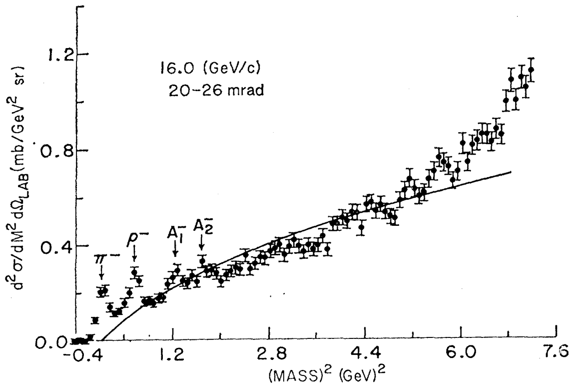

FIG. 3. Missing-mass spectrum for $\pi^- p \to pX^-$. The solid line is the fit from the multi-Regge factorizable model Eq. (16).

trajectory that can be exchanged is the Δ trajectory. (The ω-angle-independent model can be formulated for an exchanged particle with spin by assuming that only one helicity amplitude is dominant.[14]) Consequently, we obtain, for $\pi^- + p \to p + X^-$ in the laboratory frame,

$$\frac{d\sigma}{d\Omega dM^2}=\phi\times 16\pi\Delta\left(\frac{s}{M^2}\text{GeV}^2,m_\pi^2,m_N^2\right)\times\frac{d\sigma^{\pi^- p}}{du}\left(\frac{s}{M^2}\text{GeV}^2,t\right)\frac{A_{\bar{\Delta}p}(M^2)}{g_{N\pi\bar{\Delta}}^2}, \quad (16)$$

$$\phi\simeq|\bar{q}|/8(2\pi)^3 M_N^2|\bar{p}_\pi|,$$

where $A_{\bar{\Delta}p}(M^2)$ is the absorptive part of physical $\bar{\Delta}p$ scattering amplitude and $g_{N\pi\bar{\Delta}}^2$ is the coupling constant.

[14] T. W. B. Kibble, Phys. Rev. 131, 2282 (1963).

This prediction is compared with the experimental result of Anderson *et al.*,[8] and the fit to the experiment is shown in Fig. 3. We have parametrized the data for $d\sigma/du$ for backward[15] $\pi^- p$ scattering with $\alpha_\Delta(t)=0.049+0.76t$, and also used $g_{\pi N\bar{\Delta}}^2=30$ GeV^2. The absorptive part is given by a sum of a Pomeranchon term and an "average" meson term of the form $M^2[\sigma_P+\sigma_M(M^2/1\ \text{GeV}^2)^{\alpha_M(0)-1}]$ [we have used $\alpha_P(0)=1$ and $\alpha_M(0)\simeq 0.7$]. In analogy to the $\bar{p}p$ total cross section, we use $\sigma_P\simeq 45$ mb and $\sigma_M\simeq 120$ mb, although the fit is not sensitive to this particular choice.[16]

We have shown that the multi-Regge integral equation is convenient for describing the missing-mass spectrum in addition to its previous use for the two-body absorptive part (total cross section). The additional observables in the missing-mass spectrum provide a more demanding test, since Regge behavior is required in M^2 as well as in s. This test has so far been met in the experiment analyzed above and in other experiments.[7] Using a simple factorizable model for the double Regge residue, we find that the integral equations can be used as a practical method to calculate the magnitudes of the amplitudes, in reasonable agreement with experiment. Further missing-mass experiments should prove most useful toward constructing and testing more realistic multiperipheral models.

We wish to acknowledge the stimulating remarks and encouragement of Professor M. L. Goldberger. One of us (C.I.T.) wishes to thank the Aspen Center for Physics for their hospitality.

[15] C. C. Shih, Phys. Rev. Letters 22, 105 (1969).

[16] The discrepancy at large M^2 seems to indicate the gradual importance of the "central" diagram of Eq. (10).

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$\gamma\pi \to \pi\pi$ in the Veneziano Model and the Lifetime of the Neutral Pion

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The amplitude for the process $\gamma\pi \to \pi\pi$ given in terms of a five-point generalized Veneziano function is shown to remove an inconsistency in the application of the Veneziano method to the process $\pi+N \to \pi+N+\gamma$ in which the first process is known to dominate. A determination of the relevant residue function consistent with other information leads to a value of the neutral pion lifetime in good agreement with experiment, provided use is made of the idea of effective width of the ρ meson.

THE difficulty of writing down the Veneziano amplitude for the process

$$\gamma(p_1)+\pi_\alpha(p_2) \to \pi_\beta(p_3)+\pi_\gamma(p_4) \quad (1)$$

has been pointed out by several authors.[1,2] The matrix element of the reaction is written in the form

$$M_{fi}=\epsilon_{\alpha\beta\gamma}\epsilon^{\mu\nu\lambda\sigma}\epsilon_\mu p_{2\nu}p_{3\lambda}p_{4\sigma}A(s,t,u), \quad (2)$$

where the Mandelstam variables s, t, and u are connected by the relation $s+t+u=3m_\pi^2$. Since only isoscalar photons can contribute, the process is identical to the one originally considered by Veneziano, i.e., $\omega\pi \to \pi\pi$, provided m_ω is taken to be zero. But in order to eliminate undesirable poles with even angular momenta, Veneziano required in his process the con-

[1] G. S. Iroshnikov, Y. P. Nikitin, and A. S. Chernov, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 10, (1969) [Soviet Phys. JETP Letters 10, 95 (1969)].

[2] I. Raszillier and D. H. Schiller, Academia Republicu Socialiste Romania Report, Institutul De Fizica, Bucharest, 1969 (unpublished).

dition $\alpha_\rho(s)+\alpha_\rho(t)+\alpha_\rho(u)=2$ which, however, becomes in the present case $\alpha_\rho(s)+\alpha_\rho(t)+\alpha_\rho(u)=\frac{3}{2}$, since $\alpha_\rho(m_\pi^2)=\frac{1}{2}$. In order to avoid this difficulty, one may try[1] to use the Virasoro[3] amplitude

$$A(s,t,u)=\frac{\beta\Gamma(\frac{1}{2}-\frac{1}{2}\alpha(s))\Gamma(\frac{1}{2}-\frac{1}{2}\alpha(t))\Gamma(\frac{1}{2}-\frac{1}{2}\alpha(u))}{\Gamma(1-\frac{1}{2}\alpha(s)-\frac{1}{2}\alpha(t))\Gamma(1-\frac{1}{2}\alpha(t)-\frac{1}{2}\alpha(u))\Gamma(1-\frac{1}{2}\alpha(u)-\frac{1}{2}\alpha(s))}, \quad (3)$$

which has the correct asympotic form as $m_\omega\rightarrow 0$.

The difficulty with this form of the amplitude, however, is that it leads to a contradiction[2] when applied to the process $\pi+N\rightarrow\pi+N+\gamma$, which is known to be dominated by the process (1). Blokhintseva *et al.*, have analyzed their experimental differential cross section for the latter process in terms of the dominant pion-exchange diagram, using for this purpose the dispersion-theoretic calculation on the process (1). The amplitude for (1) from ρ dominance in dispersion theory is given by

$$A(\nu)=C\frac{(\frac{9}{16}+\frac{3}{4}\gamma+\nu_R)^2-a(\nu+\frac{9}{16})^2}{[\nu+\gamma(\nu_R+9/8)^{1/2}+\nu_R+9/8](\nu-i\gamma\nu^{1/2}-\nu_R)}, \quad (4)$$

where C and a are two constants, $\nu(s)=\frac{1}{4}(s-4m_\pi^2)$, $\nu_R=\nu(m_\rho^2)$, and $\gamma=0.4$. Blokhintseva *et al.* have shown that their experimental data can be understood only if $C^2=1\pm0.2$.

However, if we normalize the amplitude (4) by equating its residue at the ρ pole to the corresponding residue from the Feynman diagram, we obtain

$$g_{\rho\pi\pi}g_{\gamma\rho\pi}/m_\pi=6.55C(1.09-a). \quad (5)$$

Although the coupling constant $g_{\rho\pi\pi}^2/4\pi$ is known to be 2.5, the other constant $g_{\gamma\rho\pi}^2/4\pi$ is not so well determined. Taking $g_{\gamma\rho\pi}=0.67$ GeV^{-1} corresponding to $\Gamma_{\rho\rightarrow\pi\gamma}\sim600$ keV,[5] we find that $a\simeq1$.

Using now the Virasoro amplitude (3), one can easily obtain by equating the residue at the ρ pole with that of the Feynman diagram the following relationship:

$$-\beta\frac{2(m_\rho^2-m_\pi^2)}{\Gamma(\frac{3}{4})m_\pi^2}=\frac{g_{\rho\pi\pi}g_{\gamma\rho\pi}}{m_\pi}=6.55(1.09-a)C. \quad (6)$$

A second relation connecting β, C, and a can be obtained by going to the point $\nu=0$ or $s=4m_\pi^2$ so that

$$8.618\beta=-C(0.955-0.0055a). \quad (7)$$

These two relations give a value $a=0.31$, in contradiction to what we found before.

We wish to point a way out of this difficulty. Recently, Cooper[6] has employed an elegant spurion technique to write down the Veneziano-type amplitude for the process (1) by considering the generalized Veneziano representation for the process $K\bar{K}\rightarrow3\pi$ given by Bardakci and Ruegg.[7] Essentially, the method consists in replacing the $K\partial_\mu\bar{K}$ system by the isoscalar current S_μ and in assuming an unknown trajectory γ_1 in the spurion channel. The amplitude for the process $S_\mu+\pi\rightarrow\pi+\pi$ is then given by $A(s,t,p_1^2)$+five other cyclic terms in s, t, and u with

$$A(s,t,p_1^2)=C'\int_0^1\int_0^1 du_1du_4\, u_1^{-\alpha_\omega(p_1^2)}(1-u_1)^{-\gamma_1}\times u_4^{-\alpha_\rho(s)}(1-u_4)^{-\alpha_\rho(t)}(1-u_1u_4)^{\alpha_\rho(t)-1}. \quad (8)$$

The amplitude for the process $\omega\pi\rightarrow\pi\pi$ is obtained by taking $p_1^2=m_\omega^2$, and it is identical to Veneziano's original amplitude, whereas the amplitude for the process (1) is obtained if we set $p_1^2=0$. From the expansion

$$A(s,t,p_1^2)=C'\sum_n(-1)^n\times\frac{B(1-\gamma_1,\,1-\alpha_\omega(p_1^2)+n)B(1-\alpha_\rho(s)+n,\,1-\alpha_\rho(t))}{\alpha_\rho(t)B(n+1,\,\alpha_\rho(t)-n)}, \quad (9)$$

it is clear that the amplitude for process (1) contains an infinite number of beta functions.

The advantage of using the amplitude (9) given by Cooper is that it enables us to remove the contradiction mentioned after Eq. (7). With Cooper's amplitude for process (1), Eqs. (6) and (7) will be replaced by

$$2C'B(1-\gamma_1,\,1-\alpha_\omega(0))/\alpha'=6.55C(1.09-a) \quad (10)$$

and

$$2[A(4m_\pi^2,\,-\tfrac{1}{2}m_\pi^2,\,0)+A(-\tfrac{1}{2}m_\pi^2,\,-\tfrac{1}{2}m_\pi^2,\,0)+A(-\tfrac{1}{2}m_\pi^2,\,4m_\pi^2,\,0)]=-C(0.955-0.0052a). \quad (11)$$

It is clear now that because of the presence of the infinite series[8] in (11), the determination of the two parameters C and a is not possible and this may indeed be the origin of the contradiction found by Raszillier and Schiller.

What is required is obviously an accurate determination of the parameters C, γ_1, and $\alpha_\omega(0)$. By going to the ρ pole, one can easily establish the following relationship[9]:

$$B(1-\gamma_1,\,1-\alpha_\omega(0))=\frac{g_{\gamma\rho\pi}}{g_{\omega\rho\pi}}\frac{2\gamma_\omega}{e}, \quad (12)$$

[3] M. A. Virasoro, Phys. Rev. **177**, 2309 (1969).
[4] T. D. Blokhintseva, A. V. Klavtsov, and S. G. Sherman, Yadern. Fiz. 8, (1968) [Soviet J. Nucl. Phys. **8**, 928 (1968)].
[5] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).
[6] F. Cooper, Phys. Rev. D (to be published).
[7] K. Bardakci and G. Ruegg, Phys. Letters **28B**, 342 (1968).

[8] The second and higher terms involve gamma functions of negative arguments when we take $\alpha_\rho(t)$ at $t=-\frac{1}{2}m_\pi^2$, but we can still use the relation $\Gamma(1+z)=z\Gamma(z)$ for the evaluation of the terms [see E. T. Whittaker and G. N. Watson, *A Course on Modern Analysis* (Cambridge U. P., London, 1963), p. 243].
[9] The various vertices were defined as

$\rho_\mu^a\rightarrow\pi^b(k_1)+\pi^c(k_2)$: $\epsilon_{abc}g_{\rho\pi\sigma}(k_1-k_2)_\mu\epsilon^\mu$;
$\rho_\mu^a\rightarrow\pi^b(k_1)+\gamma(k_2)$: $\delta_{ab}g_{\gamma\rho\pi}\epsilon_{\mu\nu\lambda\sigma}\epsilon^\mu(k_1+k_2)\epsilon^\nu(k_2)k_1^\lambda k_2^\sigma$;
$\omega_\mu\rightarrow\pi^a(k_1)\rho_\nu^b(k_2)$: $\delta_{ab}g_{\omega\rho\pi}\epsilon_{\mu\nu\lambda\sigma}\epsilon^\mu(k_1+k_2)\epsilon^\nu(k_2)k_1^\lambda k_2^\sigma$.

where γ_ω is the γ-ω coupling given by $\gamma_\omega^2/4\pi=3.2$ and $e^2=1/137$. From Regge-pole fits, Hite[10] has shown that

$$0.21\leqslant\alpha_\omega(0)\leqslant 0.47\,,$$

but, using the values of the coupling constants given before, we find that the only reasonable solution of (12) is

$$\alpha_\omega(0)\simeq 0.23 \quad\text{and}\quad \gamma_1\simeq 0.$$

Values of $\alpha_\omega(0)>0.3$ seem to be completely ruled out.

The product $C'B(1-\gamma_1,\,1-\alpha_\omega(0))$ is related to the coupling constants by the relation

$$2C'B(1-\gamma_1,\,1-\alpha_\omega(0))=\alpha' g_{\rho\pi\pi}g_{\gamma\rho\pi}\,,$$

and with the coupling constants chosen, it comes out to be

$$C'B(1-\gamma_1,\,1-\alpha_\omega(0))=1.659\ \text{GeV}^{-3}.$$

We will now use this value to calculate the lifetime of the neutral pion.

Let us define the amplitude for $\pi^0(p)\to\gamma(q)+\gamma(k)$ decay by

$$F(q^2)=\epsilon^{\alpha\beta\mu\nu}q_\alpha k_\beta\epsilon_\mu(q)\epsilon_\nu(k)f(q^2)\,,$$

with $p^2=m_\pi^2$ and $k^2=0$. The mean lifetime is then given by

$$\tau=64\pi/f^2(0)-m_\pi^3. \tag{13}$$

From dispersion theory, Wong[11] has shown that

$$f(q^2)=\frac{e}{48\pi^2}\int_{4m_\pi^2}^{\infty}\frac{(t-4m_\pi^2)^{3/2}}{t^{1/2}(t-q^2)}F_\pi^\dagger(t)M_1(t)dt\,, \tag{14}$$

where $F_\pi^\dagger(t)$ is the Hermitian conjugate of the pion form factor $F_\pi(t)$ and $M_1(t)$ is the P-wave amplitude for the process (1). Assuming that this P wave for (1) is dominated by the ρ pole, we have from Cooper

$$M_1(t)=\frac{4C'B(1-\gamma_1,\,1-\alpha_\omega(0))}{\alpha'(m_\rho^2-t-im_\rho\Gamma_{\text{eff}})}\,, \tag{15}$$

where we have used $\text{Im}\alpha_\rho(t)=\alpha' m_\rho\Gamma_{\text{eff}}$ and Γ_{eff} is the effective ρ width to two-pion decay.

This idea of the effective ρ width has been introduced by Gerstein *et al.*[12] in the study of the pion form factor $F_\pi(t)$ in a model of duality and Regge asymptotic behavior. They have obtained an effective form factor $F_\pi(t)$ near $t\sim m_\rho^2$ which we can write in the form

$$F_\pi(t)=[m_\rho\Gamma_{\text{eff}}/6\pi(\alpha' m_\rho\Gamma_\rho)^2](t-m_\rho^2-im_\rho\Gamma_{\text{eff}})^{-1}\,, \tag{16}$$

where $\Gamma_{\text{eff}}=(\sqrt{2}-1)^{1/2}\Gamma_\rho=0.65\Gamma_\rho$. Using (14)–(16), we then find that the lifetime is given by

$$\tau=\frac{\pi^3 m_\rho^2(m_\rho^2-4m_\pi^2)^3(g_{\rho\pi\pi}^2/4\pi)^2}{4e^2m_\pi^3(m_\rho^2-m_\pi^2)^4(g_{\gamma\rho\pi}/g_{\rho\pi\pi})^2}$$
$$\simeq 0.9\times 10^{-16}\ \text{sec}\,, \tag{17}$$

which is of the right order of magnitude of the experimental value[13] $\tau=(0.56\pm0.06)\times10^{-16}$ sec. This close agreement may be regarded as essentially originating from the effective-width idea.

Lastly, we wish to remark that an alternative way to normalize the matrix element (2) is to go to the symmetry point $s=t=u=m_\pi^2$ and write, from symmetry considerations,[2]

$$A(m_\pi^2,m_\pi^2,m_\pi^2)=\Lambda e/m_\pi^3.$$

At the symmetry point we have, from Cooper's amplitude,

$$A(m_\pi^2,m_\pi^2,m_\pi^2)=6C'\sum_n(-1)^n\frac{B(1-\gamma_1,\,1-\alpha_\omega(0)+n)B(1-\alpha_\rho(m_\pi^2)+n,\,1-\alpha_\rho(m_\pi^2))}{\alpha_\rho(m_\pi^2)B(n+1,\,\alpha_\rho(m_\pi^2)-n)}$$
$$=6C'B(1-\gamma_1,\,1-\alpha_\omega(0))B(\tfrac12,\tfrac12)+\text{higher terms}.$$

It is interesting to note that with the value of $C'B$ quoted before, the first term on the right-hand side requires a value of Λ of the order of unity. Although a value of Λ of the order of unity has been used by Wong and Okubo and Sakita[14] and others, "hard"-pion techniques seem to require[15] a value of $\Lambda\simeq 0.03$. As Iroshnikov *et al.* have pointed out, the dynamical mechanism capable of leading to such a large deviation at small s, t, and u from the value that follows from simple dynamical considerations is not clear.

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[10] G. Hite, Rev. Mod. Phys. **41**, 669 (1969).
[11] H. Wong, Phys. Rev. **121**, 289 (1961).
[12] I. S. Gerstein, K. Gottfried, and K. Huang, Phys. Rev. Letters **24**, 294 (1970).
[13] G. Bellettini, C. Bemporad, P. L. Braccini, C. Bradaschia, L. Foa, K. Lubelsmeyer, and D. Schmitz, Nuovo Cimento **66A**, 243 (1970).
[14] H. Wong, Phys. Rev. Letters **5**, 70 (1960); S. Okubo and B. Sakita, *ibid.* **11**, 50 (1963); K. Kawarabayashi and M. Suzuki, *ibid.* **16**, 255 (1960); **16**, 384 (1960).
[15] A. Chatterjee, Phys. Rev. **170**, 1578 (1968).