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Distributions of Charged Pions*

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Statistical considerations are used to derive a new distribution for charged pions in multiparticle production reactions. The distribution is in good agreement with experiment.

MOST inelastic reactions at available accelerator energies involve primarily the emission of pions. One can argue that the gross features (multiplicities) of the events should be independent of the specific production mechanisms, which suggests that one should approach the problem on a statistical basis. There are several over-all constraints that have to be obeyed by the system, namely, momentum, isospin, and charge conservation. Since experimentally the emitted pions occupy a small fraction of the available phase space, we expect momentum conservation to be a weak constraint. By summing over all neutral pions, we may expect the constraint of total isospin conservation on the distribution of charged pions to also be weak. We are thus left with the obvious constraint of charge conservation. In this paper we address ourselves to the problem of building a statistical distribution of pions that conserves charge. We will see that the resulting new type of distribution agrees well with experiment.

It is quite straightforward to arrive at the desired distribution. If the pions are emitted independently, one would expect a Poisson distribution for each kind of pion. Because of the charge constraint, we ask for the conditional probability of emitting n positive and n negative pions simultaneously. If the Poisson distributions for the positive and negative pions is given by

$$P_n{}^{(\pm)}=e^{-x_\pm}(x_\pm)^n/n!\,, \tag{1}$$

the resulting distribution for n charged pairs is

$$P_n=\frac{1}{J_0(2ix)}\frac{x^{2n}}{(n!)^2}, \tag{2}$$

where $x^2=x_+x_-$. It follows that

$$\langle n\rangle=\sum_n nP_n=-ix\frac{J_1(2ix)}{J_0(2ix)}, \tag{3}$$

which gives a one-to-one correspondence between $\langle n\rangle$ and x. We find also

$$\langle n^2\rangle=x^2, \tag{4}$$

$$\sigma=\langle n^2\rangle-\langle n\rangle^2=x^2\left(1+\frac{J_1{}^2(2ix)}{J_0{}^2(2ix)}\right). \tag{5}$$

For high values of n, one can use Stirling's approximation to show that

$$P_n\to\frac{1}{J_0(2ix)}\frac{(2x)^{2n}}{(2n)!(\pi n)^{1/2}}, \tag{6}$$

which differs slightly from a Poisson distribution in $2n$.

In Fig. 1 we compare the predictions of Eq. (2) with the data compiled by Wang.[1] This is a compilation of many $\pi^\pm p$, pp, and nn inelastic production experiments below 27 BeV. The number of charged prongs (n_c) should be related to our n by $n_c=2n+2$ (in the case of nn collisions, $n_c=2n$). The data are assembled in a way that tests just the character of the distribution, namely, it is a plot of the probability for a certain n_c to occur provided $\langle n_c\rangle$ is given. Hence there is no free parameter to be adjusted. Wang tried to fit the data with two of the distributions shown in Fig. 1: $W^{\rm I}$ is a Poisson distribution in $\frac{1}{2}(n_c-2)$ and $W^{\rm II}$ is built of the even terms in a Poisson distribution in n_c-2. The data points seem to follow a universal curve that is not very well reproduced by either $W^{\rm I}$ or $W^{\rm II}$. Although $W^{\rm I}$ fits the low-n_c and low-$\langle n_c\rangle$ region, it fails at higher n_c and higher $\langle n_c\rangle$. We note that the curve of distribution (2) does depict correctly the experimental behavior.

In view of the success of distribution (2), we mention at this point that in plotting all the experiments together, we are closer to the case of a statistical ensemble. One may expect that some remnants of the momentum and isospin constraints are still left in any particular type of experiment. We anticipate that higher statistics experiments will show deviations from universal curves for individual reactions.

The agreement achieved in Fig. 1 raises the question whether this can serve as proof that all the reactions are mainly of one type, namely, $A+B\to A+B$ +pions, where obviously the pion cloud is neutral. In order to answer that, we look for the probability $P_n{}^{(q)}$ of finding $n+q$ positive and n negative pions. This will then correspond to the expected behavior from a cloud of pions of over-all charge q. Following a similar line of reasoning to the one used above, we find

$$P_n{}^{(q)}=P_{n+q}{}^{(-q)}=\frac{i^q}{J_q(2ix)}\frac{x^{2n+q}}{n!(n+q)!}, \tag{7}$$

where

$$\langle n\rangle_q=-ixJ_{q+1}(2ix)/J_q(2ix). \tag{8}$$

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[1] C. P. Wang, Phys. Rev. **180**, 1463 (1969).

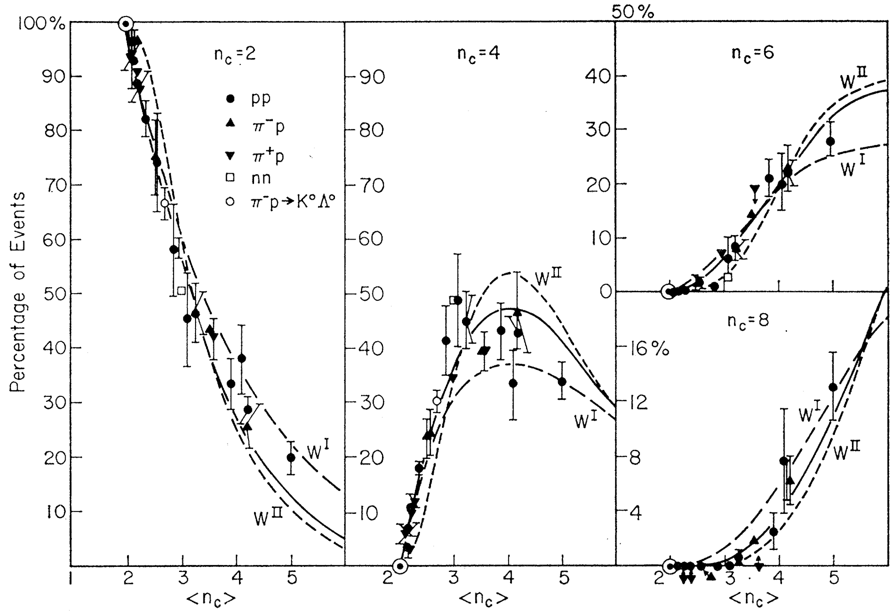

FIG. 1. Compilation of inelastic production data and its comparison with theoretical distributions. The data, as well as the fits W^{I} and W^{II}, are taken from Wang's paper (Ref. 1). The solid curve corresponds to our distribution, Eq. (2).

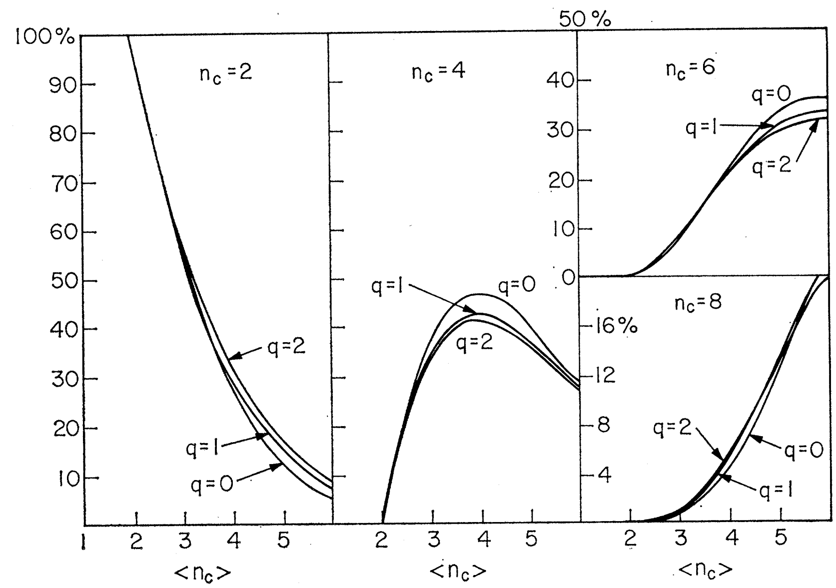

FIG. 2. Comparison of various $P^{(q)}$ distributions.

In Fig. 2 we plot the predictions of $P^{(0)}$, $P^{(1)}$, and $P^{(2)}$ in the same way as in Fig. 1. It turns out that they all coincide in the region where most data points are available. This may even be the reason for the universal character of the experimental data. For example, in $\pi^- p$ reactions, one finds outgoing "leading" particles π^- and p following the initial momenta of the incoming ones, and a cloud of pions with relatively low momenta in the center-of-momentum coordinates.[2] This cloud of pions should fit into the $P^{(0)}$ description. However, as the multiplicity increases, the leading π^- loses momentum and eventually it will be indistinguishable from the π^- particles in the cloud. Thus one should perhaps expect a smooth transition from $P^{(0)}$ to $P^{(1)}$.

We conclude that the statistical approach followed here leads to successful agreement with experiment.[3] It is desirable to have higher-precision experiments to check the systematic trends of the data as well as

[2] See, e.g., R. Honecker *et al.*, ABBCCHW Collaboration, Nucl. Phys. **B13**, 571 (1969).

[3] The success of this statistical approach does not necessarily disprove other more detailed models for many-particle productions. We first arrived at this distribution by considering a model for the production of pions in coherent states which we intend to explore in a future publication.

possible deviations from a universal curve. It is an open and intriguing question whether the statistical character of the distributions continues to prevail at higher energies or whether it undergoes some systematic changes.

Note added in proof. The distribution $P^{(0)}$ has been suggested by H. A. Kastrup, Nucl. Phys. **B1**, 309 (1967). We thank Dr. Kastrup for bringing this to our attention.

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Evaluation of the $\pi\pi$ Scattering Lengths Using On-Mass-Shell Pions*

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We have evaluated the $\pi\pi$ scattering lengths, using current algebra but without the use of a power-series expansion or extrapolation of the scattering amplitude. We have used the usual Lehmann-Symanzik-Zimmermann reduction in terms of an axial-vector current $J=A+c\partial\phi$, where A is the usual axial-vector current, c is the Goldberger-Treiman constant, and ϕ is the pion field. The scattering amplitude is decomposed into four terms, three of which are due to the equal-time commutators; two of these three are evaluated using the σ model in order to show that the usual current algebra of the current A holds for the current J. The other terms are evaluated in terms of single-particle intermediate states; we show that among these states, only s waves contribute at threshold. Assuming that the ϵ resonance is the only s wave dominating the low-energy ($\pi\pi$) scattering, we find a relation connecting the form factors arising from the equal-time commutators of the current J to the ϵ-pion coupling constant. Finally, we obtain the scattering lengths corresponding to isospins 0 and 2, and ϵ resonance width 200 MeV, as $a_0=0.278m^{-1}$ and $a_2=-0.044m^{-1}$, where m is the pion mass.

I. INTRODUCTION

BOTH experimental data and theoretical arguments have been extensively used in the discussion of the $\pi\pi$ scattering lengths.[1–7] Low-energy $\pi\pi$ scattering has been studied by many authors, using dispersion relations, phase-shift analysis, and the current algebra. Owing to the lack of adequate, accurate experimental data, the results of these calculations cannot be compared with well-established experimental numbers. Even so, the smallness of the scattering lengths as obtained by Weinberg[3] from a soft-pion treatment seems valid.

However, some plausible arguments raise questions concerning this calculation. The main objections to the application of the soft-pion treatment,[8] which have been pointed out by some of the previous authors, may be summarized as follows: First, there is the well-known partial conservation of axial-vector current (PCAC) assumption of the scattering amplitude with respect to k^2, for $0\leq k^2\leq m^2$, where m and k are the pion mass and four-momentum,[9] respectively. Yet this assumption, as pointed out by Sucher and Woo,[10] contradicts the result of the power-series expansion of the amplitude which is involved in applying the soft-pion limit in $\pi\pi$ scattering. Also, it is known that the results of Weinberg's soft-pion treatment do not satisfy the Adler sum rule[11] without some additional assumption concerning the $\pi\pi$ scattering ranges.[5]

As a contribution to the clarification of these points,

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[1] Since 1960 many authors have studied the ($\pi\pi$) scattering lengths. We start with the work of J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. **128**, 1881 (1962), and the the references given therein.

[2] See, e.g., Ref. 1, and Heinz J. Rothe, Phys. Rev. **140**, B1421 (1965), where the scattering lengths (in units of m^{-1}) for isospins 0, 1, and 2 are claimed to be $a_0=1.7$, $a_1=0.4$, and $a_2=-0.4$. See also S. H. Patil, *ibid.* **179**, 1405 (1969), where it is claimed that $a_0=-1.2$ and $a_2=-0.4$.

[3] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[4] N. N. Khuri, Phys. Rev. **153**, 1477 (1967).

[5] F. T. Meiere and M. Sugawara, Phys. Rev. **153**, 1702 (1967); **153**, 1709 (1967). In this reference the authors have a special prescription for the use of Adler sum rule. Their results which satisfy this sum rule are very close to those obtained in the present paper, and differ by about 25% with those obtained in Ref. 4.

[6] Haruichi Yabuki, Progr. Theoret. Phys. (Kyoto) **39**, 118 (1968).

[7] J. R. Fulco and D. Y. Wong, Phys. Rev. Letters **19**, 1399 (1967).

[8] By soft-pion treatment or technique we mean the complete procedure and the treatment used in Ref. 3.

[9] Y. Nambu, Phys. Rev. Letters **4**, 380 (1959); M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

[10] J. Sucher and Ching-Hung Woo, Phys. Rev. Letters **18**, 723 (1967).

[11] S. Adler, Phys. Rev. **140**, B736 (1965).