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Effect of a Transverse Momentum Distribution in the Parton Model*

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The parton model for the inelastic lepton-nucleon scattering is generalized to include a realistic momentum distribution of the partons. In this formalism each parton is given a component of momentum which is orthogonal in a four-vector sense to the nucleon momentum. An approximation scheme is developed to take into account the effect of this orthogonal (transverse) momentum distribution of the partons. It is found to generate an additional scale-invariant contribution to the structure function $\nu W_2(\nu,Q^2)$ as well as a non-scale-invariant contribution, as is expected. The scale-invariance-breaking term is found to be a power series in Q^2/ν^2 and vanishes as ν or Q^2 goes to infinity for a fixed $\omega=2M\nu/Q^2$. The effect of the transverse momentum distribution is explicitly displayed and discussed for a few momentum distribution functions. The data on inelastic e-p scattering are then analyzed for any significant deviations from scale invariance, and it is concluded that there is evidence for such deviations. On the assumption that the systematic errors in the data are small, we make some fits which display the dependence on Q^2. Finally, an attempt is made to fit the data with the formalism developed in the initial part of the paper. For this purpose use is made of a model for the partons, proposed previously by the authors to explain the data in the approximation that scale invariance is satisfied. Some comments are made on the properties of partons which are necessary to fit the data and on the present status of scale-invariance breaking.

I. INTRODUCTION

The parton concept[1] has recently been[2,3] of great utility in the description of high-energy inelastic lepton-nucleon processes. The original formulations of this model, however, involve a simplifying assumption to test the usefulness of the main concepts. It was assumed, for example, that the four-momentum of each parton is proportional to that of the proton, and it is this assumption which leads to the result of scale invariance.[4] The purpose of this paper is to study the effects of a realistic momentum distribution of the partons, since such a distribution will give rise to some breaking of scale invariance, which seems to be observed in the inelastic e-p scattering data.

In a previous paper[2] we showed that a parton model can be constructed to explain the inelastic e-p scattering data in the approximation that scale invariance is satisfied. The formalism that we shall develop here will be quite independent of that work, but for the purpose of comparing our formalism with the data we shall use it as a first approximation.

The plan of this paper is as follows. In Sec. II we develop a formalism which introduces a component of the four-momentum of each parton orthogonal to the four-momentum of the proton. We then introduce an approximation procedure which is simple to deal with in computations. The scale-invariance-breaking term in this formalism is explicitly displayed, and discussed. Section III is devoted to the discussion of various specific models for the transverse momentum distribution of the partons. (By "transverse momentum" we mean that part of the four-momentum of the parton which is orthogonal to the four-momentum of the proton.) In Sec. IV the data on inelastic e-p scattering are analyzed for any significant deviations from scale invariance. Then we use our formalism to test the various hypotheses concerning the transverse momentum distribution of the partons.

Notation and Kinematics. In Fig. 1 we show the kinematic configuration. We define

$$\nu=P\cdot q/M\,,$$
$$Q^2=-q^2\,,$$
$$\omega=1/X=2M\nu/Q^2\,.$$

The inelastic differential cross section is given in terms of the structure functions W_2 and W_1 by

$$\frac{d^2\sigma}{d\Omega dE'}=\frac{\alpha^2}{4E^2\sin^4(\frac{1}{2}\theta)}\times[W_2\cos^2(\tfrac{1}{2}\theta)+2W_1\sin^2(\tfrac{1}{2}\theta)]\,,\quad(1.1)$$

where E and E' are the initial and final electron energies in the laboratory frame, and θ is the lab angle through which the electron is scattered.

The structure functions W_1 and W_2 are functions only of ν and Q^2.

II. GENERAL FORMALISM

The basic idea of the parton model is that at large c.m. energies of the e-p system the proton may be thought of as being composed of fundamental non-interacting structureless constituents, called partons,

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[1] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969); R. P. Feynman, in *Proceedings of the Third Topical Conference on High Energy Collisions of Hadrons, Stony Brook, New York, 1969* (Gordon and Breach, New York, 1969).

[2] C. W. Gardiner and D. P. Majumdar, Phys. Rev. D 2, 151 (1970).

[3] S. D. Drell, D. Levy, and T. M. Yan, Phys. Rev. Letters **22**, 744 (1969); Phys. Rev. **187**, 2159, 1969; Phys. Rev. D 1, 1035 (1970); D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. **B14**, 337 (1969); C. H. Llewellyn Smith, CERN Report No. TH 1097 (unpublished); T. Muta, KUNS Report No. 165, Kyoto University, Japan (unpublished); A. Niegawa, KUNS Report No. 172, Kyoto University, Japan (unpublished).

[4] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

from which electrons scatter elastically and incoherently. We write the four-momentum of the ith parton as

$$p_\mu{}^i = x^i P_\mu + k_\mu{}^i, \tag{2.1}$$

where P is the four-momentum of the proton and $P\cdot k_i = 0$. In the formulation of the parton model as given by Bjorken and Paschos,[1] the momentum k_i is set equal to zero. In our treatment we shall assume that k_i is small but nonzero. We shall specify more precisely what we mean by "small" later in this section.

The requirement that the momentum of the proton be given by the sum of the four-momenta of the partons then requires that

$$\sum_{i=1}^{N} x^i = 1, \quad \sum_{i=1}^{N} k^i = 0. \tag{2.2}$$

The parton models which neglect k^i also make the assumption that the x^i are positive (so that partons have positive energy) with the consequence that (2.2) restricts $x^i \leqslant 1$. This restriction defines a certain phase space for the x^i, and this phase space seems to be very useful in the interpretation of the inelastic e-p scattering data.[2] We shall therefore preserve this feature in our treatment, and from now on require that $0 \leqslant x^i \leqslant 1$.

We shall do most of our computations in the rest frame of the proton, in which $x^i M$ is the total energy of the ith parton, and $k^i = (0, \mathbf{k}^i)$, so that $\mathbf{k}^i$ is the three-momentum of the parton. However, the formalism is covariant and the result will therefore be independent of frame.

A. Infinite-Momentum Frame

We shall indicate here how our formalism can be related to an infinite-momentum-limit formulation. Let the boost which takes place in the xt plane, and relates the rest frame of the proton to the infinite-momentum frame, be

$$\Lambda_B = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}, \tag{2.3}$$

where $\alpha^2 - \beta^2 = 1$, and $\beta = |\mathbf{P}|/M$, where $|\mathbf{P}|$ is the magnitude of the three-momentum of the proton along the x direction in the infinite-momentum frame. It is usual, and convenient in most formalisms, for q to have no x component in the infinite-momentum frame. This means that the xt plane in the rest frame of the proton must be chosen differently for each $|\mathbf{P}|$, so that in this frame

$$\begin{aligned} q &= (\nu, -\nu\beta/\alpha, \mathbf{q}_\perp), \\ P &= (M, 0, \mathbf{0}), \end{aligned} \tag{2.4}$$

whereas in the infinite-momentum frame

$$\begin{aligned} q &= (\nu/\alpha, 0, \mathbf{q}_\perp), \\ P &= (M\alpha, M\beta, \mathbf{0}). \end{aligned} \tag{2.5}$$

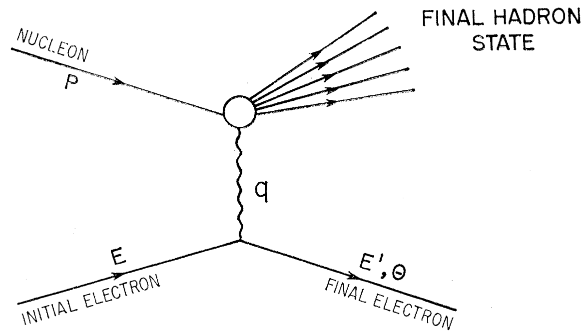


FIG. 1. Kinematics of inelastic electron nucleon scattering.

If a parton has four-momentum $P = xp + k$, and this is written in the rest frame of the proton as

$$P = (xM, k_{\shortparallel}, \mathbf{k}_\perp), \tag{2.6}$$

it becomes in the infinite-momentum frame

$$P = (xM\alpha + k_{\shortparallel}\beta,\ xM\beta + k_{\shortparallel}\alpha,\ \mathbf{k}_\perp). \tag{2.7}$$

As $|\mathbf{P}| \to \infty$, $\alpha \to \beta$, so that

$$P \to (\alpha(xM + k_{\shortparallel}), \alpha(xM + k_{\shortparallel}), \mathbf{k}_\perp), \tag{2.8}$$

that is,

$$P \to (x + k_{\shortparallel}/M)P + (0, 0, \mathbf{k}_\perp). \tag{2.9}$$

We see then that the quantity x used in the treatments of Bjorken and Paschos[1] and Drell *et al.*[3] is not strictly analogous to the x used here, but that the correspondence is really to $x + k_{\shortparallel}/M$. This leads us to expect that an additional contribution to the scale-invariant limit of νW_2 will arise from the transverse momentum distribution. Our detailed calculations, later in this section, will confirm that this is true.

B. Development of Formalism

Following the method of Bjorken and Paschos,[1] we find that the contribution to the structure function W_2 from a single parton of momentum given by Eq. (2.1) and charge Q_i is

$$W_2{}^i = Q_i{}^2 (x^i/\nu)\delta(x_i - Q^2/2M\nu + k^i\cdot q/P\cdot q). \tag{2.10}$$

If we now let $f_N(x,k)$ be the probability density for a parton in an N-parton state to have momentum $xP + k$, then the structure function will be given by

$$\nu W_2(\nu, Q^2) = \sum_N P(N)\langle \sum Q^2\rangle_N \int_0^1 dx \int d^4k \times \delta(P\cdot k) M f_N(x,k)\, \delta\!\left(x - \frac{Q^2}{2M\nu} + \frac{k\cdot q}{P\cdot q}\right). \tag{2.11}$$

In this equation $P(N)$ is the probability of a proton's being in an N-parton state, $\langle \sum Q^2\rangle_N$ is the average value of the sum of the squared charges in an N-parton configuration, and the normalization of $f_N(x,k)$ is given by

$$\int_0^1 dx \int d^4k\ \delta(k\cdot P) M f_N(x,k) = 1. \quad (2.12)$$

Equation (2.11) is manifestly covariant, but is more conveniently evaluated in the rest frame of the proton, in which case it becomes

$$\nu W_2(\nu,Q^2) = \sum_N P(N)\langle\sum Q^2\rangle_N \int d^3\mathbf{k}\left(\frac{Q^2}{2M\nu}+\frac{\mathbf{k}\cdot\mathbf{q}}{P\cdot q}\right) \times f_N\left(\frac{Q^2}{2M\nu}+\frac{\mathbf{k}\cdot\mathbf{q}}{P\cdot q},\mathbf{k}\right). \quad (2.13)$$

In the limit that $f_N(x,\mathbf{k})$ is a δ function in $\mathbf{k}$, one obtains the original scale-invariant result. We note also that since spins are implicitly summed over, $f_N(x,\mathbf{k})$ can depend only on $\mathbf{k}$ through $\mathbf{k}^2$, and further defining

$$F(x,\mathbf{k}^2) = \sum_N P(N)\langle\sum Q^2\rangle_N x f_N(x,\mathbf{k}), \quad (2.14)$$

we can derive that

$$\nu W_2 = 2\pi \int k^2 \sin\theta\, d\theta dk \times F\left(X + k\cos\theta\,\frac{(Q^2+\nu^2)^{1/2}}{M\nu}, k^2\right), \quad (2.15)$$

where $X = Q^2/2M\nu$.

We see here the existence of a scale-invariant contribution from the transverse momentum distribution, as follows. First, notice that

$$1+Q^2/\nu^2 = 1+4M^2X^2/Q^2, \quad (2.16)$$

so that as $Q^2\to\infty$ at fixed X, we get a scale-invariant limit for (2.14) of

$$\nu W_2 \to 2\pi \int k^2 \sin\theta\, dk d\theta\, F\left(X+\frac{k\cos\theta}{M}, k^2\right). \quad (2.17)$$

Since this is the limit in which scale invariance is expected to hold, the deviation from this scale-invariant limit will be small when

$$k[(1+Q^2/\nu^2)^{1/2}-1] \ll MX, \quad (2.18)$$

i.e.,

$$k \ll \tfrac{1}{2}Q^2[(\nu^2+Q^2)^{1/2}-\nu]. \quad (2.19)$$

This can be reduced to

$$\nu/k + Q^2/4k^2 \gg 1, \quad (2.20)$$

which can be interpreted as either $k\ll\nu$ or

$$k \ll \tfrac{1}{2}\sqrt{(Q^2)}, \quad (2.21)$$

since ν and Q^2 are both positive in the kinematic region being considered. We wish to obtain a theory with only a small amount of breaking of scale invariance, and Eq. (2.21) is the condition for this to be true.

We also see that the breaking of scale invariance comes from the factor $(1+Q^2/\nu^2)^{1/2}$, which in the experiments of Bloom *et al.*[5] varies between 1.0 and 1.1. To the extent that this variation may be regarded as negligible, Eq. (2.15) depends only on X. This shows that the breaking of scale invariance is expected to be small for these experiments, as is indeed observed.

C. Approximation Scheme

In all the cases we shall deal with, $F(x,k^2)$ has a singularity at $x=0$, but not at any other value of x. Thus if

$$k(1+Q^2/\nu^2)^{1/2} < MX = Q^2/2\nu, \quad (2.22)$$

we may make a Taylor expansion of (2.15) about X, and carry out the integration, obtaining

$$\nu W_2 = \sum_{m\text{ even}} \frac{4\pi}{(m+1)!}\left[\frac{(1+Q^2/\nu^2)^{1/2}}{M}\right]^m \times \int dk\, k^{m+2} F^{(m)}(X,k^2), \quad (2.23)$$

where

$$F^{(m)}(x,k^2) = \frac{\partial^m}{\partial x^m}F(x,k^2). \quad (2.24)$$

We see that there is a scale-invariant contribution from every term of (2.23), as expected. For a simple analysis of the data, we keep only the first two terms, so that the formula reduces to

$$\nu W_2 = F(X) + \left(1+\frac{Q^2}{\nu^2}\right)\frac{2\pi}{3M^2}\int k^4 dk\, F^{(2)}(X,k^2), \quad (2.25)$$

where $F(x)$ is given by

$$F(x) = \sum_N P(N)\langle\sum Q^2\rangle_N x f_N(x), \quad (2.26)$$

and

$$f_N(x) = \int d^3k\, f_N(x,k). \quad (2.27)$$

III. PARTON MOMENTUM DISTRIBUTIONS

In order to analyze the data, one must have a specific model for the partons, including one for their transverse momentum distribution. To investigate possible models, we first write

$$f_N(x,k^2) = f_N(x)\phi_N(x,k^2), \quad (3.1)$$

where $f_N(x)$ is the probability density for a parton to have longitudinal momentum xP, and $\phi_N(x,k^2)$ is the probability density for a parton with longitudinal momentum xP to have transverse momentum k. (We

[5] E. D. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969).

expect only a dependence on k^2 since this represents a spherically symmetric distribution in the rest frame of the proton.) Following the initial work of Bjorken and Paschos,[1] we assume that we can take

$$f_N(x)=(N-1)(1-x)^{N-2}, \qquad (3.2)$$

which can be derived by assuming a constant joint distribution in the x_i for the N partons. A further reason for taking (3.2) is, as we have shown in a previous paper,[2] that in the approximation that scale invariance is true, such a distribution can fit the inelastic e-p scattering data rather well.

For $\phi_N(x,k^2)$ we do not have any particular knowledge except that it is expected to fall off rapidly for large values of k^2. As an initial simplification we assume that ϕ_N is the same for all N, since this allows us to factorize it outside the N summation, and thus make calculations tractable. We shall explore the following three simple distributions for $\phi_N(x,k^2)$.

A. Phase Space

Let us first study the effect of a pure phase space for the transverse momentum distribution. This means that $\phi_N(x,k^2)$ is assumed to be independent of k^2. Here we shall require that the momentum of each parton be timelike, although it is not entirely clear that this is necessary. Thus we set

$$\phi_N(x,k^2)=(3/4\pi M^3x^3)\theta(M^2x^2-k^2). \qquad (3.3)$$

One then obtains from Eq. (2.24)

$$\nu W_2(\nu,Q^2)=F(X)+\tfrac{1}{10}\left(1+\frac{Q^2}{\nu^2}\right)\frac{d^2}{dX^2}[X^2F(X)]. \qquad (3.4)$$

The expression for $F(X)$ is now identical to the original scale-invariant result for $\nu W_2(\nu,Q^2)$ predicted by the parton model in the absence of a transverse momentum distribution. Notice that for the timelike partons the θ function occurs inside the double derivative in Eq. (2.25).

B. Momentum Distribution

We shall next consider a Gaussian function for the transverse momentum distribution given by

$$\phi_N(x,\mathbf{k}^2)=[(2\pi)^{3/2}\sigma^3]^{-1}\exp(-\mathbf{k}^2/2\sigma^2). \qquad (3.5)$$

In this case we shall make no restriction on the range of $|\mathbf{k}|$, which means that spacelike partons can occur with significant probability if x is sufficiently small. The equation for the structure function is then given by

$$\nu W_2(\nu,Q^2)=F(X)+\tfrac{1}{2}\left(1+\frac{Q^2}{\nu^2}\right)\frac{\sigma^2}{M^2}\frac{d^2}{dX^2}F(X). \qquad (3.6)$$

This expression for νW_2 differs from Eq. (3.4) mainly by the absence of X^2 before $F(X)$.

If we want to restrict the partons to be timelike, then we shall have

$$\phi_N(x,\mathbf{k}^2)=\exp(-\mathbf{k}^2/2\sigma^2)\theta(M^2x^2-\mathbf{k}^2)\bigg/\left[4\pi\int_0^{Mx}dk\,k^2\exp(-k^2/2\sigma^2)\right] \qquad (3.7)$$

and

$$\nu W_2=F(X)+\frac{1}{6M^2}\left(1+\frac{Q^2}{\nu^2}\right)\frac{d^2}{dX^2}\times\left[F(X)\int_0^{MX}dk\,k^4\exp(-k^2/2\sigma^2)\bigg/\int_0^{MX}dk\,k^2\exp(-k^2/2\sigma^2)\right]. \qquad (3.8)$$

C. Velocity Distribution

Here we consider a Gaussian distribution of the form

$$\phi_N(x,\mathbf{k}^2)=[(2\pi)^{3/2}\sigma^3x^3]^{-1}\exp(-\mathbf{k}^2/2\sigma^2x^2). \qquad (3.9)$$

We call this a velocity distribution since it is a simple three-dimensional Gaussian in the velocity $\mathbf{k}/Mx$ of the partons. In this case, if σ is very much smaller than M, there is a very small probability of finding a parton with spacelike four-momentum, and we may safely ignore such a possibility. We now find that

$$\nu W_2=F(X)+\tfrac{1}{2}\left(1+\frac{Q^2}{\nu^2}\right)\frac{\sigma^2}{M^2}\frac{d^2}{dX^2}[X^2F(X)]. \qquad (3.10)$$

Notice that Eq. (3.4) is of the same form as Eq. (3.10), with $\sigma=M/\sqrt{5}$.

IV. ANALYSIS OF INELASTIC e-p SCATTERING DATA

This section has two aims, the first of which is to study the available data on inelastic e-p scattering for significant deviations from scale invariance. The second is to see to what extent we can interpret the data in terms of our models for transverse momentum distributions.

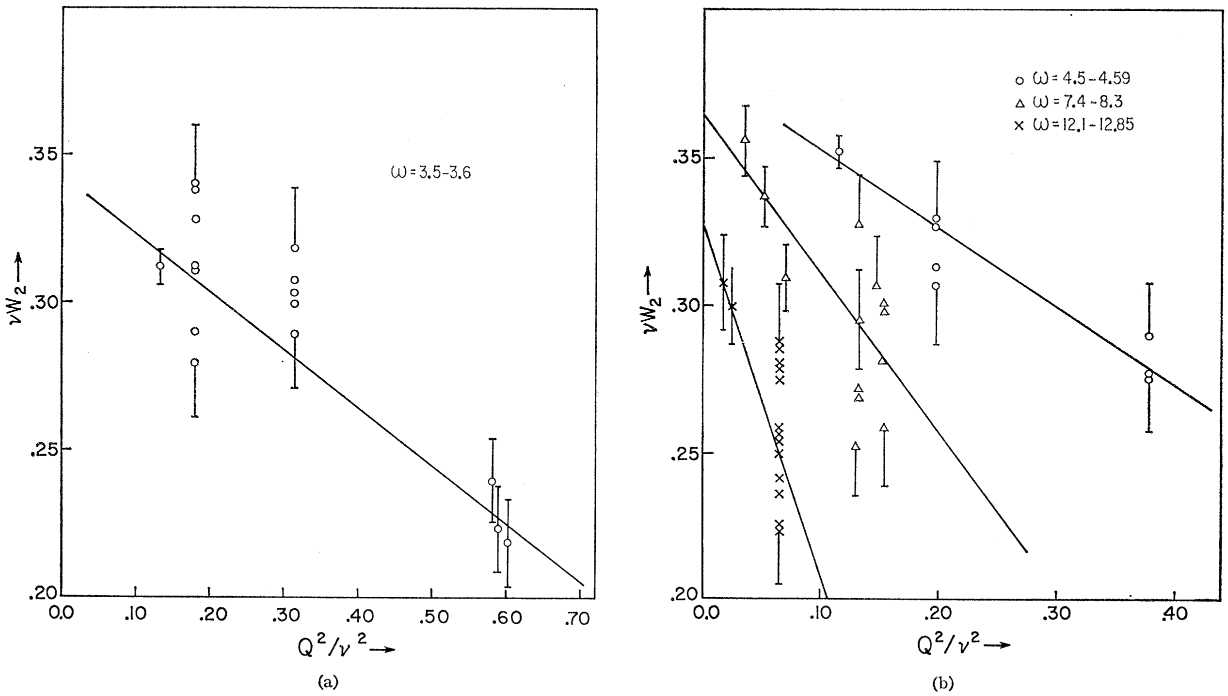
A. Deviations from Scale Invariance

In order to choose some physically significant parametrization we note that our theory predicts that, to a first approximation, the structure function νW_2 is given by an equation of the form

$$\nu W_2=a(\omega)+(Q^2/\nu^2)\,b(\omega). \qquad (4.1)$$

Thus, we have plotted,[6] in Fig. 2, νW_2 as a function of

[6] Here we have used the detailed data for 6° measurements reported in M. Breidenbach, thesis, MIT, 1970 (unpublished). [See also E. D. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969).] To analyze these data, we assumed that R, the ratio of longitudinal to transverse photoabsorption cross section, is zero, since all recent measurements show that this ratio is certainly very small (see, e.g., R. E. Taylor, Ref. 8). We have also analyzed the data with $R=0.5$ and found that the data are not sufficiently changed to affect our conclusions.



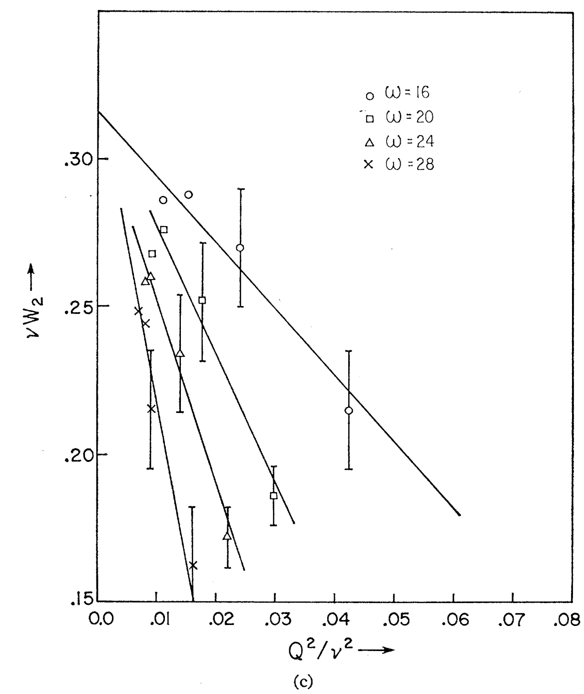


FIG. 2. Plots of νW_2 against Q^2/ν^2 for various values of ω. The points are taken from Ref. 6. Some higher ω points are obtained by interpolation, and only representative error bars are given.

Q^2/ν^2 for a few fixed values of ω. For the largest values of Q^2/ν^2 there is a definite decrease of νW_2 and, within errors, the data are roughly consistent with a linear relationship of the form given by Eq. (4.1). By drawing (by eye) a straight line through each set of points, we have derived values of $a(\omega)$ and $b(\omega)$, which are plotted in Fig. 3. The slope $b(\omega)$ appears to increase approximately quadratically at large ω. One should note, however, that

$$Q^2/\nu^2=4M^2/Q^2\omega^2\,, \tag{4.2}$$

and so as ω increases, this factor cancels the increase in $b(\omega)$.

In order to put this analysis on a sounder basis, we have made a least-squares fit to the data, of the form

$$\nu W_2=F(X)+A+BX+(C+DX)/Q^2\,, \tag{4.3}$$

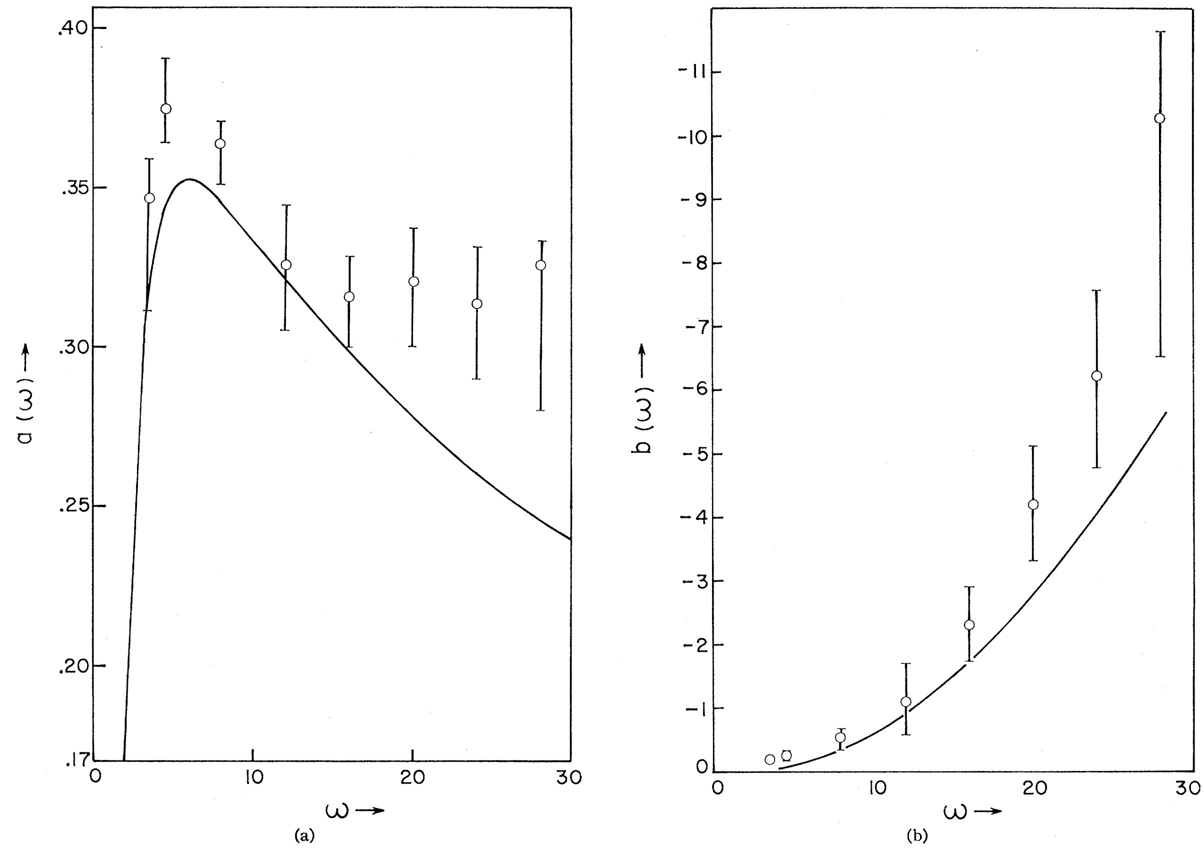


FIG. 3. Plots of $a(\omega)$ and $b(\omega)$ from the linear fits of Fig. 2 (points with error bars) and from the fit to the data given by Eq. (4.6) (solid line).

where $F(X)$ is taken to be our[2] previous good fit to νW_2 in the scale-invariant approximation, and is given by

$$F(X)=2\delta[(1-X)^2-X(\delta'+2)(\ln X+1-X)], \quad (4.4)$$

with $\delta=0.042$ and $\delta\delta'=1-4\delta$. (The physical significance of δ and δ' is explained in Ref. 2.) In this computation we have used the data published in Ref. 5, and assumed that R is zero. For the 56 points quoted in this reference we obtain a fit with a χ^2 of 104.2 and coefficients given by

$$\begin{aligned} A&=0.0174\pm0.0006\,,\\ B&=-0.0861\pm0.0008\,,\\ C&=-0.0268\pm0.0002\ \mathrm{GeV}^2\,,\\ D&=0.0517\pm0.0027\ \mathrm{GeV}^2\,. \end{aligned} \quad (4.5)$$

A fit to νW_2 with only scale-invariant functions of the form $F(X)+A+BX+CX^2+DX^3$ gives $\chi^2=184.4$, which is significantly higher than that given by Eq. (4.3). In Fig. 4 we have plotted the fit given by Eq. (4.5) and the data of Breidenbach[6] for four different incoming energies of the electron. For comparison, the scale-invariant fit given by Eq. (4.4) is also plotted on each of these plots. From these plots one can easily notice how the data vary systematically as the energy changes and thus reveal a dependence on Q^2.

By the use of Eq. (4.2), we can reduce Eq. (4.3) to the form (4.1). $a(\omega)$ and $b(\omega)$ are then given by

$$a(\omega)=F(X)+A+BX$$

and (4.6)

$$b(\omega)=(C+D/\omega)\omega^2/4M^2\,.$$

We have plotted $a(\omega)$ and $b(\omega)$ as given by this fit in Fig. 3. The reader will see that the main features of our previous analysis by graphs are reproduced in this analysis, although the stringent condition of a four-parameter fit forces there to be some disagreement. Notice also that our graphical analysis was done with the detailed 6° data of Ref. 6, while the fit was done with the data of Ref. 5, which has some 10° and some 6° data. The substantial agreement between the two analyses is therefore particularly significant.

Thus we conclude that the data as published do give a significant Q^2 dependence, particularly the data for 7 and 10 GeV, and we agree with Nauenberg[7] that there is a substantial non-scale-invariant term in νW_2. It has

[7] M. Nauenberg, Phys. Letters **24**, 625 (1970).

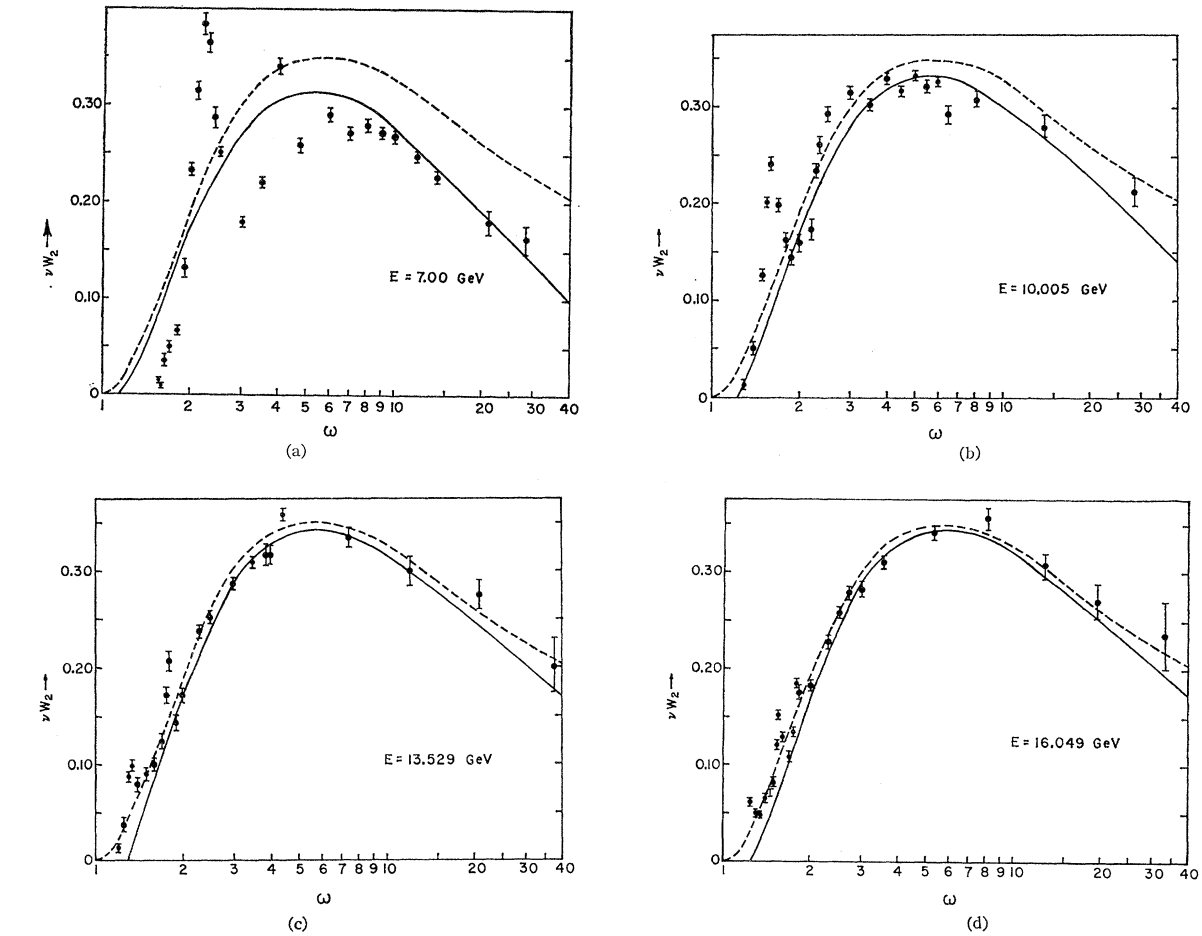


FIG. 4. Plots of the fit given by Eq. (4.5) (solid line) and the data of Breidenbach (Ref. 6) for different energies of the incoming electron. (Some of the points are obtained by averaging over a number of adjacent points.) For comparison, the scale-invariant fit given by (4.4) is also plotted (dashed line) on each of these plots.

been pointed out by Taylor[8] that systematic errors, which may amount to as much as 10%, could be responsible for the Q^2 dependence observed. We would like to point out that to produce the Q^2 dependence observed over the full range of energies measured, it would be necessary for both systematic and random errors to conspire maximally in order to produce an apparent Q^2 dependence as large as that observed. It is clear that a reduction of the systematic error is of paramount importance in ascertaining the status of any breaking of scale invariance.

TABLE I. Fits with different distributions.

Distribution	δ	$\delta\delta'+4\delta$	σ (GeV)	χ^2
Scale invariant	0.0377 ± 0.004	0.9235 ± 0.0078	0	351
Momentum dist.	0.0561 ± 0.004	0.9053 ± 0.0076	0.394 ± 0.001	341
Velocity dist.	0.000 ± 0.004	1.000 ± 0.0082	0.200 ± 0.001	228

[8] R. E. Taylor, in Proceedings of the International Conference on Expectations for Particle Reactions at the New Accelerators, Wisconsin, 1970 (unpublished).

B. Fits with Transverse Momentum Distributions

We have attempted fits with a momentum distribution and a velocity distribution by using the form of $F(X)$ given in Eq. (4.4). The formulas are specified in Sec. III. We have three physically meaningful parameters δ, δ', and σ with which to fit the data. The results of the fits are shown in Table I, where we have also shown a fit in which σ is set equal to zero. (The parameters for this fit are slightly different from those in our previous paper[2] since we are not using quite the same data.) We note that none of these is a very good fit.

It should be remembered that our formalism gives a scale-invariant contribution from the transverse momentum distribution about ten times as large as the term breaking scale invariance, so that the values of the parameter σ in our fits are determined more by this part than by the breaking of scale invariance.

It may be worthwhile to point out that with the velocity distribution our best fit[9] is with $\delta=0$, which corresponds to a theory (see Ref. 2) with all neutral partons, except for the first four, the sum of the squares of whose charges is one. This could be given by three quarks and a neutral parton (a "gluon") or a singly charged parton and three neutral partons. The momentum distribution, however, agrees with our earlier result[2] that the mean square charge distribution of the partons in the cloud is small but nonzero. We also observe that although the charge distribution of the parton cloud, δ, may vary with the transverse momentum distribution, the relation $\delta\delta'=1-4\delta$, which was conjectured before[2] on physical grounds, is obeyed to a good approximation.

An analysis of the functions $a(\omega)$ and $b(\omega)$, as defined in Sec. IV A, shows that both of the transverse momentum distributions produce far too little breaking of scale invariance. For the velocity distribution $b(\omega)$ goes to zero at higher ω, and is positive, while for the momentum distribution $b(\omega)$ is small and negative, and increases linearly at high ω. There may be four reasons for this behavior: First, there may be contributions from higher-order terms in our expansion, which would predict a nonlinear dependence of νW_2 on Q^2/ν^2. In that case the above predictions of the first-order formalism will, naturally, not work. (The data are consistent with certain nonlinear dependences on Q^2/ν^2, so this possibility is permissible.) Second, assuming that such higher-order terms do not occur, we may have the wrong sort of transverse momentum distribution. We note that to produce the observed form of scale-invariance breaking in this formalism, we would need a transverse momentum distribution with a larger mean square transverse momentum at smaller x. Third, there is the possibility that the initial form of $F(\omega)$ chosen is of the wrong form, and that a fit with many more parameters is needed to obtain a realistic theory. Fourth, when systematic errors are corrected, scale invariance may not be broken as much as it appears to be now, and in this case one of these distributions may produce the correct Q^2 dependence. Confirmation of any of these possibilities must await a more precise determination of systematic errors and the collection of more data at larger values of ω.

[9] A better fit can be obtained by allowing negative δ and δ', but this then makes $\langle\sum Q^2\rangle_N$ negative. So we have chosen fits with non-negative δ and δ'.

V. CONCLUSION

In this paper we have developed a formalism in which the partons do not have simply a fraction of the momentum of the proton, but have in addition some transverse momenta. We believe that the concept of the transverse momentum of partons must be meaningful in any theory which has a parton limit, although there may be no one-to-one correspondence between other theories and ours. We have seen that the transverse momentum distribution gives a considerable scale-invariant contribution to the structure function νW_2 as well as a term which breaks scale invariance. The scale-invariance-breaking term is found to be a power series in Q^2/ν^2 and vanishes as ν or Q^2 goes to infinity for a fixed ω.

By interpreting the data as having negligible systematic errors, we have found that there is a measurable amount of scale-invariance breaking, much more, in fact, than the two transverse momentum distributions we have considered seem to be able to predict. This is observed particularly at large ω. It seems certain that some transverse momentum distributions can be arranged to produce such an effect. However, we think that the particular transverse momentum distributions we have chosen are quite reasonable and perhaps something more fundamental may be taking place at large ω.

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