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Neutral-Pion Decay and the Weak-Coupling Limit*

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An exact version of Pagels's sum rule for neutral-pion decay is derived for massless pions. It is inconsistent, however, in the weak-coupling limit. If we modify this limit by including the σ meson, then the sum rule is consistent and also yields the physically reasonable relation $\kappa_p = -\kappa_n$.

I. INTRODUCTION

THE electromagnetic decay of $\pi^0 \to 2\gamma$ has aroused much concern lately owing to the null result of current algreba.[1] Adler[2] has recently shown that a model-dependent extension of the neutral PCAC (partially conserved axial-vector current) relation gives a reasonable correction to the lowest-order triangle-graph Steinberger calculation[3] of F_π.

On the other hand, Goldberger and Treiman,[4] Pagels,[5] and Abarbanel and Goldberger[6] have derived sum rules for F_π based on the gauge invariance of nucleon Compton scattering instead of on current algebra or PCAC. Fox and Freedman[7] have rederived the Pagels result from finite-energy sum rules in order to tighten Pagels's original arguments and to point out the sum rule's conspiratorial nature. Nevertheless, such sum rules are still suspect until tested against the weak-coupling ($g_{\pi NN}$ small) perturbation answer of Steinberger.[3]

It is to this end that we reconsider the Pagels sum rule. Moreover, by taking pions to be massless, we rederive in Sec. II the Pagels sum rule independently of conspiracy. This is analogous to the statement that the Goldberger-Treiman relation $gf_\pi = mg_A$ is exact in the limit of zero pion mass.[8]

In Sec. III we show that this massless-pion Pagels sum rule is not consistent with what is usually thought of as the weak-coupling limit (WCL). The problem is the anomalous magnetic moment of the proton κ_p which occurs in the Pagels sum rule. However, inclusion of the σ meson in the WCL leads to $\kappa_p = -\kappa_n$, and then the sum rule gives the known Steinberger result.

In the conclusion, we stress the importance of this WCL and test the various sum rules for F_π against it, as well as against the data.

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[1] J. Bell and R. Jackiw, Nuovo Cimento **60A**, 47 (1969).
[2] S. L. Adler, Phys. Rev. **177**, 2426 (1969).
[3] J. Steinberger, Phys. Rev. **76**, 180 (1949).
[4] M. L. Goldberger and S. B. Treiman, Nuovo Cimento **9**, 451 (1958).
[5] H. Pagels, Phys. Rev. **158**, 1566 (1967).
[6] H. D. I. Abarbanel and M. L. Goldberger, Phys. Rev. **165**, 1594 (1968); see also S. R. Choudhury and R. Rajaraman, *ibid.* **169**, 1218 (1968).
[7] G. C. Fox and D. Z. Freedman, Phys. Rev. **182**, 1628 (1969).
[8] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); S. Weinberg, *ibid.* **16**, 879 (1966); S. Mandelstam, Phys. Rev. **168**, 1884 (1968). For a general review of current algebra using the massless-pion technique see J. D. Bjorken and M. Nauenberg, Ann. Rev. Nucl. Sci. **18**, 229 (1968).

II. SUM RULE FOR MASSLESS-PION DECAY

We begin by deriving the zero-pion-mass Pagels sum rule. In the Appendix we review the kinematics of nucleon Compton scattering, extract the nucleon Born contributions, and write down the Regge behavior of the various invariant amplitudes. Using the amplitudes of Ref. 9, we note that the asymptotic behavior of $\tilde{A}_4$, $\tilde{A}_5$, and $\tilde{A}_6$ indicates that the combination

$$A_P^{(v)} \equiv 2\nu \tilde{A}_{45}^{(v)} + 2mt\tilde{A}_6^{(v)} \tag{1}$$

(where $\tilde{A}_{45} = m\tilde{A}_4 + 4\tilde{A}_5$) is dominated only by the pseudoscalar-pion trajectory

$$A_P^{(v)}(\nu,t) \to 2m\beta_{6\pi} t \nu^{\alpha_\pi(t)} \quad \text{as} \quad \nu \to \infty\,. \tag{2}$$

Hence, when $t=0$ ($\cos\theta_s = 1$), even though $\alpha_\pi(t=m_\pi^2=0) = 0$, $A_P^{(v)}$ still vanishes for large ν. Because $\tilde{A}_4$, $\tilde{A}_5$, and $\tilde{A}_6$ are kinematic-singularity-free (KSF) and A_P is even in ν, we have

$$A_P^{(v)}(0,0) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'} \,\mathrm{Im} A_P^{(v)}(\nu',0)\,. \tag{3}$$

No separate s- and u-channel pole terms are needed on the right-hand side of Eq. (3). This is because the nucleon Born graphs [Figs. 1(a) and 1(b)] give, from Eq. (1) and the Appendix,

$$A_P^{(v)N}(\nu,t) = \tfrac{1}{2}e^2 m(1+\kappa_p)\frac{t}{(s-m^2)(u-m^2)} + \frac{e^2}{2m}[\kappa_p + \tfrac{1}{2}(\kappa_p^2 - \kappa_n^2)]\,, \tag{4}$$

and at $t=0$ the pole term of Eq. (4) vanishes.

On the other hand, we can evaluate $A_P^{(v)}(0,0)$ by considering the Mandelstam double dispersion relations

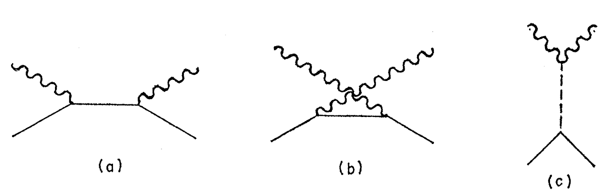

FIG. 1. Born graphs for $N\gamma \to N\gamma$.

[9] H. F. Jones and M. D. Scadron, Nucl. Phys. **B10**, 17 (1969); also see Imperial College Report No. ICTP/67/26, 1968 (unpublished).

for $\tilde{A}_{45}$ and $\tilde{A}_6$ and thus indirectly for A_P through Eq. (1). The t-channel pion Born graph [Fig. 1(c)], as well as the direct- and crossed-channel nucleon Born graphs, will contribute to A_P. Separating off the continuum amplitudes $\tilde{A}_{45}{}^c$ and $\tilde{A}_6{}^c$, we write

$$A_P(\nu,t)=2\nu\tilde{A}_{45}{}^{\text{Born}}+2mt\tilde{A}_6{}^{\text{Born}}+2\nu\tilde{A}_{45}{}^c+2mt\tilde{A}_6{}^c. \quad (5)$$

Clearly the second term of Eq. (4) contributes to Eq. (5) even at $\nu=t=0$.

We proceed to calculate the t-channel pion Born graph. We use the Goldberger-Treiman choice for the pion decay amplitude F_π, so that the $\gamma(k)+\pi\to\gamma'(k')$ vertex is[10] $-F_\pi\epsilon^{\mu *}(k')\epsilon^\nu(k)\epsilon_{\mu\nu}(k'k)$. From the definition of the covariants in the Appendix, we see that the pion Born graph contributes only to $\tilde{A}_6$,

$$\tilde{A}_6{}^{(v)\pi}(\nu,t)=-\frac{1}{4m}\frac{gF_\pi}{m_\pi{}^2-t}. \quad (6)$$

Next we form $A_P{}^{(v)\pi}$ from Eq. (1):

$$A_P{}^{(v)\pi}(\nu,t)=-\tfrac{1}{2}gF_\pi\frac{t}{m_\pi{}^2-t}. \quad (7)$$

However, in our world $m_\pi=0$, so Eq. (7) becomes

$$A_P{}^{(v)\pi}(\nu,t)=\tfrac{1}{2}gF_\pi \quad (8)$$

for all t, including $t=0$. If we keep $m_\pi\neq 0$ and evaluate the numerator of A_P instead of $\tilde{A}_6$ at the pole, $t/(m_\pi{}^2-t)\to m_\pi{}^2/(m_\pi{}^2-t)$ in the spirit of Hearn and Leader[11] or Pagels,[5] we also obtain a nonvanishing result at $t=0$, but with opposite sign as Eq. (8). This limit is nonuniform if $m_\pi\to 0$ before $t\to 0$. Moreover, A_P is not one of our six fundamental invariant amplitudes; hence setting $t=m_\pi{}^2$ in the numerator of Eq. (7) is an incorrect procedure for us.[12]

Finally, we set $t=0$ and let $\nu\to 0$ in Eq. (5). Because $\tilde{A}_{45}{}^c$ and $\tilde{A}_6{}^c$ are KSF in ν and t, only the Born terms survive and

$$A_P{}^{(v)}(0,0)=(e^2/2m)[\kappa_p+\tfrac{1}{2}(\kappa_p{}^2-\kappa_n{}^2)]+\tfrac{1}{2}gF_\pi. \quad (9)$$

Comparing Eqs. (3) and (9), we see that

$$-\tfrac{1}{2}gF_\pi=(e^2/2m)[\kappa_p+\tfrac{1}{2}(\kappa_p{}^2-\kappa_n{}^2)]-\frac{2}{\pi}\int_{\nu_0}^{\infty}\frac{d\nu'}{\nu'}\text{Im}A_P{}^{(v)}(\nu',0), \quad (10)$$

which is the zero-pion-mass Pagels sum rule. We stress that the Reggeized pion need not conspire to obtain this result if $m_\pi=0$. The conspiracy condition of Freedman[7,13] is manifested by the structure of Eq. (1) with $B_4=-A_P$ and $B_6=4m\tilde{A}_{45}$ (see the Appendix). For future reference, we also note the Drell-Hearn sum rule,[14] which in terms of our amplitudes is the statement that $\tilde{A}_4{}^{(v)}$ is superconvergent,[13,9] so that

$$-\frac{e^2}{8m^2}(\kappa_p{}^2-\kappa_n{}^2)=\frac{2}{\pi}\int_{\nu_0}^{\infty}d\nu'\,\text{Im}\tilde{A}_4{}^{(v)}(\nu',0). \quad (11)$$

III. WEAK-COUPLING LIMIT

Now we investigate Eq. (10) in the WCL. In this limit, the coupling strength g is assumed to be small enough so that ordinary perturbation theory is valid. For the moment, we neglect the continuum integral in Eq. (10) (we shall return to it later), and the term $\kappa_p{}^2-\kappa_n{}^2$, it being of a higher order in g than κ_p. Then we have

$$-gF_\pi=(e^2/m)\kappa_p \quad (12)$$

to lowest order in g. On the other hand, the lowest-order graph for neutral-pion decay is the triangle graph, Fig. 2, first calculated by Steinberger,[3] giving for zero-mass pions

$$-gF_\pi=(e^2/m)\alpha_\pi/\pi, \quad (13)$$

where $\alpha_\pi=g^2/4\pi$. Comparing Eq. (12) with Eq. (13), we see that

$$\kappa_p=\alpha_\pi/\pi \quad (14)$$

to lowest order in g.

If, instead, we consider the usual perturbation-theory graphs for the anomalous magnetic moment of the proton,[15] Figs. 3(a) and 3(b), we obtain a different

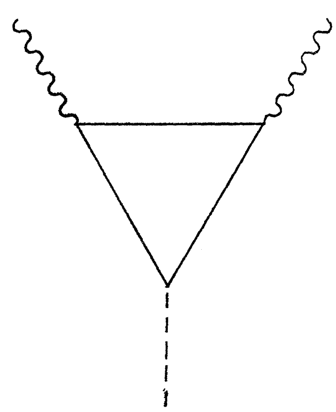

FIG. 2. Triangle graph for $\pi^0\to 2\gamma$.

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[10] We use $\epsilon_{\mu\nu}(k'k)=\epsilon_{\mu\nu\alpha\beta}k'^{\alpha}k^{\beta}$ with metric $g_{\mu\nu}=(1,-1-1-1)$ and $\epsilon_{0123}=1$. Our convention at the pion-nucleon vertex for the Hamiltonian density is $g\bar{\psi}\gamma_5\psi\phi$, where

$$i\gamma_5=\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

in the Pauli representation and $g_{\pi^0pp}=-g_{\pi^0nn}=g$. Thus the $\pi^0\to\gamma'(k')+\gamma(k)$ amplitude is $S_{fi}=-i(2\pi)^4\delta^4(P_{fi})\epsilon_\mu{}^*(k')\epsilon_\nu{}^*(k)\times\epsilon^{\mu\nu}(k'k)$.

[11] A. Hearn and E. Leader, Phys. Rev. **126**, 789 (1962).

[12] The sign change in the pole term for $m_\pi\neq 0$ versus $m_\pi=0$ is similar to the PCAC statement $\partial\cdot A=cm_\pi{}^2\phi$, where $m_\pi=0$ implies that the pion pole occurs on the left-hand side with opposite sign in the $q^2=0$ limit.

[13] D. Z. Freedman, Phys. Rev. **168**, 1822 (1968); S. R. Choudhury and D. Z. Freedman, *ibid.* **168**, 1739 (1968).

[14] S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966).

[15] B. Fried, Phys. Rev. **88**, 1142 (1952); see also H. A. Bethe, S. Schweber, and F. DeHoffman, *Mesons and Fields* (Row and Peterson, Evanston, Ill., 1956), Vol. II, p. 292.

answer than Eq. (14). For massless pions, the pion current, Fig. 3(a), contributes $\alpha_\pi/2\pi$, whereas the proton current, Fig. 3(b), contributes $-\alpha_\pi/4\pi$. Hence in this limit $\kappa_p \to \alpha_\pi/4\pi$ rather than α_π/π.

The apparent inconsistency can be resolved by including a σ meson in the WCL. This can be justified when we realize that the Pagels sum rule is primarily a consequence of the low-energy theorems, and therefore of gauge invariance.

Adler and Dothan[16] have shown that the Ward identities arising from the local commutation relations $[V,V]=V$ and $[V,A]=A$ also follow from gauge invariance (and massless leptons). Moreover, massless pions are consistent with axial-vector current conservation. A field-theoretic model which respects axial-vector current conservation and the local commutation relations is the σ model.[17]

In this model, the neutral scalar σ couples to the nucleon with the same strength as do pions, $g\bar{N}(\sigma+\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})N$. It also couples directly to pions and other σ's through another independent coupling constant λ, which is essentially the four-pion contact interaction. We stress, however, that the σ model is not really a model in the WCL because λ can be computed

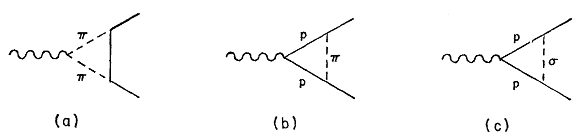

FIG. 3. Nucleon anomalous magnetic moment graphs.

in terms of g to renormalize the logarithmically divergent box graph,[18] so that $\lambda\sim O(g^4)$. Moreover, the σ mass is related to the π mass by[17,19]

$$m_\sigma^2 = m_\pi^2 - \frac{2\lambda}{(g/m)^2}. \qquad (15)$$

Thus, for $m_\pi=0$, the σ mass is zero to order g.

Returning to the WCL, we include zero-mass σ's to the same order as zero-mass π's and keep the σ mass corrections only in higher order. Consequently, we add the graph of Fig. 3(c) to our previous WCL graphs for the proton anomalous magnetic moment, Figs. 3(a) and 3(b). A simple calculation shows that Fig. 3(c) adds an amount $-\alpha_\pi/4\pi+\alpha_\pi/\pi$ to κ_p. Hence, to lowest order in the WCL,

$$\kappa_p = \left(\frac{\alpha_\pi}{2\pi}\right)_{3a} + \left(-\frac{\alpha_\pi}{4\pi}\right)_{3b} + \left(-\frac{\alpha_\pi}{4\pi}+\frac{\alpha_\pi}{\pi}\right)_{3c} = \frac{\alpha_\pi}{\pi}, \qquad (16)$$

which is consistent with the Pagels-Steinberger limit, Eqs. (12)-(14).

[16] S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966).
[17] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
[18] L. M. Brown (private communication).
[19] See also S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41**, 531 (1969). We follow their notation.

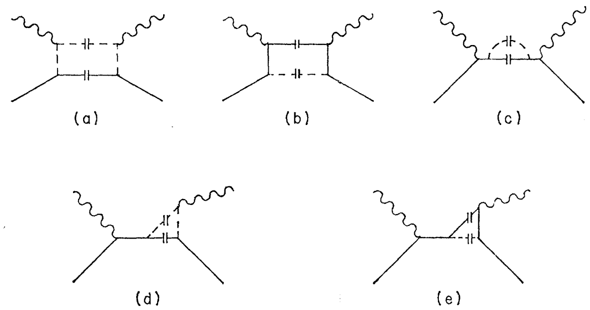

FIG. 4. Higher-order dispersion graphs for $N\gamma \to N\gamma$.

Now we return to a more careful analysis of the dispersive integral in Eq. (10). In fact, the "box" graphs having πN and σN intermediate states, Figs. 4(a)–4(e), are of lowest order $O(e^2g^2)$. Nevertheless, they do not contribute to the amplitudes $\mathrm{Im}A_P^{(v)}(\nu, t=0)$. For example, Fig. 4(a) occurs with π^+ intermediate states for external protons and π^- intermediate states for external neutrons, which means that $\mathrm{Im}(A^p-A^n)\sim g_{\pi^+np}^2 - g_{\pi^-pn}^2=0$. Other graphs are not as trivial, so we use the identity (HL means Hearn-Leader, see Appendix)

$$\tfrac{1}{4}\,\mathrm{Tr}\{\gamma_5(\not{p}'+m)M_{\mu\nu}(\not{p}+m)\} = \epsilon_{\mu\nu}(k'k)A_3^{\mathrm{HL}} - (4m\nu/tP'^2)(P_\mu'N_\nu+N_\mu P_\nu')A_6^{\mathrm{HL}}, \qquad (17)$$

which allows us to pick off the amplitude $A_3^{\mathrm{HL}}=-A_P$. The only contributions from Fig. 4(b) which survive the trace operation of Eq. (17) are of similar structure for the three meson exchanges σ, π^0, and π^+. But then $\mathrm{Im}A_P^p\sim g_{\sigma pp}^2+g_{\pi^0pp}^2=2g^2$ and $\mathrm{Im}A_P^n\sim g_{\pi^+pn}^2=2g^2$, so again $\mathrm{Im}A_P^{(v)}=0$. Finally, when we consider the even-crossing property of A_P, the remaining graphs, Figs. 4(c)–4(e), contribute terms containing at least one factor of t and therefore vanish at $t=0$.

To further appreciate the significance of the WCL with the σ included, we calculate the anomalous magnetic moment of the neutron κ_n in this limit. Now the σ does not interact with a neutron and charged proton current as would be necessary for Fig. 3(c). The usual[15] charged-pion ($m_\pi=0$) contribution of Figs. 3(a) and 3(b) to κ_n is

$$\kappa_n = \left(-\frac{\alpha_\pi}{2\pi}\right)_{3a} + \left(-\frac{\alpha_\pi}{2\pi}\right)_{3b} = -\frac{\alpha_\pi}{\pi}. \qquad (18)$$

So in the WCL we find that

$$\kappa_p = -\kappa_n, \qquad (19)$$

which, of course, is very nearly true in the real world ($\kappa_p=1.79$, $\kappa_n=-1.91$). The Drell-Hearn sum rule, Eq. (11), gives us an indication that Eq. (19) is indeed true in the WCL. We assume that $\mathrm{Im}A_4^{(v)}=0$ to order $\kappa^2\sim O(g^4)$. This would be very difficult to verify because the relevant graphs are of sixth order $O(e^2g^4)$. If this assumption is correct, then Eq. (11) implies that $\kappa_p^2=\kappa_n^2$. This sign could be checked because Eq. (19)

implies $2\kappa^{(s)}=\kappa_p+\kappa_n=0$, and the Bég[20] version of the isoscalar Drell-Hearn sum rule replaces $\kappa_p^2-\kappa_n^2$ in Eq. (11) by $(\kappa^{(s)})^2$.

IV. CONCLUSION

We have shown that there does exist a consistent weak-coupling-limit perturbation theory for massless pions. One can in fact test various strong-interaction sum rules against this WCL in much the same way one can check that a sum of graphs to a given order in e must be gauge invariant.

In particular, we have investigated the neutral-pion-decay sum rule of Pagels. The WCL indicates why the Steinberger calculation corresponding to a π^0 decay width ($\Gamma_\pi=m_\pi^3F_\pi^2/64\pi$) of $\Gamma_\pi\sim15$ eV for $\alpha_\pi\sim15$ is in the neighborhood of the real-world result,[21] $\Gamma_\pi=7.37\pm1.5$ eV. Instead, following Pagels and keeping only the first term in the real-world version of Eq. (10) yields Eq. (12) with $\kappa_p=1.79$. This gives the correct sign for F_π, as determined by Okubo[22] and by Gilman,[23] but implies $\Gamma_\pi\sim2$ eV. In order that $\Gamma_\pi\sim8$ eV, it would seem that the real-world continuum integral of Eq. (10) must be of the same order of magnitude as the Born term $e^2\kappa_p/m$. However, according to Fox and Freedman,[7] the continuum integral is small; alternatively, one might extrapolate the pion pole, Eq. (7), from $t=m_\pi^2=(140\text{ MeV})^2$ to $t=0$ in such a way as to pick up the needed factor of 2.

In contrast, the neutral-pion-decay superconvergence sum rule of Abarbanel and Goldberger[6] seems more compatible with neglecting real-world continuum integrals, giving $-gF_\pi=2e^2\kappa_p/m$ and hence $\Gamma_\pi\sim8$ eV. Yet this pole term alone is not consistent with the WCL.

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APPENDIX

We briefly review the kinematics of nucleon Compton scattering.[9,24,25] The covariant M function is defined from the S matrix as

$$S_{fi}=\delta_{fi}+i(2\pi)^4\delta^4(p'+k'-p-k)\epsilon_\mu{}^*(k')\bar{u}(p')\times M^{\mu\nu}u(p)\epsilon_\nu(k).$$

$M_{\mu\nu}$ can be developed into six invariant amplitudes $\tilde{A}_i(\nu,t)$,

$$M_{\mu\nu}=\sum_i^6\tilde{A}_i(\nu,t)\tilde{K}_{\mu\nu}{}^i,$$

where the $\tilde{A}_i(\nu,t)$ are free of kinematic singularities and zeros in $\nu=P\cdot Q=\frac{1}{4}(s-u)$ and in $t=\Delta^2$, with $P=\frac{1}{2}(p'+p)$, $Q=\frac{1}{2}(k'+k)$, $\Delta=p'-p=k-k'$, and $s=(p+k)^2$, $u=(p-k')^2$. The six covariants $\tilde{K}_{\mu\nu}{}^i$ can be taken to be[9]

$$\tilde{K}_{\mu\nu}{}^1=tg_{\mu\nu}',\quad \tilde{K}_{\mu\nu}{}^2=tP_\mu'P_\nu'-\tfrac{1}{2}(tP^2+4\nu^2)g_{\mu\nu}',$$
$$\tilde{K}_{\mu\nu}{}^3=t(P_\mu'\gamma_\nu'+\gamma_\mu'P_\nu')-4\nu g_{\mu\nu}'Q,$$
$$\tilde{K}_{\mu\nu}{}^4=4P_\mu'P_\nu'-2\nu(P_\mu'\gamma_\mu'+\gamma_\nu'P_\nu')+\nu(\gamma_\mu'Q\gamma_\nu'-\gamma_\nu'Q\gamma_\mu')+\tfrac{1}{2}tg_{\mu\nu}'Q,$$
$$\tilde{K}_{\mu\nu}{}^5=t[\gamma_\mu',\gamma_\nu']+8(mQ-\nu)g_{\mu\nu}',$$
$$\tilde{K}_{\mu\nu}{}^6=t[\gamma_\mu'Q\gamma_\nu'-\gamma_\nu'Q\gamma_\mu']=4m\gamma_5\epsilon_{\mu\nu}(k'k),$$

[where $\epsilon_{\mu\nu}(k'k)=\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta$]. These covariants are essentially those of Yamamoto[24] and of Bardeen and Tung.[25] The prime on the covariants indicates the gauge projection operator $g_{\mu\nu}'=g_{\mu\nu}-k_\mu k_\nu'/k'\cdot k$, which guarantees gauge invariance of $\tilde{K}_{\mu\nu}$ and hence of $M_{\mu\nu}$.[9,25] The isotopic decomposition is $\tilde{A}_i(\nu,t)=\tilde{A}_i{}^{(s)}(\nu,t)I+\tilde{A}_i{}^{(v)}(\nu,t)\tau_3$, so that $\tilde{A}_i{}^{(s)}=\frac{1}{2}(\tilde{A}_i{}^p+\tilde{A}_i{}^n)$ and $\tilde{A}_i{}^{(v)}=\frac{1}{2}(\tilde{A}_i{}^p-\tilde{A}_i{}^n)$.

Writing the sum of the s- and u-channel nucleon Born terms as

$$\tilde{A}_i{}^N=e^2c_i/(s-m^2)(u-m^2),$$

we have

$$c_1=-m\kappa-\frac{\mu\kappa}{4m}t,\quad c_2=-\frac{2\kappa}{m},\quad c_3=\mu+\frac{\kappa^2}{8m^2}t,$$
$$c_4=\frac{\kappa^2}{m^2}\nu,\quad c_5=-\frac{\mu\kappa}{2m}\nu,\quad c_6=\tfrac{1}{2}\mu+\frac{\kappa(\mu-\frac{1}{2}\kappa)}{8m^2}t,$$

where κ and μ are the anomalous and total magnetic moment of the proton or neutron in units of $e/2m$.

For large ν and fixed t, the Regge asympototic behavior of the $\tilde{A}_i{}^{(v)}$ is[26]

$$\tilde{A}_1{}^{(v)}\to\beta_{1\rho}\nu^{\alpha_\rho}+\beta_{1A_1}P^2\nu^{\alpha_{A1}-3},$$
$$\tilde{A}_2{}^{(v)}\to\beta_{2\rho}\nu^{\alpha_\rho-2}+\beta_{2A_1}\nu^{\alpha_{A1}-3},$$
$$\tilde{A}_3{}^{(v)}\to\beta_{3\rho}\nu^{\alpha_\rho-2}+\beta_{3A_1}P^2\nu^{\alpha_{A1}-3},$$
$$\tilde{A}_4{}^{(v)}\to\beta_{4\rho}t\nu^{\alpha_\rho-3}+\beta_{4A_1}\nu^{\alpha_{A1}-2},$$
$$\tilde{A}_5{}^{(v)}\to\beta_{5\rho}t\nu^{\alpha_\rho-1}+\beta_{5\rho}'t\nu^{\alpha_\rho-3}+\beta_{5A_1}\nu^{\alpha_{A1}-2},$$
$$\tilde{A}_6{}^{(v)}\to\beta_{6\rho}\nu^{\alpha_\rho}+\beta_{6\pi}\nu^{\alpha_\pi},$$

where

$$\tfrac{1}{2}\beta_{3\rho}=\beta_{4\rho}=-\tfrac{1}{4}m\beta_{5\rho}',\quad \beta_{5\rho}=-\tfrac{1}{4}\beta_6,$$
$$\beta_{1A_1}=\tfrac{1}{2}\beta_{2A_1}=-m\beta_{3A_1}=\tfrac{1}{2}m\beta_{4A_1}=-2\beta_{5A_1}.$$

Similar statements can be made on the $\tilde{A}_i{}^{(s)}$.

[20] M. A. B. Bég, Phys. Rev. Letters **17**, 333 (1966).
[21] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).
[22] S. Okubo, Phys. Rev. **179**, 1629 (1969).
[23] F. J. Gilman, Phys. Rev. **184**, 1964 (1969).
[24] K. Yamamoto, Phys. Rev. **169**, 1353 (1968).
[25] W. A. Bardeen and W. K. Tung, Phys. Rev. **173**, 1423 (1968).
[26] F. D. Gault and M. D. Scadron, Imperial College Report No. ICTP/67/33, 1968 (unpublished).

Finally, we relate our covariants[9] $\tilde{A}_i$ to the sets of Fox and Freedman,[7] B_i, and of Hearn and Leader,[11] A_i:

$$B_1=2(P^2\tilde{A}_2+2m\tilde{A}_3)\,,\quad B_2=2\tilde{A}_4\,,\quad B_3=4\tilde{A}_3\,,$$
$$B_4=A_3=-2(\nu\tilde{A}_{45}+mt\tilde{A}_6)=-A_P\,,$$
$$B_5=-4[2P^2(\tilde{A}_1+m\tilde{A}_3)+\nu\tilde{A}_{45}]\,,$$
$$B_6=A_4+A_5=4m\tilde{A}_{45}=4m(m\tilde{A}_4+4\tilde{A}_5)\,,$$
$$A_1+A_2=-2[t(\tilde{A}_1+m\tilde{A}_3)-2\nu\tilde{A}_{45}]\,,$$
$$A_1-A_2=-(tP^2+4\nu^2)\tilde{A}_2+4\nu\tilde{A}_{45}\,,$$
$$A_4-A_5=4(2\nu\tilde{A}_3+P^2\tilde{A}_4)\,,$$
$$A_6=t\tilde{A}_3-2\nu\tilde{A}_4\,.$$

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Low-t Theorems for Charged-Pion Photoproduction*

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Low-t theorems for charged-pion photoproduction are proven on the basis of gauge-invariance restrictions. They can be simply formulated by means of the minimal gauge-invariant extension of pion exchange (MPE). It is shown that whereas MPE leads in $\gamma N\rightarrow\pi^{\pm}N$ only to weak restrictions for $d\sigma/dt$ near the forward direction ($|t|\lesssim\mu^2$), it does actually account for all the structure observed in $\gamma p\rightarrow\pi^-\Delta^{++}$ in this t domain. Results for these reactions as well as the other charge modes of $\gamma N\rightarrow\pi^{\pm}\Delta$ are discussed and compared with experiment.

I. INTRODUCTION

THE properties of the photon lead to unique consequences for any process in which a photon participates. We know that in any such four-point function, one finds low-energy theorems in all channels in which a Born term exists[1] at the position relevant to this particular exchange. In high-energy photoproduction, one may reach very low t values; therefore, one can look for a manifestation of the low-t theorems. Alas, the low-t theorems affect only some of the amplitudes, and therefore we are not guaranteed *a priori* that their effect will be visible in the shape of the differential cross sections.

The low-t theorem is enforced by gauge invariance. Whereas, in general, any individual exchange can be written in a manifestly gauge-invariant form, that is not the case when a Born term is involved. (We mean here by Born term the exchange of a particle that appears also among the external ones.) It turns out that the sum of all possible Born terms is a gauge-invariant combination. As a matter of fact, it is usually built out of several pieces that are separately gauge invariant. The conditions of gauge invariance[1–3] may force one amplitude to have a specific value at the point where another amplitude has a pole. This behavior is exhibited in Sec. II, in which we review the situation in $\gamma N\rightarrow\pi^{\pm}N$. Thus the form of a certain set of invariant amplitudes is determined at the position of the Born term. In our case, this corresponds to the pion exchange in the t channel.

Since we cannot determine all possible amplitudes at $t=\mu^2$ on these grounds, we cannot predict in general the shape of the cross section near $t=0$. Nevertheless, we may take the following attitude: Compute $d\sigma/dt$ as if only the minimal gauge-invariant combination corresponding to a π exchange existed (the exact meaning of this minimal combination, to be denoted MPE, will be clarified in the text). Compare the result at $t=0$ with experiment. If you find an agreement in magnitude, it means that the other contributions are small and you should therefore predict the right shape near the forward direction. If, however, the result at $t=0$ does not agree with the experimental number, then all you can hope for are bounds on the rate of the variation of $d\sigma/dt$ near $t=0$. This procedure should work in the range $t\gtrsim-\mu^2$ and is simply based on the assumption that

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