$\xi_n,\xi_n^*,t_n)$ in terms of the Green's function $K^{(\Omega)}$ and the phase-space distribution function $\Phi^{(\Omega)}$:

$$N=\int\cdots\int d^2\{z_\lambda\}d^2z_0\prod_{\lambda=1}^{n}\exp[\mu\xi_\lambda{}^2+\nu\xi_\lambda{}^{*2}+(\lambda+\tfrac{1}{2})|\xi_\lambda|^2+\xi_\lambda z_\lambda{}^*-\xi_\lambda{}^*z_\lambda]$$
$$\times\prod_{\lambda=2}^{n}K^{(\Omega)}(z_\lambda,z_\lambda{}^*,t_{\lambda-1}|z_{\lambda-1}-2\nu\xi_{\lambda-1}{}^*-(\lambda+\tfrac{1}{2})\xi_{\lambda-1},\,z_{\lambda-1}{}^*+2\mu\xi_{\lambda-1}+(\lambda+\tfrac{1}{2})\xi_{\lambda-1}{}^*,t_{\lambda-1})$$
$$\times K^{(\Omega)}(z_1,z_1{}^*,t_1|z_0,z_0{}^*,t_0)\Phi^{(\Omega)}(z_0,z_0{}^*,t_0).\quad\text{(D17)}$$

The normally ordered time-ordered correlation function $\Gamma_T{}^{(N)}$ may be obtained from (D3) and (D17). We stress that in (D17) Ω is *any* mapping characterized by a filter function of the form given by (D1). With the special choice corresponding to mapping according to the antinormal rule one has $\mu=\nu=0$, $\lambda=-\frac{1}{2}$ (cf. Table IV of I) and Eqs. (D3) and (D17) may then be readily shown to give formula (5.38) derived in the text.

PHYSICAL REVIEW D VOLUME 2, NUMBER 10 15 NOVEMBER 1970

Derivation of Equal-Time Commutators Involving the Symmetric Energy-Momentum Tensor and Applications*

H. GENZ†
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

AND

J. KATZ
Purdue University, Lafayette, Indiana 47907

(Received 28 January 1970)

The use of covariance and the Jacobi identity in the study of equal-time commutators is investigated. Denoting by $T_{\mu\nu}$ the conserved and symmetric tensor density of Poincaré tranformations and by X any of the operators ϕ, $\partial_0\phi$, J_0, J_l, or J_{0l}, we use the most general form of the equal-time commutators $[iT_{0\mu}(x), X(y)]$ and $[iT_{00}(x), iT_{00}(y)]$ compatible with covariance, together with the Jacobi identities for $[[iT_{00}(x), iT_{00}(y)], X(z)]$, to derive relations between the equal-time commutators $[iT_{0m}(x), X(y)]$ and $[iT_{00}(x), Y(y)]$, where Y is any of the operators denoted by X or $\Box\phi$, $\partial^\mu\bar{\psi}\gamma_\mu$, $\partial^\mu J_\mu$, and $\partial^0 J_{0m}$. This information is first used in deriving equal-time commutators in canonical models. We then show that the assumption of $SU(2)\otimes SU(2)$ charge-current commutators together with $[A_0{}^\alpha(x),\bar{\psi}(y)]_{x_0=y_0}\propto\bar{\psi}(x)\tau^\alpha\gamma_5\delta(\mathbf{x}-\mathbf{y})$ (where $A_\mu{}^\alpha$ denotes the axial-vector current and ψ denotes a spinor field) implies (as obtained earlier by the authors under different assumptions) $[A_k{}^\alpha(x),\bar{\psi}(y)_0]_{x_0=y_0}=\frac{1}{2}\bar{\psi}(x)\gamma_5\gamma_k\tau^\alpha\delta(\mathbf{x}-\mathbf{y})+i(y-x)_k[A_0{}^\alpha(x),f_m{}^\dagger(y)\gamma_0]_{x_0=y_0}$ [where f denotes $(i\gamma^\mu\partial_\mu-m)\psi$]. For the conserved vector current an analogous relation holds. The incompatibility of field-algebra current commutators with $\int d^3x[A_k{}^\alpha(x),\bar{\psi}(y)\gamma_0]_{x_0=y_0}\propto\bar{\psi}(y)\gamma_5\gamma_k$ is noted. Taking ψ to be the nucleon field, it is shown that a certain form of the nucleon current leads to the above unless the right-hand side vanishes. Imposing this requirement, one then obtains $g_{A_1}=g_\rho$, where $g_{A_1}a_\mu{}^\alpha(x)\gamma_5\gamma^\mu(\tau^\alpha/2)\psi(x)$ $[g_\rho v_\mu{}^\alpha(x)\gamma^\mu\times(\tau^\alpha/2)\psi(x)]$ denotes the contribution of A_1 (ρ) to f_m in terms of the renormalized field $a_\mu{}^\alpha$ ($v_\mu{}^\alpha$). From this and the usual saturation of the Weinberg spectral-function sum rules by single-particle intermediate states, we obtain the universality relations $g_\rho=m_\rho{}^2/f_\rho$ and $g_{A_1}=(m_\rho/m_{A_1})^2m_{A_1}{}^2/f_{A_1}$, where f_{A_1} (f_ρ) is defined by $\rho_{A_1}(m^2)=f_{A_1}{}^2\delta(m^2-m_{A_1}{}^2)$ $[\rho_\rho(m^2)=f_\rho{}^2\delta(m^2-m_\rho{}^2)]$. For currents obeying the algebra-of-fields commutators, we obtain restrictions on Schwinger terms contained in equal-time commutators involving time derivatives of the currents. These relations show, for example, that in canonical realizations of current-field identities one needs derivative couplings of the spin-1 field.

I. INTRODUCTION

IT is generally assumed[1–7] that in relativistic local field theories a conserved and symmetric local tensor operator $T_{\mu\nu}(x)$ exists with the property that the generators of Poincaré transformations may be written as

$$P_\mu=\int d^3x\,T_{0\mu}(x)\quad(1.1)$$

and

$$M_{\mu\nu}=\int d^3x[x_\mu T_{0\nu}(x)-x_\nu T_{0\mu}(x)].\quad(1.2)$$

Denoting by ϕ, $\bar{\psi}$, J_μ, and $J_{\mu\nu}$ (defined as $J_{\mu\nu}\equiv\partial_\mu J_\nu-\partial_\nu J_\mu$) local operators with spins 0, $\frac{1}{2}$, 1, and 2, respec-

* Supported in part by the DAAD through a NATO grant and in part by the U. S. Atomic Energy Commission.

† Present address: Lush'tut für theoretische Kern physik der Universitüt Karlsnuhe, Karlsnuhe, Germany.

1 J. Schwinger, Phys. Rev. **130**, 406 (1963); Nuovo Cimento **30**, 278 (1963).

2 D. J. Gross and R. Jackiw, Phys. Rev. **163**, 1688 (1967).

3 R. Jackiw, Phys. Rev. **175**, 2058 (1968).

4 D. J. Gross and M. B. Halpern, Harvard University report, 1969 (unpublished).

5 H. Sugawara, Phys. Rev. **170**, 1659 (1968).

6 M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1969); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

7 T. K. Kuo and M. Sugawara, Phys. Rev. **163**, 1716 (1967).

tively, one finds that the equal-time commutators (ETC) between $T_{\mu\nu}$ and these operators (and between the $T_{\mu\nu}$'s themselves) are partly determined by covariance[1–4,8,9] [see Eqs. (1.14)–(1.20) and (1.24)–(1.29)]. As may be read off from Eqs. (1.14)–(1.20), the non-Schwinger terms (NST) and the first-order Schwinger terms (FOST)—the canonical terms—in the ETC $[iT_{00}(x),X(y)]$ are completely specified by covariance, whereas in the ETC $[iT_{0m}(x),X(y)]$ only the NST's are completely determined this way while the FOST's are shown to satisfy relations (1.30)–(1.35). These ETC's have some immediate applications which we discuss next.

Turning first to Eq. (1.17), we remark that it follows from this that the Gell-Mann condition[6,10]

$$\int d^3y[iT_{00}(x),J_0(y)]=\partial^\mu J_\mu(x) \tag{1.3}$$

is equivalent to[11]

$$\sum_{\alpha=2}^{N^{00}(J_0)} \frac{\partial}{\partial x_{k_1}}\cdots\frac{\partial}{\partial x_{k_\alpha}} j_{0;\{k_\alpha\}}{}^{00}(x)=0. \tag{1.4}$$

As we shall also see below, for canonical currents the noncanonical terms (NCT) $j_{0;\{k_\alpha\}}{}^{00}$ are absent[12,13] so that Eq. (1.3) holds in this case. In addition it is frequently assumed[6,14] that only the scalar part of $T_{\mu\nu}$ breaks the symmetry so that

$$\int d^3y[iT_{0m}(x),J_0(y)]=0. \tag{1.5}$$

From (1.26) we see that (1.5) is equivalent to

$$\sum_{\alpha=1}^{N^{0m}(J_0)} \frac{\partial}{\partial x_{k_1}}\cdots\frac{\partial}{\partial x_{k_\alpha}} j_{0;\{k_\alpha\}}{}^{0m}(x)=0. \tag{1.6}$$

In Sec. III it is seen that in certain models[15–18] $j_{0;\{k_\alpha\}}{}^{0m}$ vanishes so that Eq. (1.5) holds in such models. Furthermore (Sec. III), for fields ψ proportional to canonical ones, the NCT's in Eq. (1.15) are absent,[19] and thus

$$\int d^3y[iT_{00}(x),\bar\psi(y)\gamma_0]=\partial^\mu\bar\psi(x)\gamma_\mu-\tfrac{1}{2}\partial^k\bar\psi(x)\gamma_k, \tag{1.7}$$

in analogy to Eq. (1.3). [Conditions under which the additional Eqs. (3.37)–(3.42) hold are also investigated in Sec. III.]

As a consequence of Eq. (1.31) [Eq. (1.33)] the ETC of T_{0m} with fields of spin $\frac{1}{2}$ (space components of spin-1 fields) must at least have first-order ST. Since the ETC between the time-space components of the canonical energy-momentum tensor Θ_{0m} with any field which is proportional to a canonical one does not have ST, this property distinguishes the generators of local Lorentz transformations $T_{\mu\nu}$ (the symmetric energy-momentum tensor in canonical theories) from $\Theta_{\mu\nu}$. Canonical models in which both coincide therefore only contain basic fields of spin 0 (the generalization of the argument to canonical variables with spin higher than 1 should be obvious) and thus no fermion operators at all. Therefore, in the models of interest to us, one cannot assume Eq. (1.2) with $\Theta_{\mu\nu}$ replacing $T_{\mu\nu}$. However, it turns out (e.g., Sec. III) that the commutator $i[T_{0\mu}(x)-\Theta_{0\mu}(x),\ j_0(y)]$ vanishes in a large class of models and thus the calculation of $i[T_{0\mu}(x),j_0(y)]$ may be simplified by considering instead $i[\Theta_{0\mu}(x)j_0(y)]$.[20]

It is the main purpose of the present paper (Secs. II and III) to derive restrictions on the canonical and noncanonical terms in Eqs. (1.14)–(1.20) and (1.24)–(1.29). It is in view of the applications made[2–4,6,8,9,14] of these relations (see also Sec. IV and Appendix A) that a systematic investigation is desirable.

The results obtained in Secs. II and III are of different generality. Whereas in Sec. III we calculate ETC's in canonical models (the results are described in statements 1–3), Sec. II depends only on the assumed validity of the Jacobi identities[21] for $[[iT_{00}(x),iT_{00}(y)],$

[8] H. Genz and J. Katz, Nuovo Cimento **69A**, 15 (1970).

[9] H. Genz and J. Katz, Nucl. Phys. **B13**, 401 (1969).

[10] In Ref. 6 this relation has been used to derive the transformation properties of current divergences assuming the behavior of T_{00} under the chiral group. Since (Ref. 6 and, e.g., Ref. 14) this application provides possible experimental tests, it is of interest to derive further consequences of Eq. (1.3) [as done in Eq. (1.47) and in Appendix A].

[11] We denote the set $k_1,\ldots,k_\alpha$ ($\alpha\geqslant 2$) by $\{k_\alpha\}$ (where summation over repeated $\{k_\alpha\}$ is understood).

[12] The absence of NCT in Eq. (1.17) has been obtained for canonical currents in Ref. 2 by means of Schwinger's action principle (Ref. 1). Another derivation of this result has been given in Ref. 3 and in the Appendix of Ref. 9 (using the formalism of Ref. 32).

[13] We restrict our attention to the contributions of basic canonical fields with spin 0 and $\frac{1}{2}$.

[14] J. Ellis, Nucl. Phys. **B13**, 153 (1969); P. R. Auvil and N. G. Deshpande, Phys. Rev. **183**, 1463 (1969).

[15] We restrict our attention to the contributions of basic canonical fields with spins 0 and $\frac{1}{2}$ and assume a Lagrangian not involving derivatives of the fields carrying spin (see also Ref. 16).

[16] It has been shown (Ref. 17) that canonical realization of PCAC and current-field identities require couplings involving derivatives of ϕ (which we allow here). We shall see (Sec. IV) that for any (i.e., without restrictions on the basic fields) Lagrangian realization of these identities derivatives of the spin-1 field are also present in the interaction Lagrangian.

[17] H. Genz and J. Katz, Nuovo Cimento (to be published).

[18] In Ref. 3 this conclusion has been obtained for canonical theories involving only basic fields of spin 0 (i.e., no fermions at all).

[19] This conclusion has been obtained in Ref. 8 for theories fulfilling the condition of Ref. 15.

[20] Of course, $\Theta_{\mu\nu}$ may be used in any canonical theory as long as it is not interpreted as a generator of local Lorentz transformations (see Refs. 3, 8, and 9 for examples).

[21] We will assume in this paper that the equal-time limits considered exist and that the Jacobi identities employed are valid. [See J. Katz and J. Langerholc, Phys. Rev. **184**, 1577 (1969), for a discussion of equal-time limits and their possible nonexistence.] Occasionally we shall also assume the associative law. For brevity we shall refer to the consequences of assuming the Jacobi identities for $[[iT_{00}(x),iT_{00}(y)],X(z)]$ without imposing any restrictions on the possible NCT in the ETC $[iT_{00}(x),X(z)]$ and $[iT_{00}(x),iT_{00}(y)]$ as "consequences of covariance."

$X(z)]$ (where X denotes any of the operators ϕ, $\bar{\psi}\gamma_0$, J_0, J_l, J_{0l}, or $\partial_0\phi$) and on the transformation properties of $T_{\mu\nu}$ and X under Lorentz transformations. The results of Sec. II are then used in obtaining[22] some of the consequences discussed in Sec. III [Eqs. (3.20), (3.24), and (3.25)], but we would like to illustrate here possible applications by deriving the commutator $[iT_{00}(x),J_{0l}(y)]$ in the Sugawara model,[5] in which we have

$$\int d^3y(y-z)_m[T_{0n}(y),J_l(z)]=g_{ln}J_m(z). \qquad (1.8)$$

Then we use (2.21d) [and the absence of NCT in Eqs. (1.17) and (1.18) for the Sugawara model] to see that at most NCT of second order contributes to the commutator under discussion. Using (1.8) and (B4), we obtain

$$[iT_{00}(x),J_{0l}(y)] =\partial^0 J_{0l}(x)\delta(\mathbf{x}-\mathbf{y})+J_{kl}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \qquad (1.9)$$

(Of course, the above result would also follow by direct calculation, a procedure which involves ambiguities due to products of fields at a point which is avoided by the derivation presented above.)

Absence of NCT in Eq. (1.21) may be made plausible by assuming Schwinger's action principle,[1] which may be used to obtain

$$[iT_{00}(x),\, iT_{00}(y)] =iT_{0k}(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})+iT_{0k}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \qquad (1.10)$$

However, our results do not depend on this assumption.

Next we would like to make explicit[23] the consequences which Eqs. (1.1) and (1.2) together with the transformation properties of X have for the commutators $[iT_{0\mu}(x),X(y)]$. To describe a possible derivation, we consider the commutators involving $\bar{\psi}\gamma_0$. Assuming only existence of the equal-time limit, we may write

$$[iT_{00}(x),\bar{\psi}(y)\gamma_0]=\chi(x)\delta(\mathbf{x}-\mathbf{y})+\chi_k(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) + \sum_{\alpha=2}^{N^{06}(\psi)}\chi_{\{k_\alpha\}}{}^{00}(y)\frac{\partial}{\partial x_{k_1}}\cdots\frac{\partial}{\partial x_{k_\alpha}}\delta(\mathbf{x}-\mathbf{y}). \qquad (1.11)$$

Note the particular choice of the arguments of the ST in the above equation. This may always be achieved and will prove convenient in what follows. From the Heisenberg equation of motion [using Eq. (1.1)], we find

$$\partial_0\bar{\psi}(l)\gamma_0=\chi(y)-\partial^k\chi_k(y). \qquad (1.12)$$

Writing Eq. (1.11) for $y_\mu=0$, multiplying by x_k, and integrating over $\mathbf{x}$, we find by use of Eq. (1.2) from the known transformation properties of ψ under boosts

$$\chi_k(y)=\tfrac{1}{2}\bar{\psi}(y)\gamma_k, \qquad (1.13)$$

which determines the CT in Eq. (1.15). Applying the same reasoning to the other operators denoted by X, one obtains the results[23] (see also Refs. 2–4, 8, and 9)

$$[iT_{00}(x),\phi(y)]=\partial_0\phi(x)\delta(\mathbf{x}-\mathbf{y}), \qquad (1.14)$$

$$[iT_{00}(x),\bar{\psi}(y)\gamma_0] =\partial^\mu\bar{\psi}(x)\gamma_\mu\delta(\mathbf{x}-\mathbf{y})+\tfrac{1}{2}\bar{\psi}(x)\gamma_k\overleftrightarrow{\partial}^k\delta(\mathbf{x}-\mathbf{y}), \qquad (1.15)$$

where we have defined

$$\overleftrightarrow{\ }_{\mu}=\partial_\mu-\partial_\mu. \qquad (1.16)$$

One also obtains

$$[iT_{00}(x),J_0(y)] =\partial^\mu J_\mu(x)\delta(\mathbf{x}-\mathbf{y})+J_k(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) \qquad (1.17)$$

[which has been discussed in Ref. 1 (second entry) as well as in Ref. 24],

$$[iT_{00}(x),J_l(y)]=J_{0l}(x)\delta(\mathbf{x}-\mathbf{y}) -J_0(x)\frac{\partial}{\partial x^l}\partial(\mathbf{x}-\mathbf{y}), \qquad (1.18)$$

$$[iT_{00}(x),J_{0l}(y)]=\partial^0 J_{0l}(x)\delta(\mathbf{x}-\mathbf{y}) +J_{kl}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}), \qquad (1.19)$$

$$[iT_{00}(x),\partial_0\phi(y)]=\Box\phi(x)\delta(\mathbf{x}-\mathbf{y}) +\partial_k\phi(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}), \qquad (1.20)$$

and

$$[iT_{00}(x),iT_{00}(y)]=iT_{0k}(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) +iT_{0k}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \qquad (1.21)$$

NST are absent in Eq. (1.21) since $\partial^\mu T_{\mu\nu}=\partial^\mu T_{\nu\mu}=0$. From the transformation properties of $T_{00}(x)$, it also follows that

$$(\partial/\partial x_{\{k\alpha\}})t_{00;\{k_\alpha\}}{}^{00}(x)=0 \qquad (1.22)$$

and

$$\alpha(\partial/\partial x_{\{k\alpha\}})t_{00;\{k_\alpha\}}{}^{00}(x)=0. \qquad (1.23)$$

[22] An analogous derivation of Eqs. (3.24) and (3.25) was given in Ref. 3.

[23] We have not written explicitly the contributions of ST of at least second order. As in Eq. (1.11) they are understood to be written with arguments at y. For example, their contribution to Eqs. (1.17) and (1.18) is given by $j_{\mu;\{k_\alpha\}}{}^{00}(y)(\partial/\partial x_{\{k_\alpha\}})\delta(x-y)$.

[24] H. Pagels, University of North Carolina report (unpublished); (private communication).

We next use Eq. (1.1) for $\mu=m$ to obtain[23]

$$[iT_{0m}(x),\phi(y)]=\partial_m\phi(x)\delta(\mathbf{x}-\mathbf{y})+\phi_k{}^{0m}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\,, \quad (1.24)$$

$$[iT_{0m}(x),\bar{\psi}(y)\gamma_0]=\partial_m\bar{\psi}(x)\gamma_0\delta(\mathbf{x}-\mathbf{y})+\chi_k{}^{0m}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\,, \quad (1.25)$$

$$[iT_{0m}(x),J_0(y)]=-J_0(x)\frac{\partial}{\partial x^m}\delta(\mathbf{x}-\mathbf{y})+j_{0;k}{}^{0m}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\,, \quad (1.26)$$

$$[iT_{0m}(x),J_l(y)]=\partial_m J_l(x)\delta(\mathbf{x}-\mathbf{y})+j_{l;k}{}^{0m}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\,, \quad (1.27)$$

$$[iT_{0m}(x),J_{0l}(y)]=\partial_m J_{0l}(x)\delta(\mathbf{x}-\mathbf{y})+j_{0l;k}{}^{0m}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\,, \quad (1.28)$$

and

$$[iT_{0m}(x),\partial_0\phi(y)]=-\partial_0\phi(x)\frac{\partial}{\partial x^m}\delta(\mathbf{x}-\mathbf{y})+\phi_{0;k}{}^{0m}(y)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\,. \quad (1.29)$$

Using Eq. (1.2), we further obtain

$$-\phi_k{}^{0m}(y)+\phi_m{}^{0k}(y)=0\,, \quad (1.30)$$

$$-\chi_n{}^{0m}(y)+\chi_m{}^{0n}(y)=\tfrac{1}{2}\bar{\psi}(y)\gamma_0\gamma_m\gamma_n-\tfrac{1}{2}\bar{\psi}(y)\gamma_0 g_{mn}\,, \quad (1.31)$$

$$-j_{0;k}{}^{0m}(y)+j_{0;m}{}^{0k}(y)=0\,, \quad (1.32)$$

$$-j_{l;k}{}^{0m}(y)+j_{l;m}{}^{0k}(y)=g_{lk}J_m(y)-g_{lm}J_k(y)\,, \quad (1.33)$$

$$-j_{0l;k}{}^{0m}(y)+j_{0l;m}{}^{0k}(y)=g_{lk}J_{0m}(y)-g_{lm}J_{0k}(y)\,, \quad (1.34)$$

and

$$-\phi_{0;k}{}^{0m}(y)+\phi_{0;m}{}^{0k}(y)=0\,. \quad (1.35)$$

Before discussing applications of the results described above, we proceed to introduce our basic assumptions and notations concerning ETC between currents and fields. We will restrict our attention to chiral $SU(2)\otimes SU(2)$ and assume the usual ETC between charge densities. The currents $A_\mu{}^\alpha(x)$ and $V_\mu{}^\alpha({}^a)$ ($\alpha=1$–3) will be denoted by $J_\mu{}^a(x)$ ($a=1$–6) with $J_\mu{}^a(x)=V_\mu{}^a(x)$ for $a=1$–3 and $J_\mu{}^a(x)=A_\mu{}^{a-3}(x)$ for $a=4$–6. The structure constants e^{abc} are then defined by

$$[J_0{}^a(x),J_0{}^b(y)]=ie^{abc}J_0{}^c(x)\delta(\mathbf{x}-\mathbf{y})\,. \quad (1.36)$$

For the fermion field, we will occasionally assume[25]

$$[A_0{}^\alpha(x),\bar{\psi}(y)]=-r_A\bar{\psi}(x)\gamma_5\tau^\alpha\delta(\mathbf{x}-\mathbf{y})\,. \quad (1.37)$$

It follows[26] from this (by an appropriate choice of the phase of A), assuming the usual ETC between charge densities, that

$$[J_0{}^a(x),\bar{\psi}(y)]=\tfrac{1}{2}\bar{\psi}(x)\Gamma^a\delta(\mathbf{x}-\mathbf{y})\,, \quad (1.38)$$

with

$$\Gamma^a=\tau^a \quad \text{for} \quad a=1\text{–}3$$
$$=\gamma_5\tau^{a-3} \quad \text{for} \quad a=4\text{–}6\,. \quad (1.39)$$

Turning next to the applications of Eqs. (1.14)–(1.21), we note that the connection between usual ETC of charge densities, ST in $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$, and current-albegra commutators has already been partly discussed.[2,3,7] The discussion given in Refs. 2 and 3 made use of Eq. (1.17) and assumed for the main conclusions that NCT were absent, while in Ref. 7 use was made of Lorentz invariance, and it was assumed that the ETC occurring in that derivation ($T_{\mu\nu}$ was not used in Ref. 7) contain at most a FOST. It was then shown[2,3,7] that usual current-algebra commutators follow, provided that

$$(x-y)_m[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]=0\,, \quad (1.40)$$

i.e., the above ETC contains no ST.

In Refs. 8 and 9 it was shown that in certain models it follows from Eq. (1.37) that[27]

$$[J_m{}^a(x),\bar{\psi}(y)\gamma_0]=\tfrac{1}{2}\bar{\psi}(x)\Gamma^a\gamma_m\delta(\mathbf{x}-\mathbf{y})+i(y-x)_m[J_0{}^a(x),f_m{}^\dagger(y)\gamma_0]\,, \quad (1.41)$$

where f_m is defined by

$$f_m(x)=(i\gamma^\mu\partial_\mu-m)\psi(x) \quad (1.42)$$

for any m. [As noted in Ref. 9, Eq. (1.41) may be obtained for conserved currents, using direct consequences of covariance,[26] from the Heisenberg equation of motion. Also for conserved currents the x-integrated Eq. (1.41) is a simple consequence of the Heisenberg equation of motion.] Absence of ST in the ETC's $[J_k{}^a(x),\bar{\psi}(y)\gamma_0]$ and $[\partial^\mu J_\mu{}^a(x),\bar{\psi}(x)]$ was also derived in Refs. 8 and 9.[27] This result may be combined with Eq. (1.41) to see that $[J_0{}^a(x),f^\dagger(y)\gamma_0]$ contains at most a FOST.

[25] This commutator has frequently been used in the literature [Refs. 8, 9, and 26] and no contradictions with experiment have been found. [See S. Weinberg, Phys. Rev. **166**, 1568 (1968), for another proposal.] See also the following: J. Rothleitner, Nucl. Phys. **B3**, 89 (1967); M. Sugawara, Phys. Rev. **172**, 1423 (1968); M. K. Banerjee and C. A. Levinson, University of Marlyand Technical Report No. 857 (unpublished); A. M. Gleeson, Phys. Rev. **149**, 1242 (1969); H. Genz, J. Katz, and S. Wagner, Nuovo Cimento **64A**, 218 (1969); H. Genz, Phys. Rev. D **1**, 659 (1970).

[26] H. Genz and J. Katz, Nuovo Cimento **64A**, 291 (1969).

[27] To obtain the results given in Refs. 8 and 9 it was assumed in Ref. 8 that NCT were absent in the ETC $[i\Theta_{00}(x),J_0{}^b(y)]$ and $[i\Theta_{00}(x),\psi(y)\gamma_0]$. The absence of these terms was derived in Ref. 8 for canonical currents and for fields proportional to canonical ones in Lagrangian field theories. In Ref. 9, the absence of NCT in $[iT_{00}(x),J_0{}^b(y)]$ and $[iT_{00}(x),\bar{\psi}(y)\gamma_0]$ was assumed.

It is the first purpose of the applications made in Sec. IV to derive Eq. (1.41) from covariance and Eq. (1.37) alone and to discuss the dependence of the results of Refs. 8 and 9 on the absence of NCT in Eqs. (1.15) and (1.17) (as is the case for certain models discussed in Sec. III).

We next illustrate[28] applications of Eqs. (1.14)–(1.21) by considering the Jacobi identity for $[iT_{00}(x),[J_0{}^a(y),J_0{}^b(z)]]$.[2] We thus write

$$\begin{aligned}[][J_0{}^a(y),\partial^\mu J_\mu{}^b(z)]\delta(\mathbf{x}-\mathbf{z})-[J_0{}^b(z),\partial^\mu J_\mu{}^a(x)]\delta(\mathbf{x}-\mathbf{y})\\ -ie^{abc}\partial^\mu J_\mu{}^a(z)\delta(\mathbf{x}-\mathbf{y})\delta(\mathbf{y}-\mathbf{z})\\ =[J_0{}^b(z),J_k{}^a(x)]\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})-[J_0{}^a(y),J_k{}^b(x)]\\ \times\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{z})-ie^{bac}J_k{}^c(x)\delta(\mathbf{y}-\mathbf{z})\\ \times\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{z})+Z(x,y,z).\end{aligned}\quad(1.43)$$

In the above equation, we have denoted by Z the sum of terms which depend on $j_{0;\{k_\alpha\}}{}^{00}$. Owing to covariance, we have

$$\int d^3x\,Z(x,y,z)=\int d^3x\,x_kZ(x,y,z)=0.\quad(1.44)$$

Note that if one assumes Eq. (1.3), then one may also write

$$\int d^3yd^3z\,Z(x,y,z)=0.\quad(1.45)$$

Next we multiply Eq. (1.43) by $(x-y)_m$, integrate over $\mathbf{x}$ and $\mathbf{z}$, and use Eq. (1.44) to obtain (as a result of covariance, and the ETC between charge densities only)

$$\begin{aligned}[][Q^b(y_0),J_m{}^a(y)]=ie^{bac}J_m{}^c(y)\\ +\int d^3z(y-z)_m[J_0{}^a(y),\partial^\mu J_\mu{}^a(z)].\end{aligned}\quad(1.46)$$

Assuming Eq. (1.3), we then obtain from Eqs. (1.43) and (1.45)

$$\begin{aligned}[][Q^a(x_0),\partial^\mu J_\mu{}^b(x)]-[Q^b(x_0),\partial^\mu J_\mu{}^a(x)]\\ =ie^{abc}\partial^\mu J_\mu{}^c(x).\end{aligned}\quad(1.47)$$

The above relation has been derived in Ref. 7 by use of Lorentz covariance and the assumption that at most a FOST is present in the ETC's $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$ and $[J_0{}^a(x),J_k{}^b(y)]$. In Ref. 3 it was obtained assuming absence of NCT in Eq. (1.17). Our derivation shows that it is a simple consequence of Eq. (1.3).

From Eqs. (1.40) and (1.46), evidently the usual charge-current commutators follow.[2,3,7] From covariance we derive in the Appendix the absence of ST in $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$ for currents which obey field-algebra commutators with charge densities. Also in the Appendix the usual[2] symmetry relations for the FOST in $[J_0{}^a(x),J_m{}^b(y)]$ are obtained from assuming at most a FOST in this commutator. The Appendix, in which we employ the methods of Refs. 2 and 3, is independent of NCT in Eq. (1.17) and contains also a discussion of the further consequences of Eq. (1.3). This investigation is motivated by noting that only for canonical currents absence of NCT [in Eqs. (1.17) or (3.4)] has been obtained.[2,3,8,9]

It is the main purpose of Sec. IV to investigate consequences of Eq. (1.41) for ETC between currents and fermion fields. It is argued in that section[29] that large effects due to the interaction term in Eq. (1.41) are to be expected, in contrast to Eq. (1.46) in which these effects are expected to be small. The relaton in Eq. (1.41) shows that it is in fact because of the interaction of the spin-$\frac{1}{2}$ field that deviations from the quark-model result for $[J_k{}^a(x),\bar\psi(y)\gamma_0]$ are possible (as pointed out in Ref. 9). Since proportionality of the NST of this ETC to $\bar\psi\Gamma^a\gamma_m$ is incompatible[9] with commutativity of the space components of the currents, we immediately see that the algebra of field-current commutators is exlcuded if the fermion field is free. In order to investigate the compatibility of Eq. (1.37) with field-algebra commutators, we present in Sec. IV the following model for the nucleon current:

$$\begin{aligned}f_m(x)=\{c_\pi P(\phi(x))\\ +[c_VV_\mu{}^\alpha(x)+c_AA_\mu{}^\alpha(x)\gamma_5]\gamma^\mu\tau^\alpha\}\psi(x),\end{aligned}\quad(1.48)$$

which may be interpreted by use of current-field identities. [In Eq. (1.48), $P(\phi(x))$ denotes an arbitrary polynomial of the pion field with the right quantum numbers.] If algebra-of-fields current commutators are assumed, together with Eq. (1.48), then the second term on the right-hand side of Eq. (1.41) (the interaction term) is proportional to the first term. Thus field-algebra current commutators are compatible with Eq. (1.41) [a consequence of (1.37)] and Eq. (1.48) only if the right-hand side of Eq. (1.41) vanishes, which yields the relations (4.18)–(4.31).

Therefore, Eqs. (1.41) and (1.48) suggest that[30]

$$[J_k{}^a(x),\bar\psi(y)]=0\quad(1.49)$$

in case of algebra-of-fields commutators. (Note that the above equation is also a consequence of the canonical

[28] This derivation differs from that given in Ref. 2 in that Eq. (1.40) is not assumed and from that given in Ref. 3 in that we allow for the possible presence of NCT in Eq. (1.17).

[29] A different discussion is given in R. Jackiw, CERN Report No. 1065 (unpublished).

[30] Note that Eq. (1.49) is the simplest possibility to express $[J_k{}^a(x),\bar\psi(y)\gamma_0]$ as a linear form in $\bar\psi$ and its space derivatives which is compatible with rotational invariance and field-algebra commutators.

rules in case of canonical realizations of current-field identities.[31])

In the remaining part of Sec. IV consequences of Eqs. (1.15) and (1.17) are first discussed when they are combined with Eqs. (1.37) and (1.49) and finally the consequences of Eqs. (1.17) and (1.18) for currents obeying field-algebra commutators are obtained. The main results are Eqs. (4.40) and (4.42), which are obtained without any assumption about the NCT in (1.17) and (1.18). We would like to note here that Eq. (4.40) shows that in canonical realizations of current-field identities, one needs derivative couplings involving the spin-1 field.[16,17,31]

II. CONSEQUENCES OF COVARIANCE

In the present section[21] we assume the Jacobi identities involving $[[iT_{00}(x),iT_{00}(y)],X(z)]$ and utilize Eqs. (1.14)–(1.22) and (1.24)–(1.29) to obtain relations connecting the ST in $[iT_{0m}(x),X(y)]$ with the NCT in $[iT_{00}(x),Y(y)]$. In the above, X (Y) denotes any of the operators ϕ, $\bar{\psi}\gamma_0$, J_0, J_l, J_{0l}, or $\partial_0\phi$ ($\partial^\mu\bar{\psi}\gamma_\mu$, $\partial^\mu J_\mu$, J_{0m}, $\partial^0 J_{0l}$, or $\Box\phi$). Our present considerations are model independent since we only make some rather general assumptions about the existence of equal-time limits and the validity of the Jacobi identity. Since all the relations below are obtained by analogous manipulations, we shall only choose the commutators involving ψ to illustrate the calculations and merely give the results for the other cases.

We start by writing the following Jacobi identities:

$$[[iT_{00}(x),iT_{00}(y)],X(z)] = [iT_{00}(x),[iT_{00}(y),X(z)]] - [iT_{00}(y),[iT_{00}(x),X(z)]]. \quad (2.1)$$

In this equation, X denotes any of the operators indicated above. For $X(z)=\bar{\psi}(z)$, we use (1.15) and (1.21) to rewrite this as[23]

$$\begin{aligned}
&[iT_{0k}(x),\bar{\psi}(z)\gamma_0]\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) + [iT_{0k}(y),\bar{\psi}(z)\gamma_0]\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) \\
&= [iT_{00}(x),\partial^\mu\bar{\psi}(y)\gamma_\mu]\delta(\mathbf{x}-\mathbf{z}) - \tfrac{1}{2}[iT_{00}(x),\partial^k\bar{\psi}(y)\gamma_k] \\
&\quad\times\delta(\mathbf{y}-\mathbf{z}) - [iT_{00}(y),\partial^\mu\bar{\psi}(x)\gamma_\mu]\delta(\mathbf{x}-\mathbf{z}) \\
&\quad+\tfrac{1}{2}[iT_{00}(y),\partial^k\bar{\psi}(x)\gamma_k]\delta(\mathbf{x}-\mathbf{z}) \\
&\quad+\tfrac{1}{4}\bar{\psi}(x)\gamma_l\gamma_0\gamma_k\frac{\partial}{\partial x_l}\delta(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial y_k}\delta(\mathbf{y}-\mathbf{z}) \\
&\quad-\tfrac{1}{4}\bar{\psi}(y)\gamma_l\gamma_0\gamma_k\frac{\partial}{\partial y_l}\delta(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{z}). \quad (2.2)
\end{aligned}$$

[31] T. D. Lee and B. Zumino, Phys. Rev. **163**, 1667 (1967), and references therein; R. Arnowitt, M. H. Friedman, and P. Nath, Nucl. Phys. **B10**, 578 (1969), and references therein.

For a consistency check, we first multiply the above equation by $(y-z)_m$ and integrate over $\mathbf{x}$ and $\mathbf{y}$ to obtain

$$\begin{aligned}
&\int d^3x[iT_{0m}(x),\bar{\psi}(z)\gamma_0] \\
&= -\int d^3y(y-z)_m[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu] \\
&\quad+\tfrac{1}{2}\int d^3y(y-z)_m[iT_{00}(y),\partial^k\bar{\psi}(z)\gamma_k] \\
&\quad+\tfrac{1}{4}\partial^l\bar{\psi}(z)\gamma_l\gamma_0\gamma_m+\tfrac{1}{2}\partial_m\bar{\psi}(z)\gamma_0. \quad (2.3)
\end{aligned}$$

Using once again Eq. (1.15), as well as Eq. (1.25), we then obtain, after some rearrangements,

$$\int d^3y(y-z)_m[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu\gamma_0] = -\tfrac{1}{2}(\partial^\mu\bar{\psi}(z)\gamma_\mu)\gamma_m. \quad (2.4)$$

Therefore, comparison with Eq. (1.15) shows that $\partial^\mu\bar{\psi}(z)\gamma_\mu$ transforms like a spin-$\frac{1}{2}$ field, as it should.

Employing the same reasoning as above and using Eqs. (1.14) and (1.17)–(1.21) for each of the cases $X=\phi$, J_0, J_l, J_{0l}, and $\partial_0\phi$, respectively, we then obtain the correct transformation properties for $\partial_0\phi$, $\partial^\mu J_\mu$, J_{0l}, $\partial^0 J_{0l}$, and $\Box\phi$.

We now return to Eq. (2.2) and multiply it by $(x-y)_m$ and integrate over $\mathbf{x}$, with the result[23]

$$\begin{aligned}
&-2[iT_{0m}(y),\bar{\psi}(z)\gamma_0] \\
&= \int d^3x(x-y)_m[iT_{00}(x),\partial^\mu\bar{\psi}(y)\gamma_\mu]\delta(\mathbf{y}-\mathbf{z}) \\
&\quad-\tfrac{1}{2}\int d^3x(x-y)_m[iT_{00}(x),\partial^k\bar{\psi}(y)\gamma_k]\delta(\mathbf{y}-\mathbf{z}) \\
&\quad-\tfrac{1}{2}\bar{\psi}(y)\gamma_m\gamma_0\gamma_k\frac{\partial}{\partial y_k}\delta(\mathbf{y}-\mathbf{z})-(z-y)_m \\
&\quad\times[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu]+\tfrac{1}{2}(z-y)_m \\
&\quad\times[iT_{00}(y),\partial^k\bar{\psi}(z)\gamma_k]. \quad (2.5)
\end{aligned}$$

Note that there are no contributions from the higher-order ST in Eq. (1.21). Therefore, the resulting expressions are identical to those which would be obtained by use of Schwinger's condition.

Using Eqs. (2.4) and (1.15), the above equation may then be written as

$$\begin{aligned}
&[iT_{0m}(y),\bar{\psi}(z)\gamma_0] = -\tfrac{1}{2}(y-z)_m[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu] \\
&\quad-\tfrac{1}{4}\partial^\mu\bar{\psi}(y)\gamma_\mu\gamma_m\gamma_0\delta(\mathbf{y}-\mathbf{z})+\partial_m\bar{\psi}(y)\gamma_0\delta(\mathbf{y}-\mathbf{z}) \\
&\quad-\tfrac{1}{2}\bar{\psi}(z)\gamma_0\frac{\partial}{\partial y^m}\delta(\mathbf{y}-\mathbf{z})+\tfrac{1}{8}\bar{\psi}(z)\gamma_0(\gamma_k\gamma_m-\gamma_m\gamma_k) \\
&\quad\times\frac{\partial}{\partial y_k}\delta(\mathbf{y}-\mathbf{z}). \quad (2.6)
\end{aligned}$$

Employing the same reasoning as above, the collection of formulas obtained for $X=\phi$, J_0, J_l, $\partial_0\phi$, and J_{0l} may be easily written, but we shall not do so in the present paper in order to keep its size manageable.

We next proceed to obtain the relation between the FOST in $[iT_{0m}(y),\bar{\psi}(z)\gamma_0]$ and the second order ST in $[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu]$. In order to do this, we multiply Eq. (2.6) by $(y-z)_m$ and integrate over $\mathbf{y}$. This gives

$$\int d^3y(y-z)_n[iT_{0m}(y),\bar{\psi}(z)\gamma_0] = -\tfrac{1}{2}\int d^3y(y-z)_m(y-z)_n[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu] +\tfrac{1}{2}g_{mn}\bar{\psi}(z)\gamma_0+\tfrac{1}{8}\bar{\psi}(z)\gamma_0(\gamma_m\gamma_n-\gamma_n\gamma_m) +\partial^\mu\chi_{mn}{}^{00}(z)\gamma_\mu\gamma_0. \quad (2.7)$$

The analogous results for the other cases are given in Appendix B. From these relations it may then be seen that Eqs. (1.30)–(1.35) (which have not been used in the preceding calculation) emerge upon antisymmetrization in m and n.

Next we multiply Eq. (2.2) with $(x-z)_m$, integrate over $\mathbf{x}$, and use Eq. (1.15) to obtain[23]

$$[iT_{0m}(y),\bar{\psi}(z)\gamma_0]+\tfrac{1}{2}(y-z)_m[i\partial^kT_{0k}(y),\bar{\psi}(z)\gamma_0] =\tfrac{1}{2}\partial_m\bar{\psi}(z)\gamma_0\delta(\mathbf{y}-\mathbf{z})+\tfrac{1}{8}\bar{\psi}(z)\gamma_0(\gamma_k\gamma_m-\gamma_m\gamma_k) \times\frac{\partial}{\partial y_k}\delta(\mathbf{y}-\mathbf{z}). \quad (2.8)$$

[Note that the higher-order ST's in Eq. (1.21) do not contribute to the above equation.]

Similar results may be obtained for the remaining cases by an analogous procedure. However, we only wish to note here the result

$$[iT_{0m}(y),J_0(z)]+\tfrac{1}{2}(y-z)_m[i\partial^kT_{0k}(y),J_0(z)] =\tfrac{1}{2}\partial_mJ_0(y)\delta(\mathbf{y}-\mathbf{z}), \quad (2.9)$$

since we shall make explicit use of it later. Note that the only possible NCT which may contribute to Eq. (1.9) {i.e. those of $[iT_{00}(x),J_\mu(y)]$} have not been written out explicitly for simplicity.

We would next like to obtain relations between the higher-order ST's. To achieve this, we multiply Eq. (2.8) by $(y-z)_{n_1}(y-z)_{n_2}$, integrate over $\mathbf{y}$, and use Eq. (1.25) to obtain

$$-\chi_{n_1n_2}{}^{0m}(z)+\chi_{mn_2}{}^{0n_1}(z)+\chi_{n_1m}{}^{0n_2}(z)=\int d^3x(x-z)_m \times[iT_{00}(x),\chi_{n_1n_2}{}^{00}(z)]+\chi_{n_1n_2}{}^{00}\gamma_0\gamma_m. \quad (2.10)$$

From (2.10) it is easy to derive that $\chi_{n_1n_2}{}^{0m}=0$ whenever $\chi_{n_1n_2}{}^{00}=0$. In fact, when $\chi_{n_1n_2}{}^{00}=0$, Eq. (2.10) gives

$$\chi_{n_1n_2}{}^{0m}(z)-\chi_{mn_2}{}^{0n_1}(z)-\chi_{n_1m}{}^{0n_2}(z)=0 \quad (2.11)$$

and

$$\chi_{n_1m}{}^{0n_2}(z)-\chi_{n_1n_2}{}^{0m}(z)-\chi_{mn_2}{}^{0n_1}(z)=0. \quad (2.12)$$

Adding these relations, we then obtain

$$-2\chi_{mn_2}{}^{0n_1}(z)=0, \quad (2.13)$$

the desired result.

We next derive the analogous results for the ST's of the third order by multiplying Eq. (2.8) by $(y-z)_{n_1}(y-z)_{n_2}(y-z)_{n_3}$, integrating over $\mathbf{y}$, and using Eq. (1.25). We obtain

$$-3[\chi_{n_1n_2n_3}{}^{0m}(z)-\chi_{mn_2n_3}{}^{0n_1}(z)-\chi_{n_1mn_3}{}^{0n_2}(z) -\chi_{n_1n_2m}{}^{0n_3}(z)]=3\int d^3x(x-z)_m[iT_{00}(x),\chi_{n_1n_2n_3}{}^{00}(z)] +3\chi_{n_1n_2n_3}{}^{00}(z). \quad (2.14)$$

Once again it follows from the above that $\chi_{n_1n_2n_3}{}^{00}=0$ implies that $\chi_{n_1n_2n_3}{}^{0m}=0$. To see this we write Eq. (2.28) when $\chi_{n_1n_2n_3}{}^{00}=0$. We obtain

$$\chi_{n_1n_2n_3}{}^{0m}(z)-\chi_{mn_2n_3}{}^{0n_1}(z)-\chi_{n_1mn_3}{}^{0n_2}(z) -\chi_{n_1n_2m}{}^{0n_3}(z)=0 \quad (2.15)$$

and

$$\chi_{mn_2n_3}{}^{0n_1}(z)-\chi_{n_1mn_3}{}^{0n_2}(z)-\chi_{n_1n_2m}{}^{0n_3}(z) -\chi_{n_1n_2n_3}{}^{0m}(z)=0. \quad (2.16)$$

Adding and subtracting these equations, we get

$$\chi_{n_1mn_3}{}^{0n_2}(z)+\chi_{n_1n_2m}{}^{0n_3}(z)=0 \quad (2.17)$$

and

$$\chi_{n_1'n_2'n_3'}{}^{0m'}(z)-\chi_{m'n_2'n_3'}{}^{0n_1'}(z)=0. \quad (2.18)$$

Choosing $m'=n_2$, $n_1'=n_3$, $n_2'=m$, $n_3'=n_1$, and using the symmetry of $\chi_{n_1n_2n_3}{}^{0m}$ in the lower three indices, we obtain from (2.18)

$$\chi_{n_1mn_3}{}^{0n_2}(z)-\chi_{n_1n_2m}{}^{0n_3}(z)=0, \quad (2.19)$$

which upon comparison with Eq. (2.17) shows that $\chi_{n_1n_2n_3}{}^{0m}(z)$ vanishes. Clearly an analogous reasoning may be performed for higher-order ST's but we shall not do so in this paper since the generalizations are now apparent.

Our results may be schematically expressed as

$$\int d^3x(x-y)_{n_1}\prod_{j=2}^{R}(x-y)_{n_j}[iT_{0m}(x),X(y)]$$

$$=Z_\phi(\phi_{k_1\cdots k_R}{}^{00}) \qquad \text{for } X=\phi \quad (2.20a)$$

$$=Z_\psi(\chi_{k_1\cdots k_R}{}^{00}) \qquad \text{for } X=\bar{\psi}\gamma_0 \quad (2.20b)$$

$$Z={}_{J_0}(j_{0;k_1\cdots k_R}{}^{00};j_{m;k_1\cdots k_R}{}^{00}) \qquad \text{for } X=J_0 \quad (2.20c)$$

$$=Z_{J_l}(j_{l;k_1\cdots k_R}{}^{00};j_{\phi;k_1\cdots k_R}{}^{00}) \qquad \text{for } X=J_l \quad (2.20d)$$

$$=Z_{\partial_0\phi}(\phi_{k_1\cdots k_R}{}^{00};\phi_{0;k_1\cdots k_R}{}^{00}) \qquad \text{for } X=\partial_0\phi \quad (2.20e)$$

$$=Z_{J_{0l}}(j_{0m;k_1\cdots k_R}{}^{00};j_{l;k_1\cdots k_R}{}^{00}) \qquad \text{for } X=J_{0l}, \quad (2.20f)$$

since analogous calculations may be performed for the other choices of X. In the above we have denoted by Z

those linear forms in the ST obtained following the procedure indicated. Note that they vanish whenever the ST's vanish.

To obtain analogous results for commutators involving T_{00} and Y, with $Y=\partial_0\phi$, $\partial^\mu\bar{\psi}\gamma_\mu$, $\partial^\mu J_\mu$, J_{0m}, $\Box\phi$, or $\partial_0 J_{0m}$, we return to Eq. (2.6). We mutiply this equation by

$$(y-z)_{n1}\prod_{i=2}^{R}(y-z)_{ni}\quad(R=2,3)$$

and integrate over **y**. The explicit non-ST on the right-hand side do not contribute, and we are left with a relation expressing the ST or order $R+2$ in $[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu]$ in terms of the ST order or $R+1$ in $[iT_{0m}(y),\bar{\psi}(z)\gamma_0]$ and $\chi_{k_1k_2}{}^{00},\ldots,\chi_{k_1\cdots k_{R+2}}{}^{00}$. Using Eq. (2.20), we then obtain a relation expressing the ST of order $R+2$ in $[iT_{00}(y),\partial^\mu\bar{\psi}(z)\gamma_\mu]$ in terms of $\chi_{k_1k_2}{}^{00},\ldots,\chi_{k_1\cdots k_{R+2}}{}^{00}$. Our results for the different Y may be schematically represented as

$$\int d^3x(x-y)_m(x-y)_{n1}\prod_{j=2}^{R}(x-y)_{nj}[iT_{00}(x),Y(y)]$$
$$=Z_\phi(\phi_{k_1k_2}{}^{00},\ldots,\phi_{k_1\cdots k_{R+2}}{}^{00})\quad\text{for}\quad Y=\partial_0\phi \tag{2.21a}$$
$$=Z_{\bar{\psi}}(\chi_{k_1k_2}{}^{00},\ldots,\chi_{k_1\cdots k_{R+2}}{}^{00})\quad\text{for}\quad Y=\partial^\mu\bar{\psi}\gamma_\mu \tag{2.21b}$$
$$=Z_{J_0}(j_{0;k_1k_2}{}^{00},\ldots,j_{0;k_1\cdots k_{R+2}}{}^{00};\,j_{m;k_1k_2}{}^{00},\ldots,j_{m;k_1\cdots k_{R+2}}{}^{00})\quad\text{for}\quad Y=\partial^\mu J_\mu \tag{2.21c}$$
$$=Z_{J_l}(j_{m;k_1k_2}{}^{00},\ldots,j_{m;k_1\cdots k_{R+2}}{}^{00};\,j_{0;k_1k_2}{}^{00},\ldots,j_{0;k_1\cdots k_{R+2}}{}^{00})\quad\text{for}\quad Y=J_{0m} \tag{2.21d}$$
$$=Z_{\partial_0\phi}(\phi_{k_1k_2}{}^{00},\ldots,\phi_{k_1\cdots k_{R+2}}{}^{00};\,\phi_{0;k_1k_2}{}^{00},\ldots,\phi_{0;k_1\cdots k_{R+2}}{}^{00})\quad\text{for}\quad Y=\Box\phi \tag{2.21e}$$
$$=Z_{J_{0l}}(j_{0l;k_1k_2}{}^{00},\ldots,j_{0l;k_1\cdots k_{R+2}}{}^{00};\,j_{l;k_1k_2}{}^{00},\ldots,j_{l;k_1\cdots k_{R+2}}{}^{00})\quad\text{for}\quad Y=\partial^0 J_{0l}. \tag{2.21f}$$

The explicity form of the Z's (which vanish whenever the ST's vanish) may be obtained by performing the manipulations described above, taking into account the explicit forms of the equations used.

III. COMMUTATORS IN CANONICAL THEORIES

We obtain in this section some equal-time commutators of $T_{0\mu}$ with currents and fields in canonical theories with basic canonical fields of spins 0 and $\frac{1}{2}$. We will sometimes also assume that the interaction Lagrangian does not contain derivatives of the spin-$\frac{1}{2}$ field. If canonical variables of higher spin are present, a generalization of our derivations under this assumption requires absence of derivatives of any field carrying spin from the interaction Lagrangian. In obtaining these commutators we will also make use of the information obtained in Sec. II.

We start with some remarks concerning the canonical energy-momentum tensor $\Theta_{\mu\nu}$ and the symmetric energy-momentum tensor $T_{\mu\nu}$ (the generator of local Lorentz transformations). The canonical tensor is given by Noether's theorem as

$$\Theta_{\mu\nu}(x)=\frac{\partial L}{\partial(\partial_\mu\phi_a(x))}\partial_\nu\phi_a(x)-g_{\mu\nu}L(x)\,, \tag{3.1}$$

while the symmetric energy-momentum tensor $T_{\mu\nu}$ is defined by

$$T_{\mu\nu}{}^\dagger(x)=\Theta_{\mu\nu}(x)-\partial^\lambda f_{\lambda\mu\nu}(x)\,. \tag{3.2}$$

We employ the formalism of Ref. 32. However, we choose the adjoint of the $T_{\mu\nu}$ given there as our symmetric energy-momentum tensor. This will prove convenient and is possible even if $T_{\mu\nu}$ is not Hermitian, since the non-Hermitian parts cannot contribute to Eqs. (1.1) and (1.2).

The canonical energy-momentum tensor $\Theta_{\mu\nu}$ has the advantage that equal-time commutators such as[33]

$$[i\Theta_{00}(x),\psi(y)]=\partial_0\psi(x)\delta(\mathbf{x}-\mathbf{y})\,, \tag{3.3}$$

$$[i\Theta_{00}(x),J_0(y)]=\partial^\mu J_\mu(x)\delta(\mathbf{x}-\mathbf{y})+J_k(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\,, \tag{3.4}$$

$$[i\Theta_{0m}(x),\psi(y)]=\partial_m\psi(x)\delta(\mathbf{x}-\mathbf{y})\,, \tag{3.5}$$

and

$$[i\Theta_{0m}(x),J_0(y)]=-J_0(x)\frac{\partial}{\partial x^m}\delta(\mathbf{x}-\mathbf{y})\,, \tag{3.6}$$

are readily calculated, while if we consider the analogous commutators with $T_{\mu\nu}$ replacing $\Theta_{\mu\nu}$, we note that $[iT_{00}(x),\psi(y)]$ is different[8] from $[i\Theta_{00}(x),\,\psi(y)]$ [see Eq. (3.15) below], $[iT_{00}(x),J_0(y)]$ is the same[34] as $[i\Theta_{00}(x),\,J_0(y)]$, and $[iT_{0m}(x),\psi(y)]$ and $[iT_{0m}(x),\,J_0(y)]$ are in general not completely determined unless additional assumptions are made.[34] Incidentally we also

[32] G. Källén, *Quantenelectrodynamik Handbuch der Physik, Bd. V/1* (Springer-Verlag, Berlin, 1958).

[33] See Ref. 32 (and Ref. 9) for a derivation of Eq. (3.3). In Refs. 3 and 9, Eq. (3.4) was obtained, Eq. (3.5) obviously holds, and Eq. (3.6) was derived in Ref. 3.

[34] This will become apparent after reading this section.

note that[9]

$$[i\Theta_{00}(x),\bar{\psi}(y)\gamma_0]=\partial^\mu\bar{\psi}(x)\gamma_\mu\delta(\mathbf{x}-\mathbf{y})+\bar{\psi}(x)\gamma_k\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \quad (3.7)$$

Evidently Eqs. (3.3) and (3.7) may not hold with T_{00} replacing Θ_{00} owing to covariance [i.e., Eq. (1.2)]. Since for any canonical variable ϕ_a one derives[9,32]

$$[i\Theta_{00}(x),\phi_a(y)]=\partial_0\phi_a(x)\delta(\mathbf{x}-\mathbf{y}), \quad (3.8)$$

one finds from covariance that

$$M_{0i}=-\int d^3x\, x_i\Theta_{00}(0,\mathbf{x}) \quad (3.9)$$

can hold only if all the basic canonical variables have vanishing spin. Similarly Eq. (3.5) with Θ_{0m} replaced by T_{0m} would be in contradiction with Eq. (1.31). These facts have already been discussed in the Introduction. It should also be noted that even though the equal-time commutators involving $T_{0\mu}$ are in general different from those involving $\Theta_{0\mu}$, they give the same results in some instances.[8,9,35]

We will next derive $\chi^{00}=0$ without making any assumptions about derivative couplings. To this end we note that for spin $\frac{1}{2}$, $S_{\mu\nu;\alpha\beta}$ [Eq. (4.18) of Ref. 32] is given by

$$S_{\mu\nu;\alpha\beta}=\tfrac{1}{4}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu). \quad (3.10)$$

Thus we may write [Eq. (4.19) of Ref. 32]

$$f_{m00}(x)=\tfrac{1}{4}\pi(x)(\gamma_0\gamma_m-\gamma_m\gamma_0)\hat{\psi}(x), \quad (3.11)$$

where π is canonically conjugate to $\hat{\psi}$:

$$\pi(x)=\frac{\partial L}{\partial(\partial_0\hat{\psi}(x))}. \quad (3.12)$$

In the above, $\hat{\psi}$ denotes the canonical field to which ψ is assumed to be proportional. We also note that if derivative couplings involving $\hat{\psi}$ are absent, one has

$$\frac{\partial L}{\partial(\partial_m\hat{\psi}(x))}=\pi(x)\gamma_m\gamma_0. \quad (3.13)$$

Now, from Eq. (3.11) one easily obtains (using the antisymmetry of f in the first two indices)

$$\partial^\lambda f_{\lambda 00}(x)=\partial^m f_{m00}(x)=\tfrac{1}{4}\partial^m[\pi(x)(\gamma_0\gamma_m-\gamma_m\gamma_0)\hat{\psi}(x)].$$

Then from Eqs. (3.2) and (3.3) one obtains

$$[iT_{00}^\dagger(x),\gamma_0\psi(y)]=\partial^\mu\gamma_\mu\psi(x)\delta(\mathbf{x}-\mathbf{y})+\tfrac{1}{2}\gamma_k\psi(x)\frac{\overleftrightarrow{\partial}}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) \quad (3.14)$$

or

$$[iT_{00}(x),\bar{\psi}(y)\gamma_0]=\partial^\mu\bar{\psi}(x)\gamma_\mu\delta(\mathbf{x}-\mathbf{y})+\tfrac{1}{2}\bar{\psi}(x)\gamma_k\frac{\overleftrightarrow{\partial}}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \quad (3.15)$$

Next, for the time component of a canonical current defined by

$$J_0{}^a(x)=-i\pi^b(x)\phi^c(x)F_{bc}{}^a, \quad (3.16)$$

where $F_{bc}{}^a$ are the structure constants of the group considered, we use Eqs. (3.4) and (3.16) to obtain (J_0 is Hermitian)

$$[iT_{00}(x),J_0(y)]=\partial^\mu J_\mu(x)\delta(\mathbf{x}-\mathbf{y})+J_k(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \quad (3.17)$$

{To derive the above equation it is sufficient to realize that $[\partial^\mu f_{\mu 00}(x),J_0(y)]$ contains at most a FOST, and thus the result follows by covariance.}

Next we assume the absence of derivative couplings involving $\hat{\psi}$ and obtain from Eqs. (3.10) and (3.13) [using Eq. (4.18) in Ref. 32]

$$\partial^\mu f_{0\mu n}(x)=\tfrac{1}{8}\partial^m[\pi(x)(\gamma_m\gamma_n-\gamma_n\gamma_m)\hat{\psi}(x)+4g_{mn}\pi(x)\hat{\psi}(x)]. \quad (3.18)$$

Since the above expression contains only canonica variables, we may calculate $[iT_{0m}(x),\bar{\psi}(y)\gamma_0]$. We obtain

$$[iT_{0n}(x),\bar{\psi}(y)\gamma_0]=\tfrac{1}{2}\partial_n\bar{\psi}(x)\gamma_0\delta(\mathbf{x}-\mathbf{y})+\tfrac{1}{8}\bar{\psi}(y)(\gamma_m\gamma_n-\gamma_n\gamma_m)\gamma_0\frac{\partial}{\partial x_m}\delta(\mathbf{x}-\mathbf{y})-\tfrac{1}{2}\bar{\psi}(x)\gamma_0\frac{\partial}{\partial x^n}\delta(\mathbf{x}-\mathbf{y}). \quad (3.19)$$

Now, since because of Eqs. (2.20b), (2.21b), (3.15), and (3.19), at most a second-order ST is contained in $[iT_{00}(x),\bar{f}_m(y)\gamma_0]$ [f_m has been defined in Eq. (1.42)] and at most a FOST in $[iT_{0m}(x),\bar{\psi}(y)\gamma_0]$, we obtain from Eq. (2.7) and covariance that Eq. (3.19) is equivalent to

$$[iT_{00}(x)f_m(y)\gamma_0]=\partial^\mu f_m(x)\gamma_\mu\delta(\mathbf{x}-\mathbf{y})+\tfrac{1}{2}\bar{f}_m(x)\gamma_k\frac{\overleftrightarrow{\partial}}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \quad (3.20)$$

Therefore, Eq. (3.20) is derived for canonical theories which do not involve derivatives of $\hat{\psi}$ in the interaction Lagrangian.

Our next task is to determine the commutators $[iT_{00}(x),J_k(y)]$, $[iT_{0k}(x),J_0(y)]$, and $[iT_{00}(x),\partial^\mu J_\mu(y)]$ in canonical theories under the assumption that the

[35] Of course, $\Theta_{\mu\nu}$ is defined only in canonical theories and the more general results of covariance follow only from the commutators involving $T_{\mu\nu}$.

interaction does not contain derivatives of the fermion field. We will make use of Eq. (3.6), obtained in Ref. 3, and show under the present assumptions that the additional terms in the definition of T_{0m} do not contribute so that

$$[iT_{0m}(x),J_0(y)]=-J_0(x)\frac{\partial}{\partial x^m}\delta(\mathbf{x}-\mathbf{y}). \quad (3.21)$$

The relations in Sec. II will then be used to derive $[iT_{00}(x),J_k(y)]$ together with further commutators.

The derivation of Eq. (3.21) from Eqs. (3.6) and (3.18) is a straightforward calculation. Using the associative law and defining Γ_{mn} by

$$\Gamma_{mn}=\gamma_m\gamma_n-\gamma_n\gamma_m, \quad (3.22)$$

we may write

$$[T_{0m}^\dagger(x)-\Theta_{0m}(x),J_0^e(y)]$$
$$=\tfrac{1}{8}\frac{\partial}{\partial x_m}\{[\pi_a{}^\alpha(x)\psi_a{}^\beta(x),\pi_c{}^\delta(y)\psi_d{}^\delta(y)]\Gamma_{mn}{}^{\alpha\beta}F_{cd}{}^e$$
$$+4g_{mn}[\pi_a{}^\alpha(x)\psi_a{}^\alpha(x),\pi_c{}^\delta(y)\psi_d{}^\delta(y)]F_{cd}{}^e\}=0. \quad (3.23)$$

We next note that because of Eq. (2.9) it follows from the absence of ST of higher order than 1 in Eq. (3.21) and from Eq. (3.17) that

$$[iT_{00}(x),J_m(y)]=J_{0m}(x)\delta(\mathbf{x}-\mathbf{y})-J_0(x)\frac{\partial}{\partial x^m}\delta(\mathbf{x}-\mathbf{y}). \quad (3.24)$$

Note that in obtaining Eq. (3.24) from (3.21) the special form of the FOST was not needed. Therefore, Eq. (3.24) depends only on the absence of ST's of order higher than 1 in Eq. (3.21) and on the absence of NCT's in Eq. (3.17). Now, because of Eqs. (2.20c) and (2.20d) the absence of ST's of order higher than 1 in Eq. (3.21) and in the commutator $[iT_{0l}(x),J_m(y)]$ follows from Eqs. (3.17) and (3.24). It also follows from these equations [using Eqs. (2.21c) and (2.21d)] that no ST's of order higher than 2 are present in the commutators of T_{00} with $\partial^\mu J_\mu$ and with J_{0m}. Thus [Eq. (2.20a)] in this case ST of order higher than 2 are also absent in $[iT_{0m}(x),\partial^\mu J_\mu(y)]$.

Next, assuming Eqs. (3.17) and (3.24), we show that Eq. (3.21) and

$$[iT_{00}(x),\partial^\mu J_\mu(y)]=\partial_0\partial^\mu J_\mu(x)\delta(\mathbf{x}-\mathbf{y}) \quad (3.25)$$

are equivalent. Then, since Eqs. (3.17) and (3.21) are derived for certain[15] models and since, as shown above, Eq. (3.24) follows from Eq. (3.21), this establishes the validity of Eq. (3.25) in these models. To prove the equivalence, we note that Eq. (3.21) follows from Eq. (3.25) since [Eq. (2.20c)] at most a FOST is contained in this commutator which [Eq. (B3)] is as given in Eq. (3.21). Assuming Eq. (3.21), we first note [Eq. (2.21c)] that at most a second-order ST is contained in $[iT_{00}(x),\partial^\mu J_\mu(y)]$. This ST also vanishes [Eq. (B3)], and Eq. (3.25) follows from covariance. Thus, Eq. (3.25) is indeed a consequence of Eqs. (3.17) and (3.21).

Furthermore, we note that as soon as Eq. (3.25) is established, the absence of ST's of order higher than 2 in $[iT_{0l}(x),\partial_0\partial^\mu J_\mu(y)]$ [Eq. (2.21a)] and of order higher than 1 in $[iT_{0m}(x),\partial^\mu J_\mu(y)]$ [Eq. (2.20a)] follows [note also Eq. (B2)]. In addition, from Eqs. (3.17) and (3.24) it may be seen that ST's of order higher than 2 are absent in $[iT_{00}(x),\partial^\mu J_\mu(y)]$ [Eq. (2.21c)] and in $[iT_{00}(x),J_{0l}(y)]$ [Eq. (2.21d)] and that ST's of order higher than 1 are absent in $[iT_{0m}(x),J_l(y)]$ [Eq. (2.20d)]. Note also that the relation

$$\int d^3y(y-z)_n[iT_{0m}(y),J_l(z)]=g_{ln}J_m(z)-\tfrac{1}{2}\int d^3y(y-z)_m(y-z)_n\times[iT_{00}(y),J_{0l}(z)] \quad (3.26)$$

[Eq. (B4)] connects the ST in $[iT_{0m}(y),J_l(z)]$ with those in $[iT_{00}(y),J_{0l}(z)]$.

We would like to investigate next scalar fields $\phi(x)$ which are proportional to canonical ones. For example, the divergence of the axial-vector current in Lagrangian models of partial conservation of axial-vector current (PCAC) has this property. Since, for such fields,[9,32]

$$[i\Theta_{00}(x),\phi(y)]=\partial_0\phi(x)\delta(\mathbf{x}-\mathbf{y}) \quad (3.27)$$

and

$$[i\Theta_{00}(x)-iT_{00}^\dagger(x),\phi(y)]=0, \quad (3.28)$$

we have

$$[iT_{00}(x),\phi(y)]=\partial_0\phi(x)\delta(\mathbf{x}-\mathbf{y}). \quad (3.29)$$

If the coupling does not contain derivatives of the fields carrying spin, we have (since ϕ is Hermitian)

$$[iT_{0m}(x),\phi(y)]=\partial_m\phi(x)\delta(\mathbf{x}-\mathbf{y}). \quad (3.30)$$

[The absence of ST's of order higher than 1 is already a consequence of Eqs. (2.20a) and (3.29).] From Eqs. (2.21a) and (3.29) we learn that at most a second-order ST is contained in $[iT_{00}(x),\partial_0\phi(y)]$ and that this term vanishes due to (3.29), (3.30), and (B2). Thus it follows (using covariance) that

$$[iT_{00}(x),\partial_0\phi(y)]=\Box\phi(x)\delta(\mathbf{x}-\mathbf{y})+\partial_k\phi(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \quad (3.31)$$

This relation, which may have been obtained easily from the formulas given in the Appendix of Ref. 9 if $\partial_0\hat{\phi}$ is canonically conjugate to $\hat{\phi}$ (i.e., if derivative couplings involving $\hat{\phi}$ are absent) now is seen to be valid even when the coupling contains derivatives of ϕ.

Using Eqs. (B2), (2.20a), and (3.29), one may evidently also derive Eq. (3.30) from (3.31). From (3.29), (3.31), (2.20e), and (2.21e) it follows that ST's of order higher than 2 (1) are absent in the ETC $[iT_{00}(x),\Box\phi(y)]$ ($[iT_{0k}(x),\partial_0\phi(y)]$). From Eqs. (3.31) and (B5) we thus obtain equivalence of the relations

$$[iT_{0m}(x),\partial_0\phi(y)]=-\partial_0\phi(x)\frac{\partial}{\partial x^m}\delta(\mathbf{x}-\mathbf{y}) \quad (3.32)$$

and

$$[iT_{00}(x),(\Box+m^2)\phi(y)] =(\Box+m^2)\partial_0\phi(x)\delta(\mathbf{x}-\mathbf{y}). \quad (3.33)$$

Note that Eq. (3.32) is a consequence of the canonical rules in case that derivative couplings are completely absent.

The following statements summarize the content of the above discussion:

Statement 1a. Assume Eq. (3.15). Then (1) there are no ST's of order higher than 1 in $[iT_{0m}(x),\bar{\psi}(y)]$; (2) Eqs. (3.19) and (3.20) are equivalent.

Statement 1b. For a nucleon field proportional to a canonical field $\hat{\psi}$, Eq. (3.15) holds. If the interaction Lagrangian does not contain derivatives of $\hat{\psi}$ then, in addition, Eq. (3.19) holds [and consequently Eq. (3.20) holds].

Statement 2a. Assume Eq. (3.17) and at most a FOST in $[iT_{0m}(x),J_0(y)]$. Then Eq. (3.24) follows.

Statement 2b. Assume Eqs. (3.17) and (3.24). Then (1) Eqs. (3.21) and (3.25) are equivalent; (2) ST's of order higher than 1 are absent in the commutators $[iT_{0m}(x),J_0(y)]$ and $[iT_{0m}(x),J_l(y)]$. (3) ST's of order higher than 2 are absent in the commutators $[iT_{00}(x),\partial^\mu J_\mu(y)]$, $[iT_{00}(x),J_{0m}(y)]$, and $[iT_{0k}(x),\partial^\mu J_\mu(y)]$.

Statement. 2c. If $J_\mu(x)$ denotes a canonical current, Eq. (3.17) holds. If the interaction Lagrangian does not contain derivatives of $\hat{\psi}$, Eq. (3.21) holds [and consequently Eqs. (3.24) and (3.25) also hold].

Statement 3a. Assume Eq. (3.29). Then (1) Eqs. (3.30) and (3.31) are equivalent; (2) there are no ST's of order higher than 1 (2) in the ETC $[iT_{0k}(x),\partial_0\phi(y)]$ ($[iT_{00}(x),\Box\phi(y)]$).

Statement 3b. Assume Eqs. (3.29) and (3.30). Then Eqs. (3.32) and (3.33) are equivalent.

Statement 3c. If $\hat{\phi}(x)$ is a canonical spin-0 field, Eq. (3.29) holds. If the coupling does not contain derivatives of $\hat{\psi}$, Eq. (3.30) [and consequently (3.31)] holds. If all derivative couplings are absent, Eq. (3.32) [and consequently (3.33)] also holds.

Finally, we note relations analogous to Eq. (1.3) which may also be obtained. This relation itself (as noted in the Introduction) follows from Eq. (3.17) and has thus been derived for canonical currents. Analogously we may derive Eq. (1.7) from Eq. (3.15) (which holds for all spinor fields ψ proportional to canonical fields). If the coupling does not contain derivatives of $\hat{\psi}$, we may write [from Eq. (3.20) and statement 1b]

$$\int d^3y[iT_{00}(x),\bar{f}_m(y)\gamma_0]=\partial^\mu\bar{f}_m(x)\gamma_\mu-\tfrac{1}{2}\partial^k\bar{f}_m(x)\gamma_k. \quad (3.34)$$

From Eqs. (3.21), (3.24), (3.25), and with the assumption of Eq. (3.34), we have (using statement 2c)

$$\int d^3y[iT_{0m}(x),J_0(y)]=0, \quad (3.35)$$

$$\int d^3y[iT_{00}(x),J_m(y)]=J_{0m}(y), \quad (3.36)$$

and

$$\int d^3y[iT_{00}(x),\partial^\mu J_\mu(y)]=\partial_0\partial^\mu J_\mu(x). \quad (3.37)$$

If ϕ is proportional to a canonical field, it follows from Eq. (3.29) and statement 3c that

$$\int d^3y[iT_{00}(x),\phi(y)]=\partial_0\phi(x). \quad (3.38)$$

If the coupling does not contain derivatives of $\hat{\psi}$, it follows from statement 3c and Eqs. (3.30) and (3.31)

$$\int d^3y[iT_{0m}(x),\phi(y)]=\partial_m\phi(x) \quad (3.39)$$

and

$$\int d^3y[iT_{00}(x),\partial_0\phi(y)]=\Box\phi(x). \quad (3.40)$$

If the coupling does not contain any derivatives of the canonical variables, we obtain from Eqs. (3.32), (3.33), and statement 3c

$$\int d^3y[iT_{0m}(x),\partial_0\phi(y)]=0 \quad (3.41)$$

and

$$\int d^3y[iT_{00}(x),(\Box+m^2)\phi(y)]=(\Box+m^2)\partial_0\phi(x). \quad (3.42)$$

IV. SELECTED APPLICATIONS

Applications of the relations obtained in the preceding sections may be distinguished as to whether the result depends on absence of NCT or as to whether it depends on the specific form of the FOST in Eqs. (1.24)–(1.29) obtained in canonical theories. Those applications which only depend on general assumptions such as existence of equal-time limits and Jacobi identities will as such be of a much higher generality than the others.

First let us consider the applications to the ETC assumed in Eq. (1.37). We then note[26] the validity of Eq. (1.38) [assuming the usual $SU(2)\otimes SU(2)$ commutators between charges and currents] and write the Jacobi identity for $[iT_{00}(x),[J_0{}^a(y),\bar{\psi}(z)\gamma_0]]$, making use of Eqs. (1.15) and (1.17):

$$[J_k{}^a(x),\bar{\psi}(z)\gamma_0]\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})=\tfrac{1}{2}\hat{\partial}^\mu\bar{\psi}(x)\Gamma^a\gamma_\mu\delta(\mathbf{x}-\mathbf{z})\delta(\mathbf{y}-\mathbf{z})+\tfrac{1}{4}\bar{\psi}(x)\Gamma^a\gamma_k\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{z})\delta(\mathbf{y}-\mathbf{z})+\tfrac{1}{4}\bar{\psi}(y)\Gamma^a\gamma_k\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})$$
$$\times\delta(\mathbf{x}-\mathbf{z})-\tfrac{1}{4}\bar{\psi}(y)\Gamma^a\gamma_k\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{z})\delta(\mathbf{y}-\mathbf{z})-[\partial^\mu J_\mu{}^a(x),\bar{\psi}(z)\gamma_0]\delta(\mathbf{x}-\mathbf{y})$$
$$-[J_0{}^a(y),\partial^\mu\bar{\psi}(x)\gamma_\mu]\delta(\mathbf{x}-\mathbf{z})+Z(x,y,z)\,. \quad (4.1)$$

In the above equation $Z(x,y,z)$ denotes the sum of the contributions from NCT; it has the property stated in Eq. (1.44). We have also defined

$$\hat{\partial}_\mu=(\partial_0,\tfrac{1}{2}\partial_k)\,. \quad (4.2)$$

In Refs. 8 and 9 this equation with $Z=0$ has been obtained if NCT are absent. Multiplying Eq. (4.1) by $(x-y)_m$, integrating over $\mathbf{x}$, and using Eq. (1.44), we obtain

$$[J_m{}^a(y),\bar{\psi}(z)\gamma_0]=\bar{\psi}(y)(\tfrac{1}{2}\Gamma^a)\gamma_m\delta(\mathbf{y}-\mathbf{z})+i(z-y)_m[J_0{}^a(y),f_m{}^\dagger(y)\gamma_0]\,, \quad (4.3)$$

where f_m has been defined in Eq. (1.42). Note that Eq. (4.3) has been obtained in Refs. 8 and 9 assuming absence of NCT and is thus seen to hold independent of this assumption. We next discuss the dependence of the further results of Refs. 8 and 9 on the model-dependent assumptions made there.

We multiply Eq. (4.1) by $(x-z)_m$ and integrate over $\mathbf{x}$ to obtain

$$[J_m{}^a(y),\bar{\psi}(z)\gamma_0]+(y-z)_m\frac{\partial}{\partial y_k}[J_k{}^a(y),\bar{\psi}(z)\gamma_0]=(y-z)_m[\partial^\mu J_\mu{}^a(y),\bar{\psi}(z)\gamma_0]\,. \quad (4.4)$$

Integrating the above equation over $\mathbf{y}$, we obtain

$$\int d^3y(y-z)_m[\partial^\mu J_\mu{}^a(y),\bar{\psi}(z)\gamma_0]=0\,, \quad (4.5)$$

i.e., the above ETC has no FOST if one writes its ST with arguments z. This result has been obtained in Ref. 26 by a more explicit use of covariance. Multiplying Eq. (4.4) by $(y-z)_n$ and integrating over $\mathbf{y}$, we obtain

$$\int d^3y(y-z)_n[J_m{}^a(y),\bar{\psi}(z)\gamma_0]=-\int d^3y(y-z)_m(y-z)_n[\partial^\mu J_\mu{}^a(y),\bar{\psi}(z)\gamma_0]\,. \quad (4.6)$$

Thus for conserved currents FOST are absent in $[J_m{}^a(y),\bar{\psi}(z)\gamma_0]$. Furthermore it may be seen by multiplying Eq. (4.4) with $(y-z)_{n_1}\cdots(y-z)_N$, integrating over $\mathbf{y}$, and using a little algebra, that for conserved currents also no ST of higher order are contained in this commutator. This result has also been obtained in Ref. 26 by a direct use of covariance. In Ref. 9 this result has been used to obtain Eq. (4.3) for the conserved vector currents. We note in passing that the $\mathbf{x}$-integrated Eq. (4.3), for conserved currents is a simple consequence of the Heisenberg equation of motion.

Applying the manipulation described above to Eq. (4.6) for $\partial^\mu J_\mu\neq 0$ relations between ST are obtained. To obtain the most powerful results of Refs. 8 and 9, one must assume the validity of Eqs. (3.15) and (3.17). Then $Z=0$ in Eq. (4.1) and integration over $\mathbf{y}$ sohws the absence of ST's in the ETC $[\partial^\mu J_\mu{}^a(x),\bar{\psi}(y)\gamma_0]$. Multiplying Eq. (4.1) by $(x-z)_m(x-y)_n$, integration over $\mathbf{y}$ shows then also the absence of ST's in the ETC $[J_k{}^a(x),\bar{\psi}(y)\gamma_0]$ for nonconserved currents.

It is evident that Eq. (1.37) is a natural assumption for current-field commutators in a model in which ψ is proportional to a canonical field and $J_\mu{}^a(x)$ is a canonical current since this equation is then a formal consequence of the canonical rules and the associative law. Since formal agreement with current-algebra commutators might therefore be expected, no such direct formal argument for algebra-of-fields commutators exists, and it is not at all clear from the outset if the assumed current-field commutator (1.37) would be in formal agreement with algebra-of-fields commutators. The answer depends on the ETC between space components of the currents and the fermion field. Since[8,9] proportionality of $\int d^3x[J_m{}^a(x),\bar{\psi}(y)\gamma_0]$ to $\bar{\psi}(y)\Gamma^a\gamma_m$ (the quark-model result) excludes (using the Jacobi identity for $\int d^3xd^3y[[J_m{}^a(x),J_n{}^b(y)],\bar{\psi}(z)\gamma_0]$) commutativity between the space components of the currents, Eq. (4.3) shows that for a free fermion field the field-algebra commutators are in fact excluded. However, one expects[29] in Eq. (4.3) large effects due to the interaction term [we shall exhibit below a model for the nucleon currents for which the right-hand side of Eq. (4.3) vanishes], in sharp distinction from Eq. (1.46) in which the deviation from the twice-integrated current-algebra commutators is due to ST's in $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$, a term which is usually assumed to arise only in electromagnetic or weak interactions. Incidentally, note that the associative law and canonical rules do not allow for a ST in the ETC between the time component of a canonical current and a canonical field. The presence of such terms[3] in $[J_0{}^a(x),\partial^\mu A_\mu{}^\beta(y)]$ in case of minimal electromagnetic coupling shows an immediate conflict between formal reasoning, PCAC, and minimal electromagnetic coupling.

We next investigate

$$[A_k{}^\alpha(x),\bar\psi(y)\gamma_0]=0 \tag{4.7}$$

which, using the Jacobi identity involving

$$[[J_0{}^a(x),J_k{}^b(y)],\bar\psi(z)\gamma_0], \tag{4.8}$$

is equivalent to

$$[V_k{}^\alpha(x),\bar\psi(y)\gamma_0]=0. \tag{4.9}$$

In order to motivate the above choice, we first mention that Eqs. (4.7) and (4.9) are simple consequences of the canonical rules if current-field identities and PCAC hold. Next note that the above choice is the simplest possibility to express current-field commutators as linear forms in ψ and its space derivatives which is compatible with the algebra of fields. As another justification of Eqs. (4.7) and (4.9) we will give a model for f_m for which these equations hold.

Consider the part of the nucleon current f_m which may be written as

$$f_m(x)\equiv(i\gamma^\mu\partial_\mu-m)\psi(x)=\{P(\phi(x)) +[C_VV_\mu{}^\alpha(x)+c_AA_\mu{}^\alpha(x)\gamma_5]\gamma^\mu\tau^\alpha\}\psi(x), \tag{4.10}$$

where $P(\phi(x))$ denotes any polynomial in the pion field with the right quantum numbers. Concerning the part of the nucleon current not contained in Eq. (4.10), it will be sufficient for our conclusion to assume that its equal-time commutator with $J_0{}^a$ contains no FOST.

We assume field-algebra commutators for the currents and define the c-number ST by

$$[J_0{}^a(x),J_k{}^b(y)]=ie^{abc}J_c{}^k(x)\delta(\mathbf{x}-\mathbf{y}) +ic\delta^{ab}\frac{\partial}{\partial x^k}\delta(\mathbf{x}-\mathbf{y}). \tag{4.11}$$

Next we remark that owing to Eq. (4.11), PCAC, Eq. (1.46), and the absence of ST's of order higher than 1 [36] in the ETC $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$, we may write

$$\int d^3x(x-y)_m[J_0{}^a(x),\phi^b(y)]=0. \tag{4.12}$$

We define normalized π, A_1, and ρ fields by[31]

$$\partial^\mu A_\mu{}^\alpha(x)=m_\pi{}^2f_\pi\phi^\alpha(x), \tag{4.13}$$

$$a_\mu{}^\alpha(x)=f_{A_1}{}^{-1}[f_\pi\partial_\mu\phi^\alpha(x)+A_\mu{}^\alpha(x)], \tag{4.14}$$

and

$$v_\mu{}^\alpha(x)=f_\rho{}^{-1}V_\mu{}^\alpha(x). \tag{4.15}$$

[36] This fact is derived in Appendix A of the present paper (statement A1) using the methods of Refs. 2 and 3.

Next we integrate Eq. (4.3) over $\mathbf{y}$ and obtain

$$\int d^3y[J_m{}^a(y),\bar\psi(z)\gamma_0]=\bar\psi(z)(\tfrac{1}{2}\Gamma^a)\gamma_m +\int d^3y\, i(z-y)_m[J_0{}^a(y),f_m{}^\dagger(z)\gamma_0]. \tag{4.16}$$

Upon use of Eqs. (4.10), (4.12), and (4.16), it follows that

$$\int d^3x[J_m{}^a(x),\bar\psi(y)\gamma_0]=(-c_Jc+\tfrac{1}{2})\bar\psi(y)\Gamma^a\gamma_k. \tag{4.17}$$

Using the Jacobi identity for $\int d^3xd^3y[[J_m{}^a(x),J_n{}^b(y)], \bar\psi(z)\gamma_0]$, one sees that Eq. (4.17) is incompatible with the assumed commutativity of the space components of the currents unless

$$2cc_V=1 \tag{4.18}$$

and

$$2cc_A=1. \tag{4.19}$$

We now use Eqs. (4.13)–(4.15) to express the nucleon current in Eq. (4.10) in terms of the normalized π, A_1, and ρ fields as

$$f_m(x)=\{P(\phi(x))+[-g_\pi\partial_\mu\phi^\alpha(x)\gamma_5+g_\rho{}^\alpha V_\mu{}^\alpha(x) +g_{A_1}a_\mu{}^\alpha(x)\gamma_5]\gamma^\mu(\tfrac{1}{2}\tau^\alpha)\}\psi(x), \tag{4.20}$$

where we have defined

$$g_X=2c_Jf_X, \tag{4.21}$$

i.e.,

$$g_\pi f_\pi{}^{-1}=1/c, \tag{4.22}$$

$$g_{A_1}f_{A_1}{}^{-1}=1/c, \tag{4.23}$$

and

$$g_\rho f_\rho{}^{-1}=1/c. \tag{4.24}$$

Comparing Eqs. (4.23) and (4.24), we obtain

$$g_{A_1}f_\rho=g_\rho f_{A_1}. \tag{4.25}$$

A more detailed result is obtained if one saturates the vacuum expectation value of (4.11) for $a=1$–3 by the ρ-meson intermediate state. Then

$$c=f_\rho{}^2m_\rho{}^{-2} \tag{4.26}$$

which, combined with Eq. (4.24), gives

$$g_\rho=m_\rho{}^2f_\rho{}^{-1}. \tag{4.27}$$

Combining this with Eq. (4.25) we have

$$g_{A_1}=f_{A_1}(m_\rho{}^2/f_\rho{}^2). \tag{4.28}$$

Assuming the validity of the usual saturation of the Weinberg spectral-function sum rules,[37] one has

$$|f_{A_1}|=|f_\rho|. \tag{4.29}$$

[37] S. Weinberg, Phys. Rev. Letters 18, 607 (1967).

Thus Eqs. (4.25) and (4.28) may be written as

$$|g_{A_1}|=|g_\rho| \tag{4.30}$$

and

$$|g_{A_1}|=m_\rho^2/f_\rho=(m_\rho/m_{A_1})^2 m_{A_1}^2|f_{A_1}|^{-1}. \tag{4.31}$$

Having discussed a nucleon current such that Eqs. (4.7) and (4.9) hold, we would next like to obtain the consequences of covariance for this case. From Eq. (4.3) we obtain

$$\bar\psi(y)(\tfrac{1}{2}\Gamma^a)\gamma_m\delta(\mathbf{y}-\mathbf{z}) =i(y-z)_m[J_0{}^a(y),f_m{}^\dagger(y)\gamma_0] \tag{4.32}$$

or, equivalently,

$$0=(y-z)_m[\partial^\mu J_\mu{}^a(y),\bar\psi(z)\gamma_0] =(y-z)_m[\partial^0 J_0{}^a(y),\bar\psi(z)\gamma_0], \tag{4.33}$$

i.e., the result obtained for the nonconserved axial current from assuming absence of NCT in Eqs. (1.15) and (1.17) now holds due to covariance even if NCT are present in these equations.

Next we use Eqs. (1.15) and (1.18) to write the Jacobi identity involving $[iT_{00}(x),[J_l{}^a(y),\bar\psi(z)\gamma_0]]$ as

$$\begin{aligned}[iT_{00}(x),[J_l{}^a(y),\bar\psi(z)\gamma_0]]&=[J_l{}^a(y),\partial^\mu\bar\psi(x)\gamma_\mu]\delta(\mathbf{x}-\mathbf{z})\\ &-\tfrac{1}{2}\frac{\partial}{\partial x_k}[J_l{}^a(y),\bar\psi(x)\gamma_k]\delta(\mathbf{x}-\mathbf{z})+\tfrac{1}{2}[J_l{}^a(y),\bar\psi(x)\gamma_k]\\ &\times\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{z})+[J_{0l}{}^a(x),\bar\psi(z)\gamma_0]\delta(\mathbf{x}-\mathbf{y})-\bar\psi(x)(\tfrac{1}{2}\Gamma^a)\gamma_0\\ &\times\delta(\mathbf{x}-\mathbf{z})\frac{\partial}{\partial x^l}\delta(\mathbf{x}-\mathbf{y})+Z(x,y,z).\end{aligned} \tag{4.34}$$

Under the present assumptions of Eqs. (4.7) and (4.9), the left-hand side as well as the $[J_l{}^a(x),\bar\psi(y)\gamma_0]$ terms on the right-hand side vanish. Multiplying the above equation with $(x-z)_m$ and integrating over $\mathbf{x}$, we thus obtain

$$(y-z)_m[J_{0l}{}^a(y),\bar\psi(z)\gamma_0]=0; \tag{4.35}$$

i.e., in canonical realizations of current-field identities with canonical fermion fields $\hat\psi$, we have

$$(y-z)_m\left[\frac{\partial L}{\partial(\partial_0 J_l(y))}-J_{0l}{}^a(y),\bar\psi(z)\gamma_0\right]=0 \tag{4.36}$$

because of the canonical rules. Multiplying Eq. (4.34) by $(x-y)_m$ and integrating over $\mathbf{x}$, we obtain a relation which is identical to Eq. (4.34) if use is made of the Heisenberg equation of motion. It reads

$$(z-y)_m[J_l{}^a(y),\partial^\mu\bar\psi(z)\gamma_\mu]=\bar\psi(z)(\tfrac{1}{2}\Gamma^a)\gamma_0\delta(\mathbf{y}-\mathbf{z}). \tag{4.37}$$

Note that as a consequence of the Heisenberg equation of motion one may also write

$$\int d^3y[J_k{}^a(y),{}^\mu\bar\psi(x)\gamma_\mu](\mathbf{z}-\mathbf{x}) =-[J_{0k}{}^a(x),\bar\psi(z)\gamma_0]. \tag{4.38}$$

Finally, we would like to obtain the restrictions which Lorentz covariance imposes on currents obeying the algebra-of-fields commutators. To this end, we first write the Jacobi identity for $[iT_{00}(x),[J_k{}^a(y),J_l{}^b(z)]]$ as

$$\begin{aligned}&[J_l{}^b(z),J_{0k}{}^a(x)]\delta(\mathbf{x}-\mathbf{y})-[J_k{}^a(y),J_{0l}{}^b(x)]\delta(\mathbf{x}-\mathbf{z})\\ &=ie^{bac}J_k{}^c(x)\delta(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial x^l}\delta(\mathbf{x}-\mathbf{z})-ie^{abc}J_l{}^c(x)\delta(\mathbf{x}-\mathbf{z})\\ &\qquad\times\frac{\partial}{\partial x^k}\delta(\mathbf{x}-\mathbf{y})+Z(x,y,z).\end{aligned} \tag{4.39}$$

Note that the c-number ST contributions have dropped. As usual, Z has the property stated in Eq. (1.44). Multiplying Eq. (4.39) by $(x-z)_m$ and integrating over $\mathbf{x}$, we obtain

$$(z-y)_m[J_l{}^b(z),J_{0k}{}^a(y)]=ie^{bac}g_{lm}J_k{}^c(y)^a(\mathbf{y}-\mathbf{z}). \tag{4.40}$$

From this we see that canonical realizations of current-field identities[31] require derivative couplings involving the vector and axial-vector fields. From Eq. (4.40) we also obtain absence of ST of order higher than 1 in the ETC $[J_l{}^b(z),J_{0k}{}^a(y)]$. Note once again that only covariance is required in this application.

Next consider the Jacobi identity for $[iT_{00}(x),[J_0{}^a(y),J_l{}^b(z)]]$, which under the present assumptions reads

$$\begin{aligned}&[\partial^\mu J_\mu{}^a(x),J_l{}^b(z)]\delta(\mathbf{x}-\mathbf{y})+[J_0{}^a(y),J_{0l}{}^b(x)]\delta(\mathbf{x}-\mathbf{z})\\ &=ie^{abc}J_{0l}{}^c(x)\delta(\mathbf{x}-\mathbf{z})\delta(\mathbf{y}-\mathbf{z})-ie^{abc}J_0{}^c(x)\frac{\partial}{\partial x^l}\delta(\mathbf{x}-\mathbf{z})\\ &\qquad\times\delta(\mathbf{y}-\mathbf{z})+ie^{abc}J_0{}^c(x)\delta(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial x^l}\delta(\mathbf{x}-\mathbf{z})\\ &\qquad+Z(x,y,z).\end{aligned} \tag{4.41}$$

We multiply the above equation by $(x-y)_m$ and integrate over $\mathbf{x}$ to obtain

$$(z-y)_m[J_0{}^a(y),J_{0l}{}^b(x)]=ie^{abc}g_{ml}J_0{}^c(y)\delta(\mathbf{x}-\mathbf{y}). \tag{4.42}$$

Multiplying Eq. (4.41) by $(x-z)_m$ and integrating over $\mathbf{x}$, we obtain

$$(y-z)_m[\partial^\mu J_\mu{}^a(y),J_l{}^b(z)]=0. \quad (4.43)$$

Equations (4.42) and (4.43) show that ST's of orders higher than 1 are absent in the ETC involved. Finally, those weaker relations which follow by use of the Heisenberg equation of motion alone may be obtained from Eqs. (4.39) and (4.41) by integration over $\mathbf{x}$ and read

$$[J_l{}^b(z),J_{0k}{}^a(y)]-[J_k{}^a(y),J_{0l}{}^b(z)]=ie^{bac}J_k{}^c(y)\frac{\partial}{\partial y^l}\times\delta(\mathbf{y}-\mathbf{z})-ie^{abc}J_l{}^c(z)\frac{\partial}{\partial z^k}\delta(\mathbf{y}-\mathbf{z}) \quad (4.44)$$

and

$$[\partial^\mu J_\mu{}^a(y),J_l{}^b(z)]+[J_0{}^a(y),J_{0l}{}^b(z)]=ie^{abc}J_{0l}{}^c(z)\times\delta(\mathbf{y}-\mathbf{z})-ie^{abc}J_0{}^c(z)\frac{\partial}{\partial z^l}\delta(\mathbf{z}-\mathbf{y}). \quad (4.45)$$

Note that for conserved currents Eqs. (4.42) and (4.45) are equivalent.

ACKNOWLEDGMENTS

One of the authors (H. G.) would like to thank Dr. Frank von Hippel for helpful discussions concerning applications of Eqs. (4.18) and (4.19) and Professor G. F. Chew and Professor J. D. Jackson for the kind hospitality extended to him at the Lawrence Radiation Laboratory. He gratefully acknowledges a NATO grant. The remaining author (J. K.) would like to thank Professor M. Sugawara for helpful discussions.

APPENDIX A

In this Appendix we would like to note more fully some consequences of Eq. (1.3) and of the Jacobi identity given in Eq. (1.43). We start by noting that as an immediate consequence of the Heisenberg equation of motion, we may write

$$[Q^a(x_0),\partial^0 J_0{}^b(x)]-\int d^3y[J_0{}^b(x),\partial^\mu J_\mu{}^a(y)]=ie^{abc}\partial^0 J_0{}^c(x), \quad (A1)$$

where charge–charge-density commutators have been assumed. Combining Eqs. (1.47) and (A1) one obtains as a consequence of assuming Eq. (1.3)

$$[Q^a(x_0),\partial^k J_k{}^b(x)]=ie^{abc}\partial^k J_k{}^c(x)-\int d^3y\{[J_0{}^b(x),\partial^\mu J_\mu{}^a(y)]-[J_0{}^b(y),\partial^\mu J_\mu{}^a(x)]\}. \quad (A2)$$

Next we compare this equation with Eq. (1.46) under the assumption that at most a second-order ST contributes to $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$. For later use we write, for any R,

$$[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]=\sigma^{ab}(x)\delta(\mathbf{x}-\mathbf{y})+\sum_{\alpha=1}^{R}\sigma_{\{k_\alpha\}}{}^{ab}(y)\frac{\partial}{\partial x_{k_1}}\cdots\frac{\partial}{\partial x_{k_\alpha}}\delta(\mathbf{x}-\mathbf{y}), \quad (A3)$$

and obtain for $R=2$, upon comparing Eqs. (1.46) and (A2), no restrictions on σ^{ab} and $\sigma_k{}^{ab}$, whereas for $\sigma_{kl}{}^{ab}$ it follows that [as a result of Eq. (1.3)]

$$\partial^k\partial^l\sigma_{kl}{}^{ab}(x)=0. \quad (A4)$$

A more involved relation would be obtained for general R.

We next obtain—using the method of Refs. 2 and 3—the consequences of covariance if at most a FOST is present in $[J_0{}^a(x),J_l{}^b(y)]$. In what follows, we do not specialize to $R=2$ in Eq. (A3). Multiplying Eq. (1.43) by $(x-y)_m$ and integrating over $\mathbf{x}$, we have

$$(z-y)_m[J_0{}^a(y),\partial^\mu J_\mu{}^b(z)]=ie_{bac}J_m{}^c(y)\delta(\mathbf{y}-\mathbf{z})-[J_0{}^b(z),J_m{}^a(y)]+[J_0{}^a(y),J_m{}^b(z)]+(z-y)_m\frac{\partial}{\partial z_k}[J_0{}^a(y),J_k{}^b(z)]. \quad (A5)$$

In the special case of $R=2$, it follows from this that [multiplying (A5) by $(z-y)_n$ and integrating over $\mathbf{z}$]

$$2\sigma_{mn}{}^{ab}(y)=S_{m;n}{}^{ba}(y)-S_{n;m}{}^{ab}(z). \quad (A6)$$

Assuming, for general R, the absence of ST's of order higher than 1 in $[J_0{}^a(y),J_k{}^b(z)]$, we multiply Eq. (A5) for $R\geqslant 2$ successively with $(z-y)_{m_1}\cdots(z-y)_{m_R},\cdots,(z-y)_{m_1}(z-y)_{m_2}$ and obtain that at most a ST of second order is contained in $[J_0{}^a(y),\partial^\mu J_\mu{}^b(z)]$. For this, therefore, Eq. (A5) holds. Especially for field-algebra commutators with only the usual first-order c-number ST, we find from this result, Eq. (A6), and Eq. (1.46), that no ST are contained in $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$.

We collect our results for the conserved vector current and the nonconserved axial-vector current in the following two statements:

Statement A1. If the usual field-algebra commutators of charge densities with currents hold, $[J_0{}^a(x),\partial^\mu J_\mu{}^b(y)]$ contains no ST's.

Statement A2. Assume

$$(y-x)_m[J_0{}^a(x),J_l{}^b(y)]=S_{l;m}{}^{ab}(x)\delta(\mathbf{x}-\mathbf{y}). \quad (A7)$$

Then $R=2$ in Eq. (A3),

$$S_{m;n}{}^{V_\alpha V_\beta}=S_{n;m}{}^{V_\beta V_\alpha}, \quad \text{(A8)}$$

and

$$S_{m;n}{}^{V_\beta A_\alpha}=S_{n;m}{}^{A_\alpha V_\beta}. \quad \text{(A9)}$$

Furthermore

$$\sigma_{mn}{}^{ab}=0 \quad \text{(A10)}$$

is equivalent to

$$S_{m;n}{}^{A_\alpha A_\beta}=S_{n;m}{}^{A_\beta A_\alpha}. \quad \text{(A11)}$$

Moreover, from Eqs. (A4) [as a result of Eq. (1.3)]

$$\partial^m\partial^n S_{m;n}{}^{A_\alpha A_\beta}=\partial^m\partial^n S_{n;m}{}^{A_\beta A_\alpha}. \quad \text{(A12)}$$

The reader should notice that Eqs. (A8), (A9), and (A11) have been obtained in Ref. 2 from different assumptions by essentially the same method. For another method to obtain analogous results see Ref. 7.

APPENDIX B

In this appendix we list the relation between the FOST in $[iT_{0m}(y),X(z)]$ and the second-order ST in $[iT_{0m}(y),Y(z)]$, with $X(z)$ and $Y(z)$ as defined in the text. We start by writing Eq. (2.7) once again:

$$\int d^3y(y-z)_n[iT_{0m}(y),\bar\psi(z)\gamma_0]=-\tfrac{1}{2}\int d^3y(y-z)_m \times(y-z)_n[iT_{00}(y),\partial^\mu\bar\psi(z)\gamma_\mu]+\tfrac{1}{2}g_{mn}\bar\psi(z)\gamma_0+\tfrac{1}{8}\bar\psi(z) \times\gamma_0(\gamma_m\gamma_n-\gamma_n\gamma_m)+\partial^\mu\chi_{mn}{}^{00}(z)\gamma_\mu\gamma_0. \quad \text{(B1)}$$

The analogous results for the other cases may be written as

$$\int d^3y(y-z)_n[iT_{0m}(y),\phi(z)] =-\tfrac{1}{2}\int d^3y(y-z)_m(y-z)_n[iT_{00}(y),\partial_0\phi(z)] +\partial_0\phi_{mn}{}^{00}(z), \quad \text{(B2)}$$

$$\int d^3y(y-z)_n[iT_{0m}(y),J_0(z)] =-\tfrac{1}{2}\int d^3y(y-z)_n(y-z)_m[iT_{00}(y),\partial^\mu J_\mu(z)] +g_{mn}J_0(z)+\partial^\mu j_{\mu;mn}{}^{00}(z), \quad \text{(B3)}$$

$$\int d^3y(y-z)_n[iT_{0m}(y),J_l(z)] =-\tfrac{1}{2}\int d^3y(y-z)_n(y-z)_m[iT_{00}(y),J_{0l}(z)] +g_{ln}J_m(z)+\partial_0 j_{l;mn}{}^{00}(z)-\partial_l j_{0;mn}{}^{00}(z), \quad \text{(B4)}$$

$$\int d^3y(y-z)_n[iT_{0m}(y),\partial_0\phi(z)] =-\tfrac{1}{2}\int d^3y(y-z)_n(y-z)_m[iT_{00}(y),\Box\phi(z)] +g_{mn}\partial_0\phi(z)+\partial_0\phi_{0;mn}{}^{00}(z)+\partial^k\partial_k\phi_{mn}{}^{00}(z), \quad \text{(B5)}$$

and

$$\int d^3y(y-z)_n[iT_{0m}(y),J_{0l}(z)] =-\tfrac{1}{2}\int d^3y(y-z)_n(y-z)_m[iT_{00}(y),\partial_0 J_{0l}(z)] -\tfrac{1}{2}g_{lm}J_{0n}(z)+\tfrac{1}{2}g_{ln}J_{0m}(z)+\partial_0 j_{0l;mn}{}^{00}(z). \quad \text{(B6)}$$