(disregarding the effect of the real part) the shrinkage pattern of the Pomeranchuck term alone. The same is true if P'-ω exchange degeneracy in the residues and trajectories is only slightly broken as in our Solution II. For $p\bar{p}$ scattering, however, *both* Regge pole contributions of order $1/\sqrt{s}$ add below the crossover point and subtract beyond it, which makes the diffraction peak on the one hand steeper in $p\bar{p}$ compared to pp and on the other hand expanding due to the *decaying* of the contributions of order $1/\sqrt{s}$ in going to higher energies. Finally, however, at sufficiently high energies also the $p\bar{p}$ differential cross section will show shrinkage according to $(\alpha_P')_{\rm eff}=\frac{1}{2}\alpha'$ as the pp diffraction peak does. The structure in $(d\sigma/dt)_{p\bar{p}}$ around $t=-0.8$ GeV^2— being an effect of the lower lying trajectories—is predicted to disappear with increasing s, whereas the shoulder in $(d\sigma/dt)_{pp}$ at $t=-1.2$ GeV^2 is connected to the Pomeranchuk contribution and will in this model develop into a more profound diffraction minimum with growing energy. We point out, however, that the curves labelled P in Figs. 2 and 3 do not represent asymptotic curves for the differential cross sections at large s but possess themselves a logarithmic energy dependence. This is implied by the statement that diffraction peaks shrink indefinitely in this model as s increases.

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Weakly Coupled Neutral Currents

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A general analysis of interactions of the current-current form involving neutral vector and axial-vector hadron and lepton currents is given and compared with present experimental information. While the restrictions placed upon such neutral current interactions by existing data are found to be rather severe, a number of theoretically attractive possibilities remain. Still consistent with experiment are models obeying the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule, a universal model based on the $SU(2)$ algebra of charges, a model with a strangeness-conserving isovector neutral hadron current and symmetrically coupled lepton currents, and models in which the neutral currents couple only in a superweak fashion. In contrast, a model which incorporates the octet rule as a built-in symmetry appears to be ruled out on the basis of present information.

I. INTRODUCTION

THE possible existence of neutral currents in elementary-particle interactions, in addition to the established charged currents which occur in the weak interactions, has long been a subject of considerable interest. The presence of weakly coupled charged currents is well confirmed in β decay, μ capture, strange-particle decay, and more recently, in high-energy neutrino interactions. In contrast, there is still no firm experimental evidence for an interaction involving neutral currents, other than the well-known electromagnetic interaction.

In many cases where one might expect to detect such interactions if they exist at all, the electromagnetic and/or strong interactions overwhelm the expected effects. However, experiments to detect certain types of neutral currents are possible, and some have indeed been performed; their failure to detect neutral currents simply places limits on the form of interaction.

Theoretically, a significant amount of motivation has accumulated over the years to suggest neutral currents may exist. For example, the approximate $\Delta I=\frac{1}{2}$ rule[1]

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observed in strangeness-changing nonleptonic decays can arise through a cancellation of the $\Delta I=\frac{3}{2}$ part of the charged current interaction by virtue of the additional neutral current contribution. To a somewhat lesser degree, the approximate octet dominance may result similarly through a partial cancellation of the **27** part of the weak Hamiltonian.[2]

Independently of these $\Delta I=\frac{1}{2}$ and octet dominance rules, there do exist neutral currents in the chiral $SU(3)\times SU(3)$ algebra proposed by Gell-Mann.[3] To verify directly all the commutation relations of this algebra, the matrix elements of both the vector and axial-vector octets of currents (actually only their associated charges) should be measurable. Since the neutral electromagnetic and charged weak currents are observable through their interactions with lepton currents, it is natural to expect that the remaining neutral currents can be probed through some similar interaction. From this point of view, the octets of vector and axial-vector currents appear as (noncanonical) coordinates of hadronic matter with their equal-time commutation relations playing the role of quantization conditions.[4]

If the neutral currents that enter in the chiral $SU(3)\times SU(3)$ algebra are indeed involved in some neutral current interaction, both CP-even and CP-odd currents can be present. Hence it is possible that such an interaction might be the origin of the CP nonconservation in the $K_L{}^0\to 2\pi$ decays, since the cross terms in the interaction Hamiltonian would be CP nonconserving.[5–7]

The purpose of this work is to present a comprehensive analysis of the possible types of weakly coupled neutral current interactions, to discuss the constraints placed on them by present experimental information, and to single out those theoretical models which are still consistent with the data. To begin we review the experimental data in Sec. II and then present a general analysis of the possible neutral current interactions in Sec. III. In Sec. IV several specific models having certain attractive features are discussed, and in Sec. V the constraints on these models placed by the current data are presented. In conclusion we summarize the results of our analysis and discuss future experiments.

II. REVIEW OF EXPERIMENTAL DATA

In this section we briefly review the experimental data coming from leptonic, semileptonic, and nonleptonic processes which bear on the existence of weakly coupled neutral current interactions. Of course, the implications of all the data presented depend very much on the particular form of the interaction assumed and will be discussed after the general theoretical analysis is presented in Sec. III.

A. Leptonic Interactions

Among processes involving only leptons, it is quite difficult to search for neutral currents, since most such processes are overwhelmed by the electromagnetic interaction or are forbidden by lepton number conservation. Exact lepton number conservation implies that some reactions can only proceed through charged currents, some only through neutral currents, some through both types of currents, and some through neither type to zeroth order in the electromagnetic interaction. Examples of each type[8] are presented in Table I.

Until recently, only muon decay was observed among the above processes. Stothers[9] has shown, however, by astrophysical arguments that the reaction $\nu_e+e^-\to\nu_e+e^-$ does exist and that limits can be placed on the square of the coupling constant:

$$g^2=10^{0\pm2}g_\beta{}^2\,, \tag{2.1a}$$

where g_β is the effective β-decay coupling constant. A new analysis by Steiner[10] indicates that one can set the upper limit

$$\sigma_{\text{expt}}<40\sigma_{V-A} \tag{2.1b}$$

with 90% confidence for the same high-energy neutrino reaction, where σ_{V-A} is the cross section predicted on the basis of a universal $V-A$ charged current interaction. The most recent report by Reines and Gurr[11] for the related reaction $\bar\nu_e+e^-\to\bar\nu_e+e^-$, where low-energy antineutrinos are obtained from a fission reaction, cites

$$\sigma_{\text{expt}}<4\sigma_{V-A} \tag{2.1c}$$

as a conservative estimate. Albright[12] has shown that

TABLE I. Charged and neutral current contributions (indicated by the symbol ×) to various leptonic reactions.

Process	Charged currents	Neutral currents
$\mu^-\to e^-+\nu_\mu+\bar\nu_e$	×	
$\nu_e+\mu^-\to e^-+\nu_\mu$	×	
$\bar\nu_e+e^-\to\mu^-+\bar\nu_\mu$	×	
$\nu_e+e^-\to\nu_e+e^-$	×	×
$\bar\nu_e+e^-\to\bar\nu_e+e^-$	×	×
$\nu_\mu+e^-\to\nu_\mu+e^-$		×
$\nu_e+\nu_\mu\to\nu_e+\nu_\mu$		×
$\nu_e+e^-\to\mu^-+\nu_\mu$		

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[4] R. F. Dashen and D. H. Sharp, Phys. Rev. **165**, 1857 (1968).
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[7] F. Zachariasen and G. Zweig, Phys. Rev. **182**, 1446 (1969).

[8] Strictly speaking, the process $\nu_e+e^-\to\nu_\mu+\mu^-$ can occur if the lepton number conservation law is multiplicative rather than additive as is usually assumed; cf. G. Feinberg and S. Weinberg, Phys. Rev. Letters **6**, 381 (1961). Although the additive law is more natural, present evidence does not preclude the multiplicative law: C. Y. Chang, Phys. Rev. Letters **24**, 79 (1970).
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Steiner's analysis can be extended to give an upper limit on the reaction $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$ as

$$\sigma_{\text{expt}} < 0.40 \sigma_{V-A}. \quad (2.2)$$

Inequality (2.2) represents the most restrictive upper limit on neutral lepton currents.

B. Semileptonic Interactions

In the case of $\Delta S = 0$ semileptonic processes, the best experimental information on neutral currents is derived from the neutrino scattering data where the following upper limits on the elastic process $\nu_\mu + p \rightarrow \nu_\mu + p$ and on the inelastic process $\nu_\mu + p \rightarrow n + \pi^+ + \nu_\mu$ are quoted[13]:

$$\sigma(\nu_\mu + p \rightarrow \nu_\mu + p)/\sigma(\nu_\mu + n \rightarrow p + \mu^-) \leq (0.12 \pm 0.06) \quad (2.3)$$

and

$$\sigma(\nu_\mu + p \rightarrow n + \pi^+ + \nu_\mu)/\sigma(\nu_\mu + p \rightarrow p + \pi^+ + \mu^-) \leq (0.08 \pm 0.04). \quad (2.4)$$

In other $\Delta S = 0$ processes not involving neutrinos, e.g., $\eta \rightarrow \pi^0 + e^+ + e^-$, the competition with electromagnetic interactions precludes one's learning anything about weakly coupled neutral currents.

Upper limits on rare kaon decay modes provide the best information regarding $|\Delta S| = 1$ semileptonic neutral current interactions.[14–19] These branching ratios are summarized in Table II. Clearly there is no such interaction comparable in strength to the usual $|\Delta S| = 1$ semileptonic weak interaction.

Hyperon decays such as $\Sigma^+ \rightarrow p e^+ e^-$ are not suppressed as are the $0 \rightarrow 0$ kaon decays, e.g., $K^+ \rightarrow \pi^+ e^+ e^-$, and occur via the usual electromagnetic interaction. In fact, hyperon decays of this type have been observed[20] with rates which are consistent with the process proceeding through a combination of the usual weak and electromagnetic interactions and hence do not present evidence for weakly coupled neutral currents.

C. Nonleptonic Interactions

All data on nonleptonic weak interactions are consistent with $|\Delta S| \geq 2$ transition rates being no larger than would be expected from second-order weak interactions of the usual strength characterized by the Fermi constant G. Further evidence against nonleptonic neutral current theories that would lead to $K^0 \leftrightarrow \bar{K}^0$ transitions comes from the very small $K_L - K_S$ mass difference Δm. Since $\Delta m \sim 10^{-5}$ eV, the effective strength of such an interaction could not be larger than about $10^{-6} G$.

TABLE II. Upper limits on the branching ratios for the rare K decay modes which may occur via neutral currents.

Decay	Branching ratio (90% confidence)	Ref.
$K_L^0 \rightarrow e^+ e^-$	$< 1.5 \times 10^{-7}$	14
$K_L^0 \rightarrow \mu^+ \mu^-$	$< 2.1 \times 10^{-7}$	14
$K_S^0 \rightarrow \mu^+ \mu^-$	$< 7.3 \times 10^{-6}$	15
$K^+ \rightarrow \pi^+ e^+ e^-$	$< 2.5 \times 10^{-6}$	16
$K^+ \rightarrow \pi^+ \mu^+ \mu^-$	$< 2.4 \times 10^{-6}$	17
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$< 1.2 \times 10^{-6}$	18
$K^+ \rightarrow \pi^+ \pi^0 e^+ e^-$	$< 8 \times 10^{-6}$	19

The $|\Delta S| = 1$ hyperon and kaon decays provide little information concerning neutral current interactions. The empirical $\Delta I = \frac{1}{2}$ rule[1] may well be due to a neutral current weak interaction in combination with the usually accepted charged current interaction. However, there remains the possibility that the $\Delta I = \frac{1}{2}$ rule is dynamical in origin[1] and not an intrinsic property of the basic weak interaction. It has also been suggested[5–7] that the observed CP nonconservation in $K_L^0 \rightarrow 2\pi$ decay arises from a neutral current interaction. While this possibility is attractive theoretically, it has not been tested experimentally due to large uncertainties in present measurements of the $K_L^0 \rightarrow \pi^0 \pi^0$ decay amplitude.[21]

In $\Delta S = 0$ parity-conserving processes, any weak interaction effects are completely masked by the strong interactions. However, in the case of the $\Delta S = 0$ parity-violating processes, there is a certain amount of data coming from nuclear reactions which offer the possibility of searching for parity-violating neutral current interactions.[22] Parity-violating circular polarizations have been observed in various γ decays, and parity violation has been detected in the angular distribution for polarized thermal neutron capture. In addition we note that the parity-forbidden decay $^{16}O^* \rightarrow {}^{12}C + \alpha$ has been searched for, and a preliminary report[23] indicates a possible effect.

III. THEORETICAL ANALYSIS

In order to have a well-defined framework in which to explore the existence of neutral currents, we shall write the weak Hamiltonian as a general linear combination

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[21] J. Steinberger, in *Proceedings of the Lund International Conference on Elementary Particles*, edited by G. von Dardel (Berlingska Doktryckeriet, Lund, Sweden, 1969), p. 41.

[22] For a recent summary, see *High Energy Physics and Nuclear Structure*, edited by S. Devons (Plenum, New York, 1970).

[23] E. Sprenkel-Segel, R. Segel, and R. Siemssen, Ref. 22, p. 763.

of products of currents and admit both $V-A$ and $V+A$ possibilities. We denote the $V-A$ and $V+A$ hadron currents of chiral $U(3)\times U(3)$ by

$$J_{\mp\lambda}{}^{(\alpha)}=(V\mp A)_\lambda{}^{(\alpha)} \tag{3.1a}$$

and the corresponding lepton currents of chiral $U(2)\times U(2)$ by

$$l_{\mp\lambda}{}^{(a)}=\tfrac{1}{2}\bar{\psi}_l\tau^{(a)}\gamma_\lambda(1\pm\gamma_5)\psi_l\,, \tag{3.1b}$$

where $\alpha=0, 1, \ldots, 8$ and $a=0, 1, 2, 3$. The lepton field operators denote the spinor

$$\psi_l(x)=\begin{pmatrix}\nu_l(x)\\ l(x)\end{pmatrix},$$

where the subscript l corresponds to an electron- or muon-type lepton, $\nu_l(x)$ represents the neutrino field, and $l(x)$ the electron or muon field.

In terms of these currents, the most general phenomenological weak current-current Hamiltonian can be written as

$$H_W=H_{NL}+H_{SL}+H_L\,, \tag{3.2a}$$

where the nonleptonic part is given by

$$H_{NL}=\frac{G}{\sqrt{2}}\sum_{\alpha,\beta}\sum_{i,j=-,+}G_{ij}{}^{(\alpha\beta)}J_{i\lambda}{}^{(\alpha)}J_{j\lambda}{}^{(\beta)}\,, \tag{3.2b}$$

the semileptonic part by

$$H_{SL}=\frac{G}{\sqrt{2}}\sum_{\alpha,a}\sum_{i,j=-,+}\sum_{l=e,\mu}g_{ij}{}^{(\alpha a)}J_{i\lambda}{}^{(\alpha)}l_{j\lambda}{}^{(a)}\,, \tag{3.2c}$$

and the leptonic part by

$$H_L=\frac{G}{\sqrt{2}}\sum_{a,b}\sum_{i,j=-,+}\sum_{l,l'=e,\mu}f_{ij}{}^{(ab)}l_{i\lambda}{}^{(a)}l_{j\lambda}{}'^{(b)}\,. \tag{3.2d}$$

In assuming this form, we have from the outset adopted both the usual additive lepton conservation law and $\mu-e$ universality.

Hermiticity of H_W and CPT invariance, which we shall assume throughout, require that the G's and f's be real and symmetric under interchange of the internal symmetry and chiral indices: $G_{ij}{}^{(\alpha\beta)}=G_{ji}{}^{(\beta\alpha)}$ and $f_{ij}{}^{(ab)}=f_{ji}{}^{(ba)}$; the g's are required to be real. Furthermore, since H_W must be charge conserving, many of the coupling coefficients vanish while others are related.

Additional restrictions on the coupling coefficients may arise in the case of certain classes of theories. For example, if the interaction is mediated by vector bosons as might be the case for the usual charged current weak interaction, then the coupling coefficients in Eq. (3.2) will factorize. In fact, there will then exist relations among the G's, g's, and f's.

The symmetry property of the G's enables us to write the scalar and pseudoscalar nonleptonic Hamiltonians as

$$H_{NL}{}^{(s)}=\frac{G}{\sqrt{2}}\tfrac{1}{2}\sum_{\alpha,\beta}\big[(G_{--}{}^{(\alpha\beta)}+G_{++}{}^{(\alpha\beta)})T_S{}^{(\alpha\beta)}(VV+AA)+(G_{-+}{}^{(\alpha\beta)}+G_{+-}{}^{(\alpha\beta)})T_S{}^{(\alpha\beta)}(VV-AA)\big] \tag{3.3a}$$

and

$$H_{NL}{}^{(ps)}=\frac{G}{\sqrt{2}}\tfrac{1}{2}\sum_{\alpha,\beta}\big[-(G_{--}{}^{(\alpha\beta)}-G_{++}{}^{(\alpha\beta)})\times T_S{}^{(\alpha\beta)}(VA+AV)+(G_{-+}{}^{(\alpha\beta)}-G_{+-}{}^{(\alpha\beta)})\times T_A{}^{(\alpha\beta)}(VA-AV)\big]\,, \tag{3.3b}$$

where the tensors symmetric in $\alpha\beta$ are defined by

$$T_S{}^{(\alpha\beta)}(VV+AA)=\{V_\lambda{}^{(\alpha)},V_\lambda{}^{(\beta)}\}+\{A_\lambda{}^{(\alpha)},A_\lambda{}^{(\beta)}\}\,, \tag{3.4a}$$

$$T_S{}^{(\alpha\beta)}(VV-AA)=\{V_\lambda{}^{(\alpha)},V_\lambda{}^{(\beta)}\}-\{A_\lambda{}^{(\alpha)},A_\lambda{}^{(\beta)}\}\,, \tag{3.4b}$$

$$T_S{}^{(\alpha\beta)}(VA+AV)=\{V_\lambda{}^{(\alpha)},A_\lambda{}^{(\beta)}\}+\{A_\lambda{}^{(\alpha)},V_\lambda{}^{(\beta)}\}\,, \tag{3.4c}$$

and the antisymmetric tensor is defined to be

$$T_A{}^{(\alpha\beta)}(VA-AV)=\{V_\lambda{}^{(\alpha)},A_\lambda{}^{(\beta)}\}-\{A_\lambda{}^{(\alpha)},V_\lambda{}^{(\beta)}\}\,. \tag{3.4d}$$

Note that the $T_S{}^{(\alpha\beta)}(VV-AA)$ and $T_A{}^{(\alpha\beta)}(VA-AV)$ tensors appear only if *both* $V-A$ and $V+A$ currents enter the nonleptonic part of H_W. It is clear that if we assume that the V and A currents belong to octets (and possibly a singlet in the vector case), the symmetric tensors transform under $SU(3)$ like **1**, $\mathbf{8}_s$, and **27** operators while the antisymmetric tensor represents a combination of $\mathbf{8}_a$, **10**, and **10*** operators; hence $H_{NL}{}^{(s)}$ contains only symmetric tensors, while $H_{NL}{}^{(ps)}$ involves both symmetric and antisymmetric tensors.

As defined above in Eqs. (3.1) and (3.2), the weak Hamiltonian couples hadron and lepton currents which are Hermitian. It is convenient to introduce charged and neutral currents and to write H_W in terms of these. For the hadron currents, we group the octet members according to I-spin, V-spin, and U-spin components as follows:

$$\begin{aligned}\alpha=\ &I+=1+i2\,, && I-=1-i2\,,\\ &V+=4+i5\,, && V-=4-i5\,,\\ &U0=3+\frac{1}{\sqrt{3}}\times 8\,, && U3=3-\sqrt{3}\times 8\,,\\ &U+=6+i7\,, && U-=6-i7\,.\end{aligned} \tag{3.5}$$

The $I\pm$ and $V\pm$ components refer to the $I=1$ and $I=\frac{1}{2}$ charged currents, while the $U0$, $U3$, $U\pm$, and 0 components constitute the neutral currents.

By analogy, the lepton currents may be grouped according to

$$a=i+=1+i2\,,\quad i-=1-i2\,,\qquad i3=3\,,\quad i0=0. \tag{3.6}$$

Again the $i\pm$ components refer to the charged lepton currents, while both 3 and 0 label the neutral currents.

The usual charged current weak Hamiltonian[24]

$$H=\frac{G}{\sqrt{2}}\tfrac{1}{2}\{\mathcal{J}_\lambda{}^{(+)},\mathcal{J}_\lambda{}^{(-)}\}\,, \tag{3.7}$$

where $\mathcal{J}_\lambda{}^{(+)}$ is the sum of the hadron current

$$J_\lambda{}^{(+)}=\cos\theta\,(V-A)_\lambda{}^{(1+i2)}+\sin\theta\,(V-A)_\lambda{}^{(4+i5)} \tag{3.8}$$

and the lepton currents $e_{-\lambda}{}^{(1+i2)}$ and $\mu_{-\lambda}{}^{(1+i2)}$ given in (3.1b), involves the components $I\pm$, $V\pm$, and $i\pm$. The nonvanishing coupling coefficients are the following:

$$\begin{aligned}
&G_{--}{}^{(I+,I-)}=\tfrac{1}{2}\cos^2\theta=G_{--}{}^{(I-,I+)}\,,\\
&G_{--}{}^{(I+,V-)}=G_{--}{}^{(V+,I-)}=\tfrac{1}{2}\sin\theta\cos\theta\\
&\qquad\qquad=G_{--}{}^{(V-,I+)}=G_{--}{}^{(I-,V+)}\,,\\
&G_{--}{}^{(V+,V-)}=\tfrac{1}{2}\sin^2\theta=G_{--}{}^{(V-,V+)}\,,\\
&g_{--}{}^{(I+,i-)}=g_{--}{}^{(I-,i+)}=\cos\theta\,,\\
&g_{--}{}^{(V+,i-)}=g_{--}{}^{(V-,i+)}=\sin\theta\,,\\
&f_{--}{}^{(i+,i-)}=f_{--}{}^{(i-,i+)}=1\,.
\end{aligned} \tag{3.9}$$

Since the charged currents are purely of the $V-A$ chirality, only the symmetric tensors enter H_{NL} and one finds

$$\begin{aligned}
H_{NL}(\text{charged currents})=\frac{G}{\sqrt{2}}\Bigg\{&\cos^2\theta\left[\frac{1}{\sqrt{5}}T_{000}{}^{(8)}-\frac{1}{\sqrt{8}}T_{000}{}^{(1)}-\frac{1}{\sqrt{6}}T_{020}{}^{(27)}+\frac{1}{\sqrt{120}}T_{000}{}^{(27)}\right]\\
&+\sin\theta\cos\theta\left[\left(\frac{3}{10}\right)^{1/2}(T_{1\frac{1}{2}-\frac{1}{2}}{}^{(8)}+T_{-1\frac{1}{2}\frac{1}{2}}{}^{(8)})+\frac{1}{\sqrt{6}}(T_{1\frac{3}{2}-\frac{1}{2}}{}^{(27)}+T_{-1\frac{3}{2}\frac{1}{2}}{}^{(27)})+\frac{1}{\sqrt{30}}(T_{1\frac{1}{2}-\frac{1}{2}}{}^{(27)}+T_{-1\frac{1}{2}\frac{1}{2}}{}^{(27)})\right]\\
&+\sin^2\theta\left[\left(\frac{3}{20}\right)^{1/2}T_{010}{}^{(8)}-\frac{1}{\sqrt{20}}T_{000}{}^{(8)}-\frac{1}{\sqrt{8}}T_{000}{}^{(1)}-\frac{1}{\sqrt{10}}T_{010}{}^{(27)}-\left(\frac{3}{40}\right)^{1/2}T_{000}{}^{(27)}\right]\Bigg\}\,,
\end{aligned} \tag{3.10}$$

in the standard notation where the subscripts are just the YII_3 labels. For the scalar part, the tensors are just the $T_S(VV+AA)$ operators while for the pseudoscalar part they represent $-T_S(VA+AV)$. Note that **1**, **8**, and **27** all contribute.

The neutral current contribution to H_W can involve the $U0$, $U3$, $U\pm$, and 0 hadron currents and the 3 and 0 lepton currents, all with unknown coupling coefficients. One can write out the tensor forms given in Eqs. (3.4) which occur in H_{NL}, and the lengthy results are summarized in Appendix A. In addition, relationships among the coupling coefficients in the Cartesian and spherical bases are given in Appendix B. The structure of the nonleptonic Hamiltonian is then written out in Appendix C for certain nonleptonic processes of interest.

The matrix elements for various decays and scatterings can be written explicitly in terms of the coupling coefficients and reduced matrix elements or form factors. For several interesting examples of leptonic and semileptonic processes involving neutral currents these formulas are collected in Appendix D.

IV. NEUTRAL CURRENT MODELS

The starting point for the general phenomenological approach presented in Sec. III was the concept of a weak-point interaction between pairs of currents of both $V-A$ and $V+A$ chiralities. In this section, we wish to pursue various attractive roles for the neutral currents and shall thereby restrict our attention to certain classes of theories which have been proposed to explain several striking features of nature. Among these, we consider the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule, octet dominance, and CP nonconservation.

A. $\Delta I<\frac{3}{2}$, $\Delta Y<2$ Rule

It has long been apparent that the weak interactions violate some of the symmetry properties exhibited by the strong interactions, i.e., strangeness and isospin conservation. On the other hand, they appear to do so in a well-defined way. In particular, the known semileptonic reactions involving charged currents in strange-particle decays exhibit to at least 10% accuracy the $\Delta I=\frac{1}{2}$ rule and the $\Delta S=\Delta Q$ rule. In fact, these features of the charged currents are neatly summarized in the Cabibbo theory by the assumption that the charged V and A hadron currents belong to two octets.

Given the above simplifications for the semileptonic reactions, it is of interest to inquire whether similar features prevail in the nonleptonic reactions. In fact, it appears that in first order the $\Delta I=\frac{3}{2}$ nonleptonic decay amplitudes are considerably smaller than their $\Delta I=\frac{1}{2}$ counterparts, the $\Delta Y=2$ transitions are nonexistent, and the octet reduced matrix elements dominate all others.

[24] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); R. E. Marshak and E. C. G. Sudarshan, in *Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, September, 1957* (Società Italiana di Fisica, Padua-Venice, Italy, 1958); Phys. Rev. **109**, 1860 (1960); N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

Let us investigate the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule for the nonleptonic weak Hamiltonian. This implies the cancellation of the $T_{020}{}^{(27)}$, $T_{1\frac{3}{2}-\frac{1}{2}}{}^{(27)}$, and $T_{-1\frac{3}{2}\frac{1}{2}}{}^{(27)}$ tensors in (3.10) through addition to H_{NL} of the neutral current contribution; furthermore, the neutral currents should introduce no $T_{21-1}{}^{(27)}$ and $T_{-211}{}^{(27)}$ tensor terms. This can be accomplished with the following constraints on the $G^{(\alpha\beta)}$'s:

$$G_{ij}{}^{(33)}=\delta_{i-}\delta_{j-}\cos^2\theta\,,\quad G_{ij}{}^{(36)}=-\delta_{i-}\delta_{j-}\sin\theta\cos\theta\,,\qquad G_{ij}{}^{(66)}=G_{ij}{}^{(77)}\,,\quad G_{ij}{}^{(37)}=G_{ij}{}^{(67)}=0\,,\tag{4.1}$$

where i and j can take on the values $-$ and $+$. Note that the above constraints can be satisfied even if $V+A$ currents do not occur. The scale for the weak neutral $V-A$ currents is set, however, by the charged $V-A$ currents.

It is now a simple matter to show that the above restrictions can be satisfied with a $V-A$ neutral current of the general form

$$J_\lambda{}^{(0)}=b_0(V-A)_\lambda{}^{(U0)}+c_0(V-A)_\lambda{}^{(U3)}+c_+(V-A)_\lambda{}^{(U+)}+c_-(V-A)_\lambda{}^{(U-)}\tag{4.2}$$

if we require $b_0+c_0=\cos\theta$, $c_+=-\sin\theta$, and $c_-=0$. This model with $b_0=3c_0=\frac{3}{4}\cos\theta$ corresponds to the old schizon model of Lee and Yang.[25] More complicated models involving more than one neutral current are also possible but will not be elaborated upon here. It is of interest to note that the nonleptonic Hamiltonian corresponding to (4.2) turns out to be

$$\begin{aligned}H_{NL}=\frac{G}{\sqrt{2}}\bigg\{&\tfrac{2}{3}(b_0+c_0)(b_0-3c_0)\left(\frac{3}{20}\right)^{1/2}T_{010}{}^{(8)}+[1+3(\cos^2\theta-\sin^2\theta)-\tfrac{4}{3}|b_0|^2-4|c_0|^2]\frac{1}{\sqrt{20}}T_{000}{}^{(8)}\\&+(b_0+c_0)(b_0-3c_0)\frac{1}{\sqrt{10}}T_{010}{}^{(27)}+3[-\tfrac{1}{2}(1+\sin^2\theta)+|b_0|^2+3|c_0|^2]\frac{1}{\sqrt{30}}T_{000}{}^{(27)}\\&-[1+\sin^2\theta+\tfrac{2}{3}|b_0|^2+2|c_0|^2]\frac{1}{\sqrt{8}}T_{000}{}^{(1)}+[\sin\theta\cos\theta-\tfrac{2}{3}c_+b_0]\left(\frac{3}{10}\right)^{1/2}(T_{1\frac{1}{2}-\frac{1}{2}}{}^{(8)}+T_{-1\frac{1}{2}\frac{1}{2}}{}^{(8)})\\&+[\sin\theta\cos\theta+c_+b_0-5c_+c_0]\frac{1}{\sqrt{30}}(T_{1\frac{1}{2}-\frac{1}{2}}{}^{(27)}+T_{-1\frac{1}{2}\frac{1}{2}}{}^{(27)})\bigg\}\,.\end{aligned}\tag{4.3}$$

From the above, it is apparent that with the choice $b_0=3c_0$ as in the schizon model,[25] the $\Delta I=1$ part of H_{NL} vanishes. On the other hand, if one chooses instead $b_0=\cos\theta$, $c_0=0$ one obtains no **27** contribution in the $\Delta Y=\pm1$ tensor terms although the **27** tensors still contribute to the $\Delta Y=0$ terms. This latter choice of parameters was suggested previously[2] as a natural (internal symmetry) way of achieving octet dominance for the strangeness-changing reactions.

B. Octet Dominance

Octet dominance[26] appears to exist in nature for the nonleptonic transitions up to some 20% accuracy. This feature can be explained either by constructing the weak Hamiltonian at the outset with this built-in symmetry and attributing the nonoctet effects to symmetry breaking, or by postulating a dynamical enhancement of the **8** reduced matrix elements relative to the **27** and possibly the **1**, **10**, and **10*** matrix elements. We shall study the first possibility here by imposing increasingly more rigid constraints on the transformation properties of the nonleptonic part of H_W so as to finally achieve octet dominance as a built-in symmetry.

If we first demand that the **27** tensor terms be absent, we must impose the following constraints on the G's in addition to those appearing in Eqs. (4.1):

$$\begin{aligned}G_{ij}{}^{(68)}&=-\frac{1}{\sqrt{3}}\delta_{i-}\delta_{j-}\sin\theta\cos\theta\,,\\G_{ij}{}^{(87)}&=0\,,\\G_{ij}{}^{(38)}&=\frac{1}{\sqrt{3}}(\delta_{i-}\delta_{j-}\sin^2\theta-G_{ij}{}^{(66)})\,,\\G_{ij}{}^{(88)}&=-\tfrac{1}{3}\delta_{i-}\delta_{j-}(\cos^2\theta-2\sin^2\theta)+\tfrac{2}{3}G_{ij}{}^{(66)}\,.\end{aligned}\tag{4.4}$$

Elimination of the **10** and **10*** tensors in turn eliminates the $\mathbf{8}_a$ tensors, so that all $G_{-+}{}^{(\alpha\beta)}$ are forced to vanish except $G_{-+}{}^{(U0,U0)}$, $G_{-+}{}^{(U3,U3)}$, $G_{-+}{}^{(U+,U-)}$, and $G_{-+}{}^{(U-,U+)}$; the antisymmetric $T_A{}^{(\alpha\beta)}(VA-AV)$ terms do not enter H_{NL} at all. Cancellation of the **1** tensors results in the vanishing of the remaining $G_{-+}{}^{(\alpha\beta)}$ and all the $G_{++}{}^{(\alpha\beta)}$ together with the following additional constraints on the $G_{--}{}^{(\alpha\beta)}$:

$$\begin{aligned}G_{--}{}^{(66)}&=G_{--}{}^{(77)}=-1\,,\\G_{--}{}^{(38)}&=(1/\sqrt{3})(\cos^2\theta+2\sin^2\theta)\,,\\G_{--}{}^{(88)}&=-\cos^2\theta\,.\end{aligned}\tag{4.5}$$

[25] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

[26] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964); B. W. Lee, *ibid.* **12**, 83 (1964); M. Gell-Mann, *ibid.* **12**, 155 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964); S. Okubo, Phys. Letters **8**, 362 (1964).

Hence the symmetric $T_S^{(\alpha\beta)}$ $(VV-AA)$ terms are also canceled out.

The final result is that H_{NL} has the following octet behavior:

$$H_{NL}=\frac{G}{\sqrt{2}}\left\{\cos^2\theta\left[\frac{5}{3}\left(\frac{3}{20}\right)^{1/2}T_{010}{}^{(8)}+\frac{5}{2}\frac{1}{\sqrt{5}}T_{000}{}^{(8)}\right]+\sin\theta\cos\theta\left[\frac{5}{3}\left(\frac{3}{10}\right)^{1/2}(T_{1\frac{1}{2}-\frac{1}{2}}{}^{(8)}+T_{-1\frac{1}{2}\frac{1}{2}}{}^{(8)})\right]+\sin^2\theta\left[\frac{10}{3}\left(\frac{3}{20}\right)^{1/2}T_{010}{}^{(8)}\right]\right\},\quad(4.6)$$

where $T^{(8)}$ denotes $T^{(8)}(VV+AA)$ for the scalar part and $-T^{(8)}(VA+AV)$ for the pseudoscalar part. It is a simple matter to show that this result can be obtained directly from the following octet weak Hamiltonian:

$$H_{NL}=\frac{G}{\sqrt{2}}\tfrac{1}{2}\sum_{\alpha,\beta}\left[(1+\sin^2\theta)d_{3\alpha\beta}+2\sin\theta\cos\theta\, d_{6\alpha\beta}+\sqrt{3}\cos^2\theta\, d_{8\alpha\beta}\right]\{(V-A)_\lambda{}^{(\alpha)},(V-A)_\lambda{}^{(\beta)}\}.\quad(4.7)$$

The coefficients of the symmetric tensors are determined completely by the charged Cabibbo currents. Note that Eqs. (4.6) and (4.7) correspond to

$$H_{NL}=\frac{G}{\sqrt{2}}\tfrac{1}{2}\left[\{J_\lambda{}^{(+)},J_\lambda{}^{(-)}\}+\tfrac{3}{4}\{(V-A)_\lambda{}^{(U0)},(V-A)_\lambda{}^{(U0)}\}+\tfrac{1}{2}(\cos^2\theta-\sin^2\theta)\{(V-A)_\lambda{}^{(U0)},(V-A)_\lambda{}^{(U3)}\}-\tfrac{1}{4}\{(V-A)_\lambda{}^{(U3)},(V-A)_\lambda{}^{(U3)}\}-\{(V-A)_\lambda{}^{(U+)},(V-A)_\lambda{}^{(U-)}\}-\sin\theta\cos\theta\{(V-A)_\lambda{}^{(U0)},(V-A)_\lambda{}^{(U+)}\}-\sin\theta\cos\theta\{(V-A)_\lambda{}^{(U0)},(V-A)_\lambda{}^{(U-)}\}\right].\quad(4.8)$$

C. *CP* Nonconservation

We next turn to the phenomenon of *CP* nonconservation observed in $K_L{}^0\to 2\pi$ decay and discuss how this can possibly be explained as a weak neutral current interaction.

Recall that by convention the Hermitian components of the hadron currents transform under *CP* as follows:

$$CPJ_{\mp\lambda}{}^{(\alpha)}(CP)^{-1}=\eta(\alpha)J_{\mp\lambda}{}^{(\alpha)},\quad \alpha=0,1,\ldots,8\quad(4.9)$$

where

$$\eta(\alpha)=\begin{cases}+1, & \alpha=0,1,3,4,6,8\\ -1, & \alpha=2,5,7.\end{cases}$$

Since the seventh component is odd under *CP* conjugation relative to the other neutral currents, any interaction containing cross terms will be *CP* nonconserving. Hermiticity and invariance of H_{NL} under *CPT* but not under *CP* require that the coupling coefficients $G_{ij}{}^{(\alpha,7)}=G_{ji}{}^{(7,\alpha)}\neq 0$ for at least one of $\alpha=0$, 3, 6, and 8. If the vector and axial-vector currents enter with the same phase, *CP* nonconservation can occur only in the $|\Delta S|=1$ transitions. For all such theories, the neutron electric dipole moment must vanish.

As an example, we construct a neutral current model of *CP* nonconservation which involves only $V-A$ currents. Consider Eq. (4.2) with at least some of the coefficients complex. This model automatically satisfies *CPT* and Hermiticity and will contain a $\Delta S=\pm1$ *CP*-nonconserving piece provided $\mathrm{Im}(c_+ + c_-{}^*)\neq 0$ and both b_0 and c_0 do not vanish. If, in addition, $c_-=0$, no $\Delta S=\pm 2$ transitions are present; this is essentially the model of Das[6] for which $b_0=\cos\theta$, $c_0=0$, and $c_+=e^{i\varphi}\sin\theta$. On the other hand, Oakes[5] maximized the *CP*-nonconserving phase by setting $b_0=0$, $c_0=\cos\varphi$, $c_+=-c_-=-i\sin\varphi$. This model[5] is a special case of the model considered by Zachariasen and Zwieg[7] in which $b_0=0$, $c_0=-\frac{1}{2}\sin\theta_N$, $c_+=\frac{1}{2}e^{-i\varphi}(1+\cos\theta_N)$, and $c_-=-\frac{1}{2}e^{i\varphi}\times(1-\cos\theta_N)$. The coupling coefficients for these three models are tabulated in Table III.

If one is to explain the observed *CP* nonconservation in $K_L{}^0$ decay as a purely weak-interaction effect, it is necessary to obtain the correct decay parameters η_{+-} and η_{00} or ϵ and ϵ' defined by[27]

$$\eta_{+-}\equiv\frac{\langle\pi^+\pi^-|H|K_L{}^0\rangle}{\langle\pi^+\pi^-|H|K_S{}^0\rangle}\simeq\epsilon_0+i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}+\frac{i}{\sqrt{2}}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0}e^{i(\delta_2-\delta_0)}\equiv\epsilon+\epsilon',\quad(4.10\text{a})$$

$$\eta_{00}\equiv\frac{\langle\pi^0\pi^0|H|K_L{}^0\rangle}{\langle\pi^0\pi^0|H|K_S{}^0\rangle}\simeq\epsilon_0+i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}-i\sqrt{2}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_0}e^{i(\delta_2-\delta_0)}\equiv\epsilon-2\epsilon',\quad(4.10\text{b})$$

TABLE III. Coupling coefficients for the *CP*-nonconserving models of Das, Oakes, and Zachariasen and Zweig.

Coupling parameters	Models: Das	Oakes	Zachariasen and Zweig
b_0	$\cos\theta$	0	0
c_0	0	$\cos\varphi$	$-\frac{1}{2}\sin\theta_N$
c_+	$e^{i\varphi}\sin\theta$	$-i\sin\varphi$	$\frac{1}{2}e^{-i\varphi}(1+\cos\theta_N)$
c_-	0	$i\sin\varphi$	$-\frac{1}{2}e^{i\varphi}(1-\cos\theta_N)$
$G^{(U0,U0)}$	$\cos^2\theta$	0	0
$G^{(U0,U3)}$	0	0	0
$G^{(U3,U3)}$	0	$\cos^2\varphi$	$\frac{1}{4}\sin^2\theta_N$
$G^{(U0,U+)}$	$\frac{1}{2}e^{i\varphi}\sin\theta\cos\theta$	0	0
$G^{(U0,U-)}$	$\frac{1}{2}e^{-i\varphi}\sin\theta\cos\theta$	0	0
$G^{(U3,U+)}$	0	$-i\sin\theta\cos\varphi$	$-\frac{1}{4}e^{-i\varphi}\sin\theta_N\cos\theta_N$
$G^{(U3,U-)}$	0	$i\sin\theta\cos\varphi$	$-\frac{1}{4}e^{i\varphi}\sin\theta_N\cos\theta_N$
$G^{(U+,U-)}$	$\frac{1}{2}\sin^2\theta$	$\sin^2\varphi$	$\frac{1}{4}(1+\cos^2\theta_N)$
$G^{(U+,U+)}$	0	$-\sin^2\varphi$	$-\frac{1}{4}e^{-2i\varphi}\sin^2\theta_N$
$G^{(U-,U-)}$	0	$-\sin^2\varphi$	$-\frac{1}{4}e^{2i\varphi_N}\sin^2\theta_N$

[27] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).

where δ_0 and δ_2 are the $I=0$ and $I=2$ π-π phase shifts at the kaon mass. With the convention $|\bar{K}^0\rangle = CP|K^0\rangle$, ϵ_0 relates the eigenvectors of the mass matrix to the CP eigenstates according to

$$|K_{S,L}^0\rangle = \frac{(1+\epsilon_0)|K^0\rangle \pm (1-\epsilon_0)|\bar{K}^0\rangle}{[2(1+|\epsilon_0|^2)]^{1/2}}.$$

The parameters ϵ and ϵ' reflect also the imaginary parts of the $I=0$ and 2 isospin decay amplitudes for the final pions.

In order to relate the parameters ϵ and ϵ' to the coupling coefficients entering H_{NL}, we first note that an approximate expression for ϵ can be obtained by retaining only the on-mass-shell 2π contribution to the mass matrix. In this approximation,[5]

$$\epsilon \simeq \epsilon_0^* \simeq \frac{1}{\sqrt{2}} \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} e^{i(\pi/4)}, \tag{4.11}$$

and hence

$$|\epsilon'/\epsilon| \simeq |\mathrm{Im}A_2/\mathrm{Im}A_0|. \tag{4.12}$$

The isospin decay amplitudes A_0 and A_2 can be found from Eqs. (3.10) and (C3). We consider only the $V-A$ current contributions and hence the symmetric tensor $T_S(VA+AV)$ part of H_W. The A_2 amplitude can arise only from the **27** tensor while the A_0 amplitude receives contributions from both **8** and **27** tensor terms. Let a_0 and a_0' represent the two $\Delta I=\frac{1}{2}$ transition amplitudes arising from the **8** and **27** part of the Hamiltonian, respectively, and a_2 the $\Delta I=\frac{3}{2}$ transition amplitude from the **27** tensor part. We then find

$$|\epsilon| \sim \frac{1}{\sqrt{2}} \left| \frac{\mathrm{Im}[\frac{4}{3}a_0 G_{--}^{(U0,U+)} - 2a_0'(G_{--}^{(U0,U+)} - 5G_{--}^{(U3,U+)})]}{(a_0+a_0')\sin\theta\cos\theta - \mathrm{Re}[\frac{4}{3}a_0 G_{--}^{(U0,U+)} - 2a_0'(G_{--}^{(U0,U+)} - 5G_{--}^{(U3,U+)})]} \right|, \tag{4.13}$$

and

$$\left|\frac{\epsilon'}{\epsilon}\right| \sim \left| \frac{a_2\,\mathrm{Im}(G_{--}^{(U0,U+)} + G_{--}^{(U3,U+)})}{\mathrm{Im}[\frac{2}{3}a_0 G_{--}^{(U0,U+)} - a_0'(G_{--}^{(U0,U+)} - 5G_{--}^{(U3,U+)})]} \right|. \tag{4.14}$$

Experimentally the situation is still not clear, but it appears that[21] $|\eta_{+-}| = (1.92\pm0.05)\times10^{-3}$ and $|\epsilon'/\epsilon| \lesssim 1/10$. If we accept these values, the three models of Oakes,[5] Das,[6] and Zachariasen and Zweig[7] require a CP-nonconserving phase angle $|\varphi| \sim 10^{-3}$.

D. Neutral Currents and Calculation of θ

The possibility of actually calculating the value of the Cabibbo angle from first principles has received much attention lately. The works of Gatto, Sartori, and Tonin (GST)[28] and Cabibbo and Maiani (CM)[29] have represented a breakthrough in this direction, though the interpretation of their procedure is open to question. They essentially demand that the strong-interaction Hamiltonian is broken by scalar terms which transform as $(3,3^*)+(3^*,3)$ and that certain quadratic divergences vanish when one carries out a self-energy calculation for any hadron.

The calculations were initially carried out in the framework of charged currents; however, they can easily be extended to include neutral currents. We refer the reader to the papers of GST[28] and CM,[29] and the letter of Albright and McGlinn[30] for details. If we again use the Cabibbo form for the charged currents and introduce only $V-A$ neutral currents, the equation determining the Cabibbo angle can be written in terms of the coupling coefficients as

$$\frac{\sin^2\theta + \frac{1}{9}G_{--}^{(U0,U0)} - \frac{2}{3}G_{--}^{(U0,U3)} + G_{--}^{(U3,U3)} + G_{--}^{(U+,U-)}}{\cos^2\theta + \frac{1}{9}G_{--}^{(U0,U0)} + \frac{2}{3}G_{--}^{(U0,U3)} + G_{--}^{(U3,U3)} + G_{--}^{(U+,U-)}} = \frac{\beta}{\gamma}, \tag{4.15}$$

where the estimate $\beta/\gamma = 0.053$ was obtained by GST and CM from a study of the chiral symmetry-breaking parameters.

It is of interest to determine the Cabibbo angle from Eq. (4.15) for each of the models of CP nonconservation discussed in Sec. IV C. By comparing Table III and Eq. (4.15), one finds for the

Oakes model: $\sin^2\theta = -0.85$,

Das model: $\sin^2\theta = -0.037$,

Zachariasen-Zweig model: $\sin^2\theta = -0.40$.

For each of these neutral current models, an unphysical result is obtained for the Cabibbo angle in contrast to the exceptionally accurate result, $\sin\theta = 0.22$, obtained

[28] R. Gatto, G. Sartori, and M. Tonin, Phys. Letters **28B**, 128 (1968); Nuovo Cimento Letters **1**, 1 (1969); **1**, 399 (1969).

[29] N. Cabibbo and L. Maiani, Phys. Letters **28B**, 131 (1968); Phys. Rev. D **1**, 707 (1970).

[30] C. H. Albright and W. D. McGlinn, Phys. Letters **29B**, 666 (1969).

with only charged currents. What is required to preserve the accuracy of the calculation are the conditions

$$|\tfrac{1}{3}G_{--}^{(88)}+\tfrac{1}{4}(G_{--}^{(66)}+G_{--}^{(77)})|\ll 1\,,$$
$$|\tfrac{1}{4}G_{--}^{(33)}-\tfrac{1}{6}\sqrt{3}G_{--}^{(38)}+\tfrac{1}{12}G_{--}^{(88)}+\tfrac{1}{4}(G_{--}^{(66)}+G_{--}^{(77)})|\lesssim 0.25\,. \qquad (4.16)$$

As pointed out earlier, the validity of the constraints (4.16) on the neutral currents hinges on the validity of the calculations of θ proposed by GST[28] and CM.[29] Since the technique employs the Bjorken criterion,[31] there are fine mathematical details which must be justified rigorously. We simply refer the reader to the literature on this subject.

E. Consistency of Theoretical Models

In the preceding subsections we have discussed various approximate empirical laws attributed by previous authors to the weak interactions and have shown how the interaction of neutral currents may account for these in a simple fashion. The question naturally arises whether all these effects can be explained by one class of neutral current models, independently of present experimental information.

To pursue this question, we consider only $V-A$ currents and refer to Eqs. (4.1), (4.4), and (4.5), where the conditions on the coupling coefficients are specified for the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule, the no-**27** rule, and the octet rule. The conditions are progressively more restrictive, and all coefficients are determined by the octet rule. It is clear from Eqs. (4.1) and (4.4) that CP nonconservation cannot be explained in the absence of the **27**, for in that case one must set $G_{--}^{(37)}=G_{--}^{(67)}=G_{--}^{(87)}=0$. The less restrictive $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule admits the possibility that $G_{--}^{(87)}\neq 0$ and hence can accommodate CP nonconservation. In this case, $\epsilon'=0$ and

$$|\epsilon|\simeq\frac{1}{\sqrt{2}}\left|\frac{[-(1/\sqrt{3})a_0+3\sqrt{3}a_0']G_{--}^{(87)}}{3(a_0+a_0')\sin\theta\cos\theta+[-(1/\sqrt{3})a_0+3\sqrt{3}a_0']G_{--}^{(86)}}\right|. \qquad (4.17)$$

For a range of values of $G_{--}^{(86)}$ up to unity, Eq. (4.17) implies $G_{--}^{(87)}\sim 10^{-3}$.

In the case of the Cabibbo-angle calculation, the octet rule fails badly, for one finds $\sin\theta=0.78$ instead of 0.23. The correct angle can be determined in the no-**27** case provided $G_{--}^{(66)}\simeq 0.15$. In the less restrictive $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule, conditions (4.16) are more easily satisfied. Hence we conclude that a theoretically consistent $V-A$ current model can be obtained which explains the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule, CP nonconservation, and yields the correct θ provided that the coupling coefficients satisfy Eqs. (4.1), (4.16), and (4.17).

F. Neutral Lepton Current Models

We close this section on neutral current models with a brief discussion of models which can be proposed for neutral lepton currents.

Three special models can be constructed for the neutral lepton currents which are somewhat natural. In place of the general phenomenological weak Hamiltonian H_L given in (3.2d), we restrict our attention to $V-A$ currents and write instead

$$H_L(\text{neutral currents})=\frac{G}{\sqrt{2}}\sum_{l=e,\mu}\sum_{l'=e,\mu}l_{-\lambda}^{(a)}l'_{-\lambda}^{(a)}\,, \qquad (4.18)$$

where the currents appearing are of one of the types given below.

1. Neutral Lepton Coupling. Here we take

$$l_{-\lambda}^{(a)}=l_{-\lambda}^{(0+)}=\tfrac{1}{2}\bar{\psi}_l(1+\tau^3)\gamma_\lambda(1+\gamma_5)\psi_l=\bar{\nu}_l\gamma_\lambda(1+\gamma_5)\nu_l\,, \qquad (4.19\text{a})$$

which couples only neutrinos.

2. Charged Lepton Coupling. Here only the electrons or muons are coupled as in the electromagnetic current according to

$$l_{-\lambda}^{(a)}=l_{-\lambda}^{(0-)}=\tfrac{1}{2}\bar{\psi}_l(1-\tau^3)\gamma_\lambda(1+\gamma_5)\psi_l=\bar{u}_l\gamma_\lambda(1+\gamma_5)u_l\,. \qquad (4.19\text{b})$$

3. Symmetric Lepton Coupling. In this case, the neutral and charged leptons are coupled symmetrically according to

$$l_{-\lambda}^{(a)}=l_{-\lambda}^{(3)}=\tfrac{1}{2}\bar{\psi}_l\tau^3\gamma_\lambda(1+\gamma_5)\psi_l=\tfrac{1}{2}\bar{\nu}_l\gamma_\lambda(1+\gamma_5)\nu_l-\tfrac{1}{2}\bar{u}_l\gamma_\lambda(1+\gamma_5)u_l\,. \qquad (4.19\text{c})$$

In terms of the general coupling coefficients $f_{ij}^{(ab)}$, Eq. (4.18) implies

$$f_{--}^{(00)}=f_{--}^{(03)}=f_{--}^{(30)}=f_{--}^{(33)}=1 \quad \text{for model 1}\,, \qquad (4.20\text{a})$$

$$f_{--}^{(00)}=-f_{--}^{(03)}=-f_{--}^{(30)}=f_{--}^{(33)}=1 \quad \text{for model 2}\,, \qquad (4.20\text{b})$$

and

$$f_{--}^{(00)}=f_{--}^{(03)}=f_{--}^{(30)}=0\,,\ f_{--}^{(33)}=1 \quad \text{for model 3}\,. \qquad (4.20\text{c})$$

We shall discuss the consequences of these models in Sec. V B. Note, however, that the semileptonic interactions can couple any type of the special lepton cur-

[31] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

rents cited above. One might try to invoke the concept of universality of the weak interactions to restrict somewhat the wide choice of models otherwise possible for the neutral currents. This does not prove to be very satisfactory, however, for the idea itself is not unambiguous.

The concept of universality as applied to the charged currents requires that both hadron currents and lepton currents have the algebraic properties of $SU(2)$ raising or lowering operators; moreover, the over-all normalization of the hadron current relative to the lepton current is fixed. One can extend the concept of universality to the neutral weak currents by again requiring that the algebraic properties of the hadron and lepton currents be identical, as is the case for the electromagnetic current. Since neither the neutral hadron current nor the neutral lepton current is known, however, neither is specified uniquely by universality.

In line with this version of universality, we note that the $SU(2)$ algebra of charges suggests that Eq. (4.19c) should be chosen for the neutral lepton current, with

$$J_{-\lambda}^{(0)}=\tfrac{1}{2}(1+\cos^2\theta)(V-A)_\lambda^{(3)}+\tfrac{1}{2}\sqrt{3}\sin^2\theta\,(V-A)_\lambda^{(8)}-\sin\theta\cos\theta\,(V-A)_\lambda^{(6)} \quad (4.21)$$

selected for the neutral hadron current. While this choice appears quite natural, it is unattractive to accept Eq. (4.21) and not Eq. (4.19c).

An alternative extension of universality requires that one obtain the neutral hadron current by an $SU(3)$ rotation of the charged current. This prescription determines the normalization of the neutral hadron current which in turn can be used to fix the properties of the neutral lepton currents. However, in general, the resulting neutral lepton current will not be normalized in the same way as the charged lepton currents.

Yet another extension of universality was proposed by Zachariasen and Zweig,[7] who require that both the charged and neutral currents satisfy the algebraic relation

$$[J,[J,J^\dagger]]=-2J. \quad (4.22)$$

This statement yields the neutral hadron currents stated earlier in Sec. IV C but suggests, in addition, that there are no neutral lepton currents.

We end this brief digression by noting that the basic ambiguity in the extension of universality to neutral currents stems in large measure from the mismatch between the $SU(3)$ hadron current algebra and the $SU(2)$ lepton current algebra.[32]

V. COMPARISON WITH EXPERIMENT

A. Limitations on Coupling Coefficients

We have considered vector and axial-vector neutral hadron currents which are members of two octets and possibly two singlets of $SU(3)$. The V and A neutral lepton currents we have discussed transform like isoscalar and isovector members of $SU(2)$; for simplicity, we assume μ-e universality and the additive law of lepton conservation. Both $V-A$ and $V+A$ neutral hadron and lepton currents are coupled together in pairs to form the nonleptonic, semileptonic, and leptonic weak interactions. The coupling coefficients are initially only restricted by general considerations such as CPT invariance, unitarity, etc. We now impose the experimental restrictions discussed in Sec. II on the general analysis given in Sec. III. These data constrain the coefficients quite severely in some cases and not at all in others.

The most severe restrictions on the neutral currents are those arising from the absence of any observed $\Delta S=\pm1$ semileptonic decays of the K mesons involving neutral lepton currents. Upper limits for the branching ratios of the K^+, $K_S{}^0$, and $K_L{}^0$ into the modes of interest[14–19] are typically 10^{-6} or less, as shown in Table II. We list below the upper limits on the coupling coefficients implied by the upper limits on the branching ratios listed in Table II.

$K_L{}^0\to\mu^+\mu^-$:

$$|(g_{--}^{(70)}-g_{--}^{(73)})-(g_{-+}^{(70)}-g_{-+}^{(73)})-(g_{+-}^{(70)}-g_{+-}^{(73)})+(g_{++}^{(70)}-g_{++}^{(73)})|<1.3\times10^{-4}, \quad (5.1)$$

$K_S{}^0\to\mu^+\mu^-$:

$$|(g_{--}^{(60)}-g_{--}^{(63)})-(g_{-+}^{(60)}-g_{-+}^{(63)})-(g_{+-}^{(60)}-g_{+-}^{(63)})+(g_{++}^{(60)}-g_{++}^{(63)})|<1.9\times10^{-2}, \quad (5.2)$$

$K^+\to\pi^+\nu_l\bar\nu_l$:

$$[|(g_{--}^{(U-,0)}+g_{--}^{(U-,3)})+(g_{+-}^{(U-,0)}+g_{+-}^{(U-,3)})|^2+|(g_{-+}^{(U-,0)}+g_{+-}^{(U-,3)})+(g_{++}^{(U-,0)}+g_{++}^{(U-,3)})|^2]^{1/2}<1.2\times10^{-3}, \quad (5.3)$$

$K^+\to\pi^+e^+e^-$:

$$[|(g_{--}^{(U-,0)}-g_{--}^{(U-,3)})+(g_{+-}^{(U-,0)}-g_{+-}^{(U-,3)})|^2+|(g_{-+}^{(U-,0)}-g_{-+}^{(U-,3)})+(g_{++}^{(U-,0)}-g_{++}^{(U-,3)})|^2]^{1/2}<2.3\times10^{-3}. \quad (5.4)$$

If only $V-A$ neutral currents occur in the weak Hamiltonian, the reality of the $g_{ij}{}^{(\alpha a)}$ in the Cartesian basis then implies that the strangeness-changing semileptonic coupling coefficients are limited by

$$|g_{--}^{(60)}|,\quad |g_{--}^{(63)}|<3.5\times10^{-3} \quad (5.5a)$$

and

$$|g_{--}^{(70)}|,\quad |g_{--}^{(73)}|<1.2\times10^{-3}. \quad (5.5b)$$

If both $V-A$ and $V+A$ neutral hadron currents exist while only $V-A$ neutral lepton currents exist, then

$$|g_{--}^{(60)}+g_{+-}^{(60)}|,\quad |g_{--}^{(70)}+g_{+-}^{(70)}|<3.5\times10^{-3}, \quad (5.6a)$$

[32] B. d'Espagnat and J. Prentki, Nuovo Cimento **24**, 497 (1962).

$$|g_{--}^{(63)}+g_{+-}^{(63)}|\,,$$
$$|g_{--}^{(73)}+g_{+-}^{(73)}|<3.5\times10^{-3}\,, \qquad (5.6b)$$
$$|g_{--}^{(60)}-g_{--}^{(63)}|\,,$$
$$|g_{+-}^{(60)}-g_{+-}^{(63)}|<1.2\times10^{-2}\,, \qquad (5.6c)$$

and

$$|g_{--}^{(70)}-g_{--}^{(73)}|\,,$$
$$|g_{+-}^{(70)}-g_{+-}^{(73)}|<2.4\times10^{-3}. \qquad (5.6d)$$

It is clear from the above that in this case the coupling of neutral *axial-vector* strangeness-changing hadron currents to $V-A$ *neutrino* currents can be large, while the other couplings must be small. If both $V-A$ and $V+A$ lepton currents exist, less can be said though (5.6a) and (5.6b) still only allow the couplings of the neutral axial-vector strangeness-changing hadron currents to be large.

The high-energy neutrino reactions restrict the coupling of weak neutral strangeness-conserving currents. As pointed out in Sec. II, both the elastic process $\nu_l+p\rightarrow\nu_l+p$ and the inelastic process $\nu_l+p\rightarrow N^{*+}+\nu_l$ are of special interest.[13] We emphasize the importance of neutrino reaction data since they provide direct tests for $\Delta S=0$ neutral currents independently of the rare kaon decay data.

First we consider the inelastic production of the $N^*(1236)$ which can only occur through the $\Delta I=1$ component of the neutral octet current in lowest order. If only $V-A$ hadron currents enter, the result is particularly simple as seen from Eqs. (D7) and (D8), and we find

$$|g_{--}^{(30)}+g_{--}^{(33)}|\lesssim1.7 \qquad (5.7)$$

for the present experimental upper limit (2.4) on the cross-section ratio

$$\sigma(\nu_\mu+p\rightarrow n+\pi^++\nu_\mu)/\sigma(\nu_\mu+p\rightarrow p+\pi^++\mu^-)\,.$$

If both $V-A$ and $V+A$ neutral hadron currents exist, one must examine the cross-section behavior more carefully in terms of the vector and axial-vector form factors in order to quote limits on the g's. We simply note here that if and when the presence of neutral currents is established, the inelastic neutrino production of N^* will help to fix the coupling strengths of the $V+A$ hadron currents.

The elastic neutrino-proton process can proceed through both $\Delta I=0$ and $\Delta I=1$ currents; hence the third and eighth components of the octets and possibly the unitary singlets can play a role. The result is particularly simple if only the third component enters. In this case, Eqs. (D5) and (D6) yield from the experimental limit given in (2.3)

$$|g_{--}^{(30)}+g_{--}^{(33)}|\lesssim1.4\,. \qquad (5.8)$$

If all the coefficients in (D6) must be considered, they can be determined by a detailed comparison of the differential and total cross sections for the elastic and quasielastic processes when and if these data become available.

We now turn our attention to the nonleptonic weak interactions. Here the theoretical implications of the data are more uncertain. The $\Delta I=\frac{1}{2}$ rule appears reasonably well satisfied in the strangeness-changing transitions to better than 10% accuracy, but it is not clear whether this is due to the absence of the $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{5}{2}$ tensors or to a dynamical suppression of the corresponding reduced matrix elements. The only evidence bearing on the strangeness-conserving nonleptonic couplings is obtained from the weak parity-violating contribution to the nuclear potential. The most accurate information[22] comes from the circular polarization measurements on the 4.82-keV γ ray emitted in the decay of ^{181}Ta. Theoretical analysis[33] indicates that the $\Delta I=1$ contribution must be suppressed by approximately $\sin^2\theta$ as in the Cabibbo theory. For the symmetric octet tensor term, this implies

$$2\left|\frac{1}{\sqrt{3}}G_{--}^{(38)}-\tfrac{1}{4}(G_{--}^{(66)}+G_{--}^{(77)})\right|\lesssim\sin^2\theta\,, \qquad (5.9)$$

whereas the coefficient of the **27** tensor in Eq. (C1) may be of order unity if the reduced **27** matrix element is dynamically suppressed to ~5% of the symmetric **8** matrix element.[26] Essentially no upper limits can be placed on the antisymmetric **8**, **10**, and **10*** tensor coefficients, since no lower limits exist for the corresponding reduced matrix elements.

The $|\Delta S|=1$ nonleptonic processes have the same ambiguities so far as the coupling coefficients of the neutral currents are concerned due to the CP-conserving part of H_W. It is clear, however, that the CP nonconservation observed in $K_L^0\rightarrow\pi\pi$ decay can arise in the weak interactions only through interference of the seventh component of the neutral current with its third or eighth component. Since the CP-nonconserving amplitude in K_L^0 decay is only about 2×10^{-3} times the CP-conserving amplitude, we can establish the followin inequalities from Eqs. (3.10) and (C3) [cf. also Eqs. (4.13) and (4.14)]:

$$|G_{--}^{(U0,7)}-G_{++}^{(U0,7)}|\lesssim0.75\times10^{-3} \qquad (5.10a)$$

from the **8** tensor term and

$$|G_{--}^{(U3,7)}-G_{++}^{(U3,7)}|\lesssim2\times10^{-3} \qquad (5.10b)$$

from the **27** tensor term if we again assume a dynamical suppression of **27** matrix elements relative to the **8**. No limits can be placed on G_{-+} and G_{+-} couplings since the antisymmetric tensor matrix elements are unknown.

The apparent absence of the $|\Delta S|=2$ transition rates in first order and the very small K_L-K_S mass difference imply that the corresponding coupling coefficients are

[33] B. H. J. McKeller, Phys. Rev. Letters **20**, 1542 (1968); **21**, 1822 (1968).

extremely small. Dynamical suppression of the **27** reduced matrix element implies that

$$|G_{ij}^{(U+,U+)}|,\quad |G_{ij}^{(U-,U-)}|\lesssim 10^{-5}. \qquad (5.11)$$

Finally we consider the purely leptonic processes. The recent result, Eq. (2.1c), obtained by Reines and Gurr[11] for low-energy elastic $\bar{\nu}_l+e^-\to\bar{\nu}_l+e^-$ scattering, allows one to place the upper limit

$$|1+\tfrac{1}{2}(f_{--}^{(00)}-f_{--}^{(33)})|<2.0, \qquad (5.12)$$

if only $V-A$ lepton currents are considered. Similarly, the upper limit, Eq. (2.2), obtained[12] for $\nu_\mu+e^-\to\nu_\mu+e^-$ from the CERN neutrino spark-chamber experiment,[34] yields

$$\tfrac{1}{2}|f_{--}^{(00)}-f_{--}^{(33)}|<0.63. \qquad (5.13)$$

B. Tests of Various Theoretical Models

1. Neutral Hadron Current Models

The present experimental information provides no firm evidence for the existence of weakly coupled neutral lepton currents. Even if there are only neutral hadron currents, the theoretical models considered must be consistent with the small $\Delta I=1$ contribution in the parity-violating $\Delta S=0$ transitions, the absence of $|\Delta S|>1$ interactions, and the small CP-nonconserving effect observed in the $K_L\to\pi\pi$ decays.

Equations (4.1), (4.4), and (4.5) reveal that the constraint (5.9) placed by the small observed $\Delta I=1$ contribution to the parity-violating $\Delta S=0$ interaction is badly violated by the octet rule (right-hand side $\sim$1.70 in place of 0.05). This strongly suggests that the apparent octet rule arises from dynamical enhancement rather than a basic symmetry of the weak interactions.[35]

Condition (5.9) can be satisfied in the case of the no-**27** rule provided $|G_{--}^{(38)}|$, $|G_{--}^{(66)}|$, $|G_{--}^{(77)}|\lesssim\sin^2\theta$. With both (5.9) and $|G_{--}^{(88)}|\lesssim\sin^2\theta$ satisfied, the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule can account for the small $\Delta I=1$ contribution to the parity-violating $\Delta S=0$ transitions and can yield nearly the correct Cabibbo angle as determined from (4.15).

If we consider instead the models of CP nonconservation introduced by Oakes,[5] Das,[6] and Zachariasen and Zweig,[7] we find that condition (5.9) is violated badly again for each case. These models also destroy the determination of θ as already mentioned in Sec. IV D. The "universal" neutral current of Eq. (4.21) fares considerably better; the $\Delta I=1$ condition of (5.9) is well obeyed, and the Cabibbo-angle calculation in Eq. (4.15) is identical to that with only charged currents present. No CP nonconservation is introduced with this type of neutral current, however.

[34] J. K. Bienlein, A. Böhm, G. von Dardel, H. Faissner, F. Ferrero, J.-M. Gaillard, H. J. Gerber, B. Hahn, V. Kaftanov, F. Krienen, M. Reinharz, R. A. Salmeron, P. G. Seiler, A. Staude, J. Stein, and H. J. Steiner, Phys. Letters **13**, 80 (1964).

[35] R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara, in *Proceedings of the International Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1966).

2. Neutral Hadron and Lepton Current Models

If neutral lepton currents which couple to the neutral hadron currents in semileptonic interactions exist, the number of restrictions on the various neutral current couplings are increased considerably. In particular, conditions (5.5)–(5.8) now must be imposed. The strangeness-changing couplings are forced to be very small due to the apparent suppression of certain K decay modes. With only $V-A$ currents, the upper limits are especially tight; cf. Eq. (5.5).

Suppose one first writes the total neutral current $\mathcal{J}_\lambda^{(0)}$ as a sum of the hadronic and leptonic parts, $\mathcal{J}_\lambda^{(0)}=J_\lambda^{(0)}+l_\lambda^{(0)}$, and then couples the total current to form the weak Hamiltonian

$$H_W=\frac{G}{\sqrt{2}}\tfrac{1}{2}[\{\mathcal{J}_\lambda^{(+)},\mathcal{J}_\lambda^{(-)}\}+\{\mathcal{J}_\lambda^{(0)},\mathcal{J}_\lambda^{(0)}\}]. \qquad (5.14)$$

With $J_\lambda^{(0)}$ given by the general form (4.2) and $l_\lambda^{(0)}$ by one of the forms in (4.19), one is immediately confronted with a conflict. The problem is that, in the notation of Eq. (4.2), one must demand

$$|c_+ + c_-|\gtrsim\sin\theta=0.23 \qquad (5.15a)$$

on the basis of any one of the models discussed in Secs. IV A and IV B, whereas the experimental K decay branching ratios require that

$$|c_+ + c_-|<1.8\times10^{-3} \qquad (5.15b)$$

from (5.5). Hence if neutral hadron currents exist, as suggested by the chiral $SU(3)\times SU(3)$ algebra, the only way that the above conflict can be avoided in these models is to assume that neutral lepton currents do not exist or that a selection rule holds which forbids the coupling of neutral hadron and lepton currents in the semileptonic interactions.

Similar remarks hold for the universal neutral current model of Eqs. (4.19c) and (4.21) and for the CP-nonconserving model of Das.[6] The model of Zachariasen and Zweig[7] avoids the conflict since neutral lepton currents do not exist in this model as a consequence of Eq. (4.22). In the case of the CP-nonconserving model proposed by Oakes, Eq. (5.15a) is not applicable; here the limits on $g_{--}^{(70)}$ and $g_{--}^{(73)}$ imposed by the K decay data imply that

$$|c_+ - c_-|=2\sin\varphi<0.6\times10^{-3}. \qquad (5.16)$$

This limit seems to require the angle φ, associated with the strangeness-changing neutral current in this model, to be too small to explain the observed CP nonconservation in $K_L\to\pi^+\pi^-$ decay. Precise data on $K_L^0\to\pi^0\pi^0$ could rigorously clarify this matter.

One appears to be led to the conclusion that CP nonconservation in $K_L^0\to 2\pi$ probably cannot be explained by weakly coupled neutral hadron currents if these currents also couple directly to neutral lepton currents at the level of the universal weak interactions.

C. Special Models Consistent with Experiment

Since the present experimental restrictions are so tight, it is of interest to inquire what types of neutral current interactions still survive the current selection criteria. Presently acceptable models fall into three categories each with one of the following features: neutral currents without semileptonic interactions, strangeness-conserving neutral hadron currents, and "superweak" interactions of neutral currents.

1. Neutral Currents without Semileptonic Interactions

If for some reason there is no coupling of neutral hadron currents to neutral lepton currents, or alternatively no neutral lepton currents exist, the upper limits on the rare K decay modes are trivially satisfied.

As noted earlier, the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule apparently yields results which are in good agreement with all the known nonleptonic transitions. The general features of this model are summarized in Eq. (4.1). One special case we can mention is the following:

$$H_L(\text{neutral currents}) = \frac{G}{\sqrt{2}}\tfrac{1}{2}[\{J_\lambda{}^{(0)},J_\lambda{}^{(0)}\}+\{K_\lambda{}^{(0)},K_\lambda{}^{(0)}\}], \quad (5.17)$$

where

$$J_\lambda{}^{(0)}=\cos\theta\,(V-A)_\lambda{}^{(3)}-\sin\theta\,(V-A)_\lambda{}^{(6)},$$
$$K_\lambda{}^{(0)}=\sin\theta\,(V-A)_\lambda{}^{(7)}+\epsilon\sin\theta\,(V-A)_\lambda{}^{(8)}.$$

To obtain the correct order of magnitude for the CP nonconservation, we require $G_{--}{}^{(87)}\simeq 10^{-3}$ or $\epsilon\simeq\sin^2\theta$. Alternatively, the universal neutral current model of Eq. (4.21) is presently acceptable for the nonleptonic interactions although it cannot explain the phenomena of CP nonconservation.

2. Strangeness-Conserving Neutral Hadron Currents

An alternative point of view which avoids the difficulty with the rare K decay modes is the assumption that the neutral hadron currents are strangeness conserving as in the electromagnetic interaction. We can speculate, by analogy with the isovector charged currents, about the choice of Eq. (4.19c) for the neutral lepton current and

$$J_{-\lambda}{}^{(0)}=(V-A)_\lambda{}^{(3)} \quad (5.18a)$$

for the neutral hadron current. The weak Hamiltonian for the leptonic interactions is then "isospin symmetric"; however, the semileptonic and nonleptonic Hamiltonians exhibit isopin symmetry only in the limit $\theta=0$.

In adopting this strangeness-conserving model of the neutral currents,[36] one gives up the idea that the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule is a built-in symmetry; moreover, there is now no way to explain CP nonconservation on the basis of $V-A$ first-class neutral currents in the weak interaction.[37] However, it is interesting to note that the $\Delta I=1$ neutral current part of the nonleptonic Hamiltonian vanishes and hence trivially satisfies the condition (5.9) in the parity-violating $\Delta S=0$ transitions. Also $\sin\theta$ obtained from (4.15) is found to be $\sin\theta=0.25$, still in good agreement with the experimental value as the correction is small with $G_{--}{}^{(33)}=1$.

The critical tests of this model occur in the semileptonic neutrino reactions $\nu_\mu+p\rightarrow\nu_\mu+p$ and $\nu_\mu+p\rightarrow n+\pi^++\nu_\mu$ and in the leptonic reactions $\nu_\mu+e^-\rightarrow\nu_\mu+e^-$ and $\bar{\nu}_e+e^-\rightarrow\bar{\nu}_e+e^-$. On the basis of this model, one expects 0.25 and 0.11 for the ratios of (2.3) and (2.4), respectively, and the coupling coefficients $f_{--}{}^{(00)}=0$ and $f_{--}{}^{(33)}=1$ to hold in (5.12) and (5.13), respectively. The present upper limits are now at these levels.

Another special case of interest is to use Eq. (4.19c) for the neutral lepton current and the isoscalar form for the neutral hadron current:

$$J_{-\lambda}{}^{(0)}=(V-A)_\lambda{}^{(8)}. \quad (5.18b)$$

Again the neutral current part of the $\Delta I=1$ nonleptonic Hamiltonian vanishes. Here, however, we find that the Cabibbo angle, Eq. (4.15), becomes

$$\frac{12\sin^2\theta+4G_{--}{}^{(88)}}{12\cos^2\theta+G_{--}{}^{(88)}}=\frac{\beta}{\gamma}, \quad (5.19)$$

and $\sin\theta$ is much different from 0.23 unless $|G_{--}{}^{(88)}|\lesssim 0.03$. This suggests that a strangeness-conserving model with (5.18a) for the neutral hadron current is much more natural than the choice (5.18b). Any sizable admixture of (5.18a) and (5.18b) will yield a rather large $\Delta I=1$ strangeness-conserving parity-violating contribution in disagreement with experiment.

3. Superweak Interactions of Neutral Currents

The third category of special models consistent with present data is, in a sense, counter to the spirit of our whole investigation. That is, one can postulate that neutral currents couple only at a "superweak" level rather than at the universal weak level; i.e., $G'\sim 10^{-6}G$, where $GM_N{}^2\sim 10^{-5}$, or for all neutral current couplings $G_{ij}{}^{(\alpha\beta)}\lesssim 10^{-6}$. In the limit $G'\rightarrow 0$, all the superweak interactions disappear and one is left only with the weak interaction of charged currents. Since the absence of neutral currents is consistent with the present data, the superweak theory of neutral currents will also be consistent with experiment. The experiments needed to specify the superweak neutral current couplings demand

[36] S. L. Glashow, J. Iliopoulos, and L. Maiani [Phys. Rev. D 2, 1285 (1970)] demonstrate that such strangeness-conserving neutral currents can be induced in a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson.

[37] CP-nonconserving variations of this model and the following model have been discussed previously. See R. J. Oakes, in *Proceedings of the Coral Gables Conference on Fundamental Interactions at High Energy, January, 1969* (Benjamin, New York, 1969). See also C. H. Albright, CERN Report No. TH-1066 (unpublished).

such highly accurate results that decades of experimentation will be required to determine them and to isolate them from other induced effects and electromagnetic corrections.

D. Future Experimental Developments

We conclude Sec. V with a discussion of experimental developments that may be anticipated in the near future and will serve to test even more critically for the existence of weakly coupled neutral currents.

We have mentioned in Sec. II some of the recent upper limits that have been placed on elastic neutrino-electron scattering. The experiment in progress by Reines and Gurr[11] looks directly for the elastic scattering by electrons of antineutrinos from a fission reactor, $\bar{\nu}_e+e^-\rightarrow\bar{\nu}_e+e^-$, and should result in an even more restrictive limit than that quoted in (2.1c) in the near future. With an isospin-symmetric lepton coupling of charged and neutral currents, the cross-section ratio should be $\frac{1}{4}$ times as large as that for $V-A$ charged currents alone. A result consistent with unity, however, does not rule out the possibility that neutral currents couple only charged or neutral leptons. The next-generation spark-chamber experiments with high-energy ν_μ beams should lower the limit presented in Eq. (2.2) and test directly for the neutral current interaction in $\nu_\mu+e^-\rightarrow\nu_\mu+e^-$.[38]

Additional experiments are needed to lower the upper limits for the $K^0\rightarrow l^+l^-$ and $K^+\rightarrow\pi^+\nu_l\bar{\nu}_l$ branching ratios down to the 10^{-8} level. It is at this level that the combination of weak interactions with charged currents and fourth-order electromagnetic interactions is expected to be responsible for the above decays. If these decay modes fail to appear above the 10^{-8} branching-ratio level, neutral currents may still exist provided the hadron currents are strangeness conserving or a selection rule holds which rules out the appearance of neutral currents in the semileptonic interactions.

Elastic and inelastic neutrino-proton scattering test for the presence of strangeness-conserving hadron currents coupled to neutrino currents. The present bubble-chamber results with high-energy neutrinos have placed upper limits on the ν-p cross sections which are just at the levels expected, so again the next-generation experiments should help to clarify the matter.

If the cross-section ratios quoted in Eqs. (2.3) and (2.4) fall much below the ratios 25 and 10%, respectively, one could infer that neutral $\Delta S=0$ hadron currents couple only to charged leptons as in the electromagnetic case. This is a rather dismal possibility since one must then search for the parity-violating effects of such currents against the background of the electromagnetic interaction.

If neutral currents fail to show up in the leptonic and semileptonic reactions discussed above, it becomes imperative to search for them in the complex nonleptonic weak interactions. As discussed in Sec. V C, the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule and the universal neutral current model are quite consistent with the present information. The $\Delta I=1$ part of the parity violation in the nuclear potential is suppressed by $\sin^2\theta$ and contributes at the same level as the Cabibbo current. The problem then remains that the calculations for the parity-violating effects are complicated by the nuclear-physics uncertainties. Similar problems arise in the calculations of the strangeness-changing nonleptonic decays since these are fairly model dependent.

Refinements of both the theoretical models and the experiments on parity-violating $\Delta S=0$ transitions may be necessary to distinguish possible neutral current effects from those caused by charged currents. If it turns out that neutral currents can couple only at the superweak level, it is doubtful whether any experiments will be able to detect their presence in the forseeable future.

VI. SUMMARY

In this phenomenological investigation, we have assumed current-current interactions for the leptonic, semileptonic, and nonleptonic Hamiltonians and have studied what role, if any, neutral currents play in the weak interactions. For the charged currents, the Cabibbo form was adopted, while for the neutral currents we considered octet and single currents of both $V-A$ and $V+A$ chiralities. Nonlocal effects, possibly introduced by massive intermediate bosons, are negligibly small so far as our study is concerned.

The experiments bearing on the existence of neutral hadron and lepton currents are numerous and already place many constraints on the forms of such interactions. Somewhat surprising is the fact that many types of experiments are presently on the verge of testing rather severely the few remaining attractive possibilities. We refer to the neutrino-lepton scattering as a test of the isospin-symmetric lepton current coupling, to neutrino-proton elastic and inelastic scattering as a test of the coupling of the neutral hadron isovector current to the neutral neutrino current, and to the parity-violating nuclear transitions as tests of the admixture of the isovector and isoscalar neutral hadron currents.

The present information on the rate K decay branching ratios severely limits the magnitude of the coupling of the strangeness-changing neutral hadron currents to the neutral lepton currents. This is a serious problem for neutral current theories of CP nonconservation in which both hadron and lepton currents interact. The parity-violating nuclear γ transitions appear to be sufficiently accurate to rule out the octet rule as a built-in symmetry of the nonleptonic weak Hamiltonian. On the other hand, the $\Delta I<\frac{3}{2}$, $\Delta Y<2$ rule is quite compatible with the present information, as is the universal neutral current model obtained from the

[38] This process can also proceed by charged lepton currents alone if mediated by second-order electromagnetic interactions, but the cross section will be suppressed considerably.

$SU(2)$ algebra of charges if neutral currents in the semileptonic interactions are ignored.

It remains to be seen whether the long-sought weakly coupled neutral currents will be detected in the near future or whether these coordinates of hadronic matter, in the chiral $SU(3)\times SU(3)$ language, are coupled only at the superweak level on the hierarchy of interactions observed in nature and hence will escape detection for a long time to come.

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APPENDIX A: TENSOR FORMS FOR NONLEPTONIC HAMILTONIAN

The direct product of two hadron currents can be written as a direct sum of irreducible tensor operators. Here we spell out the symmetric and antisymmetric tensors $T_S^{(\alpha\beta)}$ and $T_A^{(\alpha\beta)}$ defined in Eq. (3.4). The indices α, β run over the components $U0$, $U3$, $U+$, $U-$, and 0 in the spherical basis.

The symmetric tensors are given by

$$T_S^{(U0,U0)}=\frac{4}{3}\left[\left(\frac{3}{20}\right)^{1/2}T_{010}^{(8)}+\frac{1}{\sqrt{20}}T_{000}^{(8)}\right]+2\left[\frac{1}{\sqrt{6}}T_{020}^{(27)}+\frac{1}{\sqrt{10}}T_{010}^{(27)}+\frac{1}{\sqrt{30}}T_{000}^{(27)}\right]-\frac{4}{3}\frac{1}{\sqrt{8}}T_{000}^{(1)},$$

$$T_S^{(U0,U3)}=\frac{4}{3}\left[-\left(\frac{3}{20}\right)^{1/2}T_{010}^{(8)}+3\frac{1}{\sqrt{20}}T_{000}^{(8)}\right]+2\left[\frac{1}{\sqrt{6}}T_{020}^{(27)}-\frac{1}{\sqrt{10}}T_{010}^{(27)}-2\frac{1}{\sqrt{30}}T_{000}^{(27)}\right],$$

$$T_S^{(U0,U+)}=-\frac{4}{3}\left(\frac{3}{10}\right)^{1/2}T_{1\frac{1}{2}-\frac{1}{2}}^{(8)}+2\frac{1}{\sqrt{6}}T_{1\frac{3}{2}-\frac{1}{2}}^{(27)}+2\frac{1}{\sqrt{30}}T_{1\frac{1}{2}-\frac{1}{2}}^{(27)},$$

$$T_S^{(U0,U-)}=-\frac{4}{3}\left(\frac{3}{10}\right)^{1/2}T_{-1\frac{1}{2}\frac{1}{2}}^{(8)}+2\frac{1}{\sqrt{6}}T_{-1\frac{3}{2}\frac{1}{2}}^{(27)}+2\frac{1}{\sqrt{30}}T_{-1\frac{1}{2}\frac{1}{2}}^{(27)},$$

$$T_S^{(U3,U3)}=-4\left[\left(\frac{3}{20}\right)^{1/2}T_{010}^{(8)}+\frac{1}{\sqrt{20}}T_{000}^{(8)}\right]+2\frac{1}{\sqrt{6}}T_{020}^{(27)}-6\frac{1}{\sqrt{10}}T_{010}^{(27)}+14\frac{1}{\sqrt{30}}T_{000}^{(27)}-4\frac{1}{\sqrt{8}}T_{000}^{(1)},$$

$$T_S^{(U3,U+)}=2\frac{1}{\sqrt{6}}T_{1\frac{3}{2}-\frac{1}{2}}^{(27)}-10\frac{1}{\sqrt{30}}T_{1\frac{1}{2}-\frac{1}{2}}^{(27)},$$

$$T_S^{(U3,U-)}=2\frac{1}{\sqrt{6}}T_{-1\frac{3}{2}\frac{1}{2}}^{(27)}-10\frac{1}{\sqrt{30}}T_{-1\frac{1}{2}\frac{1}{2}}^{(27)},$$

$$T_S^{(U+,U+)}=2T_{21-1}^{(27)},$$

$$T_S^{(U+,U-)}=-2\left[\left(\frac{3}{20}\right)^{1/2}T_{010}^{(8)}+\frac{1}{\sqrt{20}}T_{000}^{(8)}\right]+2\frac{1}{\sqrt{10}}T_{010}^{(27)}-3\frac{1}{\sqrt{30}}T_{000}^{(27)}-2\frac{1}{\sqrt{8}}T_{000}^{(1)},$$

$$T_S^{(U-,U-)}=2T_{-211}^{(27)},$$

$$T_S^{(0,0)}=T_{000}^{(11,1)},$$

$$T_S^{(0,U0)}=T_{010}^{(18,8)}+\frac{1}{\sqrt{3}}T_{000}^{(18,8)},$$

$$T_S^{(0,U3)}=T_{010}^{(18,8)}-\sqrt{3}T_{000}^{(18,8)},$$

$$T_S^{(0,U+)}=\sqrt{2}T_{1\frac{1}{2}-\frac{1}{2}}^{(18,8)},$$

$$T_S^{(0,U-)}=\sqrt{2}T_{-1\frac{1}{2}\frac{1}{2}}^{(18,8)}.$$

The antisymmetric tensors are given by

$$T_A^{(U0,U0)}=0,$$

$$T_A^{(U0,U3)}=-4\frac{1}{\sqrt{12}}\left(T_{010}^{(10)}+T_{010}^{(10*)}\right),$$

$$T_A^{(U0,U+)}=2\frac{1}{\sqrt{6}}\left(T_{1\frac{3}{2}-\frac{1}{2}}^{(10)}-T_{1\frac{1}{2}-\frac{1}{2}}^{(10*)}\right),$$

$$T_A^{(U0,U-)}=-2\frac{1}{\sqrt{6}}\left(T_{-1\frac{1}{2}\frac{1}{2}}^{(10)}-T_{-1\frac{3}{2}\frac{1}{2}}^{(10*)}\right),$$

$$T_A{}^{(U3,U3)}=0,$$

$$T_A{}^{(U3,U+)}=2\frac{1}{\sqrt{6}}(2T_{1\frac{1}{2}-\frac{1}{2}}{}^{(8a)}+T_{1\frac{3}{2}-\frac{1}{2}}{}^{(10)}+T_{1\frac{1}{2}-\frac{1}{2}}{}^{(10*)}),$$

$$T_A{}^{(U3,U-)}=-2\frac{1}{\sqrt{6}}(2T_{-1\frac{1}{2}\frac{1}{2}}{}^{(8a)}-T_{-1\frac{3}{2}\frac{1}{2}}{}^{(10)}-T_{-1\frac{3}{2}\frac{1}{2}}{}^{(10*)}),$$

$$T_A{}^{(U+,U+)}=0,$$

$$T_A{}^{(U+,U-)}=2\frac{1}{\sqrt{12}}(T_{010}{}^{(8a)}-T_{010}{}^{(10)}+T_{010}{}^{(10*)})-T_{000}{}^{(8a)},$$

$$T_A{}^{(U-,U-)}=0,$$

$$T_A{}^{(0,\beta)}=0,\quad \beta=U0,\ U3,\ U+,\ U,\ \text{and } 0.$$

APPENDIX B: RELATIONSHIPS AMONG COUPLING COEFFICIENTS

As pointed out in Sec. III, it is sometimes convenient to change bases from the Cartesian one to the spherical one and vice versa. We tabulate here the relationships among the coupling coefficients in the two bases.

$$G^{(U0,U0)}=\tfrac{3}{16}(3G^{(33)}+\sqrt{3}G^{(38)}+\sqrt{3}G^{(83)}+G^{(88)}),$$

$$G^{(U0,U3)}=\tfrac{1}{16}(3G^{(33)}-3\sqrt{3}G^{(38)}+\sqrt{3}G^{(83)}-3G^{(88)}),$$

$$G^{(U0,U+)}=\tfrac{1}{8}(3G^{(36)}-3iG^{(37)}+\sqrt{3}G^{(86)}-\sqrt{3}iG^{(87)}),$$

$$G^{(U0,U-)}=\tfrac{1}{8}(3G^{(36)}+3iG^{(37)}+\sqrt{3}^{(86)}+\sqrt{3}iG^{(87)}),$$

$$G^{(U3,U0)}=\tfrac{1}{16}(3G^{(33)}+\sqrt{3}G^{(38)}-3\sqrt{3}G^{(83)}-3G^{(88)}),$$

$$G^{(U3,U3)}=\tfrac{1}{16}(G^{(33)}-\sqrt{3}G^{(38)}-\sqrt{3}G^{(83)}+3G^{(88)}),$$

$$G^{(U3,U+)}=\tfrac{1}{8}(G^{(36)}-iG^{(37)}-\sqrt{3}G^{(86)}+\sqrt{3}iG^{(87)}),$$

$$G^{(U3,U-)}=\tfrac{1}{8}(G^{(36)}+iG^{(37)}-\sqrt{3}G^{(86)}-\sqrt{3}iG^{(87)}),$$

$$G^{(U+,U0)}=\tfrac{1}{8}(3G^{(63)}+\sqrt{3}G^{(68)}-3iG^{(73)}-\sqrt{3}iG^{(78)}),$$

$$G^{(U+,U3)}=\tfrac{1}{8}(G^{(63)}-iG^{(73)}-\sqrt{3}G^{(68)}+\sqrt{3}iG^{(78)}),$$

$$G^{(U+,U+)}=\tfrac{1}{4}(G^{(66)}-iG^{(67)}-iG^{(76)}-G^{(77)}),$$

$$G^{(U+,U-)}=\tfrac{1}{4}(G^{(66)}+iG^{(67)}-iG^{(76)}+G^{(77)}),$$

$$G^{(U-,U0)}=\tfrac{1}{8}(3G^{(63)}+3iG^{(73)}+\sqrt{3}G^{(68)}+\sqrt{3}iG^{(78)}),$$

$$G^{(U-,U3)}=\tfrac{1}{8}(G^{(63)}+iG^{(73)}-\sqrt{3}G^{(68)}-\sqrt{3}iG^{(78)}),$$

$$G^{(U-,U+)}=\tfrac{1}{4}(G^{(66)}-iG^{(67)}+iG^{(76)}+G^{(77)}),$$

$$G^{(U-,U-)}=\tfrac{1}{4}(G^{(66)}+iG^{(67)}+iG^{(76)}-G^{(77)}),$$

$$G^{(0,U0)}=\tfrac{1}{4}(3G^{(03)}+\sqrt{3}G^{(08)}),$$

$$G^{(0,U3)}=\tfrac{1}{4}(G^{(03)}-\sqrt{3}G^{(08)}),$$

$$G^{(0,U+)}=\tfrac{1}{2}(G^{(06)}-iG^{(07)}),$$

$$G^{(0,U-)}=\tfrac{1}{2}(G^{(06)}+iG^{(07)}),$$

$$G^{(U0,0)}=\tfrac{1}{4}(3G^{(30)}+\sqrt{3}G^{(80)}),$$

$$G^{(U3,0)}=\tfrac{1}{4}(G^{(30)}-\sqrt{3}G^{(80)}),$$

$$G^{(U+,0)}=\tfrac{1}{2}(G^{(60)}-iG^{(70)}),$$

$$G^{(U-,0)}=\tfrac{1}{2}(G^{(60)}+iG^{(70)});$$

$$g^{(U0,0)}=\tfrac{3}{4}g^{(30)}+\frac{\sqrt{3}}{4}g^{(80)},$$

$$g^{(U3,0)}=\tfrac{1}{4}g^{(30)}-\frac{\sqrt{3}}{4}g^{(80)},$$

$$g^{(U+,0)}=\tfrac{1}{2}g^{(60)}-\tfrac{1}{2}ig^{(70)},$$

$$g^{(U-,0)}=\tfrac{1}{2}g^{(60)}+\tfrac{1}{2}ig^{(70)},$$

and likewise for $g^{(\alpha,3)}$.

APPENDIX C: STRUCTURE OF NONLEPTONIC HAMILTONIAN

1. Nonleptonic Processes: $\Delta S=0$

The weak-interaction effects in the nonleptonic processes are observable only to the extent that they are not masked by the strong or electromagnetic interactions. For the $\Delta S=0$ nonleptonic processes, such an observable effect can arise in the parity-violating contribution of H_W to the nuclear potential. This small effect then manifests itself in transitions between nuclear levels in such a fashion as to render a circular polarization to the γ rays emitted from unpolarized states or to permit an α-particle emission that would be otherwise forbidden.

a. $^{181}\text{Ta}^* \rightarrow {}^{181}\text{Ta}+\gamma(482\ \text{keV})$

In this γ transition, the parity admixture in the nuclear states results in an interference between the regular $M1$ and irregular $E1$ transitions leading to a circular polarization of the γ rays. Several theoretical estimates of the effect of $H_{NL}{}^{(ps)}$ have been made, notably by McKellar[33] and by others.[22] The major contributions to the parity-nonconserving NN potential are assumed to arise from pion exchange and W-boson exchange, where the latter is expressed in terms of ρ dominance.

The calculations are very sensitive to the π and ρ contributions to the effective potential. In particular, only the $\Delta I=1$ part of $H_{NL}{}^{(ps)}$ contributes to the weak πNN vertex if CP is conserved. Present experiments imply that this contribution must be small. In the Cabibbo theory, it enters only as $\sin^2\theta$, as may be seen directly from Eq. (3.10). Any neutral current contribution must hence be similarly suppressed. Calculation shows that the neutral current contribution to the $\Delta I=1$

part of $H_{NL}{}^{(ps)}$ is given by

$$H_{NL}{}^{(ps)}(\Delta I=1)=\frac{G}{\sqrt{2}}\Big\{-[\tfrac{2}{3}\sqrt{3}(G_{--}{}^{(38)}-G_{++}{}^{(38)})-2(G_{--}{}^{(U+,U-)}-G_{++}{}^{(U+,U-)})]\left(\frac{3}{20}\right)^{1/2}T_{010}{}^{(8)}(VA+AV)$$
$$-[\sqrt{3}(G_{--}{}^{(38)}-G_{++}{}^{(38)})+2(G_{--}{}^{(U+,U-)}-G_{++}{}^{(U+,U-)})]\frac{1}{\sqrt{10}}T_{010}{}^{(27)}(VA+AV)$$
$$+2(G_{-+}{}^{(U+,U-)}-G_{+-}{}^{(U+,U-)})\frac{1}{\sqrt{12}}T_{010}{}^{(8a)}(VA-AV)+[\sqrt{3}(G_{-+}{}^{(38)}-G_{+-}{}^{(38)})$$
$$-2(G_{-+}{}^{(U+,U-)}-G_{+-}{}^{(U+,U-)})]\frac{1}{\sqrt{12}}[T_{010}{}^{(10)}(VA-AV)-T_{010}{}^{(10*)}(VA-AV)]\Big\}. \quad \text{(C1)}$$

b. $^{16}O^{*}\rightarrow{}^{12}C+\alpha$

In this reaction, one looks for α decay of the 2^- excited state in ^{16}O at 8.88 MeV. Detection of this reaction would signify a parity-violating contribution to the nuclear potential since the decay would proceed via $2^-\rightarrow 0^+ + 0^+$, a clear indication of parity nonconservation. Moreover, the transition would proceed via the $\Delta I=0$ part of the weak Hamiltonian. The contribution to $H_{NL}(\Delta I=0)$ from the charged currents enters like $\cos^2\theta$ and so represents a large effect which the neutral currents alter to some extent. Calculation again reveals that the neutral current contribution is given by

$$H_{NL}{}^{(ps)}(\Delta I=0)=\frac{G}{\sqrt{2}}\Big\{-[(G_{--}{}^{(33)}-G_{++}{}^{(33)})-(G_{--}{}^{(88)}-G_{++}{}^{(88)})-2(G_{--}{}^{(U+,U-)}-G_{++}{}^{(U+,U-)})]$$
$$\times\frac{1}{\sqrt{20}}T_{000}{}^{(8)}(VA+AV)-[\tfrac{1}{4}(G_{--}{}^{(33)}-G_{++}{}^{(33)})+\frac{9}{4}(G_{--}{}^{(88)}-G_{++}{}^{(88)})$$
$$-3(G_{--}{}^{(U+,U-)}-G_{++}{}^{(U+,U-)})]\frac{1}{\sqrt{30}}T_{000}{}^{(27)}(VA+AV)+[\tfrac{1}{2}(G_{--}{}^{(33)}-G_{++}{}^{(33)})$$
$$+\tfrac{1}{2}(G_{--}{}^{(88)}-G_{++}{}^{(88)})+2(G_{--}{}^{(U+,U-)}-G_{++}{}^{(U+,U-)})]\frac{1}{\sqrt{8}}T_{000}{}^{(1)}(VA+AV)$$
$$-(G_{-+}{}^{(U+,U-)}-G_{+-}{}^{(U+,U-)})T_{000}{}^{(8a)}(VA-AV)\Big\}. \quad \text{(C2)}$$

2. Nonleptonic Processes: $|\Delta S|=1$

Important examples of these transitions are the nonleptonic K decays such as $K\rightarrow 2\pi$, 3π and nonleptonic hyperon decays $Y\rightarrow N\pi$ or $\Xi\rightarrow\Lambda\pi$. In the Cabibbo theory with only charged currents, the weak vertices involve just the $\sin\theta\cos\theta$ terms of Eq. (3.10). To the extent that neutral currents are also present, they alter the transition amplitudes accordingly; moreover, they can provide a natural framework for CP violation in $K_L{}^0\rightarrow\pi\pi$ with a suitable choice of coefficients.

The following combination of $\Delta Y=\pm 1$ tensor operators enters into H_{NL} via the neutral currents:

$$H_{NL}(\Delta Y=\pm 1)=\frac{G}{\sqrt{2}}\Big\{G^{(U0,U+)}\left[-\frac{4}{3}\left(\frac{3}{10}\right)^{1/2}T_{1\frac{1}{2}-\frac{1}{2}}{}^{(8)}+2\frac{1}{\sqrt{6}}T_{1\frac{3}{2}-\frac{1}{2}}{}^{(27)}+2\frac{1}{\sqrt{30}}T_{1\frac{1}{2}-\frac{1}{2}}{}^{(27)}\right]$$
$$+G^{(U0,U-)}\left[-\frac{4}{3}\left(\frac{3}{10}\right)^{1/2}T_{-1\frac{1}{2}\frac{1}{2}}{}^{(8)}+2\frac{1}{\sqrt{6}}T_{-1\frac{3}{2}\frac{1}{2}}{}^{(27)}+2\frac{1}{\sqrt{30}}T_{-1\frac{1}{2}\frac{1}{2}}{}^{(27)}\right]$$
$$+G^{(U3,U+)}\left[2\frac{1}{\sqrt{6}}T_{1\frac{3}{2}-\frac{1}{2}}{}^{(27)}-10\frac{1}{\sqrt{30}}T_{1\frac{1}{2}-\frac{1}{2}}{}^{(27)}\right]+G^{(U3,U-)}\left[2\frac{1}{\sqrt{6}}T_{-1\frac{3}{2}\frac{1}{2}}{}^{(27)}-10\frac{1}{\sqrt{30}}T_{-1\frac{1}{2}\frac{1}{2}}{}^{(27)}\right]$$
$$+(G_{-+}{}^{(U0,U+)}-G_{+-}{}^{(U0,U+)})2\frac{1}{\sqrt{6}}[T_{1\frac{3}{2}-\frac{1}{2}}{}^{(10)}-T_{1\frac{1}{2}-\frac{1}{2}}{}^{(10*)}]-(G_{-+}{}^{(U0,U-)}-G_{+-}{}^{(U0,U-)})$$
$$\times 2\frac{1}{\sqrt{6}}[T_{-1\frac{1}{2}\frac{1}{2}}{}^{(10)}-T_{-1\frac{3}{2}\frac{1}{2}}{}^{(10*)}]+(G_{-+}{}^{(U3,U+)}-G_{+-}{}^{(U3,U+)})2\frac{1}{\sqrt{6}}[2T_{1\frac{1}{2}-\frac{1}{2}}{}^{(8a)}+T_{1\frac{3}{2}-\frac{1}{2}}{}^{(10)}+T_{1\frac{1}{2}-\frac{1}{2}}{}^{(10*)}]$$
$$-(G_{-+}{}^{(U3,U-)}-G_{+-}{}^{(U3,U-)})2\frac{1}{\sqrt{6}}[2T_{-1\frac{1}{2}\frac{1}{2}}{}^{(8a)}-T_{-1\frac{1}{2}\frac{1}{2}}{}^{(10)}-T_{-1\frac{3}{2}\frac{1}{2}}{}^{(10*)}]\Big\}, \quad \text{(C3)}$$

where the symmetric tensors must be summed over $T_S(VV+AA)$, $T_S(VV-AA)$, and $T_S(VA+AV)$ with the G's understood to be the combinations $(G_{--}+G_{++})$, $(G_{-+}+G_{+-})$, and $-(G_{--}-G_{++})$, respectively.

3. Nonleptonic Processes: $|\Delta S|=2$

The Cabibbo form of the weak Hamiltonian given in Eq. (3.10) contains no $\Delta S=\pm 2$ tensor components. Such components may enter through the neutral currents in the form of the **27** contributions $G^{(U+,U+)}T_{21-1}{}^{(27)}$ and $G^{(U-,U-)}T_{-211}{}^{(27)}$; however, the absence of any observed $|\Delta S|=2$ transitions such as $\Xi\to N\pi$ imposes rather severe restrictions on these coupling coefficients.

APPENDIX D: DECAY AND SCATTERING AMPLITUDES

In this appendix we write out in detail in terms of the coupling coefficients the structure of various decay and scattering amplitudes where neutral currents might play a role.

1. Leptonic Processes

The prototype for all leptonic processes is muon decay, $\mu^-\to e^-+\nu_\mu+\bar\nu_e$, which involves only charged currents. The decay matrix element can be written as

$$M(\mu^-\to e^-\nu_\mu\bar\nu_e)=\frac{G}{\sqrt2}\bar\nu_\mu\gamma_\lambda(1+\gamma_5)u_\mu\bar u_e\gamma_\lambda(1+\gamma_5)\nu_e{}^c. \quad \text{(D1)}$$

a. $\nu_\mu+e^-\to\nu_\mu+e^-$

This process occurs only via neutral currents to lowest order in G, if electromagnetic effects are ignored. To the extent that the neutrinos in question are purely left-handed, the scattering amplitude can be written as

$$\begin{aligned}M(\nu_\mu+e^-\to\nu_\mu+e^-)&=\frac{G}{\sqrt2}\tfrac12\bar u_e\gamma_\lambda[(f_{--}{}^{(00)}-f_{--}{}^{(33)})(1+\gamma_5)\\&\quad+(f_{+-}{}^{(00)}+f_{+-}{}^{(03)}-f_{+-}{}^{(30)}-f_{+-}{}^{(33)})\\&\quad\times(1-\gamma_5)]u_e\bar\nu_\mu\gamma_\lambda(1+\gamma_5)\nu_\mu. \quad \text{(D2)}\end{aligned}$$

b. $\nu_e+e^-\to\nu_e+e^-$

Here both charged and neutral currents can occur to lowest order in G. Limiting our attention to left-handed neutrinos, the matrix element in question is given by

$$\begin{aligned}M(\nu_e+e^-\to\nu_e+e^-)&=\frac{G}{\sqrt2}[\bar\nu_e\gamma_\lambda(1+\gamma_5)u_e\bar u_e\gamma_\lambda(1+\gamma_5)\nu_e\\&\quad-\tfrac12(f_{--}{}^{(00)}-f_{--}{}^{(33)})\bar u_e\gamma_\lambda(1+\gamma_5)u_e\\&\quad\times\bar\nu_e\gamma_\lambda(1+\gamma_5)\nu_e-\tfrac12(f_{+-}{}^{(00)}+f_{+-}{}^{(03)}-f_{+-}{}^{(30}\\&\quad-f_{+-}{}^{(33)})\bar u_e\gamma_\lambda(1-\gamma_5)u_e\bar\nu_e\gamma_\lambda(1+\gamma_5)\nu_e]. \quad \text{(D3)}\end{aligned}$$

2. Semileptonic Processes: $\Delta S=0$

Here neutron β decay serves as the prototype for all semileptonic processes, and the matrix element is given by

$$M(n\to pe^-\bar\nu_e)=\frac{G}{\sqrt2}\cos\theta\times\langle p|(V-A)_\lambda|n\rangle\bar u_e\gamma_\lambda(1+\gamma_5)\nu_e{}^c, \quad \text{(D4)}$$

with

$$\langle p|(V-A)_\lambda|n\rangle=\bar u_p\left(F_1{}^V\gamma_\lambda-\frac{\Delta\mu}{2m}F_2{}^V\sigma_{\lambda\rho}q_\rho+g_AF_1{}^V\gamma_\lambda\gamma_5-iF_2{}^A\gamma_5q_\lambda\right)u_n,$$

where $g_A\simeq1.23$ and $q_\lambda=(p-n)_\lambda$. The above matrix element is conveniently written in terms of F- and D-type (antisymmetric and symmetric) reduced matrix elements for the general charged current transition according to

$$\begin{aligned}\langle B_\alpha|(V-A)_\lambda{}^{(+)}|B_\beta\rangle&=\bar u_\alpha\Big[f_{\alpha\beta}F\left(F_1{}^V\gamma_\lambda-\frac{\Delta\mu}{2m}F_2{}^V\sigma_{\lambda\rho}q_\rho\right)\\&\quad+(f_{\alpha\beta}'F'+d'_{\alpha\beta}D')\left(F_1{}^A\gamma_\lambda-i\frac{F_2{}^A}{g_A}q_\lambda\right)\gamma_5\Big]u_\beta, \quad \text{(D5)}\end{aligned}$$

where for neutron decay, $f_{pn}=f_{pn}'=d_{pn}'=1$, and the best present values are $F=1$, $F'=0.49$, and $D'=0.74$.

a. $\nu_l+p\to\nu_l+p$

This elastic scattering process can proceed in first-order weak interactions solely by neutral currents. The matrix element can be written in terms of (D5) as follows:

$$M(\nu_l+p\to\nu_l+p)=\frac{G}{\sqrt2}\bar u_p[\quad]u_p\bar\nu_l\gamma_\lambda(1+\gamma_5)\nu_l, \quad \text{(D6)}$$

with

$$\begin{aligned}4f_{pp}&=\xi(g_{--}{}^{(30)}+g_{--}{}^{(33)}+g_{+-}{}^{(30)}+g_{+-}{}^{(33)})\\&\quad+\sqrt3(g_{--}{}^{(80)}+g_{--}{}^{(83)}+g_{+-}{}^{(80)}+g_{+-}{}^{(83)}),\\4f_{pp}'&=\xi(g_{--}{}^{(30)}+g_{--}{}^{(33)}-g_{+-}{}^{(30)}-g_{+-}{}^{(33)})\\&\quad+\sqrt3(g_{--}{}^{(80)}+g_{+-}{}^{(83)}-g_{+-}{}^{(80)}-g_{+-}{}^{(83)}),\\4d_{pp}'&=\xi(g_{--}{}^{(30)}+g_{--}{}^{(33)}-g_{+-}{}^{(30)}-g_{+-}{}^{(33)})\\&\quad-(1/\sqrt3)(g_{--}{}^{(80)}+g_{--}{}^{(83)}-g_{+-}{}^{(80)}-g_{+-}{}^{(83)}),\end{aligned}$$

and $\xi=+1$. We have considered only left-handed neutrinos, but allowed for the possibility of both $V-A$ and $V+A$ neutral hadron currents.

b. $\nu_l+n\to\nu_l+n$

For this elastic process, the matrix element is analogous to (D6) with the replacement $\xi=-1$.

Additional $\Delta S=0$ semileptonic processes of interest are weak-pion production. This proceeds predominantly through N^* production and subsequent decay, such that the matrix element for $\nu_l+n\to N^{*+}+l^-$ may be written as

$$M(\nu_l+n\to N^{*+}+l^-)$$
$$=\frac{G}{\sqrt{2}}\bar{u}_\rho\left\{\left[F_1{}^V\delta_{\rho\lambda}+\frac{F_2{}^V ip_{1\rho}\gamma_\lambda}{M+M^*}+\frac{F_3{}^V p_{1\rho}(p_1+p_2)_\lambda}{(M+M^*)^2}+\frac{F_4{}^V p_{1\rho}(p_1-p_2)_\lambda}{(M+M^*)^2}\right]\gamma_5+\left[F_1{}^A\delta_{\rho\lambda}+\frac{F_2{}^A ip_{1\rho}\gamma_\lambda}{M+M^*}+\cdots\right]\right\}u_n\bar{u}_l\gamma_\lambda(1+\gamma_5)\nu_l. \quad \text{(D7)}$$

c. $\nu_l+p\to N^{*+}+\nu_l$

This process occurs in lowest order via neutral currents with a matrix element analogous to (D7) given by

$$M(\nu_l+p\to N^{*+}+\nu_l)$$
$$=-\frac{G}{2\sqrt{2}}\bar{u}_\rho\{(g_{--}{}^{(30)}+g_{--}{}^{(33)}+g_{+-}{}^{(30)}+g_{+-}{}^{(33)})\times[F_1{}^V\delta_{\rho\lambda}+\cdots]\gamma_5+(g_{--}{}^{(30)}+g_{--}{}^{(33)}-g_{+-}{}^{(30)}-g_{+-}{}^{(33)})[F_1{}^A\delta_{\rho\lambda}+\cdots]\}u_p\bar{\nu}_l\gamma_\lambda(1+\gamma_5)\nu_l. \quad \text{(D8)}$$

Note that this process is sensitive only to the isovector neutral current, unlike the elastic process of parts *a* and *b*.

3. Semileptonic Processes: $|\Delta S|=1$

Two examples involving charged currents are the $K_{\mu 2}$ and K_{e3} decays. Their matrix elements can be expressed as

$$M(K^+\to\mu^+\nu_\mu)=\frac{G}{\sqrt{2}}\sin\theta\, F_K\bar{\nu}_\mu\gamma\cdot p_K(1+\gamma_5)v_\mu, \quad \text{(D9)}$$

and

$$M(K^+\to\pi^0e^+\nu_e)=\frac{G}{\sqrt{2}}\sin\theta\, 2f_+\bar{\nu}_e\gamma\cdot p(1+\gamma_5)v_e. \quad \text{(D10)}$$

a. $K_L{}^0\to l^++l^-$

To lowest order in G, this type of process can occur via the neutral axial-vector current with the general matrix element

$$M(K_L{}^0\to l^+l^-)=\frac{G}{\sqrt{2}}\frac{1}{2\sqrt{2}}F_K\bar{u}_l i\gamma\cdot p_K[(g_{--}{}^{(70)}-g_{--}{}^{(73)}-g_{+-}{}^{(70)}+g_{+-}{}^{(73)})(1+\gamma_5)-(g_{++}{}^{(70)}-g_{++}{}^{(73)}-g_{-+}{}^{(70)}+g_{-+}{}^{(73)})(1-\gamma_5)]v_l. \quad \text{(D11)}$$

Only the seventh component of the axial-vector currents enter since $K_L{}^0$ is taken to be CP odd.

b. $K^+\to\pi^++\nu_l+\bar{\nu}_l$

This decay mode can occur by lowest-order weak interactions involving strangeness-changing neutral currents. The matrix element is found to be

$$M(K^+\to\pi^+\nu_l\bar{\nu}_l)=\frac{G}{\sqrt{2}}\sqrt{2}f_+\bar{\nu}_l\gamma\cdot p_K(g_{--}{}^{(U-,0)}+g_{--}{}^{(U-,3)}+g_{+-}{}^{(U-,0)}+g_{+-}{}^{(U-,3)})(1+\gamma_5)\nu_l{}^c, \quad \text{(D12)}$$

where only left-handed neutrinos are considered. In computing the rate, one should sum over both the electron- and muon-type neutrinos since they are indistinguishable so far as the K^+ decay is concerned.

c. $K^+\to\pi^++e^++e^-$

This charged lepton counterpart to the above process occurs in lowest-order weak interactions via the matrix element

$$M(K^+\to\pi^+e^+e^-)$$
$$=\frac{G}{\sqrt{2}}\sqrt{2}f_+\bar{u}_e\gamma\cdot p_K[(g_{--}{}^{(U-,0)}-g_{--}{}^{(U-,3)}+g_{+-}{}^{(U-,0)}-g_{+-}{}^{(U-,3)})(1+\gamma_5)+(g_{-+}{}^{(U-,0)}-g_{-+}{}^{(U-,3)}+g_{++}{}^{(U-,0)}-g_{++}{}^{(U-,3)})(1-\gamma_5)]v_e. \quad \text{(D13)}$$