place the small production cross section of the above object (which is nearly 1% of that of the $\rho^0$ meson) and its small width are both consistent with its being the $V^0$ meson, since both are proportional to $g_V^2/4\pi \simeq 10^{-2}$. Thus if the above effect is confirmed, it appears to be a likely candidate for the $V^0$ meson proposed here. In this case its width should lie in the region of 1 MeV and the effect should be absent in the $(\pi^0\pi^0)$ system. A careful study of the $(\pi\pi)$ system in the above mass region should thus be very useful.

**ACKNOWLEDGMENTS**

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# Meson-Baryon Interactions with Broken $SU(3)$ and the Baryon Spectrum in Relativistic Quantum Mechanics*

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The multichannel relativistic Schrödinger equation is solved for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ partial-wave amplitudes and their Regge recurrences with an energy-dependent potential obtained by computing the baryon-exchange contribution to the pseudoscalar-meson–baryon interaction. As discussed previously, the model yields the usual $\frac{3}{2}^+$ decuplet and predicts in addition a 27-dimensional representation and a radially excited decuplet in this partial wave. It is shown that in the range of parameters which fit the usual decuplet, there are also decuplet orbital excitations in the $\frac{7}{2}^+$ and $\frac{11}{2}^+$ partial-wave amplitudes which correspond to the known experimental resonances. The $\frac{1}{2}^+$ octet is obtained as deeply bound states, and a second $\frac{1}{2}^+$ octet or a $\overline{10}$ representation is predicted, depending on whether the $F/D$ ratio is less than or greater than 0.34. In addition, an orbital excitation of the octet occurs in the $\frac{5}{2}^+$ partial-wave amplitude at very high energies. The $P$-wave phase shifts are in qualitative agreement with experiments, but there are deviations for the $P_{33}$ effective range near threshold and for the $P_{13}$ phase shifts at higher energies.

## I. INTRODUCTION

In this paper we study the baryon spectrum by solving the multichannel relativistic Schrödinger equation with a potential obtained by computing the baryon-exchange contribution to pseudoscalar meson-baryon scattering. We perform the off-shell extrapolation in such a way that no cutoff is needed. $SU(3)$ relations are assumed for the coupling constants and physical masses for the input particles. Hence our calculations depend on two parameters: the pion-nucleon coupling constant and the $F/D$ ratio. As discussed in Refs. 1–3, the relativistic Schrödinger multichannel equation may be used in dynamical calculations, since the principles of relativistic invariance, unitarity, and analyticity or causality are satisfied and there are no difficulties in dealing with the multichannel problem. In fact, this equation might even be preferred to other techniques based on the $N/D$ method since it includes iterations of the potential.

Following Gell-Mann's pioneering eightfold-way approach,[4] most papers on baryon resonances deal only with their group-theoretical classification and do not contain any detailed dynamics. On the other hand, dynamical calculations (such as Chew's first calculation of the $N_{3/2}^*$ resonance[5]) deal usually with only one channel or introduce a second channel purely phenomenologically[6] and thus neglect any internal symmetry group. Obviously, many features of the physical baryon spectrum only become clear if one studies models containing both dynamics and an internal symmetry group (as first discussed in Ref. 7). For example, $SU(3)$ group theory tells us in our case (interaction of two octets) that resonances or bound states may be present

* Work supported in part by the Deutsches Elektronen-Synchrotron DESY, Deutsche Forschungsgemeinschaft, and the U. S. Atomic Energy Commission.

[1] J. Katz, Nuovo Cimento **58A**, 125 (1968).
[2] J. Katz and S. Wagner, Phys. Rev. **188**, 2196 (1969).
[3] F. Coester, Helv. Phys. Acta **38**, 7 (1965).
[4] M. Gell Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1967).
[5] G. F. Chew, Phys. Rev. **129**, 2363 (1963); G. F. Chew and F. E. Low, *ibid.* **101**, 1571 (1956).
[6] F. Gutbrod, DESY Report No. 69/22 (unpublished); see also Ref. 9.
[7] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

in any of the irreducible representations **1**, $\mathbf{8}_s$, $\mathbf{8}_a$, **10**, $\overline{\mathbf{10}}$, and **27**. However, the dynamics of the pseudoscalar $B$-$M$-$B$ interaction excludes **8** and $\overline{\mathbf{10}}$ representations for spin $\frac{3}{2}^+$ as well as decuplets for $\frac{1}{2}^+$. This is because the forces which lead to those representations are repulsive. Baryon-exchange forces, which are supposed to be the dominant ones for spin $\frac{3}{2}^+$, are stronger in the 10- than in the 27-dimensional representations. Hence, the decuplet resonances should appear at lower energies, whereas a 27-plet of resonances is expected at higher energies. Thus our results agree with those of the quark model with $qqq$ baryons[8] when the lowest states are considered [i.e., those which belong to a 56-dimensional $SU(6)$ representation]. However, our predictions for higher resonances are not in agreement with the quark model since, for instance, for spin $\frac{3}{2}^+$ the resonances above the decuplet are predicted in the quark model to occur in an octet representation and not as 27-plets. We want to note that these predictions of the quark model are taken from purely group-theoretical arguments or an oversimplified dynamics like a nonrelativistic harmonic-oscillator potential, whereas a relativistic treatment of the three-body problem should be carried out in the quark model. Finally, we want to mention that in a model with broken symmetry such as the one we want to discuss here, the supermultiplets of higher resonances are extremely broad (as appears for most of the exotic resonances belonging to the $\frac{3}{2}^+$ **27**) or they appear at very high energies as occurs for certain $F/D$ values for the $\frac{1}{2}^+$ $\overline{\mathbf{10}}$ representation.[9] In addition it may also happen that some states disappear for certain values of the cutoff parameter as has been discussed in Ref. 1.

## II. RELATIVISTIC SCHRÖDINGER EQUATION

Our calculational method consists in solving a multichannel relativistic Schrödinger equation. Its properties and our numerical methods have been already discussed in I.[10] For the reader's convenience we shall briefly review our notation.

The relativistic Schrödinger equation may be written as

$$K'^J(q_f,q_i)=V'^J(q_f,q_i)-\frac{P}{\pi}\int_{\Delta_m}^{\infty}V'^J(q_f,q)\times\frac{d(E_q+\omega_q)}{\sqrt{s}-E_q-\omega_q}K'^J(q,q_i)\,, \quad (1)$$

where $\Delta_m$ is the lowest threshold, $\sqrt{s}$ denotes the total energy in the c.m. frame, and where the momenta of the particles in the final, initial, and intermediate states are denoted by $q_f$, $q_i$, and $q$, respectively. In addition, $V'^J$ denotes the potential and $K'^J$ denotes the $K'$ matrix (as discussed, for example, in Ref. 11). Once the $K'$ matrix is obtained, the eigenphases may be calculated by computing its eigenvalues and setting them equal to $\tan\delta_\alpha$, where $\delta_\alpha$ denotes the $\alpha$th eigenphase. To solve the coupled system of integral equations given above, we make the transformation

$$(E_q+\omega_q)_\alpha=\sigma\frac{x_\alpha}{1-x_\alpha}+\Delta_\alpha\,,\quad \sqrt{s}=\sigma\frac{z_\alpha}{1-z_\alpha}+\Delta_\alpha\,, \quad (2)$$

where $\sigma$ is a scale factor chosen to make the integrand peak around the middle of the interval of integration and where $\Delta_\alpha$ is the threshold for the corresponding channel $\alpha$.

To take care of the principal-value singularity in Eq. (1), we rewrite it as

$$U'^J(x,z)=V'^J(x,z)-\frac{1}{\pi}\int_0^1\left[V'^J(x,x')\frac{dx'}{z-x'}\frac{1-z}{1-x'}-V'^J(x,z)\frac{dx'}{z-x'}\right]U'^J(x',z)\,, \quad (3)$$

where

$$U'^J(x,z)=K'^J(x,z)\left[1-\frac{1}{\pi}\int_0^1\frac{dx'}{z-x'}K'^J(x',z)\right]^{-1}. \quad (4)$$

Our potential is the baryon-exchange contribution to the pseudoscalar meson baryon interaction as given by the baryon-exchange Feynman graph. It may be written as

$$V_{l\pm}'=-\frac{C_{\alpha,\alpha'}}{4\pi}\left[\frac{(b_0+m_b)(d_0+m_d)}{(b_0+a_0)(d_0+c_0)}\right]^{1/2}\times(|\mathbf{b}||\mathbf{d}|)^{1/2}(X_l+Y_{l\pm1})\,. \quad (5)$$

In the above, $C_{\alpha,\alpha'}$ stands for the $SU(3)$ contribution to the potential and $a$, $b$, $c$, and $d$ denote the energy-momenta of the incoming and outgoing pseudoscalar mesons and baryons. In addition, we have defined

$$X_l=[-A_l-\tfrac{1}{2}B_l(2\sqrt{s}-m_b-m_d)]\,, \quad (6)$$

$$Y_l=[A_l-\tfrac{1}{2}B_l(2\sqrt{s}+m_b+m_d)]\times\left[\frac{(b_0-m_b)(d_0-m_d)}{(b_0+m_b)(d_0+m_d)}\right]^{1/2}, \quad (7)$$

$$A_l=\left(m_e-\frac{m_b+m_d}{2}\right)(-1)^{l+1}\frac{1}{2|\mathbf{b}||\mathbf{d}|}Q_l(z)\,, \quad (8a)$$

and

$$B_l=(-1)^{l+1}\frac{1}{2|\mathbf{b}||\mathbf{d}|}Q_l(z)\,, \quad (8b)$$

[8] M. Gell-Mann, Phys. Rev. Letters **8**, 214 (1964); G. Zweig, CERN Report, 1964 (unpublished); R. Dalitz, in Proceedings of the Oxford International Conference, 1965 (unpublished); in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 215.

[9] R. Gastman and F. Halzen, University of Louvain, Belgium, report, 1968 (unpublished).

[10] From here on we shall denote our previous paper, Ref. 2, as I.

[11] R. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **3**, 307 (1960).

TABLE I. $\frac{3}{2}^+$ decuplet states. Energies and widths (in parentheses) are given in MeV for the experimental and calculated decuplet states. (b.st.≡bound state.) Columns 4–7 contain the calculated values for the usual decuplet for $g^2/4\pi=38$, 60, and 80 and the last two columns contain the values for the orbital excited decuplet for $g^2/4\pi=38$ and 80. Additional values may be found in Fig. 1 for the parameters $g^2/4\pi=38$ and $f=0.4$ and for other values of $g^2$ and $f$ in Ref. 1.

| | $\frac{3}{2}^+$ resonances and bound states (energies and widths in MeV) | | | | | |
|---|---|---|---|---|---|---|
| | | Lower decuplet | | | Orbital excited decuplet | |
| | Experiment | $g^2/4\pi=38$ | $g^2/4\pi=60$ | $g^2/4\pi=80$ | $g^2/4\pi=38$ | $g^2/4\pi=80$ |
| Δ | 1236 (120) | 1236 (176) | 1147 (50) | 1113 (20) | 3182 ( 655) | 2560 ( 700) |
| Σ* | 1385 ( 36) | 1444 ( 83) | 1367 (40) | 1326 (21) | 3532 (1661) | 2870 (1100) |
| Ξ* | 1530 (7.3) | 1627 ( 30) | 1556 (14) | 1516 (7.2) | 3782 (1090) | 3120 ( 930) |
| Ω⁻ | 1672 (b.st.) | 1801 (b.st.) | 1727 (b.st.) | 1683 (b.st.) | 4076 (1400) | 3400 ( 510) |

with

$$z=\frac{1}{2|\mathbf{b}||\mathbf{d}|}[b^2+d^2-\tfrac{1}{4}(a_0-b_0-d_0+c_0)^2+m_e^2], \quad (9)$$

which follows from the relativistic kinematics of the pseudoscalar $B$-$P$-$B$ interaction and is given in I in more detail. In Eqs. (6) and (7), we have made the replacement

$$(a_0+b_0+c_0+d_0)\to 2\sqrt{s}, \quad (10)$$

where $\sqrt{s}$ is the total c.m. energy. This allows solutions of Eq. (1) without introducing an arbitrary cutoff parameter. To justify this replacement we merely note that the Schrödinger equation is an off-shell equation and that the potential, however, is known from Feynman graphs only on the mass shell, which is not changed by the condition (10). Note also that our way of performing the off-mass-shell extrapolation is symmetric in initial and final particle momenta. The coupling constant will be taken as an adjustable parameter when considered off the mass shell. These modifications do not violate $SU(3)$ and do not introduce unphysical parameters into the theory, as would be the case if one introduced a cutoff.

In the case of perfect symmetry, the potential may be developed into the normalized eigenstates of the irreducible $SU(3)$ representations

$$C_{\alpha,\beta}=-\tfrac{4}{3}(4f^2+10f-5)|\mathbf{1}\rangle\langle\mathbf{1}|-2(4f^2-2f+1)|\mathbf{8}_s\rangle\langle\mathbf{8}_s| +\tfrac{2}{3}(4f^2+10f-5)|\mathbf{8}_a\rangle\langle\mathbf{8}_a|-(8/3)(2f^2-f-1) \times|\mathbf{10}\rangle\langle\mathbf{10}|+(8/3)(4f^2-5f+1)|\overline{\mathbf{10}}\rangle\langle\overline{\mathbf{10}}| +\tfrac{4}{3}(4f^2-2f+1)|\mathbf{27}\rangle\langle\mathbf{27}|. \quad (11)$$

The coefficients of $C_{\alpha\beta}$ are plotted in Fig. 1 as functions of the $F/D$ ratio

$$f=g_F/(g_F+g_D). \quad (12)$$

A positive sign of a coefficient $C_{\alpha\beta}$ means attraction in case of spin $\frac{3}{2}^+$ and repulsion for $\frac{1}{2}^+$ and vice versa. Hence for physical values $0.25\leq f\leq 0.5$, the forces in the $\frac{3}{2}^+$ decuplet representation are the strongest ones, whereas the forces for the 27-plet are less than half as strong, and a singlet may be present for $f<0.4$. The forces leading to $\mathbf{8}_s$, $\mathbf{8}_a$, or $\overline{\mathbf{10}}$ representations are repulsive for spin $\frac{3}{2}^+$.

In the case of broken $SU(3)$ symmetry (i.e., physical input masses), the coefficients $C_{\alpha,\beta}$ are exhibited for the baryon-exchange contribution in I at the end of Sec. III. Note that symmetry breaking does not change the above conclusions since it leads just to a splitting of the resonances belonging to a supermultiplet which is degenerate for perfect symmetry.

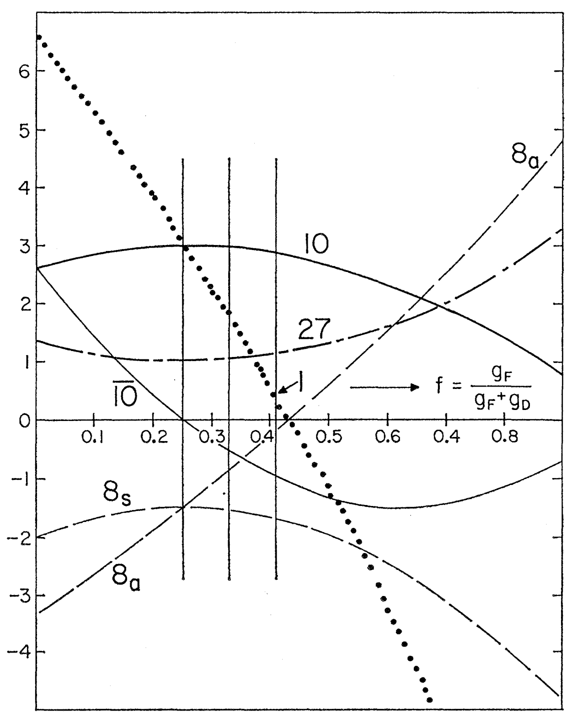


FIG. 1. $SU(3)$ decomposition of potential versus $F/D$ ratio $f$. We plot in this figure the expansion coefficients of the BPB potential into the irreducible $SU(3)$ representations which are indicated on the graph [see Eq. (11)]. For spin $\frac{3}{2}^+$ a positive sign of a coefficient means attraction, a negative sign repulsion.

## III. RESULTS OF CALCULATION AND COMPARISON WITH EXPERIMENTS

### A. $\frac{3}{2}^+$ Partial-Wave Amplitude and Its Orbital Excitations

A detailed discussion of the $\frac{3}{2}^+$ partial-wave amplitude was already given in Ref. 1 and in I. We include here

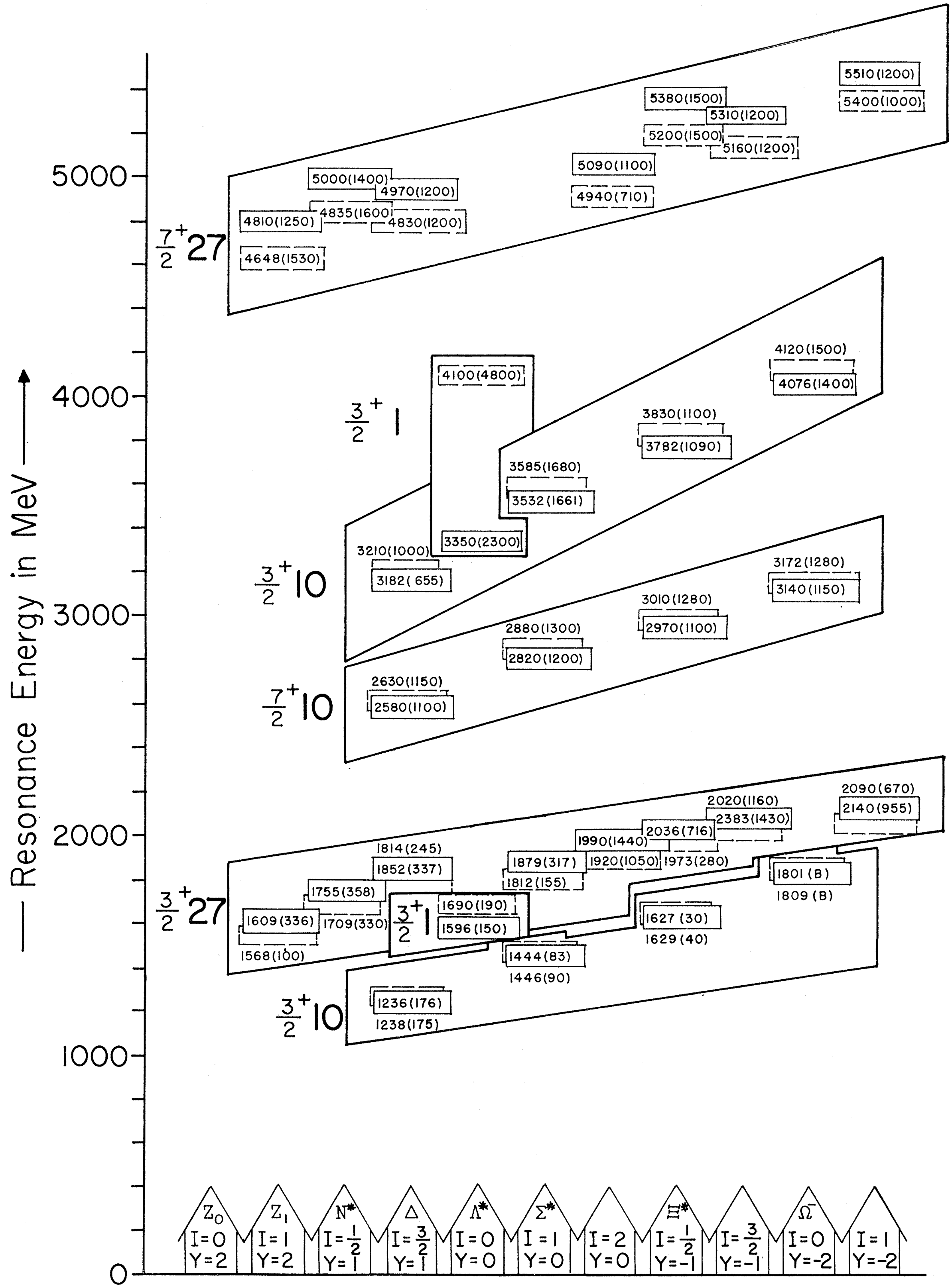


FIG. 2. Calculated baryon spectrum for spin $\frac{3}{2}^+$ and its Regge recurrences. Resonance energies and widths (in parentheses) are given in MeV. The $SU(3)$ quantum numbers are indicated at the bottom. The coupling constant is chosen as $g^2/4\pi=38$ for which $\Delta(1236)$ appears at the experimental energy. The figures inside the boxes marked with solid lines are attributed to a value $f=0.33$ of the $F/D$ ratio. The figures inside or beside the dashed boxes are attributed to $f=0.4$. Except for the resonances given in this figure, there are no further ones at higher energies or for higher spins for the chosen value of the coupling constant.

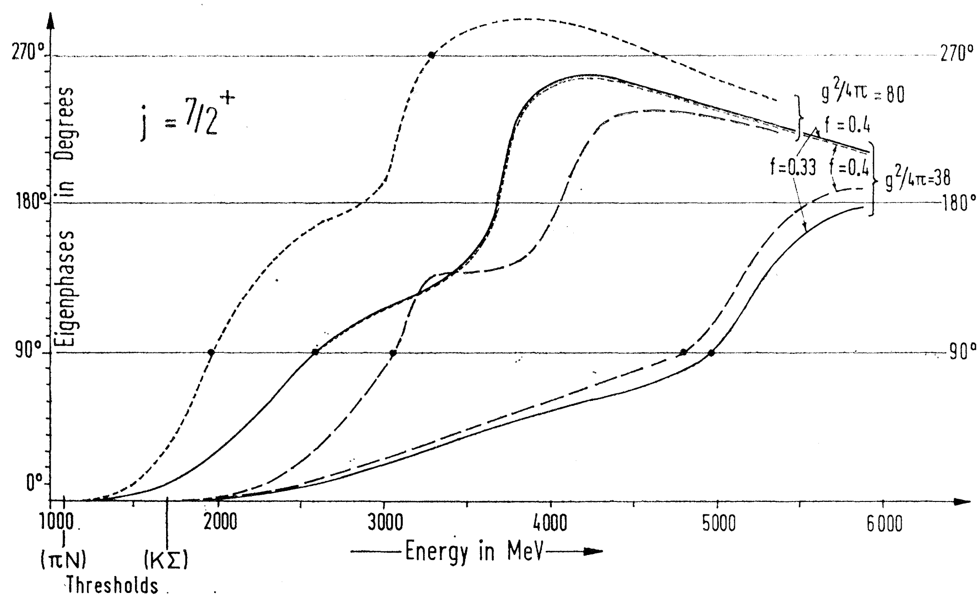


FIG. 3. Eigenphases for the $\frac{7}{2}^+$ $\Delta$ states. The eigenphases are given for $F/D$ values of $f=0.33$ (solid lines) and for $f=0.4$ (dashed lines). The coupling constant $g^2/4\pi=38$ fits $\Delta_{3/2}(1236)$ and $g^2/4\pi=80$ fits $\Delta_{5/2}(1950)$.

these results in Fig. 2 and in Table I. In addition we give the results for an $F/D$ ratio $f=0.4$ (Fig. 2) and exhibit the eigenphases for the orbitally excited resonances (Figs. 3 and 4).

### *1. $\frac{3}{2}^+$ Decuplet*

It was shown in I that the decuplet appears as the lowest $\frac{3}{2}^+$ supermultiplet and that the pion-nucleon coupling constant ought to be chosen as $g^2/4\pi=38$ within this model in order to obtain $N_{3/2}^*$ at the experimental energy of 1236 MeV. In addition, it was noted that these results are practically independent of the $F/D$ ratio since according to Fig. 1 the $\frac{3}{2}^+$ decuplet potential strength is practically independent of the value of $f$. Similar qualitative results (but with a somewhat stronger $f$ dependence) were obtained using the $N/D$ approach[12] and a static-meson theory with recoil terms,[13] both without cutoffs and with a larger value of the coupling constant. In agreement with the nonrelativistic Chew-Low theory,[5] $N/D$ calculations, and Bethe-Salpeter calculations,[6,14] the decuplet may also be obtained with a physical coupling constant $g^2/4\pi=14.6$ if one introduces an adjustable cutoff parameter.[1,15] As in Refs. 7, 12, and 13, the other members of the decuplet appear with energies about 100 MeV larger than the experimental ones and with somewhat larger widths. We note that the experimental widths may be obtained provided we increase the value of the coupling constants so that these resonances appear at about the experimental values (see Table I). The $Y^*$ resonance requires $g^2/4\pi=60$ to appear at 1385 MeV with a width of about 55 MeV. The $\Xi^*$ needs $g^2/4\pi=70$ to appear at 1530 MeV with a width of about 12 MeV. Finally, the $\Omega^-$ requires $g^2/4\pi=90$ in order to appear as a bound state at an energy of 1672 MeV. Furthermore, we note that if one increases the coupling constant to 50 (rather than 38), the width of the $N_{3/2}^*$ resonance decreases from 170 to 120 MeV, whereas the resonance energy only shifts from 1236 to 1180 MeV.

We remind the reader that we used $SU(3)$ relations among coupling constants which of course are not necessarily valid in a broken-symmetry model with unequal input masses. Indeed, there are indications that the coupling constants $g_{\pi\Sigma\Lambda}$ and $g_{\pi\Sigma\Sigma}$ are considerably larger than the ones obtained from $SU(3)$ relations among pseudoscalar octet couplings.[16,17] This is in agreement with the larger value of $g^2/4\pi$ which we need for the $Y^*$ resonance. For the two other decuplet resonances, namely, $\Xi^*$ and $\Omega^-$, there is no experimental information available since they essentially correspond to $\pi\Xi\Xi$ and $\bar{K}\Sigma\Xi$ couplings, respectively. The latter (i.e., $g_{\bar{K}\Sigma\Xi}^2/4\pi$) would be equal to $g_{\pi NN}^2/4\pi$ if $SU(3)$ relations were valid. Thus, $SU(3)$ requires that the same coupling constant which is relevant for $N_{3/2}^*(1236)$ should also be relevant for $\Omega^-(1672)$ if baryon exchange is assumed to be the dominant contribution in either case (the other coupling constant which is relevant for

[12] K. C. Wali and R. L. Warnock, Phys. Rev. **135**, B1358 (1964); R. C. Slansky, LRL Report No. UCRL 17450, 1967 (unpublished).
[13] L. B. Redei, Nucl. Phys. **B10**, 419 (1969); and University of Umeå, Sweden, report, 1969 (unpublished).
[14] J. A. Tijon and H. M. Nieland, University of Nijmegen, Netherlands, report, 1969 (unpublished).
[15] J. Katz and S. Wagner, DESY Report No. 70/21 (unpublished).
[16] N. M. Quinn, M. Restignoli, and G. Violini, Fortschr. Physik **17**, 467 (1969).
[17] G. Ebel, H. Pilkuhn, and F. Steiner, University of Karlsruhe, Germany, report, 1969 (unpublished).

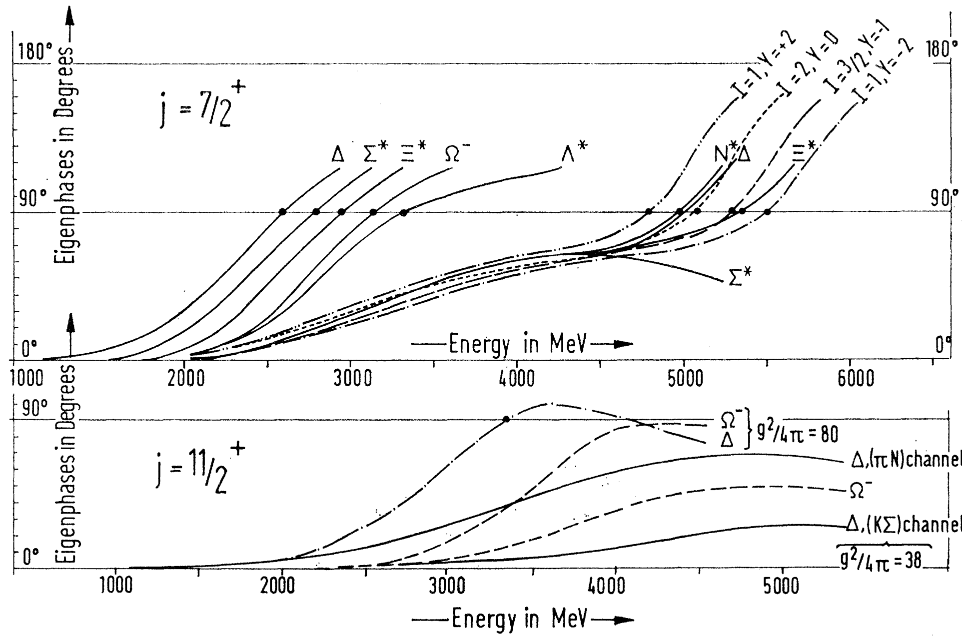


FIG. 4. Eigenphases for the $\frac{7}{2}^+$ and $\frac{11}{2}^+$ baryon states for $g^2/4\pi=38$ $f=0.33$, and in addition for $g^2/4\pi=80$ in the case of the $\frac{11}{2}^+$ partial wave. The $SU(3)$ states are indicated. For the multichannel states, only the attractive eigenphases which are mentioned in Tables I and II are included. The others are all repulsive.

$\Omega^-$, i.e., $g_{\bar{K}\Lambda\Xi}{}^2/4\pi$, is unimportant). If one considers the threshold differences in the two cases, different values of the $g_{\pi NN}{}^2/4\pi$ and $g_{K\Sigma\Xi}{}^2/4\pi$ coupling constants do not seem unreasonable.

In summary, once the coupling constants are better known, the baryon-exchange force may well be sufficient to explain the $\frac{3}{2}^+$ decuplet.

### 2. $\frac{3}{2}^+$ *27-Plet*

Several hundred MeV above the decuplet (in the region where second $\frac{3}{2}^+$ resonances have been established experimentally) a second supermultiplet is obtained in the 27-dimensional $SU(3)$ representation. In I we have given the resonance energies and widths of the 27-plet for $g^2/4\pi=38$ and values of the $F/D$ ratio $f=0.33$, 0.25, and 0.50 as well as for $g^2/4\pi=100$ and $f=0.33$ (see Figs. 3 and 14 and Table II of I). We concluded there that the choice $g^2/4\pi=38$ and $f=0.4$ is most appropriate to fit the decuplet as well as the experimental second resonances. We add here the results for this choice of parameters in Fig. 2. Table III contains in addition the results for the choice $g^2/4\pi=80$ and $f=0.33$. We recall that $g^2/4\pi=38$ fits $\Delta(1236)$, whereas $g^2/4\pi=80$ is required to make $\Xi^*(1530)$ and $\Omega^-(1672)$ appear at about the correct experimental energies.

A detailed discussion of these resonances and their comparison with experiments has already been given in I. The pion-nucleon resonances $P_{13}(1860)$ and $P_{33}(1690)$ which have been obtained in phase-shift analyses,[18] as well as some doubtful candidates for $Y^*$ and $\Xi$ resonances (whose spin-parity assignment are not well established), were assumed to belong to the 27-plet. The situation has not improved very much, but we add in Table III some further possible candidates.[19] At that time, we had to mention that there was no indication for a $\frac{3}{2}^+$ $\Lambda$ state. Now there is a candidate, $\Lambda(1860)$, which, however, still needs confirmation. So far we have only discussed those resonances which are also predicted by the quark model.[8] We remind the reader here that the quark model expects two $Y^*$'s and two $\Xi^*$'s belonging to the octet with $P_{13}(1860)$ and the decuplet with $P_{33}(1690)$, respectively, whereas we predict only one of each. However, some of the 27-plet exotic resonances which are excluded in the usual quark model with only $qqq$ baryon states are also essentially excluded in our model, namely, in contrast to the nonexotic resonances mentioned above, three of the exotic ones appear with extremely broad widths.

There is, however, one important exception: the exotic $Z_1$ resonance with the quantum numbers $I=1$, $Y=+2$. This resonance appears in our model at an energy of about 1600 MeV with a width of about 300 MeV as a member of the $\frac{3}{2}^+$ 27-plet. Considerable experimental study has been devoted in order to find $Z$ resonances in the $K^+p$ beam. Ever since the so-called

[18] A. Donnachie, R. G. Kirsopp, and L. Lovelace, CERN Report No. TH-838 (unpublished).

[19] The data are taken from the latest values which have been reported by the Particle Data Group, and have been given to us by Dr. P. Söding, Lawrence Radiation Laboratory and DESY.

TABLE II. $\frac{7}{2}^+$ and $\frac{11}{2}^+$ decuplet states. Energies and widths (in parentheses) are given in MeV for the experimental and calculated decuplet resonances, for $g^2/4\pi=38$ and 80 and for $f=0.33$.

| | $\frac{7}{2}^+$ decuplet resonances (lower decuplet) | | | | $\frac{11}{2}^+$ decuplet resonances | | |
|---|---|---|---|---|---|---|---|
| | Experiment | $g^2/4\pi=38$ | $g^2/4\pi=80$ | Orb. excited $g^2/4\pi=80$ | Experiment | $g^2/4\pi=38$ | $g^2/4\pi=80$ |
| $\Delta$ | 1950 (220) | 2580 (1100) | 1930 (400) | 3200 (370) | 2420 (370)[a] | none | 3300 (1700) |
| $\Sigma^*$ | 2250 (200)[a] | 2820 (1200) | 2160 (430) | 3410 (410) | 2595 (140)[a] | none | 3560 (1800) |
| $\Xi^*$ | | 2970 (1100) | 2320 (380) | 3610 (300) | ... | none | 3760 (1800) |
| $\Omega^-$ | ... | 3140 (1150) | 2460 (330) | 3820 (210) | ... | none | 3990 (1800) |

[a] Spin and/or parity not established experimentally.

Cool[20] bump at an energy of 1900 MeV with a width of about 200 MeV was first found in the $I=1$ $K^+p$ cross section, some papers in recent years have mentioned evidence[20–26] for $Z$ resonances while others gave evidence[27–29] against their existence. The recent CERN[26] phase-shift analysis has also one solution with a $Z_1$ resonance in the $\frac{3}{2}^+$ partial wave at an energy of about 2200 MeV. Recently, a series of experiments was carried out by Cool *et al.*[29] at Brookhaven with improved accuracy, showing indubitable $Z_1$ resonances at energies of 1900, 2190, and 2505 MeV. In the literature[20–27] the $Z_0(1860)$ enhancement is sometimes interpreted as a $K\Delta$ threshold effect. We believe that a resonance interpretation is a more natural one, since there is no principal distinction between a resonance and a threshold effect if it appears as a bump in the cross section with a phase shift passing 90°. This was already pointed out by Lovelace.[30–32]

So far we have not paid very much attention to the exact energies of the resonances which are predicted by our model in comparison with the experimental ones. This is so because we expect deviations of the numerical values of the coupling constants from their perfect symmetry limit to cause considerably changes which are of crucial importance for the $Z$ resonances. The input potential is determined by the $KN\Lambda$ and $KN\Sigma$ coupling constants, the first being of the order of magnitude of $g_{\pi N}{}^2/4\pi$ while the second one is about zero if $SU(3)$ relations are fulfilled. However, there are experimental indications[16,17] that the coupling constant $g_{KN\Lambda}{}^2$ is nearly half as large. In this case our model would expect $Z$ resonances at such high energies and with such broad widths (compare Fig. 4 in I) that one should not find them at all.

In our model, a value of the coupling constant $g_{KN\Lambda}{}^2/4\pi=10$ instead of 14.5 is required in order to make the $Z_1$ resonance belonging to the $\frac{3}{2}^+$ 27-plet appear at an energy of about 1900 MeV with a width of about 300 MeV.[30] On the other hand, if the $\bar{K}\Xi\Xi$ coupling is really so strong as the experimental $\Omega^-$ expects it to be, a rather narrow $\Omega_1$ resonance should occur instead of the second $\Omega_0{}^-$ which is predicted in the quark model.[8] Since there is no indication of any $\Omega$ resonance as yet, neither prediction can be compared with experiment.

Concluding this section, we note that our model predicts a complete 27-plet of resonances in the whole region of the parameters $38\leq g^2/4\leq 90$ and $0.25\leq f \leq 0.5$ (which fit the usual $\frac{3}{2}^+$ decuplet).

### 3. Radially Excited $\frac{3}{2}^+$ States

In the quark model,[8] higher resonances are often discussed in terms of radial excitation in analogy to

[20] R. L. Cool *et al.*, Phys. Rev. Letters **17**, 102 (1966); R. J. Abrams *et al.*, *ibid.* **19**, 259 (1967).

[21] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

[22] D. V. Bugg, R. S. Gilmore, M. A. Knight, D. C. Salter, G. H. Stafford, E. J. N. Wilson, J. D. Davies, J. D. Dowell, P. M. Hattersley, R. J. Homer, A. W. O'Dell, A. A. Carter, R. J. Trapper, and K. F. Riley, Phys. Rev. **168**, 1466 (1968).

[23] J. Tyson, J. S. Greenberg, V. W. Hughes, D. C. Lu, R. C. Minehart, S. Mori, and J. E. Rothberg, Phys. Rev. Letters **19**, 255 (1967); S. Mori, J. S. Greenberg, V. W. Hughes, D. C. Lu, J. E. Rothberg, and P. A. Thompson, Phys. Letters **28B**, 152 (1968).

[24] A. S. Carroll, J. Fischer, A. Lundby, R. H. Phillips, C. L Wang, F. Lobkowicz, A. C. Melissinos, Y. Nagashima, C. A. Smith, and S. Tewksbury, Phys. Rev. Letters **21**, 1282 (1968).

[25] R. W. Bland, M. G. Bowler, J. L. Brown, G. Goldhaber, S. Goldhaber, V. H. Seeger, and G. H. Trilling, Phys. Rev. Letters **18**, 1077 (1967); A. A. Hirata, C. G. Wohl, G. Goldhaber, and G. H. Trilling, *ibid.* **21**, 1485 (1968).

[26] S. Anderson, C. Duam, F. C. Erne, J. P. Lagnaux, J. Sens, F. Udo, and F. Wagner, Phys. Letters **30B**, 56 (1969).

[27] S. Anderson, C. Daum, F. C. Erne, J. P. Lagnaux, J. Sens, and F. Udo, Phys. Letters **28B**, 611 (1969).

[28] G. Bassompierre, Y. Goldschmidt-Clermont, A. Grant, V. P. Henri, R. Jennings, B. Jongejans, C. Linglin, F. Muller, J. M. Perreau, R. Sekulin, W. Dabaere, J. Debaisieux, P. Dufour, F. Grad, J. Heughebaert, L. Paper, P. Peeters, F. Verbeure, and R. Windmolders, Phys. Letters **27B**, 468 (1968).

[29] R. W. Bland, M. G. Bowler, J. L. Brown, J. A. Kadyk, G. Goldhaber, S. Goldhaber, V. H. Seeger, and G. H. Trilling, Nucl. Phys. **B13**, 595 (1969); B. H. Hall, R. W. Bland, G. Goldhaber, and G. H. Trilling, LRL Report No. UCRL 19231, 1969 (unpublished).

[30] C. Lovelace, in *Proceedings of the Heidelberg International Conference on Elementary Particles, 1967*, edited by H. Filthuth (Interscience, New York, 1968), pp. 85 ff.

[31] According to a private communication from Dr. Ebel, University of Karlsruhe, Germany, the most probable mean experimental value obtained from the parametrization of dispersion relation and Cabibbo theory is about $g_{KN\Lambda}{}^2/4\pi=10$. This is just the value which is required in our model to produce $Z_1(1900)$ as a member of a 27-plet.

[32] After the completion of this paper, a phase-shift analysis using the new data of Ref. 30 was performed; see S. Kato *et al.*, Phys. Rev. Letters **24**, 615 (1970). These authors found evidence for the interpretation of $Z_1(1900)$ as a resonance in three out of four sets. The CERN group of F. Wagner *et al.* (unpublished) came to similar results, although they conclude that other partial waves seem to be admixed, and the resonance is a very inelastic one containing contributions from decuplet and vector-meson channels.

TABLE III. $\frac{3}{2}^+$ and $\frac{7}{2}^+$ 27-plet. Energies and widths (in parentheses) are given in MeV for the experimental and calculated resonances which are attributed to a 27-dimensional $SU(3)$ representation, for $g^2/4\pi=38$, 60, and 80 and for $f=0.33$. Additional values for $g^2/4\pi=38$ and $f=0.4$ are given in Fig. 1 and others in Ref. 1. (b.st.≡bound state.)

| 27-plet | $I$ | $Y$ | $\frac{3}{2}^+$ Experiment | Resonating channel | $\frac{3}{2}^+$ lower 27-plet resonances $g^2/4\pi=38$ | $g^2/4\pi=60$ | $g^2/4\pi=80$ | Orb. excited $\frac{3}{2}^+$ 27-plet $g^2/4\pi=80$ | $\frac{7}{2}^+$ 27-plet $g^2/4\pi=80$ |
|---|---|---|---|---|---|---|---|---|---|
| exotic | 1 | 2 | 1560[a] | $(KN)$ | 1609 ( 336) | 1727 (b.st.) | 1336 (b.st.) | 3300 (1500) | 2800 ( 930) |
| $N^*$ | $\frac{1}{2}$ | 1 | 1860 (300)[a] | $(\eta N)$ | 1755 ( 358) | none | none | 3670 (1060) | 2920 ( 770) |
| $\Delta$ | $\frac{3}{2}$ | 1 | 1690 (280)[a] | $(K\Sigma)$ | 1852 ( 337) | 1682 ( 91) | 1590 ( 96) | 3800 (6000) | 2950 ( 500) |
| $\Lambda^*$ | 0 | 0 | 1860[b] | $(\pi\Sigma)$ | 1596 ( 150)[c] | 1477 ( 70) | 1437 ( 29)[c] | 3080 ( 580) | 2440 ( 580) |
| $\Sigma^*$ | 1 | 0 | 1940 ( 90)[d] | $(\bar{K}N)$ | 1879 ( 317) | 1732 ( 78) | 1644 ( 59) | 3770 (1000) | 3038 (1360) |
| exotic | 2 | 0 | never found | $(\pi\Sigma)$ | 1990 (1440) | 1670( 540) | 1560 (320) | none | 3040 (1160) |
| $\Xi^*$ | $\frac{1}{2}$ | $-1$ | 1705[b] | $(\bar{K}\Sigma)$ | 2036 ( 716) | 1850 (122) | 1764 ( 70) | none | 3420 (1500) |
| exotic | $\frac{3}{2}$ | $-1$ | never found | $(\pi\Xi)$ | 2083 (1430) | 1790 (290) | 1712 (127) | none | 3200 (1180) |
| exotic | 1 | $-2$ | never found | $(\bar{K}\Xi)$ | 2140 ( 955) | 1910 (150) | 1847 ( 29) | none | 3340 (1180) |

[a] Nonstrange resonances which appear in $\pi N$ phase-shift analyses. Values taken from Lovelace *et al.*
[b] Doubtful resonances which still need confirmation and for which there might be other candidates in addition.
[c] The resonances with the quantum numbers $I=0$, $Y=0$ of $\Lambda$ appear considerably apart from the other **27** states depending very much on the value of the $F/D$ ratio. In fact, these resonances have to be attributed to a singlet $SU(3)$ representation, and an additional **27** $\Lambda$ state appears only for $f<0.3$.
[d] $\Sigma(1940)$ was reported as a resonance decaying into $\Sigma(1120)+\pi$ by V. E. Barnes *et al.*, in Proceedings of the Boulder Conference on Elementary Particles, Boulder, Colorado, 1969 (unpublished).

usual quantum mechanics. However, these considerations are only qualitative. In our calculations we do get radially excited supermultiplets of resonances (see Fig. 2 and Tables I–III). However, all of them appear at such high energies and with such large widths (of the order of magnitude 1000 MeV) that they certainly have nothing to do with physical resonances. The so-called radial excitations of the quark model appear in our model as incomplete higher-dimensional supermultiplets.

### 4. *Orbitally Excited $\frac{3}{2}^+$ States*

Whereas the radial excitations appear with large widths, this is not necessarily the case for the orbital excitations even at higher energies. It is well known in potential scattering that finite potentials can only produce a finite number of orbital excitations and that the more the poles migrate to the physical sheet, the stronger the potential. This is what happens also in our model. For $g^2/4\pi=38$, which was required to make $\Delta(1236)$ appear at the correct energy, there is only one orbitally excited decuplet at about 2600 MeV with a rather large width. It requires $g^2/4\pi=80$ to shift it to an energy of about 1930 MeV where the experimental $\frac{7}{2}^+\Delta$ resonance occurs (see Fig. 3). The other members of the decuplet as well as the $\Delta$ appear with a reasonable width of about 400 MeV, with a mass formula similar to the one for the usual decuplet (see Fig. 4). This is

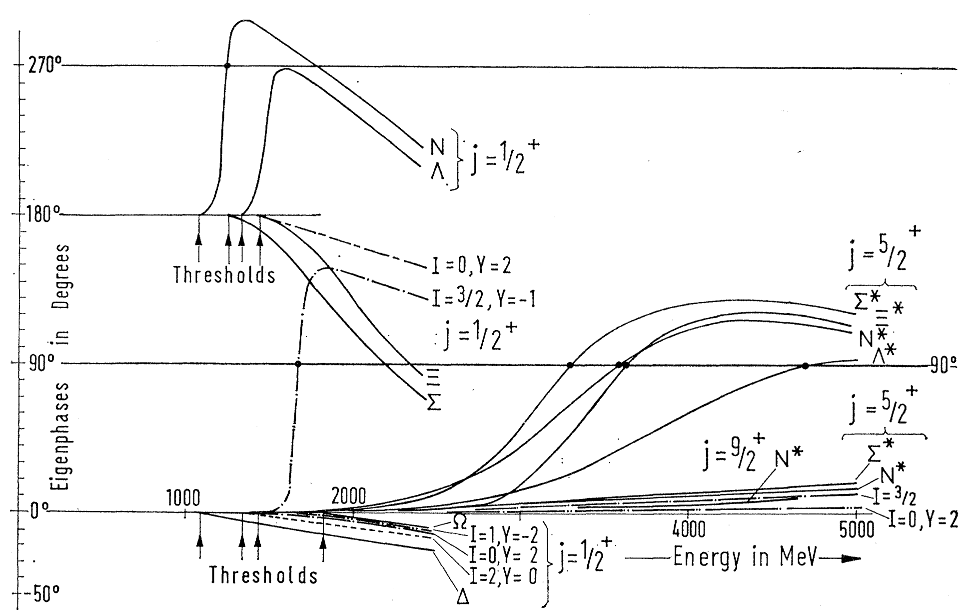


FIG. 5. Eigenphases for the $\frac{1}{2}^+$, $\frac{5}{2}^+$, and $\frac{9}{2}^+$ baryon states for $g^2/4\pi=38$ and $f=0.33$. For the multichannel states, only the attractive channels, which are given in Table III, are plotted. The others are all repulsive.

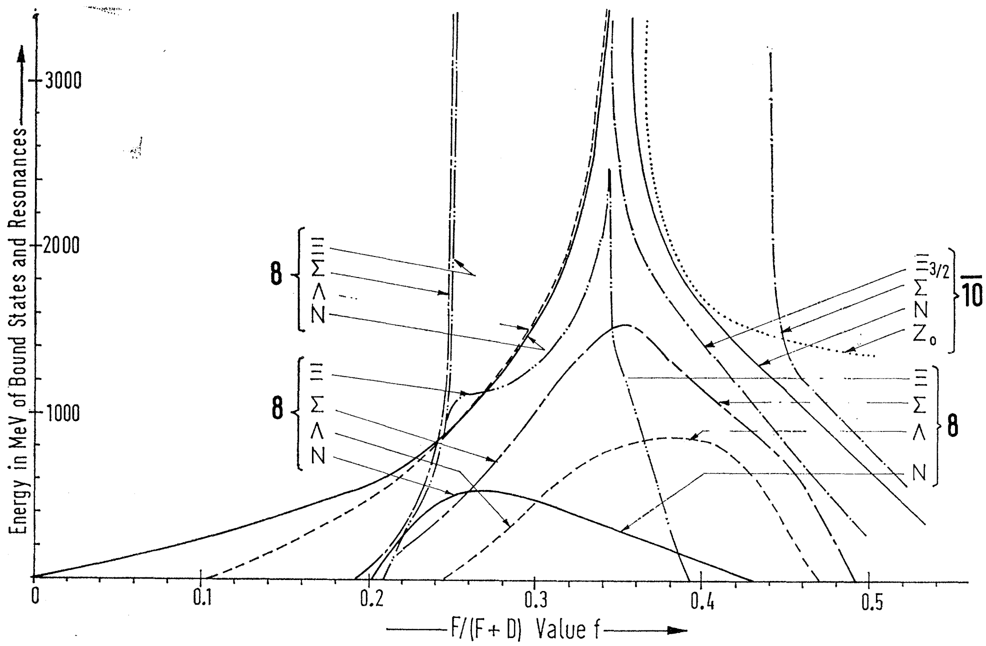


FIG. 6. Dependence of the $\frac{1}{2}^+$ states on the $F/D$ ratio $f$ for $g^2/4\pi=14.6$.

expected from the Gell-Mann–Okubo mass formula,[30] which is assumed to be spin independent. For $g^2/4\pi=80$ there is in addition a $\frac{7}{2}^+$ 27-plet and an extremely broad radially excited decuplet as well as a second recurrence of the decuplet. Most probably the second $Z_1$ resonance in experiments[29] has spin $\frac{7}{2}$ and belongs to a 27-plet. The $\frac{11}{2}^+$ decuplet, which appears at about 2400 MeV in experiments, is here obtained at about 3000 MeV with very broad widths (see Fig. 4). It would require an even stronger coupling. For a coupling which is also required for the $\Xi^*$ and $\Omega^-$ members, namely $g^2/4\pi=80$, one obtains an orbitally excited decuplet at about the experimental energies and widths. The third recurrence appears at too large energies. Thus our model yields a finite number of Regge recurrences in a rising trajectory in consistency with experiments.

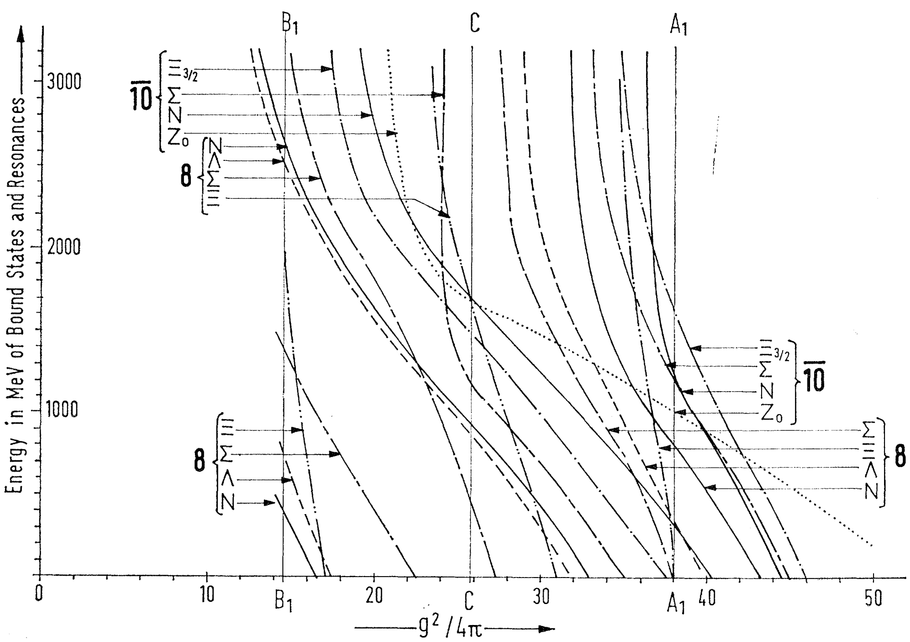


FIG. 7. Dependence of the $\frac{1}{2}^+$ states on the coupling constant $g^2/4\pi$ for $f=0.33$.

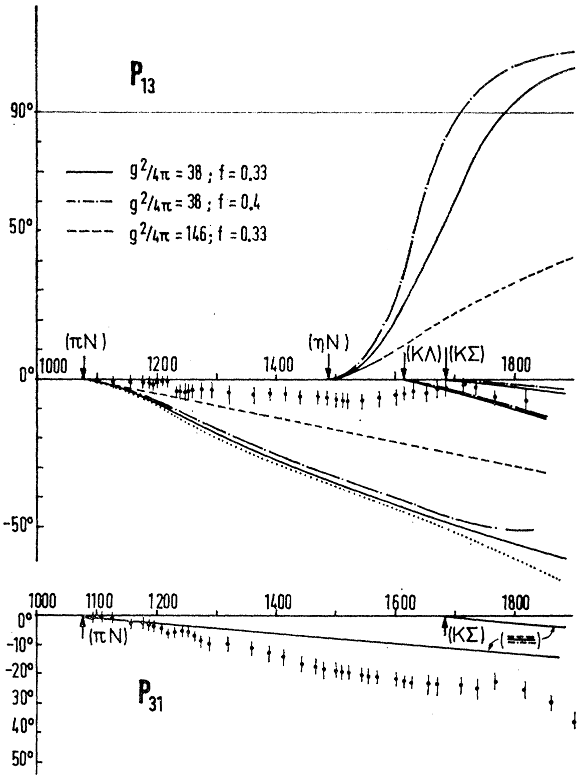


FIG. 8. $P_{31}$ and $P_{13}$ pion-nucleon eigenphase shifts and comparison with the CERN phase-shift analysis. Solid lines, $g^2/4\pi=38$, $f=0.33$; dot-dashed lines, $g^2/4\pi=38$, $f=0.4$; dashed line, $g^2/4\pi=14.6$, $f=0.33$.

### B. $\frac{1}{2}^+$ Partial-Wave Amplitude and Its Orbital Excitations

Baryon-exchange forces are generally believed to be the most important contributions to the $\frac{3}{2}^+$ partial-wave amplitude. However, at least in the one-channel $\pi N$ case other contributions such as $\frac{3}{2}^+$ decuplet exchange and perhaps exchanges of vector mesons are very important for the $\frac{1}{2}^+$ partial waves. This was discussed in our approach in Ref. 1. For completeness we give here the $\frac{1}{2}^+$ results calculated with the same potential which we chose for the $\frac{3}{2}^+$ case.

#### *1. $\frac{1}{2}^+$ Bound States and Resonances*

The results of the calculations for the $\frac{1}{2}^+$ partial-wave amplitude are given in Figs. 5–9 as well as in Table I. The nucleon cannot be obtained as a bound state or even a resonance in one-channel calculations since the nucleon exchange is the only contribution to the potential and the forces are then repulsive. Thus Chew's reciprocal bootstrap model needs $N_{3/2}{}^*$ exchange in addition. As may be seen in Fig. 6, this is of course also true in our model if one considers the case of an $F/D$ ratio $f=0$, where neither bound states nor resonances occur. However, the situation changes drastically in the multichannel case with physical $F/D$ values. Figure 7 shows that for $g^2/4\pi$ and for values of $0.25\leq f \leq 0.45$, the $N$ state as well as the other octet states appear as bound states which are even too strongly bound. In addition to the ground-state octet, there occurs a second octet of resonances for $f\leq 0.34$, and second resonances are in the $\overline{\mathbf{10}}$ representation for $f\geq 0.34$. These supermultiplets are not complete for all values of $f$. Only for $f=0.25$ is the second octet complete (since baryon exchanges are maximal for this value in the $\frac{1}{2}^+$ partial wave), whereas for larger values of $f$ the $\Sigma$ and $\Xi$ states are missing. The $\Sigma$ state belonging to the $\overline{\mathbf{10}}$ representation appears only for $f\geq 0.41$. For values of $f$ in the neighborhood of 0.33 and lower, $\Sigma$ and $\Xi$ states become maximal and appear even as resonances above threshold because of representation mixing with the higher supermultiplets.

In Fig. 7 we show the displacement of the resonance positions with increasing coupling constants for $f=0.33$. Note that the resonances appear in the representations $\mathbf{8}$, $\overline{\mathbf{10}}$, $\mathbf{8}$ with a repetition of this sequence. For any given value of the coupling constant there are two $\frac{1}{2}^+$ supermultiplets. In addition to the physical case $g^2/4\pi=14.6$, we shall consider the cases $g^2/4\pi=26$ (which is required to fit the $P_{33}$ scattering at threshold) and $g^2/4\pi=38$ [which is required to fit the resonance position of $\Delta(1236)$ in Sec. III]. One may choose the parameters as, for instance, $g^2/4\pi=26$ and $f=0.25$, so that the nucleon appears as a bound state with the physical energy 940 MeV. In this case also the other members of the octet appear at energies in the neighborhood of the physical input masses. The second nucleon then appears at about 2000 MeV. We would of course attribute the lower-octet bound states to the physical one. A unique solution for the second resonance super-

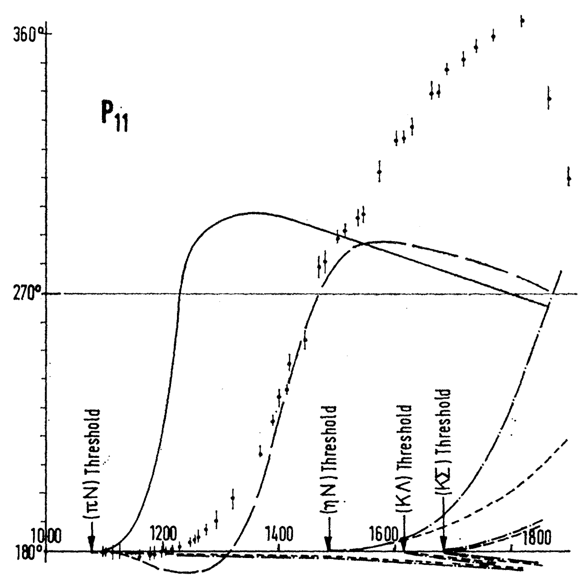


FIG. 9. $P_{11}$ phase shifts in comparison. Solid lines, $g^2/4\pi=38$, $f=0.33$; long dashed lines, $g^2/4\pi=38$, $f=0.36$; dot-dashed lines, $g^2/4\pi=14.6$, $f=0.4$; dashed lines, $g^2/4\pi=14.6$, $f=0.25$.

multiplet is not so easy, since there are two nucleon resonances, the Roper resonance $N(1460)$ and the phase-shift resonance $N(1750)$. If one notes the large energy difference between the two nucleon states which we obtain (it is at least 1 GeV), one may prefer to attribute our second nucleon to the third experimental one. In this case our model does not account for the Roper resonance. Indeed, it was obvious in some phenomenological-model calculations that the Roper resonance does not appear due to a second channel, but rather in the $N\pi\pi$ system.[9,33] Another interesting question is the occurrence of the exotic $Z_0$ resonance belonging to the $\overline{\mathbf{10}}$ representation. Our model would expect it at lower energies only for $f>0.4$. If the $KN\Lambda$ coupling constant were considerably smaller than its value derived from $SU(3)$ relations, then there would not be any $Z_0$, as has been discussed in Sec. III for the $\frac{3}{2}^+ Z_1$ resonance. In experiments many of the $\frac{1}{2}^+ Z$ resonances which were once obtained[20,28] have vanished, and even the bump[32] at about 1860 MeV, which is still mentioned in the Particle Data Group tables, is commonly explained as a $K^+\Delta$ threshold effect. Recent $K^+p$ phase-shift analyses[26] did not show any resonance behavior in the $\frac{1}{2}^+$ partial-wave amplitude. However, this question remains open.

#### 2. *Orbital Excitations*

For $g^2/4\pi=38$, an orbital excitation of the octet occurs at about 4 GeV in the $\frac{5}{2}^+$ partial wave. This can only be lowered below 2 GeV for very large values of the coupling constant.

### C. *P*-Wave Phase Shifts and Comparison with Experimental Values

In I we showed the $P_{33}$ eigenphase shift and compared it with the experimental values taken from the CERN phase-shift analyses[18] as well as the effective-range plot (see Figs. 12 and 13 in I). The over-all agreement of the $P_{33}$ phase shifts is quite good for $g^2/4\pi=38$ and rather independent of the $F/D$ value, except for energies near threshold where a coupling constant of $g^2/4\pi=26$ was required. The $P_{13}$ phase shift, which is given in Fig. 8, shows a two strong repulsion for $g^2/4\pi=38$ and it agrees better with experiments for $g^2/4\pi=14.6$, although in the latter case the $P_{13}$ resonance is displaced to very large energies. The $P_{31}$ phase shift, which is also shown in Fig. 8, agrees quite well with experiments and hardly depends on $g$ or $f$. Our model does not account for any $P_{31}$ resonance. The experimental enhancement at about 1930 MeV might well come from an $N\pi\pi$ channel as we believe is the case for the Roper resonance. In Fig. 9 we show the $P_{11}$ eigenphase shifts. It is amusing that they agree quite well for $g^2/4\pi=38$ and $f=0.36$, which we needed to fit $\Delta(1236)$, although such large values of the coupling constant should not make too much sense near threshold. Furthermore, it may just be an accident that the $P_{11}$ resonance appears at the energies of the Roper resonance since the bound state appears far too low. For physical coupling constants, the $P_{11}$ phase shift is repulsive and agrees with experiments only at very low energies. Concluding this section, we remark that all $P$-wave phase shifts agree with the CERN phase-shift analysis qualitatively ($P_{33}$ is attractive, the others repulsive at low energies) but no quantitative agreement is obtained.

## IV. COMPARISON WITH OTHER MODELS

All calculations of the $\frac{3}{2}^+$ decuplet states in models based on $SU(3)$-symmetric meson-baryon interactions arrive essentially at the same results. Namely, if the parameters are fixed so as to fit the $\Delta(1236)$ resonance position, its width then comes out too broad, and the other members of the decuplet appear with energies about 100 MeV larger than the experimental ones and with very broad widths. This occurs for baryon-exchange forces with large values of the coupling constant and without cutoffs (as in our model, the $N/D$ calculations of Ref. 12, and the solutions of the field equations of Ref. 13), or with baryon-exchange forces with physical coupling constants and a cutoff.[6,14,15] It also occurs for more complex potentials containing other forces in addition to baryon exchange, such as vector mesons, as, for instance, in Ref. 1. This might indicate that couplings derived from $SU(3)$ relations are not valid in a theory which introduces symmetry breaking by using the physically unequal octet masses. Indeed there are indications that analysis of experiments indicate large deviations of the coupling constants from their $SU(3)$-symmetry value. In contrast to the numerous papers dealing with the properties of $\Delta(1236)$ in one-channel calculations or in purely phenomenological models, there are only a few papers dealing with $SU(3)$-symmetric multichannel calculations.[34] In the $N/D$ calculations of Refs. 7 and 12, in addition to the $\frac{3}{2}^+$ decuplet, only a $\frac{3}{2}^+$ singlet was obtained, and no 27-plet. However, using $N/D$ and Balaz's procedure, the authors of Ref. 33 obtained resonances in the exotic one-channel states belonging to the 27-plet including the $Z_1$. The Bethe-Salpeter solutions have not been carried out to our knowledge for these cases. Let us also note that the same result is obtained in strong coupling,[35] i.e., there is a $\frac{3}{2}^+$ decuplet

[33] S. C. Bhargava and S. H. Patil, Phys. Rev. **182**, 1711 (1969); J. C. Pati and K. V. Vasavada, *ibid.* **144**, 1270 (1966); K. V. Vasavada, Nuovo Cimento **40A**, 1045 (1965).

[34] Among these, one should mention J. J. Brehm and J. F. Cook, Phys. Rev. **187**, 2174 (1969); P. Carruthers, *ibid.* **152**, 1345 (1966); **154**, 1399 (1968); P. Carruthers and M. Nieto, *ibid.* **163**, 1646 (1967); and E. Golowich, *ibid.* **139**, B1297 (1965).

[35] C. Goebel, Phys. Rev. Letters **16**, 1130 (1966); C. Dullemond and J. M. van der Linden, Ann. Phys. (N. Y.) **41**, 372 (1967); M. Beduar and J. Tolar, Nucl. Phys. **B5**, 255 (1968); J. Tolar, Technical University of Prague report (unpublished); G. Wentzel, Helv. Phys. Acta **41**, 1263 (1968); C. Dullemond, Phys. Rev. **180**, 1468 (1969).

and a subsequent 27-plet. Higher states, however differ from the predictions of our model.

Calculations of the $\frac{1}{2}^+$ partial wave for the octet states are at least four-channel problems in an $SU(3)$-symmetric model. Most of the calculations in the literature deal with bound states and resonances with the quantum numbers of the nucleon, and are done as a one-channel problem or with a second phenomenological channel.[5,6] $SU(3)$-symmetric dynamical calculations were carried out in relativistic quantum mechanics,[1] yielding the octet as bound states strongly bound. We arrived at the same result and obtained in addition an **8** or $\overline{\mathbf{10}}$ representation. The latter appears also in strong coupling.[35]

In conclusion, we note that in all cases where comparison is possible, the predictions of our model agree with those of other methods. Namely, the lowest $\frac{3}{2}^+$ states are a 10- and a 27-dimensional representation, whereas the lowest $\frac{1}{2}^+$ states are **8** and $\overline{\mathbf{10}}$ representations. In addition, there are radial and orbital excitations.

Group-theoretically, these results disagree with the predictions of the usual quark model which allows only **1**, **8**, and **10** representations, although both models are based on $SU(3)$. Hence it is the requirement that baryons should only occur as $qqq$ states that makes the difference. However, both kind of models, namely, the ones on hypothetical quarks and the ones based on the existing mesons and baryons, allow modifications in the direction of each other. In the quark model, one has just to allow $qqq\bar{q}q$ states to account for exotic $SU(3)$ representations. However, there might then be too many states, as in the Han-Nambu model.[36] In the model of the kind we discussed here, all exotic states except for one became very broad and were thus practically absent. The remaining $Z$ resonances would disappear if symmetry breaking of the coupling constants would indeed account for very low $KN\Lambda$ couplings, as is indicated by analysis of the $K^+p$ data. Even in this case, our model predicts only two $\frac{3}{2}^+$ $Y^*$ and $\Xi^*$ resonances, whereas the quark model needs three of each.

[36] M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965); O. W. Greenberg and C. A. Nelson, *ibid.* **179**, 1354 (1969).

## V. CONCLUSIONS

We solved the relativistic Schrödinger equation for the even-parity partial waves with a potential obtained by computing the baryon-exchange contribution to the pseudoscalar-meson–baryon interaction without introducing an arbitrary cutoff parameter. For the $\frac{3}{2}^+$ partial-wave amplitude, we obtained a decuplet and a 27-dimensional $SU(3)$ representation. Radial excitations are unimportant, and orbital excitations of the decuplet appear with rising trajectories. In the $\frac{1}{2}^+$ partial-wave amplitude, we obtained the octet as a set of bound states and a second supermultiplet in the **8** or $\overline{\mathbf{10}}$ representation depending on the $F/D$ ratio $f \gtrless 0.34$.

Group-theoretically, our results agree with other methods such as strong coupling, $N/D$ calculations, or variations of the static model with recoil terms. We disagree with some predictions of the usual quark model, since we predict exotic $Z$ resonances, and $\frac{3}{2}^+$ octets or $\frac{1}{2}^+$ decuplets do not occur in our model. The existence of exotic resonances depends crucially on the validity of the $SU(3)$ relations among coupling constants. If the low value of the $KN\Lambda$ coupling constant which was derived from some $K^+p$ data were to be confirmed, we arrive at a spectrum which agrees with the quark model. Otherwise a $\frac{3}{2}^+$ $Z_1$ resonance is predicted. If, on the other hand, the value of the $\bar{K}\Xi\Xi$ coupling constant were to be confirmed (which is required in order to obtain the usual $\Omega^-$ at the correct energy), then we would predict an $\Omega_1$ resonance where the quark model expects another $\Omega_0^-$.

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