Baryon Pole Model for S-Wave Hyperon Decay*

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The role of the decuplet particle poles in a baryon pole model for the parity-violating amplitude is investigated from the algebraic point of view. The conditions under which the baryon pole model, with decuplet poles, reproduces the results of the K^* pole model and of current algebra are analyzed.

There exist two complementary theoretical models which have been successful in explaining the empirical relations satisfied by the S-wave nonleptonic decays of the hyperons. One is the dynamical pole model[1] and the other is the Sugawara-Suzuki method which makes use of current algebra and the current $\times$current interaction.[2] The reasons for the success of the baryon pole model have been elucidated in an algebraic form by Carlstone, Rosen, and Pakvasa.[3]

The present note investigates the baryon pole model in which the decuplet poles, as well as the octet poles, are present and clarifies the role of the decuplet poles in the model. It has been shown by a phenomenological fit to the experimental data that the decuplet poles make a significant contribution to the parity-violating amplitudes.[4,5] In the following results obtained by CRP for the pole model with only the octet poles are reviewed. It is then shown that in the limit of equal masses within each of the $SU(3)$ multiplets, the algebraic results can be extended to include the decuplet poles. Further, in the limit when the amplitude $A(\Sigma^+\to n\pi^+)=A(\Sigma^+{}_+)=0$, the octet- and decuplet-pole strengths can be related and all amplitudes can be expressed in terms of two parameters corresponding to the common F/D ratio for the strong- and the weak-interaction $\bar{B}B$ vertex and an over-all normalizing factor. When the F/D ratio is assumed to be known, the amplitudes are given by just one parameter and the model reduces to the "tadpole model" in which the single parameter is assumed to be related to the $K_1{}^0\to$ vacuum transition.

The $SU(3)$-symmetric strong vertex used in the calculations is given by

$$H_s=d(\bar{B}_i{}^j\gamma_5B_k{}^i+\bar{B}_k{}^i\gamma_5B_i{}^j)P_j{}^k+f(\bar{B}_i{}^j\gamma_5B_k{}^i-\bar{B}_k{}^i\gamma_5B_i{}^j)P_j{}^k+(\lambda/m_\pi)[\bar{B}_a{}^i\partial_\mu P_b{}^j\epsilon^{kab}(\psi_{ijk})_\mu+\text{H.c.}] \quad (1)$$

and the two-body, $TL(2)$-invariant[6] weak vertex Hamiltonian is of the form

$$H_w{}^{\text{PV}}=D_w(\bar{B}_i{}^2\gamma_5B_3{}^i+\bar{B}_3{}^i\gamma_5B_i{}^2)+F_w(\bar{B}_i{}^2\gamma_5B_3{}^i-B_3{}^i\gamma_5B_i{}^2)+(\lambda_w/m_\pi)[-\partial_\mu\bar{B}_a{}^i\epsilon^{ka2}(\psi_{i3k})_\mu+\text{H.c.}]. \quad (2)$$

The term in brackets on the right-hand side of Eqs. (1) and (2) gives the octet-decuplet coupling. It is assumed that the weak-vertex Hamiltonian transforms as an octet under $SU(3)$ transformations.

The parity-violating amplitudes are obtained from the s- and u-channel pole diagrams and can be written as $A(B\to B'\pi)=A_8+A_{10}$, corresponding to octet- and decuplet-pole contributions. For octet poles, CRP note that in the limit of degeneracy for the masses of the octet baryons the amplitudes A_8, which are a sum of s- and u-channel terms, can be regrouped in the form of the anticommutator:

$$A_8\sim\bar{B}_i\{dD_k+fF_k,\, D_wD_7+F_wF_7\}_{ij}B_jP_k. \quad (3)$$

This anticommutator will not contain the tensorial characters **10** and **10*** if $f/d=F_w/D_w=\beta$, and, in the tensor notation of Okubo,[7] it can be expressed as

$$A_8\sim\mathcal{B}\{\tfrac{1}{3}(1-3\beta^2)[\mathbf{27}]_{j3}{}^{i2}-\tfrac{3}{5}(1-3\beta^2)[\delta_j{}^2D_3{}^i+\delta_3{}^iD_j{}^2]+2\beta[\delta_j{}^2F_3{}^i+\delta_3{}^iF_j{}^2]+(\tfrac{5}{6}+\tfrac{3}{2}\beta^2)\langle\bar{B}B\rangle\delta_j{}^2\delta_3{}^i\}P_i{}^j, \quad (4)$$

where $\mathcal{B}$ is the strength of the octet pole and includes the strong- and weak-vertex coupling constants. In the above expression the baryon-antibaryon coupling has only **1**, **8**, and **27** tensors. For strangeness-changing decays the singlet term is zero, and Rosen's theorem[8] allows us to conclude that the nonleptonic decay amplitudes evaluated from the above expression will satisfy the Lee-Sugawara sum rule[9]

$$\sqrt{3}A(\Sigma^+{}_0)=A(\Lambda^0{}_-)-2A(\Xi^-{}_-).$$

If the common F/D ratio, denoted by β, were chosen to be $\beta=\frac{1}{3}\sqrt{3}$, then A_8 would be given by a pure F-type $\bar{B}B$ coupling; the same coupling is obtained in the K^* pole model, in the limit of octet dominance.

Now the modifications introduced by the presence of decuplet particle poles are presented. A perusal of

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the pole terms evaluated in Refs. 4 and 5 for the decuplet-pole contribution A_{10} reveals that in the limit of mass degeneracy within each of the $SU(3)$ multiplets **8** and **10**, the decuplet contribution has the $SU(3)$ structure given by

$$A_{10}\sim \mathfrak{D}\sum_n [\bar{B}_a{}^i P_b{}^j \epsilon^{kab}(\psi(n))_{ijk}(\bar{\psi}(n))^{p2r}\epsilon_{rc3}B_p{}^c + \bar{B}_a{}^i\epsilon^{ka2}(\psi(n))_{i3k}(\bar{\psi}(n))^{pqr}\epsilon_{rcd}B_p{}^c P_q{}^d], \quad (5)$$

where $\mathfrak{D}$ is the strength of the decuplet pole and involves the strong- and weak-vertex coupling constants and the masses of the octet and decuplet particles.

The summation over the decuplet intermediate states yields

$$A_{10}\sim \mathfrak{D}\{-\tfrac{1}{4}[\mathbf{27}]_{j3}{}^{i2}+\tfrac{3}{2}(\delta_j{}^2F_3{}^i+\delta_3{}^iF_j{}^2) -(9/5)(\delta_j{}^2D_3{}^i+\delta_3{}^iD_j{}^2)+(15/8)\langle\bar{B}B\rangle\delta_j{}^2\delta_3{}^i\}P_i{}^j. \quad (6)$$

Again the $\bar{B}B$ coupling is limited to $SU(3)$ tensorial characters **1**, **8**, and **27**. The singlet term does not contribute to strangeness-changing decays and by Rosen's theorem[8] the decuplet-pole terms will satisfy the Lee-Sugawara sum rule. Since the octet-pole terms and the decuplet-pole terms independently satisfy the Lee-Sugawara sum rule, a sum of the two amplitudes will also do so. This suggests that the decuplet-pole terms could make a significant contribution to the s-wave amplitudes.

The next step is to study the relative strengths of the decuplet- and octet-pole terms. An estimate can be obtained in the symmetry limit of mass degeneracy, by demanding that the sum of the amplitudes from Eqs. (4) and (6) lead to the experimentally observed result

$$A(\Sigma^+{}_+)=0. \quad (7)$$

Since this amplitude is given only in terms of the 27-plet $\bar{B}B$ coupling, it is possible to derive the relation

$$4(1-3\beta^2)\mathfrak{B}=3\mathfrak{D}. \quad (8)$$

From Eq. (8) it is clear that the relative strengths of the two poles depend on the value of the parameter β. The choice $\beta=0.50$ leads to the relation

$$\mathfrak{B}=3\mathfrak{D}, \quad (9a)$$

and the value $\beta=\frac{2}{3}$ yields the relation

$$4\mathfrak{B}=-9\mathfrak{D}. \quad (9b)$$

Also, the "magic" value of $\beta=\frac{1}{3}\sqrt{3}$ which sets the **27** and the $\mathbf{8}_D$ couplings [see Eq. (4)] equal to zero in the octet-pole terms makes the decuplet-pole contribution zero through the relation of Eq. (8).

The S-wave amplitude is now given in terms of just one parameter, the strength of the octet pole, if use is made of Eq. (8) and it is assumed that β is known.

The decay amplitudes are evaluated from the expression

$$A=A_8+A_{10}\sim\mathfrak{B}\{[2\beta+2(1-3\beta^2)](\delta_j{}^2F_3{}^i+\delta_3{}^iF_j{}^2) -3(1-3\beta^2)(\delta_j{}^2D_3{}^i+\delta_3{}^iD_j{}^2)\}P_i{}^j. \quad (10)$$

The ratio of the symmetric $\bar{B}B$ coupling to the antisymmetric coupling in Eq. (10) is

$$-3(1-3\beta^2)/[2\beta+2(1-3\beta^2)].$$

For $\beta=\frac{2}{3}$ this ratio is 1.50 and for $\beta=0.5$ the ratio takes on the value -0.5. This value has been obtained in the limit of zero mass-splitting within the $SU(3)$ multiplets. The corresponding result from current-algebra calculations[2,10] is ≈ -0.5.

We have expressed the decuplet-pole amplitudes in the algebraic form familiar from the work of CRP and shown (i) why these pole terms satisfy the Lee-Sugawara sum rule; (ii) that these terms make a significant contribution to the nonleptonic decay amplitudes, and as evinced by the relations in Eq. (9), the decuplet-pole strength is comparable to the strength of the octet poles; and finally (iii) that the decuplet-pole strength is dependent on the F/D ratio β .It is well known[1] that in the limit of $SU(3)$ symmetry the baryon pole terms vanish. The present work shows that this is true of the decuplet-pole terms as well because of the connection between the octet and decuplet poles expressed by Eq. (8).

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