calculations, the contribution from the other heavier mesons is not negligible. The pion contribution alone gives $\mu_p = 2.04$.

In Sec. V we give the results for the remaining charged octet baryons.

V. MAGNETIC MOMENTS OF Σ^+, Σ^-, Ξ^-, AND Σ^0

The calculation of the magnetic moments of the other charged octet baryons in the lowest-order [1, 1] Padé approximation proceeds in the same way as for the proton. One obtains, in units of the proton magneton,

$$\mu_{\Sigma^+} = (M_p/M_{\Sigma^+})\{1-(1/8\pi^2)[L^{\mathrm{II}}(M_{\Sigma^+}, M_{\Sigma^0}, m_{\pi^+}) + L^{\mathrm{II}}(M_{\Sigma^+}, M_{\Lambda^0}, m_{\pi^+}) + 2L^{\mathrm{II}}(M_{\Sigma^+}, M_{\Xi^0}, m_{K^+}) - L^{\mathrm{I}}(M_{\Sigma^+}, M_{\Sigma^+}, m_{\pi^0}) - L^{\mathrm{I}}(M_{\Sigma^+}, M_{\Sigma^+}, m_{\eta^0}) - 2L^{\mathrm{I}}(M_{\Sigma^+}, M_p, m_{K^0})]\}^{-1},$$

$$\mu_{\Sigma^-} = -(M_p/M_{\Sigma^-})\{1-(1/8\pi^2)[L^{\mathrm{II}}(M_{\Sigma^-}, M_{\Sigma^0}, m_{\pi^-}) + L^{\mathrm{II}}(M_{\Sigma^-}, M_{\Lambda^0}, m_{\pi^-}) + 2L^{\mathrm{II}}(M_{\Sigma^-}, M_n, m_{K^-}) - L^{\mathrm{I}}(M_{\Sigma^-}, M_{\Sigma^-}, m_{\pi^0}) - L^{\mathrm{I}}(M_{\Sigma^-}, M_{\Sigma^-}, m_{\eta^0}) - 2L^{\mathrm{I}}(M_{\Sigma^-}, M_{\Xi^-}, m_{K^0})]\}^{-1}, \quad (16)$$

$$\mu_{\Xi^-} = -(M_p/M_{\Xi^-})\{1-(1/8\pi^2)[2L^{\mathrm{II}}(M_{\Xi^-}, M_{\Xi^0}, m_{\pi^-}) + L^{\mathrm{II}}(M_{\Xi^-}, M_{\Lambda^0}, m_{K^-}) + L^{\mathrm{II}}(M_{\Xi^-}, M_{\Sigma^0}, m_{K^-}) - L^{\mathrm{I}}(M_{\Xi^-}, M_{\Xi^-}, m_{\pi^0}) - L^{\mathrm{I}}(M_{\Xi^-}, M_{\Xi^-}, m_{\eta^0}) - 2L^{\mathrm{I}}(M_{\Xi^-}, M_{\Sigma^-}, m_{K^0})]\}^{-1},$$

where the quantities L^{I} and L^{II} are still given by Eq. (13).

From the isospin properties of the currents it follows then that[17]

$$\mu_{\Sigma^0} = \tfrac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-}). \quad (17)$$

The magnetic moment of Σ^+ is the only one of these for which experimental information is available. The latest experimental value[18] is quoted to be $\mu_{\Sigma^+} = (2.5 \pm 0.5)(e/2M_p)$. Using $\alpha = 0.725$, we obtained $\mu_{\Sigma^+} = 1.5(e/2M_p)$. The μ_{Σ^+} is also sensitive to changes in the value of α but less so than μ_p. The numerical results are given in Table II for three typical values of α in the range $0.67 \leq \alpha \leq 0.725$.

The magnetic moments of Σ^- and Ξ^- are practically independent of α in the range considered.

VI. CONCLUSIONS

The numerical results of the [1, 1] Padé approximation with $\alpha = 0.725$ are compiled in Table III. For comparison, we have given the experimental values[18] and the perfect-SU_3 and quark-model predictions.[1,2]

The [1, 1] Padé approximation gives systematically lower values for the magnetic moments than the SU_3 and quark-model predictions. The Padé and SU_3 values are, however, closer than they look from Table III since our results are given in proton magnetons whereas the SU_3 ones presumably refer to some octet average magneton. This could reduce the SU_3 values[19] by as much as 20 to 25%. It is really quite surprising that there is this qualitative agreement between the SU_3 and the [1, 1] Padé results since SU_3 is very badly broken in the Lagrangian we have been using. This can be seen by,

TABLE III. The calculated magnetic moments for $\alpha = 0.725$ ($\alpha_{\mathrm{expt}} = 0.75 \pm 0.03$) in units of proton magnetons.

	[1, 1] Padé	SU_3	Quark model	Experimental
μ_p	2.76			2.79
μ_{Σ^+}	1.48	2.79	2.79	2.5±0.5
μ_{Σ^0}	0.59	0.95	0.93	
μ_{Σ^-}	−0.31	−0.88	−0.93	
μ_{Ξ^-}	−0.21	−0.88	−0.93	

[19] M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965). Also, nonstatic corrections in the quark model seem to reduce the absolute value of the predictions for the baryon magnetic moments. See J. Franklin, Phys. Rev. **182**, 1607 (1969).

e.g., looking at the perturbation-theory results. There is a marked difference though between the [1, 1] Padé and the SU_3 values for μ_{Σ^-} and μ_{Ξ^-}. Since these are exactly the magnetic moments which were found to be insensitive to small changes in α, these can be looked upon as definite predictions of our work.

Our results are inconclusive for two reasons. One is that too little is known experimentally about the magnetic moments we have been able to calculate with the [1, 1] Padé approximant. Also, the reliability of the lowest-order Padé approximant has first to be checked. For both of these reasons it would be very informative as to the usefulness of our approach to calculate the results of the [1, 2] and [2, 1] Padé approximation. This would, apart from testing the practical convergence of the method, also give the magnetic moments of the neutral-octet baryons. The magnetic moments of the neutron and Λ^0 being known experimentally, one would then have a severe check on the theory.

Though our results are not conclusive, it seems that they are sufficiently promising to call for further work along these lines. On the positive side, we have the excellent agreement between the value $\alpha=0.725$ required to reproduce the proton magnetic moment and the experimental value $\alpha=0.75$. On the other hand, the result for Σ^+, $\mu_{\Sigma^+}=1.5$, is too low compared to the present-day experimental value $\mu_{\Sigma^+}=2.5\pm0.5$.

Extension of this work to calculate the electromagnetic form factors of the proton is in progress.

The calculation of the magnetic moments in the next-order [1, 2] and [2, 1] Padé approximation might also be undertaken in the future.

PHYSICAL REVIEW D VOLUME 2, NUMBER 1 1 JULY 1970

Linearly Rising Regge Trajectories and the Isovector Form Factor $F_1{}^V(t)$*

HÜSEYIN AKCAY†
Physics Department, Yale University, New Haven, Connecticut 06520
(Received 12 March 1970)

A simple ansatz for the isovector Dirac form factor is examined. A fit to an approximate dipole form is achieved, and deviations from this form are also accounted for. A ρ' is no longer necessary; the next vector meson used in the saturation has a mass of approximately 1.7 GeV. The fit also gives a ρNN coupling constant which has the right value, as well as providing a good value for the necleon charge radius.

I. INTRODUCTION

ONE of the major problems in electromagnetic interactions in recent years has been to find a theoretical explanation for the experimental features of the pion and nucleon electromagnetic form factors. It is difficult to understand the t dependence of the form factors on the basis of the usual pole-dominance model, which works at low momentum transfers and has approximate validity in the timelike region. Far from the vector-meson pole in the spacelike region, nucleon form factors fall off faster than implied by the pole dominance, and have approximately the dipole form. The asymptotic behavior of the pion form factor is not yet known accurately. The dipole formula, unless accompanied by a pole, is very difficult to understand theoretically. One idea is that the dipole form of the nucleon form factors might be simulated by more complicated functions obtained by saturating the form factors by an infinite number of poles. Away from the pole in the spacelike region, we might try the narrow-resonance approximation for such a string of poles. Physically, this would imply that the virtual photon can change not only into a vector meson like the ρ meson, but also into other vector mesons with higher masses and the same quantum numbers as the ρ meson.

Such a model in which the meson spectrum includes an infinity of vector mesons with the same quantum numbers as the ρ is provided by the Veneziano[1] model, devised to satisfy duality, Regge behavior in all channels, and crossing symmetry. Several authors[2] have used the spectrum provided by the Veneziano model to saturate electromagnetic form factors. To this end they have used an ansatz for the form factors being approximated by the ratio of two gamma functions with a proper normalization constant. Making this ansatz for the Sachs form factors, they obtained the following results: (1) By choosing suitable parameters that appear in the gamma functions, an approximate dipole form can be obtained at high t, although the form factor is only saturated by single poles. (2) The deviation from

* Research (Yale Report No. 2726-567) supported by the U. S. Atomic Energy Commission under Contract No. AT(30-1)2726.
† On leave from the Middle East Technical University, Ankara, Turkey.

[1] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).
[2] P. Di Vecchia and F. Drago, Nuovo Cimento Letters **1**, 917 (1969); R. Lengo and E. Remiddi, *ibid.* **1**, 922 (1969); P. H. Frampton, Phys. Rev. **186**, 1419 (1969); Y. Oyanagi, Progr. Theoret. Phys. (Kyoto) **42**, 898 (1969); Fayyazuddin and Riazuddin, Phys. Letters **28B**, 8 (1969); M. Namiki and I. Ohba, Progr. Theoret. Phys. (Kyoto) **42**, 1166 (1969).