Phases and Forward Cross Sections in Vector-Meson Photoproduction*

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Using a quark model for forward scattering and Regge-pole parametrization of meson-nucleon scattering amplitudes, $d\sigma/dt|_{t=0}$ and the phases of the forward scattering amplitudes are calculated for ρ^0, ω, and φ photoproduction and compared with the experimental data.

I. INTRODUCTION

OVER the past few years, a large number of data have been obtained on the energy dependence of the cross section for the photoproduction of the ρ^0 in the forward direction. This is especially true for photoproduction off protons.[1,2] It has been frequently pointed out that both the energy dependence of the forward cross section and the t dependence of the differential cross section for the reaction are very similar to the behavior of the differential cross sections for π^+p and π^-p elastic scattering in what is usually called the diffraction region, i.e., for incident momenta above ~ 3 GeV/c. This similarity is made quantitative in a broken-$SU(3)$ quark model,[3,4] originally applied to forward and total cross sections, but which has since been extended to differential cross sections at nonzero values of t.[5] The extension involves approximating the πp amplitudes by the square root of the differential cross sections, and therefore ignores the possibility of phase differences between the amplitudes. The resulting t dependence, however, fits the ρ-production data very well.[1,5]

In this paper, we examine the energy dependence of the forward photoproduction cross section, again using the quark model referred to above. Since we are also interested in calculating the relative phases of the photoproduction amplitudes, Regge-pole fits to πp (and KN) elastic scattering data[6,7] are used for the amplitudes instead of the square roots of cross sections. The Regge fits are obtained using polarization data as well as elastic scattering data, and therefore can be considered to be good parametrizations of the phases, as well as the magnitudes, of the scattering amplitudes.

In experiments in which lepton pairs are produced with an invariant mass near that of a resonance, one can observe the interference between purely electrodynamic amplitudes, which in principle can be calculated exactly to a given order, and resonance amplitudes, which are usually parametrized using a Breit-Wigner resonance form. Two such experiments have been performed in which the phase of the ρ-production amplitude is measured relative to the quantum electrodynamic amplitudes, one in photoproduction of electrons,[8] the other in electroproduction of muons.[9] Both experiments were done using complex nuclei for targets.

The approach taken in this paper is to use the two Regge-pole terms (the P and P') that contribute in the broken-$SU(3)$ quark model to ρ photoproduction in calculating the forward differential cross section. The relative amount of P and P' is varied and the results compared with the data.[1,2] It is found that the relative amount of P and P' predicted by the quark model and the Regge-pole fits to πp scattering[7] yields an energy dependence for the forward ρ-photoproduction cross section that compares well with the data. The energy dependence of the phase of the resulting Regge-pole amplitude is then obtained and compared with the phase determinations in the lepton-pair production experiments.[8,9] For the last comparison, there is an added complication in that the phases are obtained experimentally from production off complex nuclei. These phases are related to proton phases using standard vector-dominance and eikonal approximation methods.[10]

Recently there has been a good deal of interest in the interference of the ρ and ω amplitudes in both colliding-beam experiments[11] and production experiments.[12–15]

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[13] G. Goldhaber *et al.*, Phys. Rev. Letters **23**, 1351 (1969), $\pi^+p \to \pi^+\pi^-\Delta^{++}$; M. Abramovich *et al.*, CERN Report No. CERN D.Ph. II/Phys. 70-5 (unpublished), $\pi^-p \to p\pi^+\pi^-\pi^-$.
[14] W. W. M. Allison *et al.*, Phys. Rev. Letters **24**, 618 (1970), $\bar{p}p \to \pi^+\pi^-\pi^+\pi^-$.
[15] G. R. Allcock, summary talk at the 1969 International Conference on Electron and Photon Interactions at High Energies, Daresbury (unpublished); P. S. Biggs *et al.*, Phys. Rev. Letters **24**, 1197 (1970), $\gamma A \to Ae^+e^-$.

In fact, previous to the direct observation of ρ-ω interference, a method had been developed[16] that extracts the relative phase between the ρ and ω amplitudes by comparing the leptonic branching ratios obtained in colliding beams and pair production. The latter method, as well as the recent direct observations, have by now yielded a number of determinations of the relative ρ-ω phase, and these results can be used to obtain the phase of the ω amplitude relative to quantum electrodynamics once the ρ phase relative to quantum electrodynamics is known.[17] Using the Regge-pole model discussed above, the energy dependence of the forward ω-photoproduction cross section is calculated for various values of the phase of the ω amplitude and compared with the experimental data. Again, only the P and P' are used, as suggested by the broken-$SU(3)$ quark model. Measurements of the energy dependence of forward ω production, coupled with the results of this calculation, can be used as another method of estimating the phase of the ω amplitude.

Finally, the same methods can be applied to the φ-photoproduction amplitude. Here, the broken-$SU(3)$ quark model predicts that KN forward amplitudes contribute along with the πN amplitudes and the resulting number of Regge-pole contributions increases from two to five. Various forms of the quark model are examined and their predictions, in terms of the phase of the φ amplitude and the energy dependence of the forward cross section, are calculated. The forward cross sections are compared with the experimental data. The phase predictions are seen to depend critically on the precise form chosen for the quark-model amplitudes.

In Sec. II we review the basic elements of the broken-$SU(3)$ quark model[3] and introduce the Regge-pole parameters needed in the calculations. Sections III–V contain the calculations of the differential cross sections and amplitude phases for the ρ, ω, and φ mesons, respectively. Finally, in Sec. VI, we summarize our results. In the Appendix the phases measured in photoproduction from complex nuclei are related to phases that would be obtained in production from protons.

II. BROKEN-$SU(3)$ QUARK MODEL AND REGGE PARAMETERS

The basic assumption of Lipkin's quark model for scattering amplitudes[3] is that the forward scattering amplitude for a given reaction is the sum of all possible two-body quark-quark or quark-antiquark scattering amplitudes. Let us denote the basic triplet of quarks by $\mathcal{P}$, $\mathfrak{N}$, and λ, where $\mathcal{P}$ and $\mathfrak{N}$ are the isodoublet of strangeness zero and λ is the isosinglet of strangeness -1. Applying the Pomeranchuk theorem to forward quark scattering, one obtains

$$\mathcal{A}(\mathcal{P}\mathfrak{N})=\mathcal{A}(\mathfrak{N}\mathcal{P})=\mathcal{A}(\bar{\mathcal{P}}\mathfrak{N})=\mathcal{A}(\bar{\mathfrak{N}}\mathcal{P}) =\mathcal{A}(\mathcal{P}\mathcal{P})=\mathcal{A}(\mathfrak{N}\mathfrak{N})=P\,, \tag{2.1a}$$

$$\mathcal{A}(\lambda\mathcal{P})=\mathcal{A}(\lambda\mathfrak{N})=\mathcal{A}(\bar{\lambda}\mathcal{P})=\mathcal{A}(\bar{\lambda}\mathfrak{N})=P-S\,, \tag{2.1b}$$

$$\mathcal{A}(\mathcal{P}\bar{\mathcal{P}})=\mathcal{A}(\mathfrak{N}\bar{\mathfrak{N}})=P+\mathrm{A}\,, \tag{2.1c}$$

where $\mathcal{A}$ denotes the forward scattering amplitude, S in (2.1b) represents the $SU(3)$-symmetry breaking in strange quark scattering, and A in (2.1c) takes into account the effect of annihilation in the quark-antiquark channel. The quark compositions of the various mesons and nucleons of interest are as follows:

$$\begin{gathered}\pi^+=(\mathcal{P}\bar{\mathfrak{N}})\,,\quad \pi^-=(\bar{\mathcal{P}}\mathfrak{N})\,,\\ K^+=(\mathcal{P}\bar{\lambda})\,,\quad K^0=(\mathfrak{N}\bar{\lambda})\,,\quad K^-=(\bar{\mathcal{P}}\lambda)\,,\quad \bar{K}^0=(\bar{\mathfrak{N}}\lambda)\,,\\ p=(\mathcal{P}\mathcal{P}\mathfrak{N})\,.\end{gathered} \tag{2.2}$$

The quark-scattering model then gives the following relations:

$$\mathcal{A}(\pi^+p)=2\mathcal{A}(\mathcal{P}\mathcal{P})+\mathcal{A}(\mathcal{P}\mathfrak{N})+2\mathcal{A}(\bar{\mathfrak{N}}\mathcal{P}) +\mathcal{A}(\bar{\mathfrak{N}}\mathfrak{N})=6P+A\,, \tag{2.3a}$$

$$\mathcal{A}(\pi^-p)=2\mathcal{A}(\mathfrak{N}\mathcal{P})+\mathcal{A}(\mathfrak{N}\mathfrak{N})+2\mathcal{A}(\bar{\mathcal{P}}\mathcal{P}) +\mathcal{A}(\bar{\mathcal{P}}\mathfrak{N})=6P+2A\,, \tag{2.3b}$$

$$\mathcal{A}(K^+p)=2\mathcal{A}(\mathcal{P}\mathcal{P})+\mathcal{A}(\mathcal{P}\mathfrak{N})+2\mathcal{A}(\bar{\lambda}\mathcal{P}) +\mathcal{A}(\bar{\lambda}\mathfrak{N})=6P-3S\,, \tag{2.3c}$$

$$\mathcal{A}(K^-p)=2\mathcal{A}(\lambda\mathcal{P})+\mathcal{A}(\lambda\mathfrak{N})+2\mathcal{A}(\bar{\mathcal{P}}\mathcal{P}) +\mathcal{A}(\bar{\mathcal{P}}\mathfrak{N})=6P-3S+2A\,, \tag{2.3d}$$

$$\mathcal{A}(K^0p)=2\mathcal{A}(\bar{\lambda}\mathcal{P})+\mathcal{A}(\bar{\lambda}\mathfrak{N})+2\mathcal{A}(\mathfrak{N}\mathcal{P}) +\mathcal{A}(\mathfrak{N}\mathfrak{N})=6P-3S\,, \tag{2.3e}$$

$$\mathcal{A}(\bar{K}^0p)=2\mathcal{A}(\lambda\mathcal{P})+\mathcal{A}(\lambda\mathfrak{N})+2\mathcal{A}(\bar{\mathfrak{N}}\mathcal{P}) +\mathcal{A}(\bar{\mathfrak{N}}\mathfrak{N})=6P-3S+A\,. \tag{2.3f}$$

The relations (2.3) along with similar ones for $\mathcal{A}(K^+n)$ and $\mathcal{A}(K^-n)$ give a large number of equalities between various combinations of meson-nucleon total cross sections via the optical theorem. The energy variations of these equalities have been checked[3] and are seen to be in agreement to within 10–20%.

Using the vector-meson-dominance model, the quark-scattering model is extended to forward photoproduction of vector mesons. The quark compositions of the neutral vector mesons are

$$\begin{gathered}\rho^0=(1/\sqrt{2})(\mathfrak{N}\bar{\mathfrak{N}}-\mathcal{P}\bar{\mathcal{P}})\,,\\ \omega=(1/\sqrt{2})(\mathfrak{N}\bar{\mathfrak{N}}+\mathcal{P}\bar{\mathcal{P}})\,,\quad \varphi=(\lambda\bar{\lambda})\,,\end{gathered} \tag{2.4}$$

and the forward scattering amplitudes are given by

$$\begin{aligned}\mathcal{A}(\rho^0p)=\tfrac{1}{2}[&2\mathcal{A}(\bar{\mathcal{P}}\mathcal{P})+\mathcal{A}(\bar{\mathcal{P}}\mathfrak{N})+2\mathcal{A}(\mathfrak{N}\mathcal{P})\\ &+\mathcal{A}(\mathfrak{N}\mathfrak{N})+2\mathcal{A}(\bar{\mathfrak{N}}\mathcal{P})+\mathcal{A}(\bar{\mathfrak{N}}\mathfrak{N})+2\mathcal{A}(\mathcal{P}\mathcal{P})\\ &+\mathcal{A}(\mathcal{P}\mathfrak{N})]=6P+\tfrac{3}{2}A\,,\end{aligned} \tag{2.5a}$$

$$\mathcal{A}(\omega p)=\mathcal{A}(\rho^0p)\,, \tag{2.5b}$$

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[17] There are added complications because the ρ and ω are overlapping resonances. We include a discussion of these in Sec. III.

$$\mathcal{A}(\varphi p)=2\mathcal{A}(\bar{\lambda}\mathcal{P})+\mathcal{A}(\bar{\lambda}\mathfrak{N})+2\mathcal{A}(\lambda\mathcal{P})+\mathcal{A}(\lambda\mathfrak{N}) = 6P-6S. \quad (2.5c)$$

By comparing (2.5) with (2.3), we immediately obtain the following relations between forward scattering amplitudes:

$$\mathcal{A}(\rho^0 p)=\tfrac{1}{2}[\mathcal{A}(\pi^+p)+\mathcal{A}(\pi^-p)], \quad (2.6a)$$

$$\mathcal{A}(\omega p)=\tfrac{1}{2}[\mathcal{A}(\pi^+p)+\mathcal{A}(\pi^-p)]. \quad (2.6b)$$

We note that these relations do not depend on the Pomeranchuk theorem, i.e., the right-hand side of (2.1), but depend only on the assumption that the scattering amplitudes are the sum of two-body quark amplitudes.

Similar relations can also be obtained for the φp forward scattering amplitude. There is some ambiguity, however, in which meson-nucleon amplitudes (2.3) to choose. The most frequently used relation in the literature is[3]

$$\mathcal{A}^{(1)}(\varphi p)=2\mathcal{A}(K^+p)+\mathcal{A}(\pi^-p)-2\mathcal{A}(\pi^+p), \quad (2.7)$$

where the symmetry-breaking parameters P, S, and A have been eliminated between (2.3a)–(2.3c) and (2.5c). Since there are six relations[18] in (2.3) and only three symmetry-breaking parameters, this choice is not unique. Another choice found in the literature is[1,19]

$$\mathcal{A}^{(2)}(\varphi p)=\mathcal{A}(K^+p)+\mathcal{A}(K^-p)-\mathcal{A}(\pi^-p). \quad (2.8)$$

We shall see later that the various Regge-pole amplitudes needed to parametrize the meson-nucleon scattering amplitudes do not enter with the same coefficients in (2.7) and (2.8). Possibly a less ambiguous approach would be to ignore symmetry breaking in the Pomeranchuk theorem and retain only the assumption that the scattering amplitudes are the sum of two-body quark amplitudes. Then the simplest combinations of meson-proton scattering amplitudes to give the φp forward scattering amplitude are

$$\mathcal{A}^{(3)}(\varphi p)=\mathcal{A}(K^+p)+\mathcal{A}(\bar{K}^0p)-\mathcal{A}(\pi^+p), \quad (2.9a)$$

$$\mathcal{A}^{(4)}(\varphi p)=\mathcal{A}(K^-p)+\mathcal{A}(K^0p)-\mathcal{A}(\pi^-p), \quad (2.9b)$$

where (2.3) and (2.5c) have been used. The relations (2.9) are on the same footing as the relations (2.6) in that neither rely on the $SU(3)$-breaking assumption. In this sense, (2.9) may be considered more fundamental than (2.7) and (2.8), although we will show results for all four forms for the φp forward amplitude. We will find that although (2.7) and (2.8) agree quite well with experiment in terms of t dependence at a fixed energy[1,3] as long as the over-all normalization is left as a free parameter, the relations (2.7)–(2.9) agree only fairly well with experiment in terms of the energy dependence of the differential cross section at $t=0$.

In order to obtain a parametrization of the meson-nucleon amplitudes, we use fits to the data in terms of Regge-pole amplitudes.[6,7] Since these fits have been made using not only the differential and total cross-section data, but the polarization data as well, we can assume that the phase information contained in the Regge-pole parametrization is a good approximation to the actual phases of the meson-nucleon amplitudes. Since we are dealing with πN and KN scattering, five Regge trajectories are needed: P, P', ω, ρ, and R. The πp amplitudes have the form

$$\mathcal{A}(\pi^+p)=A_P{}^\pi+A_{P'}{}^\pi-A_\rho{}^\pi, \quad (2.10a)$$

$$\mathcal{A}(\pi^-p)=A_P{}^\pi+A_{P'}{}^\pi+A_\rho{}^\pi, \quad (2.10b)$$

and Kp amplitudes are given by

$$\mathcal{A}(K^+p)=A_P{}^K+A_{P'}{}^K-A_\omega{}^K-A_\rho{}^K+A_R{}^K, \quad (2.11a)$$

$$\mathcal{A}(K^-p)=A_P{}^K+A_{P'}{}^K+A_\omega{}^K+A_\rho{}^K+A_R{}^K, \quad (2.11b)$$

$$\mathcal{A}(K^0p)=A_P{}^K+A_{P'}{}^K-A_\omega{}^K+A_\rho{}^K-A_R{}^K, \quad (2.11c)$$

$$\mathcal{A}(\bar{K}^0p)=A_P{}^K+A_{P'}{}^K+A_\omega{}^K-A_\rho{}^K-A_R{}^K, \quad (2.11d)$$

where the Regge amplitudes at $t=0$ have the following form:

$$A_P{}^\pi=i\beta_P(k/k_0), \quad A_P{}^K=F_PA_P{}^\pi, \quad (2.12a)$$

$$A_j{}^\pi=i\beta_j[1+i\cot(\tfrac{1}{2}\pi\alpha_j)](k/k_0)^{\alpha_j}, \quad A_j{}^K=F_jA_j{}^\pi \quad (j=P',R), \quad (2.12b)$$

$$A_j{}^\pi=i\beta_j[1-i\tan(\tfrac{1}{2}\pi\alpha_j)](k/k_0)^{\alpha_j}, \quad A_j{}^K=F_jA_j{}^\pi \quad (j=\rho,\omega). \quad (2.12c)$$

The energy k_0 is chosen to be 1 GeV and α_j are the $t=0$ intercepts of the Regge trajectories. The values of α_j, β_j, and F_j are given in Table I. The set of values labeled set 1 are obtained from Ref. 7 in which πp scattering data were used. We use this set in calculating ρ^0 and ω amplitudes (2.6) since it is a more recent fit to the data than set 2, which was obtained from both πN and KN scattering data. Set 2, obtained from Ref. 6, will be used in calculating the φ amplitudes (2.7)–(2.9).

III. PHOTOPRODUCTION OF ρ^0

Combining (2.6a) and (2.10), the forward ρp scattering amplitude is given by

$$\mathcal{A}(\rho^0p)=A_P{}^\pi+C_\rho A_{P'}{}^\pi, \quad (3.1)$$

TABLE I. Regge-pole parameters.

	Set 1		Set 2		
j	α_j	β_j	α_j	β_j	F_j
P	1.0	14.5	1.00	19.6	0.901
P'	0.73	20.7	0.50	19.6	0.279
ρ	0.58	1.35	0.54	2.75	0.527
ω	···	···	0.52	6.36	1.00
R	···	···	0.32	1.75	1.00

[18] Only meson-proton amplitudes are included in (2.3) for simplicity and because we shall be interested only in photoproduction from protons.

[19] The claim is made in Ref. 1 that (2.8) is obtained without using symmetry breaking and the Pomeranchuk theorem. It can be readily seen, however, that (2.8) holds only if $2\mathcal{A}(\mathcal{PP})-\mathcal{A}(\mathcal{P}\mathfrak{N})-\mathcal{A}(\mathfrak{NN})=0$, which is only a consequence of (2.1a).

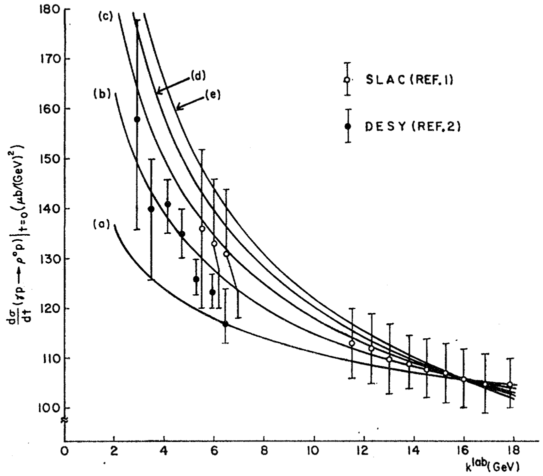

FIG. 1. Quark-model plus Regge-pole parametrization predictions for $(d\sigma/dt)(\gamma p \to \rho^0 p)|_{t=0}$ [Eq. (3.3)] compared with the experimental data of Refs. 1 and 2. The curves correspond to the sets of values for C_ρ and D_ρ in Table II.

where the factor C_ρ has been introduced as an adjustable parameter to account for possible deviations from the pure quark-model predictions. Using vector dominance, the differential cross section for ρ photoproduction at $t=0$ is proportional to $|\mathcal{A}(\rho^0 p)|^2/k^2$ and therefore

$$\frac{d\sigma}{dt}(\gamma p \to \rho^0 p)\bigg|_{t=0} = D_\rho\left[1+2b_\rho+\frac{b_\rho^2}{\sin^2(\frac{1}{2}\pi\alpha_{P'})}\right],\qquad b_\rho = C_\rho \frac{\beta_{P'}}{\beta_P}\left(\frac{k_0}{k}\right)^{1-\alpha_{P'}}, \tag{3.2}$$

where we have used (2.12) and where D_ρ is fixed by normalizing $d\sigma/dt|_{t=0}$ to the value 106 μb/GeV^2 at $k^{\text{lab}}=16$ GeV obtained in the recent experiment at SLAC.[1]

In Fig. 1, (3.2) is plotted for the values[20] of C_ρ and D_ρ shown in Table II and compared with the recent experimental data from SLAC[1] and DESY.[2,21] Acceptable fits to the data are obtained with values of C_ρ varying from 1/3 to 5/3, with the value $C_\rho=1$, corresponding to no deviation from the quark-model prediction, giving a very good fit.

TABLE II. Parameters entering in (3.2) normalizing $d\sigma/dt(\gamma p \to \rho^0 p)|_{t=0}$ to 106 μb/GeV^2 at $k^{\text{lab}}=16$ GeV.

	C_ρ	D_ρ (μb/GeV^2)
a	0.33	71
b	0.67	51
c	1.00	38
d	1.33	29
e	1.67	23

Again using vector dominance, the phase of $\mathcal{A}(\rho^0 p)$ can be directly related to the phase of the ρ-photoproduction amplitude in the forward direction. We denote the latter phase, after extracting an over-all factor of i from the amplitude, by Δ_ρ.[22] From (3.1) and (2.12) we obtain

$$\Delta_\rho = \tan^{-1}\left[\left(\frac{b_\rho}{1+b_\rho}\right)\cot(\tfrac{1}{2}\pi\alpha_{P'})\right],\qquad b_\rho = C_\rho\frac{\beta_{P'}}{\beta_P}\left(\frac{k_0}{k}\right)^{1-\alpha_{P'}}. \tag{3.3}$$

The normalization factor D_ρ does not appear in Δ_ρ. The result (3.3) is plotted in Fig. 2 for various values of C_ρ. The phase is seen to be positive and, for $C_\rho=1$, between 10° and 13°.

There are two experimental determinations of the phase of the forward ρ-photoproduction amplitude. In an electron-pair photoproduction experiment at 2.8 GeV, Asbury *et al.* obtain[8]

$$\delta_\rho = 15°\pm 25°, \tag{3.4}$$

and in a muon-pair electroproduction experiment in which the virtual photon carries an energy of 4.6±0.3 GeV, Earles obtains[9]

$$\delta_\rho = 16°\pm 22°. \tag{3.5}$$

In both experiments, a carbon target was used. The measured phases (3.4) and (3.5) must be converted to phases for photoproduction from protons and this is done using eikonal methods and the vector-meson-dominance model.[10] The details are given in the Appendix. The results are that, when substituted into

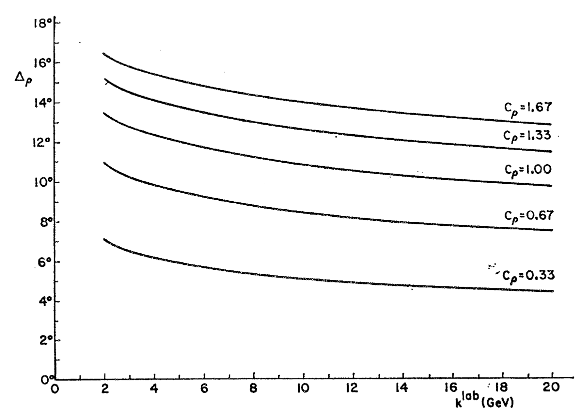

FIG. 2. Quark-model plus Regge-pole parametrization predictions for Δ_ρ, the phase of the forward ρ-photoproduction amplitude.

[20] We use the Regge-pole parameters in set 1 of Table I since they are more recent and since only parameters for the P and P' Regge poles are needed.

[21] Data on $d\sigma/dt(\gamma p \to \rho^0 p)|_{t=0}$ from other groups have not been included, in order to simplify the figure. Within errors, they all agree with the data shown in Fig. 1. A graphical summary of all existing data (except those of Ref. 2) is given in Fig. 9 of Z. G. T. Guiragossián, invited talk given at the International Seminar on Vector Mesons and Electromagnetic Interactions, Dubna, 1969, SLAC Report No. SLAC-PUB-694 (unpublished).

[22] Therefore the total phase of the ρ-photoproduction amplitude relative to quantum electrodynamic amplitudes is $ie^{i\Delta_\rho}$.

(A8) and (A9), (3.4) and (3.5) give, respectively, the proton phases

$$\Delta_\rho = 4° \pm 42°\,, \tag{3.4$'$}$$

$$\Delta_\rho = 17° \pm 39°\,. \tag{3.5$'$}$$

These results tend to indicate positive values for Δ_ρ. The curves in Fig. 2 agree with (3.4′) and (3.5′) within their rather large errors.

IV. PHOTOPRODUCTION OF ω

The quark-model prediction (2.5b) indicates that ω photoproduction should be quite similar to ρ^0 production. Unfortunately, data such as those shown in Fig. 1 for the ρ^0 do not exist over as wide an energy range for the ω, and therefore a fit determining the relative amount of P and P' Regge-pole contributions will not be attempted. Instead, we will give predictions for $d\sigma/dt|_{t=0}$ for $\gamma p \to \omega p$ for various values of Δ_ω, the phase of $\mathcal{A}(\omega p)$ (after removing a factor of i) chosen at the laboratory energy $k^{\text{lab}} = k_4 = 4$ GeV. The relevant equations are analogous to (3.1)–(3.3), where the subscripts are now changed from ρ to ω. The results are[20]

$$\frac{d\sigma}{dt}(\gamma p \to \omega p)\bigg|_{t=0} = D_\omega\left[1 + 2b_\omega + \frac{b_\omega^2}{\sin^2(\frac{1}{2}\pi\alpha_{P'})}\right],$$
$$b_\omega = \frac{\tan\Delta_\omega}{\cot(\frac{1}{2}\pi\alpha_{P'}) - \tan\Delta_\omega}\left(\frac{k_4}{k}\right)^{1-\alpha_{P'}}. \tag{4.1}$$

The normalization of $d\sigma/dt|_{t=0}$ is chosen to be 20 μb/GeV^2 at $k^{\text{lab}} = 16$ GeV.

In (4.1), b_ω is singular at

$$\Delta_\omega = \tfrac{1}{2}\pi(1 - \alpha_{P'}) \approx 24°\,. \tag{4.2}$$

Below this value, b_ω is positive and $d\sigma/dt|_{t=0}$ is a monotonically decreasing function of k^{lab}. Examples are shown in Fig. 3(a) for four values of Δ_ω. Above the value in (4.2), b_ω is negative and $d\sigma/dt|_{t=0}$ has a minimum. This behavior is shown in Fig. 3(b). For phase angles between 70° and 90°, the curves look very much like those in Fig. 3(a), but in principle can be distinguished from them by the presence of the minimum. The data in Fig. 3 have been taken from Ref. 11 and are mostly from bubble-chamber experiments. The data point at $k^{\text{lab}} \approx 5$ GeV tends to favor the curves in Fig. 3(**b**), but if that point is ignored, phase angles of the order of 18° and 80° seem to fit the data equally well.

In principle, Δ_ω can be extracted from experiments which detect ρ-ω interference, once Δ_ρ is known. There are, however, at least two sources of difficulty in determining Δ_ω. The first arises due to the fact that at low energies the ω-photoproduction amplitude from protons contains a one-pion-exchange term along with the diffractive part which is presumably described by the quark-model amplitude (2.5′). Therefore, in order

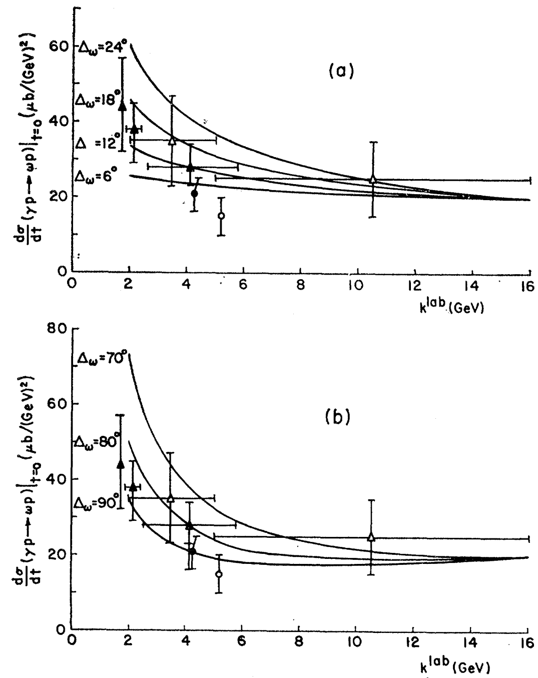

FIG. 3. The Regge-pole parametrization predictions for $(d\sigma/dt)(\gamma p \to \omega p)|_{t=0}$ for various values of Δ_ω. (a) $\Delta_\omega \leq 24°$; (b) $\Delta_\omega > 24°$. The experimental points are from Ref. 21.

to determine Δ_ω, one should make comparisons with the curves in Fig. 3 only at high energies, although agreement with the data seems to be obtained even at relatively low energies.

The second complication is due to the fact that the ω and ρ^0 are overlapping resonances.[23,24] The off-diagonal terms in the interaction Hamiltonian contain the amplitudes for the direct decay of the ρ^0 and ω into two and three pions. The magnitudes and phases of some of these amplitudes are not known with any precision. Horn has shown[24] that a variation of the 3π amplitudes within the experimental limits can cause a significant variation in the predicted shape of the cross section in the region of the ω, independent of the relative phase between the ρ^0 and ω production amplitudes.

Even if one ignores the direct decay amplitudes in the off-diagonal terms of the interaction Hamiltonian, additional phases are introduced due to the overlapping of the ω and ρ^0. Following Horn,[24] the additional phase in experiments in which the final state contains a pion pair[11–14] would be

$$\epsilon_{\pi\pi} = \pi - \tan^{-1}\left[\left|\frac{\Gamma_\omega - \Gamma_{\rho^0}}{2(m_\omega - m_{\rho^0})}\right|\right] \approx 110°\,, \tag{4.3}$$

where m_V and Γ_V are the masses and widths of the vector mesons. We emphasize that (4.3) ignores the

[23] A. Goldhaber, G. C. Fox, and C. Quigg, Phys. Letters **30B**, 249 (1969).

[24] D. Horn, Phys. Rev. D **1**, 1421 (1970).

presence of the 2π and 3π direct decay amplitudes in the Hamiltonian and is therefore only a rough estimate of the actual additional phase due to resonance overlap. We note that (4.3) agrees fairly well with the results of the colliding-beam experiment in which ρ-ω interference is observed.[11] In this experiment, two fits to the data give relative phases of $164°\pm28°$ and $150°\pm26°$. Here the observed phase is due only to resonance overlap since, in the vector-dominance model, there are no production amplitudes, the exchanged photon being directly coupled to the vector mesons.

In ρ-ω interference experiments in which the final state is a lepton pair,[15] the measured phase difference between the ω and ρ^0 amplitudes is given by[24]

$$\delta_{\omega\rho}=\tan^{-1}\left[\frac{(\gamma_\rho/\gamma_\omega+\mathrm{Re}\eta)\sin\Delta_{\omega\rho}+\mathrm{Im}\eta(\cos\Delta_{\omega\rho}\pm1)}{(\gamma_\rho/\gamma_\omega)\cos\Delta_{\omega\rho}+\mathrm{Re}\eta(\cos\Delta_{\omega\rho}+1)-\mathrm{Im}\eta(\sin\Delta_{\omega\rho})}\right], \tag{4.4}$$

where the photon vector-meson coupling is $em_V{}^2/2\gamma_V$, $\Delta_{\omega\rho}$ is the relative phase between the ω and ρ^0 production amplitudes, and

$$\eta=\frac{m_{\rho^0\omega}-\frac{1}{2}i\Gamma_{\rho^0\omega}}{m_\omega-m_{\rho^0}-\frac{1}{2}i(\Gamma_\omega-\Gamma_{\rho^0})}\approx\frac{-3}{19+54i}. \tag{4.5}$$

In (4.4) it is assumed that the ratio of the magnitudes of the ω and ρ^0 production amplitudes is equal to $\gamma_\omega{}^{-1}/\gamma_\rho{}^{-1}$. There is uncertainty in regard to which sign to use in the numerator in (4.4),[25] and we shall consider both cases. In (4.5), $m_{\rho^0\omega}$ is the mass matrix element between the ρ^0 and ω and is estimated to be approximately -3 MeV.[24] The term $\Gamma_{\rho^0\omega}$ contains the direct decay amplitudes into 2π and 3π and is neglected in (4.5). Using the $SU(6)$ value $\gamma_\rho/\gamma_\omega=\frac{1}{3}$ and (4.5) in (4.4), we give curves in Fig. 4 for $\delta_{\omega\rho}-\Delta_{\omega\rho}$, the excess relative phase measured in lepton-pair production experiments. The solid curve is for the choice of plus sign in (4.4), the dashed curve is for the choice of minus sign. We see that, within the approximations made here, there can be an excess in the measured ω-ρ phase difference of as much as $\sim20°$ due to resonance overlap.

Two electron-pair photoproduction experiments have been performed recently in which ρ-ω interference has been observed directly.[15] One experiment was done at DESY with a beryllium target and a maximum incident photon energy of 5.12 GeV. A preliminary result for the measured phase is $22°\pm25°$, which, using the results of the Appendix, converts to an effective measured ω-ρ phase difference for a proton target of

$$\delta_{\omega\rho}{}^{(1)}\approx36°\pm42°. \tag{4.6}$$

The other experiment was done at Daresbury using a carbon target with a maximum incident photon energy of 4.1 GeV. The result was a phase of $(100_{-30}^{+38})°$. For a proton target, again using the results of the Appendix, this becomes[26]

$$\delta_{\omega\rho}{}^{(2)}\approx120°\pm50°. \tag{4.7}$$

Unless there is a rapid energy or nucleon number dependence for the phase $\delta_{\omega\rho}$, which is unlikely, (4.6) and (4.7) are not consistent with one another. The discrepancy between these two results essentially points up the difficulty in making such phase measurements. Nevertheless, it may be of some interest to derive values of the ω-production phase Δ_ω from these experimental results. Using Fig. 4, we find that one must subtract either $\sim18°$ (+ sign) or $\sim3°$ ($-$ sign) from (4.6) to obtain the relative phase of the production amplitudes

$$\Delta_{\omega\rho}{}^{(1)}\approx(18° \text{ or } 33°)\pm42°, \tag{4.8}$$

or, following a similar procedure for (4.7),

$$\Delta_{\omega\rho}{}^{(2)}\approx(112° \text{ or } 104°)\pm50°. \tag{4.9}$$

If we use the value $C_p=1$ in Fig. 2, then at these energies $\Delta_\rho\approx10°$ and we obtain for the phase of the ω-production amplitude

$$\Delta_\omega{}^{(1)}\approx(28° \text{ or } 43°)\pm42° \quad \text{(DESY)}, \tag{4.10}$$

$$\Delta_\omega{}^{(2)}\approx(122° \text{ or } 114°)\pm50° \quad \text{(Daresbury)}. \tag{4.11}$$

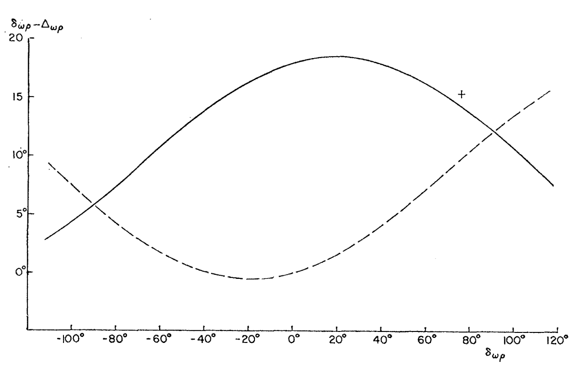

FIG. 4. Excess phase measured in ω-ρ interference experiments in which lepton pairs are produced for the two choices of sign in (4.4).

[25] The results of Ref. 24 contain the plus sign. The minus sign is given support in H. R. Quinn and T. F. Walsh, DESY Report No. DESY 70/13 (unpublished).

[26] The results of the Appendix indicate that for the conditions of the Daresbury experiment, the maximum allowed ω-ρ phase difference measured from the nucleus is 60°, which barely agrees with the lower limit on the experimental result. We are tacitly assuming throughout this paper that there are no additional phases in the photoproduction amplitude beyond those present in the vector-meson–nucleus scattering amplitude The Daresbury result tends to indicate that such additional phases may be present.

The measured phases used as input in obtaining (4.10) and (4.11) are obtained assuming no Ross-Stodolsky factor $(m_\rho/m)^4$ in the ρ amplitude. If one includes this factor, the measured phases then become $11°\pm29°$ (DESY) and $76°\pm31°$ (Daresbury). Following the same procedure as above, we obtain the following results for the phase of the ω-production amplitude:

$$\Delta_\omega^{(1)}\approx(10° \text{ or } 26°)\pm33° \quad \text{(DESY)}, \tag{4.10'}$$

$$\Delta_\omega^{(2)}\approx(122° \text{ or } 114°)\pm50° \quad \text{(Daresbury)}. \tag{4.11'}$$

We note that in Fig. 3, the curves for phase angles $\Delta_\omega\approx18°$ and $\Delta_\omega\approx80°$ fit the data quite well and it is therefore not possible with the presently available data on $d\sigma/dt|_{t=0}$ to give more support to either set of results (4.10) or (4.11).

V. PHOTOPRODUCTION OF φ

We proceed now to calculate the φ-photoproduction amplitudes (2.7)–(2.9) using the Regge parametrization (2.10)–(2.12) and set 2 of Table I. The results are

$$\mathfrak{A}^{(1)}(\varphi p)=(2F_P-1)A_P{}^\pi+(2F_{P'}-1)A_{P'}{}^\pi +(3-2F_p)A_\rho{}^\pi-2A_\omega{}^K+2A_R{}^K, \tag{5.1}$$

$$\mathfrak{A}^{(2)}(\varphi p)=(2F_P-1)A_P{}^\pi+(2F_{P'}-1)A_{P'}{}^\pi -A_\rho{}^\pi+2A_R{}^K, \tag{5.2}$$

$$\mathfrak{A}^{(3)}(\varphi p)=(2F_P-1)A_P{}^\pi+(2F_{P'}-1)A_{P'}{}^\pi -(2F_p-1)A_\rho{}^\pi, \tag{5.3a}$$

$$\mathfrak{A}^{(4)}(\varphi p)=\mathfrak{A}^{(3)}(\varphi p)+2(2F_p-1)A_\rho{}^\pi. \tag{5.3b}$$

The amplitudes in (5.1) and (5.2) depend on $SU(3)$ breaking in the right-hand side of (2.3) and (2.5c), whereas (5.3) depends only on the quark model which predicts that the forward scattering amplitude is the sum of quark-quark and quark-antiquark scattering amplitudes.

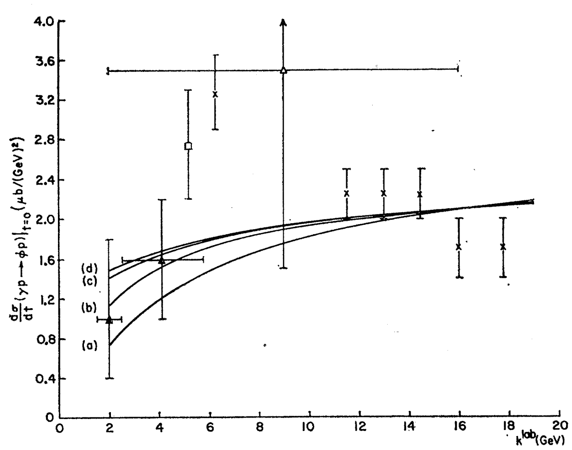

FIG. 5. Quark-model plus Regge-pole parametrization predictions for $(d\sigma/dt)(\gamma p\to\varphi p)|_{t=0}$. (a) and (b) correspond to the amplitudes (2.7) and (2.8), respectively, and (c) and (d) to (2.9a) and (2.9b), respectively. (a) and (b) rely on $SU(3)$ breaking in the Pomeranchuk theorem; (c) and (d) do not. The data are from Ref. 21.

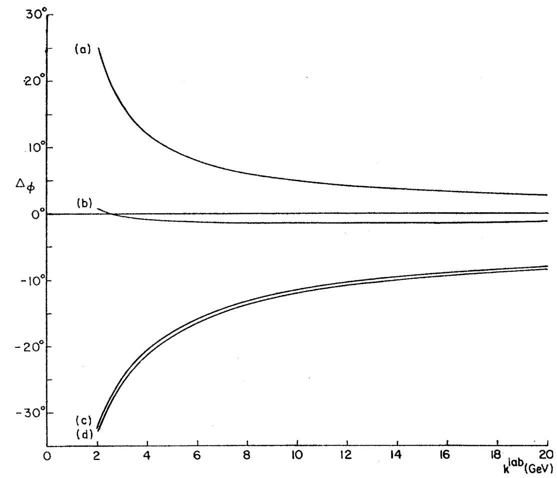

FIG. 6. Quark-model plus Regge-pole parametrization predictions for Δ_φ, the phase of the forward φ-production amplitude. The curves (a)–(d) correspond to the same amplitudes as those in Fig. 5.

The cross section $d\sigma/dt|_{t=0}$ is calculated from (5.1)–(5.3) as in the previous sections and the results are shown in Fig. 5, normalized to 2.1 μb/GeV^2 at $k^{\rm lab}=16$ GeV. Also shown are the data from Ref. 21. The results for all forms of the amplitude (5.1)–(5.3) behave differently from that for ρ^0 or ω production in that they increase monotonically with energy. The data taken as a whole seem to agree with this behavior although the recent SLAC data,[1] represented by the crosses in Fig. 5, decrease with increasing energy. This explains why the over-all normalization had to be varied with energy to obtain the good fits to the t dependence of the φ-photoproduction data in the SLAC experiment[1] using the amplitude (2.8). It seems that φ photoproduction and ρ^0 photoproduction are similar to one another in that the t dependence in both experiments is predicted quite well by the quark model.[3] However, the energy dependence of $d\sigma/dt|_{t=0}$ is not predicted nearly as well by this model for φ photoproduction as it is for ρ^0 (and possibly ω) photoproduction.

Finally, we calculate the phases Δ_φ of the amplitudes (5.1)–(5.3) defined by

$$\mathfrak{A}^{(i)}(\varphi p)=i|\mathfrak{A}^{(i)}(\varphi p)|e^{i\Delta_\varphi}. \tag{5.4}$$

The results are shown in Fig. 6. The phase predictions depend strongly on the choice of the form of the φ-production amplitude. At high energies, the results in Fig. 6 indicate that Δ_φ should be roughly between $\pm10°$. Comparing curves (c) and (d) in Fig. 6, which correspond to amplitudes (5.3a) and (5.3b), we see that the ρ-Regge-exchange term has little effect on Δ_φ. Since one less assumption went into obtaining (5.3a) and (5.3b), i.e., the assumption of $SU(3)$ breaking in the quark model was not used, one might tend to favor curves (c) and (d) in Fig. 6 over curves (a) and (b). This would indicate that the phase of the φ-production amplitude, after removing a factor of i, would be negative with respect to quantum electrodynamic amplitudes at high energies.

Since the φ resonance overlaps the tail of the ρ^0, there is the possibility of obtaining Δ_φ experimentally by looking for ρ-φ interference effects. Up to the present, no experiment has been done to measure directly the relative phase of the φ and ρ^0 production amplitudes, $\Delta_{\varphi\rho}$. However, it is possible to get an indication of the value of $\Delta_{\varphi\rho}$ by comparing B_p, the branching ratio into lepton pairs obtained in lepton-pair photoproduction experiments where production amplitudes are present, with B_c, the branching ratio obtained with colliding beams where production amplitudes are not present.[16] Present indications are that the ratio B_p/B_c is less than unity,[27] although the errors are of the order of 40%. This would indicate[16] that $\Delta_{\varphi\rho}$ lies between $\sim -30°$ and $-180°$[28] and, if Δ_ρ is of the order of 10°, this in turn gives a range of $\sim -20°$ to $\sim -170°$ for Δ_φ. This result essentially agrees with the prediction of curves (c) and (d) in Fig. 6 that the phase of the φ amplitude relative to quantum electrodynamics is negative.

VI. SUMMARY

We have calculated $d\sigma/dt|_{t=0}$ for vector-meson photoproduction from protons and the phase of the forward production amplitudes as a function of energy using a model which relates the forward scattering amplitude to the sum of all possible quark-quark and quark-antiquark forward scattering amplitudes and, in turn, relates vector-meson–proton scattering to various meson-proton scattering amplitudes. Regge-pole parametrizations are then used for the meson-proton amplitudes.

A good fit is obtained to $d\sigma/dt|_{t=0}$ for ρ^0 photoproduction with a prediction that the phase of the forward production amplitude at high energies should be $\sim 10°$ relative to quantum electrodynamics. In ω photoproduction, good fits are obtained to $d\sigma/dt|_{t=0}$ with phases of the order of 20° and 80°, each of which agrees roughly with the results of one of two separate experiments on ρ-ω interference in electron-pair production.

The situation is less clear in the case of φ photoproduction in that a number of forms of the scattering amplitude are possible, depending on whether or not one includes $SU(3)$ breaking in the quark model. The data on $d\sigma/dt|_{t=0}$ for φ photoproduction are not in complete agreement with the predictions, although there is some experimental evidence for an increase in the cross section with increasing energy, a behavior which is common to all forms of the amplitude. The phase predictions depend strongly on the form chosen for the scattering amplitude. The forms that do not include the $SU(3)$-breaking assumption give phase predictions that lie within the range allowed by a comparison of the presently available branching ratios.

ACKNOWLEDGMENT

The author is grateful to Professor Roy Weinstein for suggesting this work and for many helpful comments while it was in progress.

APPENDIX: PHASE OF ρ PHOTOPRODUCTION FROM COMPLEX NUCLEI RELATED TO PHASE OF ρ PHOTOPRODUCTION FROM PROTONS

We describe a method for extracting the phase of the ρ-photoproduction amplitude from protons from the value of the phase of the ρ-photoproduction amplitude from complex nuclei. We follow the work of Drell and Trefil[10] on coherent photoproduction. We shall assume ρ dominance and use the eikonal approximation, treating the nucleus as a homogeneous sphere of radius $R=r_0A^{1/3}$, $r_0=1.2$ F. The forward amplitude for ρ photoproduction from nuclei is then given by

$$F=\frac{3f}{R^3(Y+Z)}\left[e^{iYR}\left(\frac{R}{Y}-\frac{1}{Y^2}\right)+\frac{1}{Y^2}+e^{-iZR}\left(\frac{R}{Z}+\frac{1}{Z^2}\right)-\frac{1}{Z^2}\right], \quad \text{(A1)}$$

where

$$Y=im_\rho^2/2k\,,\quad Z=(i/2k)(m_\rho^2+2V_{\rho\rho})\,. \quad \text{(A2)}$$

In (A1), f is the forward amplitude for ρ production from an individual nucleon and, using vector dominance, is proportional to the forward ρp scattering amplitude,

$$f=(e/2\gamma_\rho)\mathcal{A}(\rho^0p)\,. \quad \text{(A3)}$$

Let us assume that, aside from an over-all factor of i, $\mathcal{A}(\rho^0p)$ contains a phase Δ_ρ, i.e.,

$$\mathcal{A}(\rho^0p)=ie^{i\Delta_\rho}|\mathcal{A}(\rho^0p)|\,. \quad \text{(A4)}$$

The forward nucleon amplitude f will also contain the phase Δ_ρ.

In (A2), $V_{\rho\rho}$ is an optical potential related to the forward ρp scattering amplitude by

$$V_{\rho\rho}=-4\pi d\mathcal{A}(\rho^0p)\,, \quad \text{(A5)}$$

where d is the nuclear density. Using the optical theorem and (A4),

$$V_{\rho\rho}=-ikd\sigma_{\rho n}e^{i\Delta_\rho}/\cos\Delta_\rho=-(ik/\mu)(1+i\tan\Delta_\rho)\,, \quad \text{(A6)}$$

[27] Photoproduction: U. Becker *et al.*, Phys. Rev. Letters **21**, 1504 (1968); C. Tank *et al.*, Bull Am. Phys. Soc. **14**, 543 (1969); also K. M. Moy, thesis, Northeastern University, 1969 (unpublished). Colliding beams: J. E. Augustin *et al.*, Phys. Letters **28B**, 517 (1969).

[28] The fact that the ϕ and ρ^0 overlap presumably contributes an insignificant amount to the measured ϕ-ρ^0 phase since the quark model suggests that the mass matrix element between the ρ^0 and ϕ is zero (Ref. 24).

where $\sigma_{\rho n}$ is the total ρ-nucleon cross section and μ is the ρ-meson mean free path, taken to be 2.4 F.

It is now a straightforward, but tedious, task to extract the phase of the amplitude in (A1) and relate it to Δ_ρ. Writing

$$F=ie^{i\delta_\rho}\rho|F|, \tag{A7}$$

we find

$$\delta_\rho=\tan^{-1}\left[\frac{-m_\rho^2\mu/k}{1+\tan^2\Delta_\rho+(m_\rho^2\mu/k)\tan\Delta_\rho}\right]+\tan^{-1}\left(\frac{T+VW+UX}{S+UW-VX}\right), \tag{A8}$$

where

$$S=\frac{4k^2}{m_\rho^4}\left(\frac{m_\rho^2R}{2k}\sin\frac{m_\rho^2R}{2k}-2\sin^2\frac{m_\rho^2R}{4k}\right),$$

$$T=\frac{4k^2}{m_\rho^4}\left(\sin\frac{m_\rho^2R}{2k}-\frac{m_\rho^2R}{2k}\cos\frac{m_\rho^2R}{2k}\right),$$

$$U=\frac{a}{a^2+b^2},\quad V=\frac{b}{a^2+b^2},$$

$$a=\left(\frac{1}{\mu}\right)^2-\left(\frac{m_\rho^2}{2k}+\frac{1}{\mu}\tan\Delta_\rho\right)^2,$$

$$b=-\frac{2}{\mu}\left(\frac{m_\rho^2}{2k}+\frac{1}{\mu}\tan\Delta_\rho\right),$$

$$W=e^{-R/\mu}[(1+R/\mu)\cos g+g\sin g]-1,$$

$$X=e^{-R/\mu}[g\cos g-(1+R/\mu)\sin g],$$

$$g=\left(\frac{m_\rho^2}{2k}+\frac{1}{\mu}\tan\Delta_\rho\right)R. \tag{A9}$$

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Implications of Full Causality for Neutrino and Other Particle Production Rates

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If the correct physical theory satisfies full causality, then, in our neighborhood of the universe, the total production rate for certain particles (e.g., neutrinos) may be different from what one would normally expect.

THE purpose of this paper is to describe a certain effect which may change the production rates for some particles, away from the value which one would normally expect. Generally speaking, this effect should be more pronounced for the production of those particles which have a longer mean free path in material (e.g., gravitons or low-energy neutrinos).

The effect is to be expected, if the correct theory of elementary particles satisfies full causality.

Before explaining how the effect arises, we recall some definitions and a few simple results, which will be needed later.

The principle of full casality states[1] that an effect may both precede and follow its cause, with (roughly speaking) the same probability. The exact definition is given in Ref. 1, and for a simple special case it will be stated below. This principle is to be compared with the principle of retarded causality, according to which no effect can precede its cause. We say that a theory satisfies full causality, if the theory contains the prescription that boundary conditions are to be imposed according to the principle of full causality.

[1] Paul L. Csonka, Phys. Rev. **180**, 1266 (1969).

To illustrate, consider the idealized case when a point source at $\mathbf{x}=0$ emits a spinless, massless particle with energy ω, into a state with zero angular momentum relative to the source. If the theory satisfies full causality, then the amplitude of the emitted particles, at time t and point $\mathbf{x}$, will be

$$\Psi_\omega(t,\mathbf{x})=Ce^{-i\omega t}\tfrac{1}{2}[e^{i\omega r}/r+e^{-i\omega r}/r], \tag{1}$$

where $r\equiv|\mathbf{x}|$. This expression is to be contrasted with $(C/r)e^{-i\omega t}e^{i\omega r}$, which is the expression for the amplitude when retarded causality holds. When the source does not emit monoenergetic particles, but emits wave packets of particles at times near $t=0$, then the amplitude $\Psi(t,\mathbf{x})$ is an integral over ω of $\Psi_\omega(t,\mathbf{x})$, and contains two terms. One of the terms is an integral containing $e^{i\omega r}$, and it describes particles emitted "towards the future": It represents a wave packet moving away from $\mathbf{x}=0$ at times $t>0$, i.e., after emission has taken place. The other term is an integral containing $e^{-i\omega r}$, and describes particles emitted "towards the past": It represents a wave packet moving towards $\mathbf{x}=0$, *before* emission has taken place. Particles described by the second term are not usually observed.