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Deuteron-Deuteron Scattering at 2.2 and 7.9 GeV/c; Theoretical Interpretation

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We extend Franco's treatment of dd elastic scattering by including the quadrupole deformation of the deuteron. We present the results of numerical computations of the cross sections at 2.2 and 7.9 GeV/c, where comparison with experimental results is possible.

THE Glauber multiple scattering theory was applied by Franco[1] to dd scattering and qualitative predictions under a set of very restrictive assumptions were obtained. Recent experiments[2] allow one to test quantitatively the Glauber multiple scattering theory as applied to dd elastic collisions. Other formalisms have already been shown to be inadequate for explaining the features of the elastic cross section.[2] In this article we develop formulas that include the quadrupole deformation of the deuteron and compute the differential cross section at 2.2 and 7.9 GeV/c using realistic form factors to describe the deuteron. Furthermore, we allow the np and pp scattering amplitudes to be different where so indicated by experiment. The effect of the quadrupole deformation of the deuteron is of particular interest since in pd and πd scattering it has been shown to eliminate a dip in the differential cross section which would exist were the deuteron spherical. A similar dip is predicted for dd elastic scattering in the absence of the quadrupole deformation.[1]

When one includes the quadrupole deformation of the deuteron, the elastic scattering amplitude depends on the initial and final spin projections of the colliding deuterons. Thus we may write the scattering amplitude as

$$A_{pp'rr'}(\mathbf{\Delta})=\sum_{i=1,4}\alpha_{pp'rr'}{}^{(i)}(\mathbf{\Delta}),$$

where $\hbar\mathbf{\Delta}$ represents the momentum transfer in the center-of-mass system of the deuterons, p and p' represent the spin projections of the projectile before and after the collision, r and r' are the analogous projections for the target, and $\alpha_{pp'rr'}{}^{(i)}(\mathbf{\Delta})$, $i=1, 2, 3, 4$, are the single, double, triple, and quadruple scattering amplitudes, respectively. We take the spin projections along the incident beam direction. The quadruple scattering amplitude is the highest order of multiple scattering which occurs because the assumption of small-angle internal collisions inherent in the Glauber approach does not allow higher-order scattering processes.

The differential cross section is given in terms of the scattering amplitudes by

$$\frac{d\sigma}{dt}(\mathbf{\Delta})=\tfrac{1}{9}\sum_{pp'rr'}|A_{pp'rr'}(\mathbf{\Delta})|^2,\quad \text{with}\quad t=\hbar^2|\Delta|^2.$$

In deriving expressions for the multiple scattering amplitudes, we assume charge symmetry of nuclear forces so that the charge exchange amplitude may be expressed in terms of the np and pp amplidues. We also assume spin independence of the nucleon-nucleon interaction, which simplifies the algebra enormously; furthermore, no complete data exist on the full spin dependence of the nucleon-nucleon amplitude at the energies of interest. The assumption is fully justified in the limit of very high energies.

The multiple scattering amplitudes can be written as

$$\alpha_{pp'rr'}{}^{(1)}(\mathbf{\Delta})=S_{pp'}(\tfrac{1}{2}\mathbf{\Delta})S_{rr'}(\tfrac{1}{2}\mathbf{\Delta})4f(\mathbf{\Delta}),\tag{1}$$

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$$\alpha_{pp'rr'}{}^{(2)}(\mathbf{\Delta})=\frac{i\hbar}{\pi^{3/2}}S_{rr'}(\tfrac{1}{2}\mathbf{\Delta})\int S_{pp'}(\mathbf{q}')[f(\tfrac{1}{2}\mathbf{\Delta}+\mathbf{q}')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')-3g(\tfrac{1}{2}\mathbf{\Delta}+\mathbf{q}')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')]d^2q'$$
$$+\frac{i\hbar}{\pi^{3/2}}S_{pp'}(\tfrac{1}{2}\mathbf{\Delta})\int S_{rr'}(\mathbf{q}')[f(\tfrac{1}{2}\mathbf{\Delta}+\mathbf{q}')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')-3g(\tfrac{1}{2}\mathbf{\Delta}+\mathbf{q}')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')]d^2q'$$
$$+\frac{i\hbar}{\pi^{3/2}}\int S_{pp'}(\mathbf{q}')S_{rr'}(\mathbf{q}')[f(\tfrac{1}{2}\mathbf{\Delta}+\mathbf{q}')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')+3g(\tfrac{1}{2}\mathbf{\Delta}+\mathbf{q}')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')]d^2q',\quad (2)$$

$$\alpha_{pp'rr'}{}^{(3)}(\mathbf{\Delta})=\frac{-\hbar^2}{\pi^3}\int S_{pp'}(\mathbf{q}')S_{rr'}(\mathbf{q}'')[f(\mathbf{q}'+\mathbf{q}'')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}'')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')$$
$$-6g(\mathbf{q}'+\mathbf{q}'')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}'')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')+3f(\mathbf{q}'+\mathbf{q}'')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}'')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}')]d^2q'd^2q'',\quad (3)$$

$$\alpha_{pp'rr'}{}^{(4)}(\mathbf{\Delta})=\frac{-i\hbar^3}{8\pi^{9/2}}\int S_{pp'}(\mathbf{q}')S_{rr'}(\mathbf{q}'')[f(\mathbf{q}''')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}''-\mathbf{q}''')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}'-\mathbf{q}''')$$
$$\times f(\mathbf{q}'+\mathbf{q}''+\mathbf{q}''')+21g(\mathbf{q}''')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}''-\mathbf{q}''')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}'-\mathbf{q}''')g(\mathbf{q}'+\mathbf{q}''+\mathbf{q}''')$$
$$+6f(\mathbf{q}''')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}''-\mathbf{q}''')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}'-\mathbf{q}''')f(\mathbf{q}'+\mathbf{q}''+\mathbf{q}''')$$
$$-12f(\mathbf{q}''')f(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}''-\mathbf{q}''')g(\tfrac{1}{2}\mathbf{\Delta}-\mathbf{q}'-\mathbf{q}''')g(\mathbf{q}'+\mathbf{q}''+\mathbf{q}''')]d^2q'd^2q''d^2q''',\quad (4)$$

where $S_{mm'}(\mathbf{\Delta})$ is the deuteron form factor evaluated between states with spin projections m and m', and $f(\mathbf{\Delta})$ and $g(\mathbf{\Delta})$ are related to the forward pp and np elastic amplitudes by

$$f(\mathbf{\Delta})=\tfrac{1}{2}[f_{pp}(\mathbf{\Delta})+f_{np}(\mathbf{\Delta})]$$

and

$$g(\mathbf{\Delta})=\tfrac{1}{2}[f_{pp}(\mathbf{\Delta})-f_{np}(\mathbf{\Delta})].$$

The deuteron form factor is defined by

$$S_{mm'}(\mathbf{\Delta})=\int\psi_{m'}{}^*(\mathbf{r})e^{i\Delta\cdot\mathbf{r}}\psi_m(\mathbf{r})d^3r,$$

where $\psi_m(\mathbf{r})$ is the deuteron wave function with spin projection m, in our case along the incident beam direction. It can be shown that in the small-angle approximation the form factor vanishes between states that differ in spin projection by one unit because the argument of $S_{mm'}$ is always transverse to the quantization direction. It follows from Eqs. (1)–(4) that the amplitudes for which either the projectile or the target deuteron change the projection of their spin by one unit vanish. The form factors for the allowed transitions are then

$$S_{00}(\mathbf{\Delta})=S_0(\Delta)-S_2(\Delta)/\sqrt{2},\quad (5)$$

$$S_{11}(\mathbf{\Delta})=S_{-1-1}(\mathbf{\Delta})=S_0(\Delta)+S_2(\Delta)/\sqrt{8}\quad (6)$$

for $\Delta m=0$, and

$$S_{+1-1}(\mathbf{\Delta})=-(3/\sqrt{8})S_2(\Delta)e^{+2i\phi},\quad (7)$$

$$S_{-1+1}(\mathbf{\Delta})=-(3/\sqrt{8})S_2(\Delta)e^{-2i\phi}\quad (8)$$

for $\Delta m=2$, where ϕ is the azimuthal angle of the momentum transfer vector about the incident beam direction and $S_0(\Delta)$ and $S_2(\Delta)$ are the usual spherical and quadrupole form factors of the deuteron.[3] We have evaluated them using the Moravcsik III approximation to the Gartenhaus wave function[4] with a 7% D-wave admixture, and the results are shown in Fig. 1(a).

The character of the form factor allows one to classify the different transitions into three families. The first, labeled ($\Delta m=0$, $\Delta m=0$), contains the transitions in which both the target and the projectile deuteron do not change the projection of their spins. The second family, labeled ($\Delta m=\pm2$, $\Delta m=0$), contains the transitions in which one of the deuterons changes the projection of its spin by $\Delta m=\pm2$. The third family, labeled ($\Delta m=\pm2$, $\Delta m=\pm2$), contains the transitions in which both deuterons change the projection of their spin by $\Delta m=\pm2$. We shall illustrate the discussion of the properties of each family with the calculations performed at 7.9 GeV/c, which use nucleon-nucleon amplitudes to be discussed below.

Figure 1(b) shows the three distinct partial cross sections $|A_{pp'rr'}(\mathbf{\Delta})|^2$ for the ($\Delta m=0$, $\Delta m=0$) family of transitions. The amplitudes for the transitions in this family contain the product of two spherical form factors, and hence yield the dominant partial cross sections. The differences in the partial cross sections with the family are due to the different ways in which the quadrupole form factor enters the calculation of the form factors in Eqs. (5) and (6). The fact that the interference minimum for the partial cross sections in this family lies at different values of t contributes to washing out the interference minimum in the over-all cross section.

The two distinct cross sections for the ($\Delta m=\pm2$, $\Delta m=0$) family are shown in Fig. 1(c). In this family

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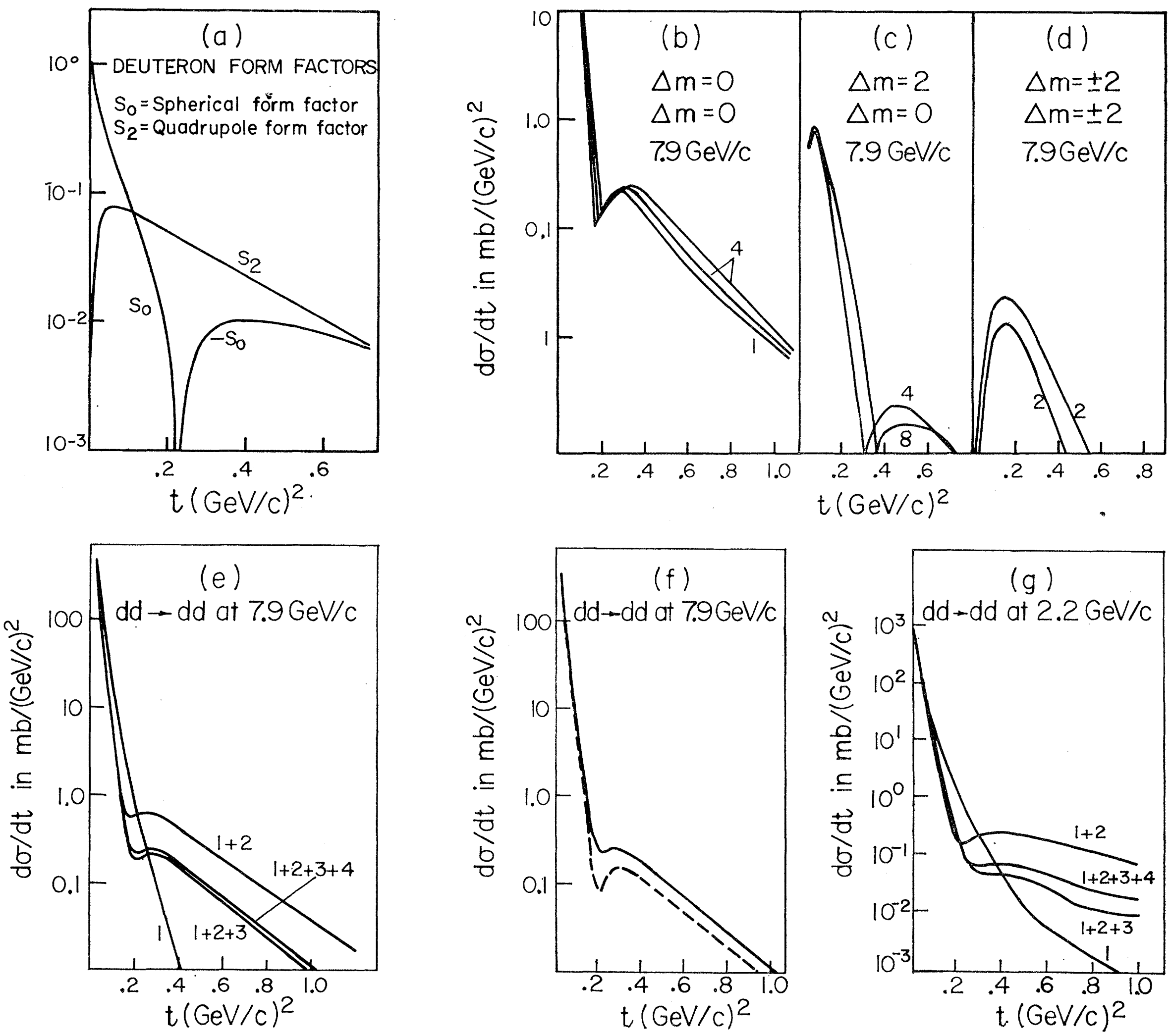

FIG. 1. Deuteron-deuteron elastic scattering. (a) Spherical and quadrupole form factors $S_0(\Delta)$ and $S_2(\Delta)$, respectively. (b)–(d) Partial cross sections for the families of transitions at 7.9 GeV/c; the numbers indicate the multiplicity of each family. (e) Elastic differential cross section at 7.9 GeV/c as progressively higher orders of multiple scattering are included. The curves are labeled by the orders included: 1, 2, 3, and 4 mean single, double, triple, and quadruple scattering, respectively. (f) Effect of letting the ratio of real to imaginary parts of the nucleon-nucleon amplitude be zero. The solid curve is the same as the curve labeled 1+2+3+4 in (e). The broken curve has the same parameters except that the ratio of real to imaginary parts of the nucleon-nucleon amplitude has been set to zero. (g) Differential cross section at 2.2 GeV/c in a form analogous to (e).

the product of a spherical form factor and a quadrupole form factor enter the calculation of the amplitudes. Since the contributions of the quadrupole form factor in the multiple scattering integrals are smaller than the spherical form factor, the amplitudes for this family are generally smaller than the amplitudes for the family ($\Delta m=0$, $\Delta m=0$). Because the slope of the quadrupole form factor is less steep than the slope of the spherical form factor, the interference minimum occurs at larger values of the momentum transfer. From the shape of the partial cross sections shown in Fig. 1(c), one concludes that a further filling of the interference minimum occurs when the partial cross sections in this family are added to the ($\Delta m=0$, $\Delta m=0$) family.

The amplitudes for the family ($\Delta m=\pm 2$, $\Delta m=\pm 2$) contain the product of two quadrupole form factors and are generally much smaller than the amplitudes for the other families. Their contribution to the over-all cross section is negligible.

The most obvious effect of the quadrupole deformation of the deuteron is the almost complete elimination of the interference minimum in the differential cross section. The extent to which a small minimum remains also depends on the ratios of real to imaginary parts of the nucleon-nucleon amplitudes, as we shall now see.

In the calculations that we have performed, we have used the standard high-energy parametrization of the nucleon-nucleon amplitudes:

$$f_{nn'}(\mathbf{\Delta})=[\sigma_{nn'}/4(\sqrt{\pi})\hbar](i+\alpha_{nn'})\exp(-\tfrac{1}{2}B_{nn'}\Delta^2),$$

where $\sigma_{nn'}$ is the nn' total cross section, $\alpha_{nn'}$ is the ratio of real to imaginary parts, which is assumed to be independent of Δ, and $B_{nn'}$ are the slopes of the nn' differential cross sections. The strength of the test of the Glauber theory depends on how well the above parametrization describes the nucleon-nucleon scattering data.

We find that the parametrization is adequate for the calculations at 7.9 GeV/c. The nucleon-nucleon scattering parameters at 3.95 GeV/c are nearly equal. We have taken them as equal in the calculation since the errors introduced by this assumption are smaller than the statistical errors in present dd scattering data. The values of the parameters are

$$\sigma_{pp}=\sigma_{np}=42.3 \text{ mb} \quad (\text{Ref. } 5),$$
$$\alpha_{np}=\alpha_{pp}=-0.45 \quad (\text{Ref. } 6),$$
$$B_{pp}=B_{np}=6.7(\text{GeV}/c)^{-2}\hbar^2 \quad (\text{Ref. } 7).$$

The equality of the np and pp scattering amplitudes implies that the charge-exchange amplitude vanishes. This is a good approximation; the charge-exchange cross section is small at this energy and the charge-exchange amplitude must enter at least twice for the small dd collision to be elastic. The results are shown in Fig. 1(e). One observes a very sharp forward peak that is primarily due to the single scattering amplitude, but which is affected by the sequential double scattering. There is a break in the cross section and then a less sharp region due to simultaneous and higher-order multiple scattering processes. A small dip occurs in the differential cross section in the region of the break. To show how the ratio of real to imaginary parts affects the region of the break, we show in Fig. 1(f) the calculations performed with the nonphysical value $\alpha_{nn'}=0.0$ for the nucleon-nucleon amplitude at 3.95 GeV/c. We see that the interference minimum is larger than for $\alpha_{nn'}=-0.45$. If the quadrupole deformation did not exist, there would be a true zero in the differential cross section when $\alpha_{nn'}=0.0$.

Similar calculations have been performed at 2.2 GeV/c. Here we have chosen the parameters so as to fit the pp and np forward scattering amplitudes. The values of the parameters were taken as $\sigma_{pp}=30$ mb, $\sigma_{np}=35$ mb (Ref. 5), $B_{pp}=0.85$ (GeV/c)$^{-2}\hbar^2$ (Ref. 8), $B_{np}=1.94$ (GeV/c)$^{-2}\hbar^2$ (Ref. 9). The ratios of real to imaginary parts were found by computing $\alpha_{nn'}{}^2$ so as to satisfy the optical theorem using the total cross sections at $t=0$. Dispersion-relation calculations for the spin-independent part of the nucleon-nucleon amplitude[6] give us the sign of $\alpha_{nn'}$ and are consistent with the result obtained by the procedure outlined above. The values used are $\alpha_{pp}=0.5$ and $\alpha_{np}=-0.1$. The parameters chosen fit the forward scattering data well. The charge-exchange cross section, however, cannot be reproduced using the forward scattering parameters. This is an indication that the high-energy parametrization at these low energies is an oversimplification. Spin effects which have not been included should be considered in a better parametrization of the nucleon-nucleon scattering data.

The results of the computations in Fig. 1(g) show features similar to those exhibited by the differential cross section at 7.9 GeV/c. The forward peak at 2.2 GeV/c is less steep than that at 7.9 GeV/c, a fact directly attributable to the less sharp nucleon-nucleon slopes. A similar remark applies to the region beyond the break. Again the effect of the quadrupole deformation is to fill in the minimum in the differential cross section which in its absence would reach 0.08 mb/(GeV/c)^2 instead of 0.8 mb/(GeV/c)^2. Finally, we checked that our results are insensitive to possible refinements in the choice of the deuteron form factor at large t values. Even a pure exponential form factor with no D wave already gives the general shape of the differential cross section, except in the dip region. We thus agree with the results obtained in the theoretical study of πd scattering by Michael and Wilkin[10] who show that the choice of any reasonable form factor does not affect the differential scattering cross section.

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