mation of neglecting terms in J_μ proportional to q/M_p, we get only final states with $j=j_e=j_\nu=\frac{1}{2}$. It is easy to see by conservation of angular momentum that in the transition $J_I=N\pm\frac{1}{2}\to J_F=N\mp\frac{1}{2}$ $(J_F=J_I\mp 1)$, we have only one hyperfine state $J=J_F\pm\frac{1}{2}$. Since there can be no interference, Eq. (10) follows immediately. Hence, we find for any pure Gamow-Teller transition that

$$|D^{\rm MAG}|\lesssim|D^{\rm COUL}|(|g|/|f_M|)(R/Z)\,, \qquad (11)$$

where $R\sim O(W/M_p)$. $D^{\rm MAG}$ can be neglected compared to $D^{\rm COUL}$ as long as $|f_M|$ is not too small compared to $|g|$.[7] If we use experiments on $1\to 0$ transitions as a guide, we expect[8] that $|f_M|\gtrsim|g|$ and $|D^{\rm MAG}|\ll|D^{\rm COUL}|$.

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[7] There will also be a contribution $D^{\rm MAG}\sim D^{\rm COUL}(R/Z)$ from weak magnetism Gamow-Teller interference and another contribution $D^{\rm MAG}\sim D^{\rm COUL}(|g|/|f_M|)R/(AZ)$ from the bottom components of the nuclear spinors. These may also be neglected.

[8] We expect that $|f_M/g|$ for fermion transitions is of order $|F_M/F_A|$ (see Ref. 1) for boson transitions. Experimentally we have $|F_M/F_A|\approx 4.7$ for B^{12}, $N^{12}\to C^{12}+e+\nu_e$; $|F_M/F_A|\approx 40$ for $P^{32}\to S^{32}+e^-+\nu_e$; and $|F_M/F_A|\approx 6$.

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Chiral-Symmetry-Breaking Corrections to the $K\to 3\pi$ Soft-Pion Theorem

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First-order corrections, due to chiral symmetry breaking, to the $K\to 3\pi$ soft-pion theorem are calculated. The corrections turn out to be independent of any assumption of specific properties of the chrial-symmetry-breaking Hamiltonian.

THE success of soft-pion theorems[1] derived from the hypothesis of partially conserved axial-vector current (PCAC) and current algebra has an elegant and simple explanation in the notion that there exists an underlying broken $SU(3)\otimes SU(3)$ symmetry for strong interactions.[2] In this approach the low-energy (or soft-pion) theorems, which are only approximate in the real world, would become exact in the symmetry limit where axial-vector currents are conserved and the pseudoscalar-meson masses vanish. Such a symmetry does not manifest itself in the multiplets of particles, as does $SU(3)$, but through the appearance of eight Goldstone bosons (massless in the symmetry limit). The language of approximate symmetry is useful in that it not only gives a precise meaning to PCAC but can provide a scheme for keeping track of corrections to PCAC approximation. The picture that emerges is that the hadronic Hamiltonian can be decomposed as follows:

$$H=H_0+\epsilon H'\,,$$

where H_0 is invariant under $SU(3)\otimes SU(3)$, and H' breaks the symmetry. ϵ is small so that symmetric-theory predictions approach the real world. In the symmetry limit when $\epsilon\to 0$, the axial-vector currents are conserved, and the symmetry is realized by the appearance of eight massless pseudoscalar mesons. Further, the basic symmetry is broken according to the pattern

$$SU(3)\otimes SU(3)\to SU(2)\otimes SU(2)\to SU(2)\,.$$

Assuming that major deviations from the predictions of symmetric theory may be computed by working to the lowest order in ϵ, there have been computed two types of corrections to the results of the symmetric theory, namely, those which are independent of any specific

[1] See, for example, S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968); R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), where references to original literature can be found.

[2] The original suggestion that PCAC is related to broken chiral symmetry is due to Nambu and his collaborators. See Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962), and earlier papers quoted therein. The first paper relating the modern work on current algebras to chiral symmetry seems to be S. Weinberg, Phys. Rev. Letters **16**, 163 (1966). See also S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 253, where other references can also be found. This point of view has recently been clearly stated by R. F. Dashen, Phys. Rev. **183**, 1245 (1969); R. F. Dashen and M. Weinstein, *ibid.* **183**, 1261 (1969); **188**, 2330 (1969).

assumption of the properties of ϵH_1, and those which depend on the assumption that ϵH_1 is a particular component[3] of the $(3,3^*)+(3^*,3)$ representation of $SU(3)\otimes SU(3)$. In the first category, there is a theorem of Dashen and Weinstein[4] giving a relation between the K_{l3} form factors which hold to order ϵ. So far, this is the only calculation which falls into the first category. In the second category, Dashen and Weinstein[5] have also derived a sum rule relating the corrections to Goldberger-Treiman–type relations for $N\to N+e+\nu$, $\Lambda\to N+e+\nu$, and $\Sigma\to N+e+\nu$. The sum rule is correct to first order in symmetry breaking, but is difficult to test, as there is as yet no accurate experimental information about the various coupling constants involved in the sum rule. Into the second category also falls the corrections calculated by de Alwis[6] for K_{l4} form factors. In this note we consider first-order corrections, due to chiral symmetry breaking, to the soft-pion theorem on $K\to 3\pi$ decay[7] and show that these can be calculated without making any specific assumption on the transformation properties of ϵH_1. We follow a method which we[8] utilized in giving an alternative proof to the theorem of Dashen and Weinstein on K_{l3} form factors.

We begin by defining the quantities

$$M_{i\lambda}=i\int d^4x\, e^{-ik\cdot x}\langle\beta|T[A_{i\lambda}(x)H_w^{\text{p.c.}}(0)]|K_2^0\rangle$$
$$=i\Gamma_{i\lambda}+\frac{i(f_\pi/\sqrt{2})k_\nu}{k^2+m_\pi^2}\Gamma_i, \tag{1a}$$

$$i\int d^4x\, e^{-ik\cdot x}\langle\beta|T[\partial_\lambda A_{i\lambda}(x)H_w^{\text{p.c.}}(0)]|K_2^0\rangle$$
$$=\frac{f_\pi}{\sqrt{2}}\frac{m_\pi^2}{k^2+m_\pi^2}\Gamma_i+\tilde{\Gamma}_i, \tag{1b}$$

where $\tilde{\Gamma}_i$ is clearly $O(\epsilon')=O(m_\pi^2)$, ϵ' being a parameter which determines the strength of $SU(2)\otimes SU(2)$ symmetry breaking, and

$$\Gamma_i=\langle\beta\pi_i(k)|H_w^{\text{p.c.}}|K_2^0\rangle. \tag{1c}$$

Now we have the Ward identity

$$ik_\lambda M_{i\lambda}=i\int d^4x\, e^{-ik\cdot x}\langle\beta|T[\partial_\lambda A_{i\lambda}(x)H_w^{\text{p.c.}}(0)]|K_2^0\rangle$$
$$+i\int d^4x\, e^{-ik\cdot x}\delta(x_0)\langle\beta|[A_{i0}(x),H_w^{\text{p.c.}}(0)]|K_2^0\rangle, \tag{2}$$

which gives us the low-energy theorem

$$-k_\lambda\Gamma_{i\lambda}=(f_\pi/\sqrt{2})\Gamma_i+\tilde{\Gamma}_i+i\int d^4x\, e^{-ik\cdot x}\delta(x_0)$$
$$\times\langle\beta|[A_{i0}(x),H_w^{\text{p.c.}}]|K_2^0\rangle. \tag{3}$$

We now consider the process

$$K_2^0(K)\to\pi^+(q_1)+\pi^-(q_2)+\pi^0(q_3)$$

and define the variables

$$s_i=-(K-q_i)^2,\quad s_0=\tfrac{1}{3}(s_1+s_2+s_3)=\tfrac{1}{3}m_K^2+m_\pi^2. \tag{4}$$

We shall take H_w to be of the current-current form and assume octet dominance. We first take $i=3$ so that $k=q_3$, $\beta=\pi^+(q_1)+\pi^-(q_2)$, and obtain from (3)

$$-q_{3\lambda}\Gamma_{3\lambda}=(f_\pi/\sqrt{2})\langle\pi^+(q_1)+\pi^-(q_2)+\pi^0(q_3)|H_w^{\text{p.c.}}(0)|K_2^0\rangle+\tilde{\Gamma}_3$$
$$+\tfrac{1}{2}i\langle\pi^+(q_1)+\pi^-(q_2)|H_w^{\text{p.v.}}|K_1^0\rangle, \tag{5}$$

where we have ulitized the equal-time commutation relation

$$\delta(x_0)[A_{i0}(x),H_w^{\text{p.c.}}(0)]=[F_i^5,H_w^{\text{p.c.}}]\delta^4(x)$$
$$=[F_i,H_w^{\text{p.v.}}]\delta^4(x) \tag{6}$$

and

$$F_3|K_2^0\rangle=-\tfrac{1}{2}|K_1^0\rangle,\quad F_3|2\pi(I=0)\rangle=0. \tag{7}$$

Now from Lorentz covariance we can write the most general form for $\Gamma_{3\lambda}$ as

$$\Gamma_{3\lambda}=[F_1(s_1,s_2,s_3)q_{1\lambda}+F_2(s_1,s_2,s_3)q_{2\lambda}+F_3(s_1,s_2,s_3)q_{3\lambda}]. \tag{8}$$

We thus obtain from (5)

$$\langle\pi^+\pi^-\pi^0|H_w^{\text{p.c.}}(0)|K_2^0\rangle+\frac{i}{\sqrt{2}f_\pi}\langle\pi^+\pi^-|H_w^{\text{p.v.}}(0)|K_1^0\rangle$$
$$=\frac{\sqrt{2}}{f_\pi}\tilde{\Gamma}_3+\frac{1}{\sqrt{2}f_\pi}[(s_2-2m_\pi^2)F_1(s_1,s_2,s_3)$$
$$+(s_1-2m_\pi^2)F_2(s_1,s_2,s_3)+2m_\pi^2F_3(s_1,s_2,s_3)]. \tag{9}$$

We use linear expansion for $\langle\pi^+\pi^-\pi^0|H_w^{\text{p.c.}}(0)|K_2^0\rangle$:

$$A^{+-0}(s_1,s_2,s_3)\equiv\langle\pi^+\pi^-\pi^0|H^{\text{p.c.}}(0)|K_2^0\rangle$$
$$=A^{+-0}(0)\left[1-\frac{\sigma^{+-0}}{m_\pi^2}(s_3-s_0)\right], \tag{10a}$$

where $A^{+-0}(0)$ is the value of the amplitude at the symmetric point. We define

$$i\langle\pi^+\pi^-|H_w^{\text{p.v.}}(0)|K_1^0\rangle=-A(K_1^0\to\pi^+\pi^-). \tag{10b}$$

We select the point

$$s_1=m_\pi^2,\quad s_2=2m_\pi^2,\quad s_3=m_K^2$$

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[3] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1969); S. L. Glashow and S. Weinberg, Phys. Rev. Letters **22**, 224 (1968).

[4] R. F. Dashen and M. Weinstein, Phys. Rev. Letters **22**, 1337 (1969).

[5] R. F. Dashen and M. Weinstein, Phys. Rev. **188**, 2330 (1969).

[6] S. P. de Alwis, Phys. Rev. D **1**, 2131 (1970).

[7] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966). For other references see the second book quoted in Ref. 1.

[8] Fayyazuddin and Riazuddin, Phys. Rev. D **1**, 361 (1970); **1**, 2716(E) (1970).

so that the right-hand side of (9) is clearly of order m_π^2 or ϵ'. Thus we obtain from (9)

$$A^{+-0}(0)\left[1-\frac{1}{3}\frac{\sigma_{+-0}}{m_\pi^2}(2m_K^2-3m_\pi^2)\right] - \frac{1}{\sqrt{2}f_\pi}A(K_1^0\to\pi^+\pi^-)=\alpha O(\epsilon'). \quad (11)$$

Taking $i=1+i2$ so that $k=q_2$, $\beta=\pi^+(q_1)+\pi^0(q_3)$, we obtain from (3) a relation similar to (5). [Note that the equal-time commutator gives here a matrix element of the form $\langle\pi^+\pi^0(I=2)|[F_1+iF_2, H_w^{\mathrm{p.v.}}(0)]|K_2^0\rangle$, which is zero, since we are assuming octet dominance for H_w.]

$$\langle\pi^+\pi^-\pi^0|H_w^{\mathrm{p.c.}}(0)|K_2^0\rangle = \frac{1}{f_\pi}\tilde{\Gamma}_{1+i2}+\frac{1}{2f_\pi}[(s_3-2m_\pi^2)G_1(s_1,s_2,s_3) +2m_\pi^2G_2(s_1,s_2,s_3)+(s_1-2m_\pi^2)G_3(s_1,s_2,s_3)]. \quad (12)$$

Selecting now the point

$$s_1=2m_\pi^2,\quad s_2=m_K^2,\quad s_3=m_\pi^2,$$

we obtain from (12)

$$A^{+-0}(0)\left[1+\frac{1}{3}\frac{\sigma_{+-0}}{m_\pi^2}m_K^2\right]=\beta O(\epsilon'). \quad (13)$$

Subtracting (13) from (11),

$$A^{+-0}(0)\left[-\sigma^{+-0}\frac{m_K^2-m_\pi^2}{m_\pi^2}\right] - \frac{1}{\sqrt{2}f_\pi}A(K_1^0\to\pi^+\pi^-)=(\alpha-\beta)O(\epsilon'). \quad (14)$$

It is well known[1] that $A(K_1^0\to\pi^+\pi^-)$ vanishes in the exact $SU(3)$ limit. Thus the first and second terms on the left-hand side of (14) are each of order λ, where λ is a parameter which characterizes the $SU(3)$ breaking. Thus $(\alpha-\beta)$ must also be of order λ. Hence

$$A^{+-0}(0)\left[-\frac{\sigma^{+-0}}{m_\pi^2}(m_K^2-m_\pi^2)\right] - \frac{1}{\sqrt{2}f_\pi}A(K_1^0\to\pi^+\pi^-)=O(\epsilon'\lambda). \quad (15)$$

Now we know from soft-pion calculations[7] that

$$\frac{\sigma^{+-0}}{m_\pi^2}=-3\frac{1}{m_K^2}+O(\epsilon'). \quad (16)$$

Substituting in (15), we obtain

$$\frac{1}{3\sqrt{2}f_\pi}\frac{A(K_1^0\to\pi^+\pi^-)}{A^{+-0}(0)} =\frac{m_K^2-m_\pi^2}{m_K^2}+O(\epsilon'\lambda)=\left(1-\frac{m_\pi^2}{m_K^2}\right)+O(\epsilon'\lambda). \quad (17)$$

Thus Eq. (17) holds to first order in symmetry breaking and has been obtained independent of any specific assumption of the properties of ϵH_1. The quantity m_π^2/m_K^2 in (17) represents the first-order correction to the soft-pion result of Hara and Nambu.[7] This correction tends to improve the agreement with experiment. A similar correction has also been obtained by Okubo and Mathur[9] in a rather *ad hoc* manner and their argument depends on a specific assumption of the properties of ϵH_1.

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[9] S. Okubo and V. S. Mathur, Phys. Rev. D 1, 2046 (1970).