so that the function $\Delta^{1/2}(x,y,z)$ that appears in (B1) has two square-root branch points as a function of x. The denominator of (B1) has a single branch point.

Suppose y and z are positive. Then define the analytic structure of $\Delta^{1/2}(x,y,z)$ in the x plane by a branch cut between the two branch points and define $\Delta^{1/2}(x,y,z)$ to be positive for x real and $>(y+z)^2$. Then

$$\Delta^{1/2}(x,y,z)=\text{sgn}(\text{Im}x)\times i\,|\Delta^{1/2}(x,y,z)|$$

if

$$(\sqrt{y}-\sqrt{z})^2<\text{Re}x<(\sqrt{y}+\sqrt{z})^2$$

and $\text{Im}x\to 0$. Also,

$$\Delta^{1/2}(x,y,z)=-|\Delta^{1/2}(x,y,z)| \quad \text{if} \quad \text{Re}x<(\sqrt{y}-\sqrt{z})^2.$$

Also, we take $\sqrt{x}$ to be positive for $x>0$ and $\text{Im}x\to 0^+$, with a branch cut extending from 0 to $+\infty$. We extend the definition of $\Delta^{1/2}(x,y,z)$ to the cases where x, y, and z are real and of arbitrary magnitude by the rules

$$\Delta^{1/2}(x,y,z)=\Delta^{1/2}(-x,\,-y,\,-z) \tag{B3}$$

and

$$=\Delta^{1/2}(Px,Py,Pz)\,, \tag{B4}$$

where (Px,Py,Pz) is any permutation of (x,y,z). These rules, although arbitrary, have the advantage of simplicity and symmetry in dealing with all cases of physical interest, namely, those cases in which x, y, and z are all real.

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Axial-Vector K_{l4}-Decay Form Factors Based on Current Algebra

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The axial-vector K_{l4}-decay form factors given by current algebra have been reevaluated using the vector K_{l3}-decay form factors and the ratio f_K/f_π, obtained on the basis of broken chiral $SU_3\otimes SU_3$ and SU_3 symmetries. It is shown that the assumption of near constancy of these form factors leads to good agreement with the experimental data, using consistently the single Cabibbo angle $\sin\theta_V=\sin\theta_A=0.22$.

I. INTRODUCTION

EMPLOYING the soft-pion current-algebra approach, Callan and Treiman and Weinberg[1] (CTW) obtained relations between the axial-vector K_{l4}-decay form factors and the vector K_{l3}-decay form factors, all extrapolated to unphysical points in the relevant invariant variables. To compare these relations (the CTW relations) with experiment, it is thus necessary to take account of the extrapolation in going from the unphysical to the physical values of the invariant variables. With the assumption that all these form factors are constant (i.e., no extrapolation correction is required), Weinberg showed that the K_{e4}-decay rate so obtained was already in rough agreement with experiment, so that one might hope for a reasonably good agreement when some of the corrections that are needed because of the possible lack of constancy of the form factors are included.

The above current-algebra values of the K_{l4}-decay form factors depend on the values of the K_{l3}-decay form factors $F_\pm(t)$ at $t=m_K{}^2$, and the K_{l2} and the π_{l2} decay constants f_K and f_π. Recent theoretical work[2] employing the first-order broken chiral $SU_3\otimes SU_3$ and SU_3 symmetries has enabled an evaluation of the form factors $F_\pm(t)$ and the ratio f_K/f_π. In a recent work,[3] we have shown that these form factors permit a rather good description of the K_{l3}, the K_{l2}, and the π_{l2} decays based on a *single* Cabibbo angle ($\sin\theta_A=\sin\theta_V=0.22$), provided that a small $(\mu-e)$-universality violation in the strangeness-changing decays is admitted.[4] Using then these values for $F_\pm(m_K{}^2)$ and f_K/f_π, we can reevaluate the K_{e4}-decay axial-vector form factors from the CTW relations, thus taking into account perhaps the most important extrapolation correction arising from the form factors $F_\pm(t)$. Even with no further correction, the K_{l4}-decay rate is already found to be in very good agreement with experiment if we take $\sin\theta_A=\sin\theta_V=0.22$.

[1] C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966); S. Weinberg, *ibid.* **17**, 336 (1966); **18**, 1178(E) (1967); see also V. S. Mathur and L. K. Pandit, in ***Advances in Particle Physics***, edited by R. L. Cool and R. E. Marshak (Interscience, New York, 1968), Vol. II.

[2] L. K. Pandit and G. Rajasekaran, Tata Institute of Fundamental Research report, 1969 (unpublished); see also Ref. 3.

[3] S. C. Chhajlany, L. K. Pandit, and G. Rajasekaran, Nucl. Phys. B (to be published).

[4] $(\mu$-$e)$ universality has so far been tested only in the strangeness-conserving decays. For the strangeness-changing decays it is still an open experimental question. On the basis of violation of $(\mu$-$e)$ universality in the strangeness-changing decays, we have suggested a possible solution to the well-known ξ-parameter controversy arising from the conflict between μ-polarization data on the one hand, and deductions from the Dalitz-plot and the branching ratio $K_{e3}/K_{\mu3}$ on the other hand.

There exist in the literature a number of calculations[5–10] which are aimed towards improving the results of Weinberg, using the hard-pion current-algebra technique, vector and axial-vector-meson dominance, chiral dynamics, etc. In most of these (often very elaborate) theoretical works, the value of the angle used is $\sin\theta_A \approx 0.268$, implying the assumption of two angles in the Cabibbo theory. We may further point out that a number of authors even use $\sin\theta_A \approx 0.268$, while at the same time they adopt inconsistently $f_K/f_\pi \approx 1.2$ [from the experimental values of the $K_{\mu 2}$- and $\pi_{\mu 2}$-decay rates, in the conventional theory,[11] $(f_K/f_\pi)\tan\theta_A \approx 0.268$]. We find that the results of the aforementioned calculations do not agree with experiment, especially when interpreted from the point of view of the single-angle theory with $\sin\theta = 0.22$.

In the present paper, we adopt the simplifying *assumption* that the K_{l4} form factors as obtained above (from Weinberg's results, with the $F_\pm$ extrapolation taken into account) can be treated as constants right up to the physical region of the K_{l4} decays. We find that this simple approach succeeds much better in describing the data than the more elaborate calculations mentioned above.

In view of the importance of the form factors $F_\pm(t)$ and the ratio f_K/f_π to our work, Secs. II and III are devoted to a brief discussion of these quantities. Section IV contains our results on the K_{l4} decays. Finally, Sec. V gives a comparison with the previous theoretical treatments.

II. RATIO f_K/f_π

The K_{l2}- and π_{l2}-decay constants are defined by the matrix elements of the axial-vector currents

$$\langle 0|A_\mu{}^{4-i5}(0)|K^+(p)\rangle = i f_K p_\mu, \qquad (1)$$

$$\langle 0|A_\mu{}^{1-i2}(0)|\pi^+(p)\rangle = i f_\pi p_\mu. \qquad (2)$$

In the chiral-$SU_3\otimes SU_3$-symmetry limit, we assume that $f_K = f_\pi \neq 0$, whereas $m_K = m_\pi = 0$. At the level of only the exact SU_3 symmetry, $m_K = m_\pi \neq 0$ and $f_K = f_\pi$.

In the conventional one-angle Cabibbo theory with $(\mu - e)$ universality, from the experimental $K_{\mu 2}$-, $\pi_{\mu 2}$-, and K_{e3}-decay rates, one obtains[12]

$$f_K/f_\pi[-\sqrt{2}F_+(0)] \approx 1.22. \qquad (3)$$

The K_{l3}-decay form factor $F_+(t)$ is defined in Sec. III. Note that without assuming a value for the Cabibbo angle, the parameters f_K/f_π and $F_+(0)$ are not determined separately. From a purely theoretical viewpoint, the Ademollo-Gatto theorem ensures that, to the first order in the SU_3-symmetry breaking, $F_+(0)$ retains the symmetric value

$$F_+(0) = -1/\sqrt{2}. \qquad (4)$$

The question is one of ascertaining the value of f_K/f_π purely theoretically, taking account of symmetry breaking to the same order. Then only can one really check the theoretical values against the experimental determination cited in Eq. (3). Various authors have addressed themselves to this question. A summary is given below.

(i) Gell-Mann, Oakes, and Renner[13] find that to the first order in the symmetry breaking, which transforms as $(3^*,3)+(3,3^*)$,

$$f_K \approx f_\pi. \qquad (5)$$

The result is based on certain reasonable saturation approximations.

(ii) Glashow and Weinberg[14] have obtained a result by saturating two- and three-point-function Ward identities with meson poles including a strange scalar meson, called the κ. They also used certain smoothness assumptions on the corresponding form factors and the empirical result of Eq. (3) to obtain

$$f_K/f_\pi \approx 1.01. \qquad (6)$$

However, they had to invoke a nearly 17% symmetry-breaking correction to $F_+(0)$, which, in view of the Ademollo-Gatto theorem, must be attributed to the second and higher orders of the SU_3 breaking, so that $-\sqrt{2}F_+(0) \approx 0.83$ instead of 1. This would imply having to give up the treatment of symmetry breaking as a small perturbation. They had to introduce in their treatment the κ meson with $|f_\kappa/f_\pi| \approx 0.6$ and $m_\kappa \leqslant 670$ MeV, very near the $K\pi$ threshold. Experimentally no such state has yet been observed.

(iii) Lai[15] has, using spectral-function sum rules saturated by the A_1, K_A, π, and K states and employing the not-so-well-known axial-vector-meson decay parameters δ_{A_1} and δ_{K_A}, obtained $f_K/f_\pi \approx 1.23$ in an attempt to achieve consistency between Eqs. (3) and (4).

(iv) The result

$$f_K = f_\pi \qquad (7)$$

has been obtained as an exact result, correct to first order in a local SU_3-breaking Hamiltonian, by Pandit and Rajasekaran,[16] using equal-time-commutator techniques.

[5] L. J. Clavelli, Phys. Rev. **154**, 1509 (1967).
[6] D. Greenberg, Phys. Rev. **174**, 1821 (1968).
[7] A. Q. Sarker, Phys. Rev. **176**, 1959 (1968).
[8] S. N. Biswas, R. Dutt, and K. C. Gupta, Ann. Phys. (N. Y.) **52**, 366 (1969).
[9] R. Dutt, J. S. Vaishya, and K. C. Gupta, Phys. Rev. **175**, 1884 (1968).
[10] S. N. Biswas, R. Dutta, P. Nanda, and L. K. Pandit, Phys. Rev. D **1**, 1445 (1970).
[11] N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. **B6**, 255 (1968).
[12] S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. **41**, 531 (1969).
[13] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968); see also C. S. Lai, Phys. Rev. Letters **20**, 509 (1968); R. Oakes, *ibid.* **20**, 513 (1968); P. Auvil and N. Deshpande, Phys. Rev. **183**, 1463 (1969). The same result has also been obtained in a different approach by J. Sakurai, Phys. Rev. Letters **17**, 552 (1966); **17**, 1021 (1966).
[14] S. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968); see also I. Gerstein and H. Schnitzer, Phys. Rev. **175**, 1876 (1968).
[15] C. S. Lai, Nucl. Phys. **B9**, 521 (1969).
[16] L. K. Pandit and G. Rajasekaran, Nucl. Phys. **B9**, 531 (1969).

We shall *adopt* in our work the point of view that the results of Eqs. (4) and (7), which are correct to the first order in the SU_3-breaking, are reliable, and that the higher-order corrections are not significant.

The question then arises of how these results can be reconciled with the experimental value of Eq. (3). One possibility is to say that $\theta_A \neq \theta_V$, so that Eq. (3) is modified to

$$\frac{f_K}{f_\pi[-\sqrt{2}F_+(0)]}\frac{\tan\theta_A}{\sin\theta_V}\approx 1.22\,, \tag{8}$$

which is consistent with Eqs. (5) and (7) if $\sin\theta_V \approx 0.22$ and $\sin\theta_A \approx 0.268$.

The other possibility, the one we choose to follow here, is that there is a violation of $(\mu - e)$ universality in the strangeness-changing decays,[4] which takes care of the discrepancy. A treatment of this effect has been reported recently by us.[3] Let us emphasize that $(\mu-e)$-universality violation has been invoked by us for much more than this effect, namely, to understand the otherwise rather contradictory values of the ξ parameter suggested by different pieces of data on the K_{l3} decays (see Sec. III).

III. VECTOR K_{l3}-DECAY FORM FACTORS

The vector K_{l3}-decay form factors are defined by the following matrix element of the strangeness-changing vector current density:

$$\langle \pi^0(p_\pi)|V_\mu{}^{4-i5}(0)|K^+(p_K)\rangle = F_+(t)(p_K+p_\pi)_\mu + F_-(t)(p_K-p_\pi)_\mu\,,\quad t=(p_K-p_\pi)^2\,. \tag{9}$$

In the SU_3-symmetry limit, $F_+(0)=-1/\sqrt{2}$ and $F_-(t)=0$. To first order in the SU_3 breaking, we have the result of Eq. (4), $F_+(0)=-1/\sqrt{2}$.

Correct to first order in the breaking of the chiral $SU_3 \otimes SU_3$ and the SU_3 symmetries, Dashen and Weinstein[17] have recently given the result

$$\xi(0) \equiv \frac{F_-(0)}{F_+(0)} = \frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)-\left(\frac{m_K{}^2-m_\pi{}^2}{m_\pi{}^2}\right)\lambda_+\,, \tag{10}$$

where $\lambda_\pm$ are defined by the expansion

$$F_\pm(t) = F_\pm(0)\big(1+\lambda_\pm(t/m_\pi{}^2)+\cdots\big)\,. \tag{11}$$

Making use of the results of Eqs. (4), (7), and (10), we have recently obtained the form factors[2,3]

$$\begin{aligned} &F_\pm(t) = -F_\pm(0)M_V{}^2/(t-M_V{}^2)\,,\\ &F_+(0) = -1/\sqrt{2}\,,\\ &\xi(0)\equiv F_-(0)/F_+(0) = -(m_K{}^2-m_\pi{}^2)/M_V{}^2\,. \end{aligned} \tag{12}$$

Here M_V is the mass of the strange vector meson dominating the form factors. The possible contribution of a scalar meson was found to be exactly zero up to first order in the symmetry breaking. A very important feature of the form factors of Eqs. (12) and (7) is that, as they should, they analytically satisfy for $m_\pi = 0$ the soft-pion current-algebra relation of Callan and Treiman, and Mathur, Okubo, and Pandit[18] (the CTMOP relation):

$$F_+(m_K{}^2)+F_-(m_K{}^2) = -(1/\sqrt{2})(f_K/f_\pi)\,. \tag{13}$$

With $M_V = M(K^*(890))$, we obtain

$$\lambda_\pm = 0.025\,,\quad \xi(0) = -0.31\,. \tag{14}$$

We have recently shown[3] that these form factors describe all the K_{l3}-decay data quite well, provided that we again invoke the same $(\mu-e)$-universality violation[4] in the strangeness-changing decays as is required in the treatment of the K_{l2} decays based on a single-Cabibbo-angle theory.

IV. K_{l4}-DECAY FORM FACTORS

Consider the experimentally studied decay

$$K^+(p)\to\pi^+(q_1)+\pi^-(q_2)+l^+ +\nu_l\,.$$

The corresponding axial-vector and vector form factors are defined by the matrix elements:

$$\langle\pi^+(q_1)\pi^-(q_2)|A_\mu{}^{4-i5}(0)|K^+(p)\rangle = (i/m_K)\times[F_1(q_1+q_2)_\mu+F_2(q_1-q_2)_\mu+F_3(p-q_1-q_2)_\mu]\,, \tag{15}$$

$$\langle\pi^+(q_1)\pi^-(q_2)|V_\mu{}^{4-i5}(0)|K^+(p)\rangle = (1/m_K{}^3)\times F_4\epsilon_{\mu\nu\lambda\sigma}p_\nu(q_1+q_2)_\lambda(q_1-q_2)_\sigma\,, \tag{16}$$

where

$$\begin{gathered} F_i = F_i(s,t,u)\,,\quad i=1,\ldots,4\\ s=(q_1+q_2)^2\,,\quad t=(p-q_1)^2\,,\quad u=(p-q_2)^2\,. \end{gathered} \tag{17}$$

The CTW relations[1] (using soft-pion current algebra) are

$$F_1(0,m_K{}^2,m_K{}^2) = F_2(0,m_K{}^2,m_K{}^2) = \sqrt{2}(m_K/f_\pi)F_+(m_K{}^2)\,, \tag{18}$$

$$F_3(0,m_K{}^2,m_K{}^2) = -\frac{m_K f_K}{2f_\pi{}^2}\left[1+\frac{p\cdot(q_1-q_2)}{p\cdot(q_1+q_2)}\right]_{\text{soft }\pi}\,. \tag{19}$$

There is no corresponding relation for the vector form factor.[19] We shall determine it purely empirically.

In evaluating F_1 and F_2 from Eq. (18), Weinberg had used the value $F_+(m_K{}^2)\approx F_+(0)\approx -1/\sqrt{2}$. We, on the other hand, can obtain $F_+(m_K{}^2)$ from the explicit t dependence given in Eq. (12), putting $M_V = M(K^*(890))$. We find

$$F_+(m_K{}^2) = -\frac{1}{\sqrt{2}}\left(\frac{M_{K^*}{}^2}{M_{K^*}{}^2-M_K{}^2}\right) = -\frac{1.44}{\sqrt{2}}\,, \tag{20}$$

[17] R. Dashen and M. Weinstein, Phys. Rev. Letters 22, 1337 (1969).

[18] C. G. Callan and S. B. Treiman, Ref. 1; V. S. Mathur, S. Okubo, and L. K. Pandit, Phys. Rev. Letters 16, 371 (1966).

[19] It is interesting to note that the SU_3 symmetry imposes no restriction on F_4, since the corresponding matrix element of $\partial_\mu V_\mu{}^{4-i5}$ vanishes identically even when the current $V_\mu{}^{4-i5}$ is not conserved.

TABLE I. Comparison of the axial-vector form factors.

	F_1	F_2	F_3
Fit of Berends *et al.* (Ref. 21) (with $a_0=0.3/m_\pi$)	5.6±0.6	5.5±1.2	···
Weinberg	3.7	3.7	$1.85\left[1+\dfrac{p\cdot(q_1-q_2)}{p\cdot(q_1+q_2)}\right]$
Present work	5.3	5.3	$1.85\left[1+\dfrac{p\cdot(q_1-q_2)}{p\cdot(q_1+q_2)}\right]$

leading to an increase by 44% over the values used by Weinberg. Thus with $\sin\theta\approx0.22$ and neglecting any further corrections, we obtain the K_{e4}^+-decay rate (to which only F_1 and F_2 contribute significantly):

$$\Gamma(K^+\to\pi^+\pi^-e^+\nu_e)=2.60\times10^3\ \mathrm{sec}^{-1}, \tag{21}$$

to be compared with

$$\Gamma(K_{e4}^+)=1.28\times10^3\ \mathrm{sec}^{-1}\quad(\text{Weinberg}^1), \tag{22}$$

and

$$\Gamma(K_{e4}^+)=(2.6\pm0.3)\times10^3\ \mathrm{sec}^{-1}\quad(\text{experiment}^{20}). \tag{23}$$

The agreement between Eqs. (21) and (23) is very encouraging. The comparison with the result of Eq. (22) suggests that the main correction to Weinberg's form factors is to be sought in the proper evaluation of $F_+(t)$ at $t=m_K^2$.

The next point to consider is that F_1, F_2, and F_3 given by Eqs. (18) and (19) are at $s=0$, $t=u=m_K^2\approx12m_\pi^2$, whereas the physical region is given by $4m_\pi^2\leqslant s\leqslant m_K^2\approx12m_\pi^2$, $m_\pi^2\leqslant(t,u)\lesssim6m_\pi^2$. Thus there are two possibilities: Either the extrapolation to the physical region involves a large correction, so that the agreement cited above is fortuitous, or this agreement is actually significant and the form factors are essentially constant up to the physical region, so as to allow extrapolation without any significant corrections. We shall adopt the latter point of view and proceed to test it by comparison with the experimental spectra and angular distributions.

Our result for F_1, F_2, and F_3 using Eqs. (7) and (20) is given in Table I. This should be compared with the fit of Berends *et al.*[21] to the experimental data for constant form factors, normalized to $\sin\theta=0.22$, given in the table. For comparison we also include in the table the Weinberg form factors without the extrapolation correction due to the K_{l3} form factors $F_\pm$. Our result is thus seen to be in good agreement with experiment.

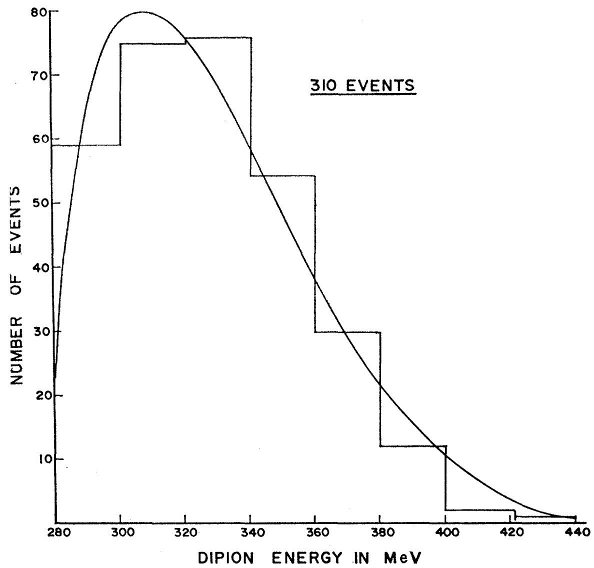

FIG. 1. Dipion energy distribution.

Using our values of the form factors F_1 and F_2, we have further calculated the following distributions and compared them with experiment.[22]

(*i*) *The dipion-energy spectrum* (Fig. 1). The experimental spectrum essentially follows the phase-space distribution, providing further confirmation of the assumed constancy of the form factors.

(*ii*) *The* $\cos\theta$ *distribution*, where θ is the angle in the dipion center-of-mass system of the π^+ with respect to the dipion line of flight in the K rest frame (Fig. 2).

(*iii*) *The* ϕ *distribution*, where ϕ is the angle between the dipion and the dilepton planes in the K rest frame (Fig. 3). This distribution is sensitive to the vector form factor F_4 whose value is not known. We may use this distribution with our values of F_1 and F_2 to estimate the value of F_4 required; the result is

$$F_4\approx-22. \tag{24}$$

It may be noted that the various theoretical attempts at calculating F_4 give generally too low a value.[23]

All the above distributions are in satisfactory agreement with experiment, except that experimentally one observes an asymmetry in the ϕ distribution about 180° which cannot be obtained from real form factors. To obtain such an asymmetry, many authors[6,21] have intro-

[20] R. Ely, Jr., *et al.*, Phys. Rev. **180**, 1319 (1969).

[21] F. A. Berends, A. Donnachie, and G. C. Oades, Nucl. Phys. **B3**, 569 (1967). The fit with $a_0=0.3/m_\pi$, but without the enhancement factors, has been chosen here; see the remarks concerning the final-state interaction after Eq. (24). A more recent fit has been given by R. Ely, Jr., *et al.*, Ref. 20. One of their fits (solution B) is roughly consistent with the fit of Berends *et al.*, used by us for comparison, although they seem to require a nonvanishing p-wave contribution to F_1. If it is firmly established that a large p-wave contribution is present, then the simple current-algebra approach will be in difficulty.

[22] Berkeley-UCL-Wisconsin collaboration, paper presented by M. J. Esten at the Physical Society Conference on Elementary Particles, London, 1967 (unpublished).

[23] Simple vector-meson pole models (Refs. 6 and 21) give $|F_4|\approx2$–3. Calculations based on a comparison with the decay $\pi^0\to2\gamma$ employing soft-pion current-algebra techniques give $|F_4|\approx3.5$; compare A. K. Mohanti and R. E. Marshak, Nuovo Cimento **52A**, 967 (1967); **54A**, 213(E) (1968). If the decay $\eta^0\to2\gamma$ is used for this comparison, one obtains $|F_4|\approx15$.

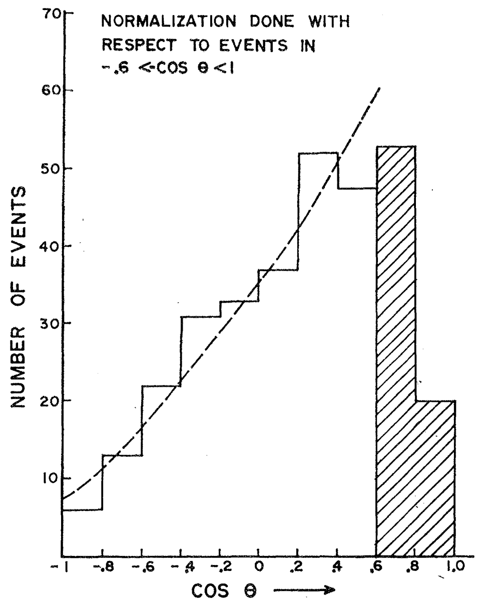

FIG. 2. cosθ distribution. (The events in the shaded region have been ignored since the depletion of events in the last bin cannot be understood.)

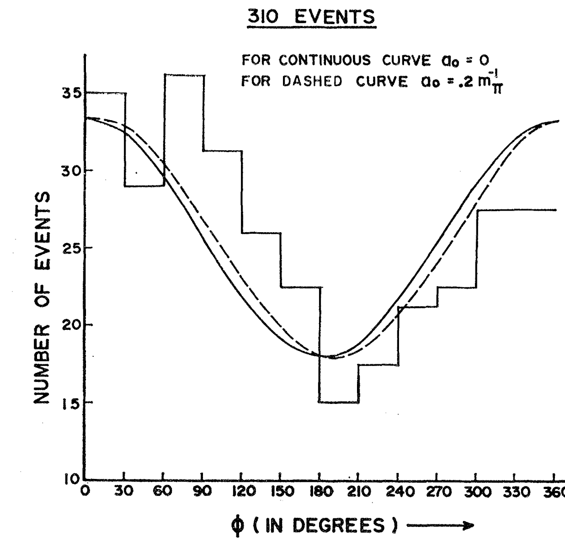

FIG. 3. ϕ distribution.

duced a final-state s-wave $\pi\pi$ interaction in an *ad hoc* manner. If we arbitrarily include such an interaction consistent with the current-algebra prediction of the $\pi\pi$ scattering length given by Weinberg[24] ($a_0 = 0.2/m_\pi$), the correct sign of the asymmetry can be obtained, though not the magnitude (see Fig. 3). To the extent that all the previous theoretical attempts have put in the final-state s-wave interaction by hand, we feel that on the whole our simplified approach fares much better (see Sec. V).

We thus see that the assumption of constant K_{l4}-decay form factors works quite well. One may wish to understand the dynamical reason for this. It should be noted that these form factors are more like scattering amplitudes with one of the external particles having a variable mass. A dynamical basis for treating such amplitudes is not well understood. Our assumption of their constancy is perhaps analogous to the case of an amplitude which is dominated by a subtraction constant. Furthermore, the possible poles due to the ρ, K^*, and K_A are all somewhat away from both the physical region and the soft-pion point. The exception is the K pole which has already been explicitly taken into account by the current-albegra treatment of Weinberg.

$K_{\mu 4}$ *Decays*

The $K_{\mu 4}$ decays have not yet been studied experimentally in sufficient detail. The only datum known is the following:

$$\Gamma(K^+ \to \pi^+\pi^-\mu^+\nu_\mu) = (1.1\pm 0.7)\times 10^3 \text{ sec}^{-1}. \quad (25)$$

With our form factors,[25] assuming $(\mu - e)$-universality and $\sin\theta = 0.22$, we obtain for this rate a value of 0.31 $\times 10^3$ sec^{-1}. In a previous work[3] on the K_{l3} and the K_{l2} decays, we have shown, as already mentioned, that a small $(\mu - e)$-universality violation in strangeness-changing decays is necessary to describe the data with the form factors given in Secs. II and III, adopting the single Cabibbo angle $\sin\theta = 0.22$. The essential effect is to increase the effective coupling constant for the μ mode with respect to that for the e mode of the decays by nearly 10–20%. A similar effect may be present for the K_{l4} decays, and it would then tend to increase the theoretical $K_{\mu 4}$ rate in the right direction, leaving the K_{e4} decays unchanged. More cannot be stated at the present stage.

V. COMPARISON WITH OTHER CALCULATIONS

To appreciate the degree of success of our simplified approach, we must compare our results with those of the previous theoretical attempts at improving the soft-pion current-algebra results of Weinberg. A brief summary of some of those results follows, which for consistency we have reevaluated on the basis of a single Cabibbo angle $\sin\theta_V = \sin\theta_A = 0.22$.

(1) Clavelli[5] attempted to obtain the axial-vector form factors by use of vector-meson dominance using gauge invariance and imposing the soft-meson current-algebra constraints. He took the matrix element of the

[24] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966). The numbers quoted in Table I from the fits of Berends *et al.* (Ref. 21) correspond to $a_0 = 0.3/m_\pi$.

[25] Although F_3 can contribute significantly to the muon mode, the current-algebra value given in Table I still remains essentially untested because of the large uncertainty in the value of $\Gamma(K_{\mu 4}^+)$.

axial-vector current to the vacuum to be dominated by the K pole. A simple consideration shows that this dynamical pole can contribute only to F_3. Thus he should have obtained $F_1=F_2=0$, whereas through an ambiguous use of gauge invariance he obtained rather large values for F_1 and F_2. His results therefore cannot be used for a meaningful comparison.

(2) Greenberg[6] has used a hard-meson current-algebra technique. He has further used dominance by the ρ, K^*, K_A, and K poles and made use of spectral-function sum rules. He also takes $f_K=f_\pi$. His form factors, which he uses as constants in the physical region, are $F_1=4.0$, $F_2=4.6$, and $F_3=1.06$. Using $\sin\theta=0.22$, the K_{e4} rate for these values is $\Gamma(K_{e4}^+)=1.6\times10^3$ sec^{-1}, which is too small. Greenberg has also included an s-wave final-state interaction to enhance this rate to 1.9×10^3 sec^{-1} (with $\sin\theta=0.22$), which is still unsatisfactory.

(3) There is a calculation, employing the hard-meson techniques of Schnitzer and Weinberg, carried out by Sarker,[7] who finds $F_1=4.8$, $F_2=2.66$, and $F_3=4.4$, taken constant over the physical region. The K_{e4} rate implied is 2.1×10^3 sec^{-1}. He had, however, inconsistently used $\sin\theta_A=0.265$ along with $f_K/f_\pi=1.28$. He further used a value 0.6 for $\xi=F_-(0)/F_+(0)$ (referring to the K_{l3}-decay form factors). The K_{l3}-decay data, on the other hand, rather suggest that ξ has a negative value.

(4) Using dispersion relations, Biswas *et al.*[8] obtained for the physical region $F_1=3.8$, $F_2=4.6$, and $F_3=4.2$. This leads to $\Gamma(K_{e4}^+)\approx1.4\times10^3$ sec^{-1}. They had again used $\sin\theta_A=0.265$ and $f_K/f_\pi=1.17$.

(5) Dutt *et al.*[9] have also calculated the form factors, using generalized Ward identities saturated by various meson poles (again taking $\sin\theta_A=0.265$, $f_K/f_\pi=1.17$). They give the results $F_1=5.85$, $F_2=9.4$, and $F_3=8.95$ at a characteristic important point of the physical region.

(6) Employing the effective-Lagrangian approach with $SU_3\otimes SU_3$ chiral gauge invariance broken by gauge-field mass terms, Biswas *et al.*[10] obtain $F_1=4.3$, $F_2=3.05$, and $F_3=2.6$ at $s=t=u=0$.

In spite of the rather detailed dynamical assumptions and elaborate calculational techniques involved, none of the above sets of form factors agree with experiment. This can readily be seen by a comparison with the values quoted in Table I, as well as the decay rates already quoted. In contrast, our approach, which is based on a very simple dynamical assumption, gives results that are in much better agreement with the experimental data.

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Optical Model of Elastic Proton-Proton Scattering*†

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The energy and momentum-transfer dependence of the differential cross section ($k_\perp\geq0.8$ GeV/c, $k_{\rm lab}\geq8$ GeV/c), the position of "breaks," and the parameter $b=[\partial\ln(d\sigma/d\Omega)/\partial t]_{t=0}$ are reproduced by a smooth absorber determined by two energy-independent parameters. We assume the absorber representing a proton flattens in a manner suggested by the Lorentz transformation.

I. INTRODUCTION

IN the following investigation, we will develop a model of elastic proton-proton scattering designed to reproduce the behavior of the spin-averaged differential cross section at high energy ($k_{\rm lab}\geq8$ GeV/c).[1] We will be particularly concerned with the properties of large-angle scattering: the s dependence, t dependence, and diffraction structure (which some authors have referred to as a series of "breaks"). For the sake of clarity, however, we will begin by discussing small-angle scattering.

In the region $|t|\leq1$ (GeV/c)^2, $k_{\rm lab}\geq8$ GeV/c, the behavior of the elastic cross section appears to be rather simple. The three quantities $\partial\sigma_{\rm tot}/\partial s$, $\sigma_{\rm el}/\sigma_{\rm tot}$, and ${\rm Re}f(0)/{\rm Im}f(0)$ are all much less than 1, and the curve $[d\sigma(s,t)/d\Omega]/[d\sigma(s,0)/d\Omega]$ does not depend on s and falls rapidly with $|t|$. These properties are neatly reproduced by Serber's optical model,[2] in which the c.m. differential cross section is derived from the scattering amplitude given by a Klein-Gordon equation governed by the absorptive potential $[i\eta\alpha V(\alpha r),0,0,0]$. For a wide variety of functions V, it is possible to reproduce the observed differential cross section in the forward region by an energy-independent choice of η and α.

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[1] For the process $k_1+k_2\to k_3+k_4$, we define $s=(k_1+k_2)^2$, $t=(k_3-k_1)^2$. In the c.m. system $k_1=(E,0,0,k)$; $k_3=(E,0,k_\perp,k-k_{\parallel})$; in the laboratory system $k_1=(E_{\rm lab},0,0,k_{\rm lab})$, $k_2=(m,0,0,0)$.

[2] R. Serber, Phys. Rev. Letters **10**, 357 (1963); Rev. Mod. Phys. **36**, 649 (1964).