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Cross Sections for Two-Pair Production at Infinite Energy*

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We give here the total cross sections for the following three high-energy processes: (i) $\gamma+\gamma\to e^++e^++e^-+e^-$, (ii) $\gamma+\gamma\to\pi^++\pi^++\pi^-+\pi^-$, and (iii) $\gamma+\gamma\to\pi^++\pi^-+e^++e^-$. Up to the eighth order, all of them approach constants in the high-energy limit. These constants are 6.5, 0.23×10^{-5}, and 0.26×10^{-3} μb, respectively, for (i), (ii), and (iii). The corresponding photon-photon elastic scattering amplitudes in the forward direction are also given. The creation of μ pairs is also briefly discussed.

I. INTRODUCTION

TOTAL cross sections for two-particle scattering are experimentally found to be approximately constant at high energies. It is therefore interesting to ask whether this behavior has a theoretical basis and, if so, whether *quantitative* predictions can be made for some of the cross sections.

For this purpose, it is best to examine the scattering processes in quantum electrodynamics and scalar electrodynamics. This is because both quantum electrodynamics and scalar electrodynamics are on a firm theoretical ground. Furthermore, perturbation calculations give good predictions because the coupling constant is small.

In this paper, we shall concern ourselves with the following three total cross sections: (i) $\gamma+\gamma\to e^++e^++e^-+e^-$, (ii) $\gamma+\gamma\to\pi^++\pi^-+\pi^++\pi^-$, and (iii) $\gamma+\gamma\to e^++e^-+\pi^++\pi^-$. Up to the eighth order in e, all of these cross sections approach constants at infinite energy, and are related to the eighth-order photon-photon scattering amplitudes in the forward direction.

II. RESULTS

A. $\gamma+\gamma\to e^++e^++e^-+e^-$

The cross section for $\gamma+\gamma\to e^++e^++e^-+e^-$ was calculated in a recent paper.[1] It is given by

$$\lim_{s\to\infty}\sigma(s)\sim(\alpha^4/36\pi m_e^2)[175\zeta(3)-38]\sim6.5\ \mu\text{b}\,, \tag{2.1}$$

where $\zeta(3)\sim1.2020569$ is the value of the Riemann ζ function at 3 and m_e is the mass of the electron. Equation (2.1) is obtained from the eighth-order amplitude for photon-photon scattering via two electron loops. Up to the eighth order, the right side of (2.1) is the largest term in the total cross section for photon-photon scattering at infinite energy. For instance, the lowest-order process possible in a photon-photon collision is $\gamma+\gamma\to e^++e^-$. The cross section for this process at high energies is[2]

$$2\alpha^2\pi s^{-1}[\ln(s/m_e^2)-2]\,, \tag{2.2}$$

which vanishes as $s\to\infty$. Thus, although (2.2) is of lower order in α, (2.1) is larger than (2.2) as the energy becomes sufficiently high. In particular, (2.1) surpasses (2.2) as $\omega\geq0.24$ BeV, where ω is the c.m. energy of the photon.

B. $\gamma+\gamma\to\pi^++\pi^++\pi^-+\pi^-$

The photon impact factor via a π loop is,[3] in the forward direction,

$$\mathcal{I}_{ij}{}^{\rho}(0,\mathbf{q}_\perp)=\tfrac{4}{3}\alpha^2\int_0^1dx\frac{x(1-x)q_{\perp i}q_{\perp j}+\delta_{ij}(x-\frac{1}{2})^2\mathbf{q}^2}{x(1-x)\mathbf{q}^2+m_\pi^2}\,, \tag{2.3}$$

where i and j denote the polarization of the incoming and the outgoing photons, respectively. In obtaining (2.3), we have set $\lambda=0$ and replaced m by m_π in (3.9) of Ref. 3, where m_π is the mass of the pion.

The scattering amplitude of $\gamma+\gamma\to\gamma+\gamma$ via two pion loops is given, in the forward direction, by

$$is(2\pi)^{-2}\int d^2q_\perp(\mathbf{q}_\perp{}^2)^{-2}\mathcal{I}_{ij}{}^{\rho}(0,\mathbf{q}_\perp)\mathcal{I}_{i'j'}{}^{\rho}(0,\mathbf{q}_\perp)$$
$$=A\delta_{ij}\delta_{i'j'}+\tfrac{1}{2}A_{\text{ex}}(\delta_{ij'}\delta_{i'j}+\delta_{ii'}\delta_{jj'}-\delta_{ij}\delta_{i'j'})\,, \tag{2.4}$$

where i and i' (j and j') are the polarization of the two incoming (outgoing) photons, respectively, and where

$$A=\frac{is}{36\pi}\frac{\alpha^4}{m_\pi^2}\int_0^1dy\int_0^1dy'\frac{(1+y^2)(1+y'^2)}{y'^2-y^2}\ln\frac{1-y^2}{1-y'^2}$$
$$=\frac{is}{144\pi}\frac{\alpha^4}{m_\pi^2}[7\zeta(3)+10] \tag{2.5}$$

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and

$$A_{\text{ex}}=\frac{is}{36\pi}\frac{\alpha^4}{m_\pi^2}\int_0^1 dy\int_0^1 dy'\frac{(1-y^2)(1-y'^2)}{y'^2-y^2}\ln\frac{1-y^2}{1-y'^2}$$
$$=\frac{is}{144\pi}\frac{\alpha^4}{m_\pi^2}[7\zeta(3)-6]. \quad (2.6)$$

In (2.5) and (2.6), $y=2x-1$ and $y'=2x'-1$. From (2.5), we get

$$\sigma_{2\gamma\to4\pi}=\frac{1}{144\pi}\frac{\alpha^4}{m_\pi^2}[7\zeta(3)+10]\sim0.23\times10^{-5}\,\mu\text{b}, \quad (2.7)$$

which is extremely small. However, the pions can interact through exchanging hadrons other than photons, and the production cross section is perhaps 10^4 times that of (2.7).

C. $\gamma+\gamma\to\pi^++\pi^-+e^++e^-$

The scattering amplitude of $\gamma+\gamma\to\gamma+\gamma$ via one pion loop and one electron loop is given, in the forward direction, by

$$is(2\pi)^{-2}\int d^2q_\perp(\mathbf{q}_\perp^2)^{-2}\mathcal{G}_{ij}{}^{\rho}(0,\mathbf{q}_\perp)\mathcal{G}_{i'j'}{}^{\gamma}(0,\mathbf{q}_\perp)$$
$$=B\delta_{ij}\delta_{i'j'}+\tfrac{1}{2}B_{\text{ex}}(\delta_{ij'}\delta_{i'j}+\delta_{ii'}\delta_{jj'}-\delta_{ij}\delta_{i'j'}), \quad (2.8)$$

where $\mathcal{G}^\gamma$ is the photon impact factor due to an electron loop, i.e.,

$$\mathcal{G}_{i'j'}{}^{\gamma}(0,\mathbf{q}_\perp)=\tfrac{2}{3}\alpha^2\int_0^1 dx$$
$$\times\frac{\mathbf{q}_\perp^2\delta_{i'j'}[3-(1-2x)^2]-4x(1-x)q_{\perp i'}q_{\perp j'}}{\mathbf{q}_\perp^2x(1-x)+m_e^2}. \quad (2.9)$$

The coefficients B and B_{ex} are given by

$$B=\frac{is\alpha^4}{9\pi}\int_0^1 dx\int_0^1 dx'\frac{[1-2x(1-x)][1+x'(1-x')]}{x(1-x)m_e^2-x'(1-x')}$$
$$\times\ln\frac{x(1-x)m_e^2}{x'(1-x')m_\pi^2} \quad (2.10)$$

and

$$B_{\text{ex}}=-\frac{2is\alpha^4}{9\pi}\int_0^1 dx\int_0^1 dx'\frac{x(1-x)x'(1-x')}{x(1-x)m_e^2-x'(1-x')m_\pi^2}$$
$$\times\ln\frac{x(1-x)m_e^2}{x'(1-x')m_\pi^2}. \quad (2.11)$$

Since $m_\pi^2/m_e^2\sim7.45\times10^4$, we shall calculate B and B_{ex} in the approximation $m_\pi^2/m_e^2\to\infty$. The amplitude B_{ex} is easy. We have

$$B_{\text{ex}}\sim\frac{2is\alpha^4}{9\pi m_\pi^2}\int_0^1 dx\int_0^1 dx'\,x(1-x)\,\ln\frac{x(1-x)m_e^2}{x'(1-x')m_\pi^2}$$
$$=\frac{is\alpha^4}{81\pi m_\pi^2}\left(6\ln\frac{m_\pi}{m_e}-1\right). \quad (2.12)$$

To obtain B, we call $\tau=m_\pi^2/m_e^2$ and make a Mellin transform of (2.10) with respect to τ. Since

$$\int_0^\infty\tau^{-\xi}(a\tau-b)^{-1}\ln(a\tau/b)\,d\tau=a^{-1+\xi}b^{-\xi}\frac{\pi^2}{\sin^2\pi\xi}, \quad (2.13)$$

we have

$$\tilde{B}(\xi)=\int_0^\infty m_e^2B(\tau)\tau^{-\xi}d\tau$$
$$=\frac{is\alpha^4}{9\pi}\frac{\pi^2}{\sin^2\pi\xi}\int_0^1 dx\int_0^1 dx'\frac{[1-2x(1-x)][1+x'(1-x')]}{[x'(1-x')]^{1-\xi}[x(1-x)]^{\xi}}$$
$$=\frac{is\alpha^4}{9\pi}\frac{\pi^2}{\sin^2\pi\xi}\frac{\Gamma^2(\xi)\Gamma^2(1-\xi)}{\Gamma(2\xi)\Gamma(2-2\xi)}\left[1-\frac{1-\xi}{3-2\xi}+\frac{\xi}{2(1+2\xi)}-\frac{(1-\xi)\xi}{2(1+2\xi)(3-2\xi)}\right]$$
$$=\frac{is\alpha^4}{9\pi}\frac{\pi^3\cos\pi\xi}{\sin^3\pi\xi}\frac{4+8\xi-5\xi^2}{(1-4\xi^2)(3-2\xi)}\sim\frac{is\alpha^4}{27\pi}\frac{1}{\xi^3}\left(4+\frac{32}{3}\xi+\frac{163}{9}\xi^2\right). \quad (2.14)$$

Thus

$$B\sim\frac{2is\alpha^4}{27\pi}\frac{1}{m_\pi^2}\left[\left(\ln\frac{m_\pi^2}{m_e^2}\right)^2+\frac{16}{3}\ln\frac{m_\pi^2}{m_e^2}+\frac{163}{18}\right], \quad (2.15)$$

and the total cross section for $\gamma+\gamma\to\pi^++\pi^-+e^++e^-$ is

$$\sigma\sim\frac{2\alpha^4}{27\pi}\frac{1}{m_\pi^2}\left[\left(\ln\frac{m_\pi^2}{m_e^2}\right)^2+\frac{16}{3}\ln\frac{m_\pi^2}{m_e^2}+\frac{163}{18}\right]$$
$$\sim0.26\times10^{-3}\,\mu\text{b}, \quad (2.16)$$

which is more than one hundred times larger than the right side of (2.7).

III. MUON PAIRS

It is not difficult to extend the above treatments to the processes with creation of muon pairs. Such processes will be discussed as follows.

(i) $\gamma+\gamma \to \mu^+ + \mu^+ + \mu^- + \mu^-$: The cross section for this process is equal to (2.1) with m_e replaced by the mass of the muon m_μ. Thus

$$\sigma(s) \sim 1.5\times 10^{-4}\,\mu\text{b}.$$

(ii) $\gamma+\gamma \to \mu^+ + \mu^- + \pi^+ + \pi^-$: The relevant quantities can be obtained from (2.8), (2.10), and (2.11) with the replacement of m_e by m_μ everywhere. Since m_π and m_μ are comparable, no further approximation can be made.

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ρ-Meson Mass Shift in Photoproduction Processes

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We argue that the ρ-meson mass shift, the mass dependence of the forward peak, and the structure of the spin-density matrix observed in photoproduction experiments are correlated and can be well parametrized utilizing crossing symmetry and t-channel kinematics.

The purpose of this note is to provide an interpretation of the experimentally observed shift to lower masses of the ρ-meson peak produced in the reaction $\gamma p \to \rho^0 p$.[1] We show that this phenomenon is correlated to the smooth flattening of the diffraction photoproduction peak when the ρ mass is increased.[2] Our interpretation provides a simple expression for the spin-density matrix which can be tested experimentally. The present work is closely connected to a recently proposed Regge-pole model for photoproduction of vector mesons.[3] Independently, such an approach is compatible with any model utilizing crossing and the t-channel kinematics.

We follow Jackson's parametrization[4] and write

$$\frac{d\sigma}{d\omega^2} = d\sigma_s(\omega^2)\frac{\omega_0\Gamma(\omega)}{(\omega_0^2-\omega^2)^2+\omega_0^2\Gamma^2(\omega)}, \tag{1}$$

where $d\sigma_s(\omega^2)$ is the corresponding production cross section for a "stable" particle having the mass ω. ω_0 is defined as the resonance mass and $\Gamma(\omega)$ is an energy-dependent width given by

$$\Gamma(\omega) = \left(\frac{\omega^2-4m_\pi^2}{\omega_0^2-4m_\pi^2}\right)^{3/2} \frac{\rho(\omega)}{\rho(\omega_0)}\Gamma(\omega_0). \tag{2}$$

It contains explicitly the threshold behavior for $\rho \to 2\pi$ and an additional empirical form factor $\rho(\omega)$ which is approximated by

$$\rho(\omega) = (\omega^2+\omega_0^2-8m_\pi^2)^{-1}. \tag{3}$$

This parametrization has been quite successful when applied to a variety of strong-interaction processes. The assumption employed is that $d\sigma_s(\omega^2) = d\sigma_s(\omega_0^2)$ and the relatively small mass shifts observed are well understood in terms of the energy-dependent width, Eq. (2).

The expression given above for $\Gamma(\omega)$ does not provide an adequate parametrization for the ρ^0 mass distribution seen in the photoreaction $\gamma p \to \pi^+\pi^- p$. The measured shift of the peak in $d\sigma/d\omega^2$ is much larger than that predicted by $\Gamma(\omega)$ alone.[1] This experimental observation calls for a more careful consideration of the mass dependence of $d\sigma_s(\omega^2)$. To this end, let us examine the parametrization of the t-channel parity-conserving

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