TABLE IV. Comparison between the results of this experiment at the (5.8°, 2.25 GeV/c) point, the results of the experiment of Cohen *et al.*, and the various theoretical predictions for the Compton contribution due to the ρ meson.

Source	Description	I $\left(\frac{\text{n6}}{(\text{MeV})\ (\text{sr}^2)}\right)$
Experiment	Cohen *et al.*	$-18.0\ \pm 8.0$
	This result	$+\ 1.76 \pm 4.7$
Theory	Jackson width, $\psi=0°$	-0.18
	Jackson width, $\psi=15°$	-0.98
	Jackson width, $\psi=-37°$	$+1.76$
	120-MeV constant width, $\psi_\rho=0°$	-0.80
	Jackson width, Ross-Stodolsky factor,[18] $\psi_\rho=-51°$	$+6.49$

nism gives at 460 MeV/c^2 an imaginary amplitude even with proposed corrections.[24]

The Compton amplitude used here conserves t-channel helicity[21] in contrast with the results for ρ photoproduction.[21,22] The 25° difference between the t-channel and s-channel axes at the extremes of our acceptance is not sufficient to disturb the helicity overlap function of Eq. (17).

[24] J. Pumplin, this issue, Phys. Rev. D 2, 1859 (1970).

These results for the interference cross section indicate a negligible Compton contribution to the electron pair yields of Ref. 6. At (5.8°, 2.25 GeV/c) the 68% confidence level limit on a dispersive Compton cross section for electron pair production is 0.01% of the Bethe-Heitler cross section.

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Diffraction Dissociation and the Reaction $\gamma p \to \pi^+\pi^- p$*

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The diffraction dissociation process for the reaction $\gamma p \to \pi^+\pi^- p$ is analyzed in detail. Much of the analysis is relevant to reactions initiated by hadrons. A procedure is suggested for adding nonresonant background (the "Söding term") to ρ^0 production without double counting, and numerical calculations are presented.

I. INTRODUCTION

In this paper I wish to consider diffractive processes in which a high-energy photon dissociates into hadrons, as a consequence of its coupling to them, and the resulting virtual hadronic system is brought onto the mass shell by elastic scattering on a target particle, which provides the necessary slight change in longitudinal momentum. In particular, I will discuss the "Drell-Söding mechanism"[1] for $\gamma p \to \pi^+\pi^- p$, illustrated in Fig. 1, and some corrections to it. Similar considerations apply to $\gamma p \to K^+K^- p$, $\gamma p \to \bar{p}pp$, etc., and also to reactions like $\gamma p \to A_2^+\pi^- p$ which involve strongly unstable particles in the final state. Diffractive processes in which the photon dissociates into more than two particles, or in which the target particle also dissociates, will not be discussed here.

The work is motivated by the experimental accessability of reactions like $\gamma p \to \pi^+\pi^- p$, together with the belief that diffractive processes will dominate them at high energy. In addition, I hope to clarify some general aspects of diffractive two-particle $\to$ three-particle processes, which are applicable as well to hadron-induced reactions like $\pi^\pm p \to \pi^\pm \rho^0 p$ or $pp \to \pi^+ np$. The reaction $\gamma p \to \pi^+\pi^- p$ is particularly suited for testing the basic idea of diffraction dissociation, because of the simplicity which results from the pion having spin zero, because of the smallness of the pion mass, which allows the propagator pole to approach very close to the physical region, and because good data can be obtained

* Work supported by the U. S. Atomic Energy Commission.
[1] S. Drell, Rev. Mod. Phys. 33, 458 (1961); P. Söding, Phys. Letters 19, 702 (1966).

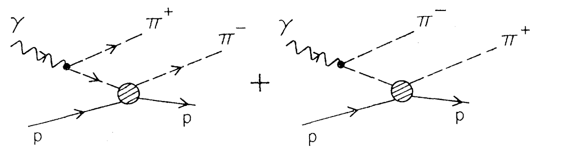


FIG. 1. Feynman diagrams for the diffraction dissociation $\gamma p \to \pi^+\pi^- p$ ("Drell-Söding mechanism"). A third diagram, which contains no pion pole, is required by gauge invariance, as discussed in the text.

for this reaction—including data with polarized photons.[2]

Much of the current interest in $\gamma p \to \pi^+\pi^- p$ centers on the copiously produced ρ^0 resonance, and on the so-far fruitless search for new resonances in the $\pi^+\pi^-$ system: in particular, the $J^P=3^-$ recurrence of ρ^0 predicted by Regge-pole theory,[3] and the $J^P=1^-$ daughter of f^0 predicted by simple forms of the Veneziano model.[4] The model diagrammed in Fig. 1 can be applied directly only to mass regions, or partial waves, of the diffractively produced system in which resonances are unimportant. Where resonances are important, the model can be looked upon as generating a "background" term, which is to be added to the resonance production amplitude, along with an appropriate correction for "double counting," as discussed in Sec. V. Alternatively, it may be possible to incorporate resonance production into the model, using the theory of final-state interactions.[5]

The possibility of diffractive production of hadrons was first set forth by Feinberg and Pomeranchuk,[6] by analogy with coherent bremsstrahlung. The idea was further developed by Good and Walker.[7] Diagrams like Fig. 1 were proposed by Drell[1] as promising contributors to secondary beams at SLAC, and were later calculated by Söding[1] for $\gamma p \to \pi^+\pi^- p$ as a background to diffractive ρ^0 production, in an attempt to understand the observed asymmetry of the ρ mass spectrum. Söding's calculation, and a refined version of it by Krass,[8] ignores the question of double counting, which is discussed in Sec. V.

The Drell-Söding mechanism can be looked upon as a diffraction dissociation process, similar to those possible in $\pi p \to \pi p p$,[9] $Kp \to \pi K^* p$,[10] and $pp \to pn\pi^+$.[11] It can also be considered as a special case of the multi-Regge model,[12] which has come to be interpreted as applying even when not all of the final subenergies are large. For $\gamma p \to \pi^+\pi^- p$, the effect of multi-Reggeism would be to introduce a factor $s'^{\alpha(t')}$ and a signature factor into the amplitude, where $s'=m_{\pi\pi}{}^2$ and t' is the squared four-momentum of the virtual pion. Because the pion has spin zero and very small mass, this has little consequence at small t'. Another effect of the multi-Regge model is to introduce form factors (residue functions) which depend on t', but that is not unique to the Regge model: Bare Feynman propagators in general must be reduced, when the propagating particle is far off the mass shell, and only low-order diagrams are being kept on the basis of "nearby singularities" arguments rather than on the basis of perturbation theory (see Sec. V).

II. KINEMATICS IN JACKSON FRAME

For the reaction $\gamma p \to \pi^+\pi^- p$, denote the four-momentum of the photon by k; of the initial and final protons by p, p'; and of the pions by q_+, q_-. The Jackson frame is defined as the rest frame of the pion pair, with the z axis in the direction of the photon:

$$\begin{aligned} k&=(0,0,k;k),\\ p&=(p_x,0,p_z;E),\\ q_+&=(q\sin\theta\cos\phi,\, q\sin\theta\sin\phi,\, q\cos\theta;\omega),\\ q_-&=(-q\sin\theta\cos\phi,\, -q\sin\theta\sin\phi,\, -q\cos\theta;\omega),\\ p'&=(p_x,0,p_z';E'), \end{aligned} \tag{1}$$

where $\omega=(q^2+m_\pi{}^2)^{1/2}$, $E=(p_x{}^2+p_z{}^2+m_p{}^2)^{1/2}$, $p_z'=k+p_z$, and energy conservation implies $p_z=E-2\omega+2\omega(\omega-E)/k$, $E'=k+E-2\omega$.

A convenient choice for the five Lorentz-invariant kinematic variables are $s=(k+p)^2$, the total c.m. energy squared; $t=(p-p')^2$, the four-momentum transfer to the proton squared; $m^2=(q_++q_-)^2$, the invariant mass of the $\pi\pi$ system squared; and θ and ϕ are the decay angles of the two-pion system in the Jackson frame. The Jackson-frame variables are related to the invariants by

$$\begin{aligned} \omega&=\tfrac{1}{2}m,\\ k&=\omega-t/4\omega,\\ E&=(s-m_p{}^2+t)/4\omega,\\ p_z&=E+(m_p{}^2-s)2/k,\\ p_x&=(E^2-p_z{}^2-m_p{}^2)^{1/2}. \end{aligned} \tag{2}$$

[2] H. Bingham, W. Fretter, K. Moffeit, W. Podolsky, M. Rabin, A. Rosenfeld, R. Windmolders, J. Ballam, G. Chadwick, R. Gearhart, Z. Guiragossián, M. Menke, J. Murray, P. Seyboth, A. Shapira, C. Sinclair, I. Skillicorn, G. Wolf, and R. Milburn, Phys. Rev. Letters **24**, 955 (1970). I wish to thank I. Skillicorn and J. Ballam for discussions about these data prior to publication.

[3] D. Crennell, P. Hough, G. Kalbfleisch, Kwan Wu Lai, J. Scarr, T. Schumann, I. Skillicorn, R. Strand, M. Webster, P. Baumel, A. Bachman, and R. Lea, Phys. Rev. Letters **18**, 323 (1967).

[4] J. Shapiro, Phys. Rev. **179**, 1345 (1969).

[5] G. Kramer and J. Uretsky, Phys. Rev. **181**, 1918 (1969), treat the ρ^0 production as entirely due to a final-state enhancement. Their model would be more in keeping with final-state interaction theory if the normalization of the resonance term were considered arbitrary (in a naive model it depends on the behavior of the $\pi\pi$ phase shift at infinite energy); their procedure for normalizing it is unconvincing, and in poor agreement with experiment. For a final-state interaction treatment of N^* production, see J. Rushbrooke, *ibid.* **177**, 2357 (1969).

[6] E. Feinberg and I. Pomeranchuk, Nuovo Cimento Suppl. **3**, 652 (1956).

[7] M. Good and W. Walker, Phys. Rev. **120**, 1857 (1960).

[8] A. Krass, Phys. Rev. **159**, 1496 (1967).

[9] S. Drell and K. Hiida, Phys. Rev. Letters **7**, 199 (1961); R. Deck, *ibid.* **13**, 169 (1964); M. Ross and Y. Yam, *ibid.* **19**, 546 (1967); **19**, 940(E) (1967).

[10] M. S. Farber *et al.*, Phys. Rev. Letters **22**, 1394 (1969).

[11] L. Resnick, Phys. Rev. **175**, 2185 (1968).

[12] E. Berger, Phys. Rev. **166**, 1525 (1968); S. Chung *et al.*, *ibid.* **182**, 1443 (1969).

In order to obtain qualitative understanding, it will be useful to consider the limit of high energy: $s=m_p^2+2m_pE_\gamma^{(\mathrm{lab})}\to\infty$. In that limit, to leading order in s,

$$\begin{aligned}&E\cong E'\cong s/4\omega\,,\\&p_z\cong p'\cong(s/4\omega)(t+4\omega^2)/(t-4\omega^2)\,,\\&p_x\cong s(-t)^{1/2}/(4\omega^2-t)\,.\end{aligned}\tag{3}$$

The momenta of the initial and final target protons become infinite and equal, while the momentum of the photon and its angle with respect to the proton direction remain finite.

III. DRELL-SÖDING AMPLITUDE

The diagrams of Fig. 1 correspond to a matrix element for $\gamma p\to\pi^+\pi^-p$ of

$$\begin{aligned}\mathfrak{M}=e[&(\epsilon\cdot q_+/k\cdot q_+)T_-(k-q_+,\,p\to q_-,\,p')\\&-(\epsilon\cdot q_-/k\cdot q_-)T(k-q_-,\,p\to q_+,\,p')+\epsilon\cdot V],\end{aligned}\tag{4}$$

where ϵ is the photon polarization vector, and T_+, T_- are pion-nucleon elastic scattering amplitudes for initial-state pions which are slightly off the mass shell. Throughout this paper, the target proton will be treated as spinless; this approximation is not necessary, but should be acceptable, since we are only interested in small momentum transfer elastic scattering from it. Corrections to Eq. (4) due to off-mass-shell effects, and other effects, are discussed in Sec. V.

A term of the form $\epsilon\cdot V$, where V is some four-vector satisfying $T_--T_++k\cdot V=0$ is required in Eq. (4), in order to make the expression invariant under gauge transformations, $\epsilon\to\epsilon+\lambda k$. At first look, V could contain terms proportional to each of the available four-vectors k, p, q_1, q_2, and p'. However, we have already let the photon couple to the pion charge in the way we want it to physically, and $\epsilon\cdot k=0$, so V must be some combination of p and p'. The combination $p-p'$ is also unacceptable since $p-p'=q_++q_--k$. In this paper, I use $V\propto p+p'$, i.e., I let the additional photon coupling be to the current of the scalar "nucleon." Adjusting the magnitude of V to provide gauge invariance,

$$\mathfrak{M}=e\left[\frac{\epsilon\cdot q_+}{k\cdot q_+}T_--\frac{\epsilon\cdot q_-}{k\cdot q_-}T_++\frac{\epsilon\cdot(p+p')}{k\cdot(p+p')}(T_+-T_-)\right].\tag{5}$$

Krass[8] chooses the transverse gauge for the photon in the center-of-momentum frame, which is equivalent to $V\propto p$. I know of no strong argument to choose between this and $p+p'$, but the difference is not very important, since the Drell-Söding process is confined to the diffractive region, where $(p-p')^2$ is small; and the two prescriptions are equal in the high-energy limit.

The pion-nucleon elastic amplitudes can be obtained from measured differential cross sections, by ignoring spin and making assumptions about the phase and off-mass-shell dependence. The effective energy and momentum transfer values can be taken from the final particles, which are on the mass shell.

For simplicity of calculation, I use the forms

$$\begin{aligned}T_\pm(k&-q_\mp,\,p\to q_\pm,\,p')\\&=2i\sigma_\pm[(q_\pm\cdot p')^2-m_\pi^2m_p^2]^{1/2}e^{B_\pm t/2},\end{aligned}\tag{6}$$

which are pure imaginary in phase, and correspond on the mass shell to constant diffraction peaks, $d\sigma/dt\propto e^{B_\pm t}$, and constant total $\pi^\pm p$ cross sections, $\sigma_\pm$. When the invariant mass of one of the πp systems is low, as happens when the angle θ is near 0 or π even at fairly high photon laboratory energies, resonances can be important, and Eq. (6) is not such a good approximation. In comparing with experiment, it would probably be best to make a cut on the data to require that both πN invariant masses be reasonably large—at least to eliminate the region of $\Delta(1236)$. One could separately ask whether diagrams like Fig. 1, calculated with detailed π-N amplitudes obtained from phase-shift analysis, would be capable of explaining nondiffractive processes such as $\gamma p\to\pi^-\Delta^{++}$, but the theoretical basis would then be somewhat different.

Since gauge invariance has been enforced, the polarization vectors can be chosen in any convenient manner. Using $\epsilon_x=(1,0,0;0)$, which corresponds to photons with electric vector lying in the scattering plane of the Jackson frame, and $\epsilon_y=(0,1,0;0)$, with electric vector perpendicular to it, the spin-averaged cross section is

$$\frac{d\sigma}{dt\,dm\,d\Omega}=\frac{q}{256\pi^4(s-m_p^2)^2}|\mathfrak{M}|^2,\tag{7}$$

where

$$|\mathfrak{M}|^2=\tfrac{1}{2}(|\mathfrak{M}_x|^2+|\mathfrak{M}_y|^2),\tag{8}$$

and in terms of the Jackson-frame variables,

$$\begin{aligned}\mathfrak{M}_x&=\frac{eq}{\omega^2(1-t/4\omega^2)}\left[\frac{\sin\theta\cos\phi}{1+(q/\omega)\cos\theta}T_++\frac{\sin\theta\cos\phi}{1-(q/\omega)\cos\theta}T_-\right.\\&\qquad\left.+\frac{2p_x\omega/q}{E+E'-p_z-p_z'}(T_+-T_-)\right],\\\mathfrak{M}_y&=\frac{eq}{\omega^2(1-t/4\omega^2)}\left[\frac{\sin\theta\sin\phi}{1+(q/\omega)\cos\theta}T_+\right.\\&\qquad\left.+\frac{\sin\theta\sin\phi}{1-(q/\omega)\cos\theta}T_-\right],\\T_\pm&=2i\sigma_\pm e^{B_\pm t/2}[(E'\omega\mp p_z'q\cos\theta\\&\qquad\mp p_xq\sin\theta\cos\phi)^2-m_\pi^2m_p^2]^{1/2}.\end{aligned}\tag{9}$$

Since the pion-nucleon invariant masses are assumed to be reasonably large, it is a good approximation to drop the mass terms which appear in the square roots in $T_\pm$: This corresponds to assuming the pions are extremely relativistic in the lab frame. Making this approximation, and noticing that q, ω, k, E', p_x, and p_z are related to s, t, and m but not to θ or ϕ, the only ϕ dependence of the matrix element is $\sin\phi$ and $\sin\phi\cos\phi$ in $\mathfrak{M}_y$, and 1, $\cos\phi$, and $\cos^2\phi$ in $\mathfrak{M}_x$. This implies that the z component of angular momentum in the Jackson frame for the final $\pi\pi$ system can be only 0, 1, or 2 for unpolarized photons, and only 1 or 2 for photons with electric vector perpendicular to the production plane. This simple result would not be affected by including form factors, since they would be functions of $(k-q_\pm)^2-m_\pi^2=2k(\omega\mp q\cos\theta)$. The origin of the $J_z=0$, 1, 2 rule is as follows: At a fixed value of momentum transfer, the diffraction scattering amplitude depends linearly on $q_\pm$, just as if it resulted from exchange of an elementary spin-1 particle—since that would also correspond to a constant cross section. The effective spin-1 particle can carry only $J_z=0, \pm1$, since it meets the photon "head on," and therefore carries no orbital contribution to J_z.

Now consider the limit of infinite energy, in order to obtain qualitative understanding of the cross section at finite energy. When $s\to\infty$,

$$T_\pm \cong \tfrac{1}{2} i\sigma_\pm s e^{B_\pm t/2}\left[1\pm\frac{q}{\omega}\frac{4\omega^2+t}{4\omega^2-t}\cos\theta \mp\frac{4q\sqrt{(-t)}}{4\omega^2-t}\sin\theta\cos\phi\right], \quad (10)$$

$$\left(\frac{2p_x}{E+E'-p_z-p_z'}\right)\cong\frac{\sqrt{(-t)}}{2\omega},$$

and the cross section $d\sigma/dt\,dm\,d\Omega$ approaches a finite limit, as expected for a diffractive process.

At $t=0$, for $s\to\infty$, the angular dependence of the propagators is canceled by that of $T_\pm$, so the amplitudes $\mathfrak{M}_x$ and $\mathfrak{M}_y$ are proportional to $\sin\theta\cos\phi$ and $\sin\theta\sin\phi$. The $\pi\pi$ state therefore has the same spin and helicity as the incident photon: $J=1$, $J_z=\pm1$. Its having the same helicity is required by angular momentum conservation. Its having the same spin comes about because the scattering amplitude is independent of the angular orientation of the $\pi\pi$ system in this limit. (If the pions are replaced by particles wlth spin, this is no longer true—e.g., in $\pi p\to\pi\rho p$, $\pi\rho$ systems with both $J=0$ and $J=1$ are obtained in the $s\to\infty$, $t\to0$ limit.) When form factors are included to reduce the amplitude when the virtual pion is far off the mass shell, additional angular dependence is introduced at high $\pi\pi$ masses, since forward-backward configurations of the $\pi\pi$ system are favored, with the backward π doing the scattering; and higher angular momentum states come in. At finite momentum transfer, states with $J\neq1$ can be produced even when form factors are unimportant, but only to the extent that $\sqrt{(-t)}$ is sizeable compared to $\omega=\frac{1}{2}m$. The magnitude of $-t$ is effectively limited by the width of the diffraction peak, so systems of moderately high mass should be produced mainly with $J=1$. When systems of very high mass are produced, form-factor effects are certain to be strong, and high angular momenta will be produced. In a numerical calculation containing a "reasonable" form factor (see Secs. V and VI), the production of $J=3$ and $J=1$ were found to be of comparable magnitude by $m=1640$ MeV, so the suppression of angular momentum change cannot account for the failure to observe g-meson photoproduction, if one assumes that the same suppression would occur in resonance production as occurs in the Drell-Söding process. [In accord with this, the $N^*(1688)$ with $J^P=\frac{5}{2}^+$ does seem to be produced in $pp\to pN^*$.[13]] States with $J=5$ and higher *are* significantly suppressed at this mass. States with even values of J are not produced at all by the Drell-Söding mechanism, if one makes the approximation that π^+p and π^-p scattering are equal, since in that case the $\pi\pi$ system must have odd charge conjugation.

Now consider the dependence on momentum transfer n the $s\to\infty$ limit, making the rather good approximation $B_+\cong B_-=B$. If m is large, so that $-t\ll\omega$, the cross section has the same t dependence as πp elastic scattering, except for effects due to form factors, which make the t dependence flatter. At low π-π masses, $d\sigma/dt\,dm\,d\Omega$ falls faster than e^{Bt}. This steepening of the diffraction peak at low masses results from the pion propagator. It can be understood using the uncertainty principle by thinking in terms of "old-fashioned perturbation theory": When the photon dissociates into a state of low invariant mass, the violation of energy conservation involved is small, so the time allowed for the dissociation is long, and the virtual state is spread out over a relatively wide range of impact parameters (in this case, spread out by $\approx\hbar/m_\pi c$); the effective size of the interaction region is therefore larger than that corresponding to the πp elastic scattering alone, so the diffraction peak is steeper. This steepening effect has been seen in $\gamma p\to\pi^+\pi^-p$ at masses below the ρ^0,[2] and in $pp\to pN^*$ for low-mass N^*'s.[13]

Now consider the dependence of the cross section on the $\pi\pi$ invariant mass for $s\to\infty$. At $t=0$, $T_\pm\cong\frac{1}{2}i\sigma_\pm s e^{B_\pm t/2}[1\pm(q/\omega)\cos\theta]$, so the matrix element contains a factor $1/m^2$. (It also contains a factor of q, but this is the usual threshold factor for scattering a p-wave system, and should not be thought of as tending to cancel the $1/m^2$.) This $1/m^2$ factor can be understood in old-fashioned perturbation theory to result from an energy denominator: the violation of energy conservation at the dissociation vertex being $m^2/2E_{\rm lab}$, as shown in Sec. IV. A factor of $1/m^2$ was assumed also to be present in the amplitude for $\gamma p-\rho^0p$ by Ross and Stodolsky,[14] and has since been controversial. It arises

[13] J. Rhode, R. Leacock, W. Kernan, R. Jespersen, and T. Schalk, Phys. Rev. **187**, 1844 (1969); W. Anderson, G. Collins, E. Bleser, T. Fujii, J. Menes, F. Turkot, R. Carrigan, Jr., R. Edelstein, N. Hien, T. McMahon, and I. Nadelhaft, Phys. Rev. Letters **19**, 198 (1967).

[14] M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966).

naturally at $t=0$ in the old-fashioned perturbation theory picture, independently of the two-pion model, although one could imagine that other dynamical effects might cancel it in the production of a wide resonance. Furthermore, the form suggested by the old-fashioned perturbation theory argument away from the forward direction is not nearly so simple as $1/m^2$: Because of the steepening of the t dependence at low masses mentioned above, it is more like $1/(m^2-t)$. This will be discussed further in Sec. IV.

If no form factors are included, $\int dt\, d\Omega(d\sigma/dt\, dm\, d\Omega)$ rises from threshold like $q^3=(\frac{1}{4}m^2-m_\pi^2)^{3/2}$, reaches a maximum at ≈ 600 MeV, and falls off for high masses like m^{-1} when $s\rightarrow\infty$. The m^{-1} falloff is unreasonably slow, and results from kinematic regions where the virtual pion is far off the mass shell. It would imply that the cross section integrated over m rises like $\ln s$. Including form factors changes the cross section only slightly at low $\pi\pi$ masses, but causes it to fall like m^{-5} at high mass, by limiting the transverse momenta of the outgoing pions. The existence of form factors also implies that the Drell-Söding mechanism is less effective in producing pairs of high-mass particles, such as K^+K^-, $p\bar{p}$, $\Delta^{++}\bar{\Delta}^{--}$, etc., than it is for producing $\pi^+\pi^-$.

IV. IN THE LAB FRAME

The angular momentum properties of the diffractively produced system are most easily analyzed in its rest frame, which I have employed up to now. More insight into the diffraction dissociation process can be obtained, however, in a frame in which the dissociating particle has an "infinite" momentum, such as the laboratory frame. In the high-energy limit, the four-momenta in the lab can be chosen as

$$k=(\mathbf{0},E_\gamma;\,E_\gamma)\,,$$
$$p=(\mathbf{0},0;\,m_p)\,,$$
$$q_+=\left(\mathbf{r}-\tfrac{1}{2}\boldsymbol{\delta},\,\eta(E_\gamma-\Delta);\,\eta(E_\gamma-\Delta)+\frac{(\mathbf{r}-\frac{1}{2}\boldsymbol{\delta})^2+m_\pi^2}{2E_\gamma\eta}\right),\quad(11)$$
$$q_-=\left(-\mathbf{r}-\tfrac{1}{2}\boldsymbol{\delta},\,(1-\eta)(E_\gamma-\Delta);\,(1-\eta)(E_\gamma-\Delta)+\frac{(\mathbf{r}+\frac{1}{2}\boldsymbol{\delta})^2+m_\pi^2}{2E_\gamma(1-\eta)}\right),$$
$$p'=(\boldsymbol{\delta},\,\Delta,\,(m_p^2+\boldsymbol{\delta}^2+\Delta^2)^{1/2})\,,$$

where $\mathbf{r}$ and $\boldsymbol{\delta}$ are two-dimensional vectors of transverse momentum, and the energies are given only to order $1/E_\gamma$. The pions have finite fractions of the infinite longitudinal momentum E_γ, given by η and $1-\eta$, where $0<\eta<1$, since Δ is of order $1/E_\gamma$. Neglecting the recoil energy, i.e., assuming $-t\ll m_p^2$,

$$m^2=[(\mathbf{r}-\tfrac{1}{2}\boldsymbol{\delta})^2+m_\pi^2]/\eta+[(\mathbf{r}+\tfrac{1}{2}\boldsymbol{\delta})^2+m_\pi^2]/(1-\eta)-\boldsymbol{\delta}^2\,,$$
$$\Delta=(m^2+\boldsymbol{\delta}^2)/2E_\gamma\,,\quad(12)$$
$$t=\boldsymbol{\delta}^2\,.$$

Writing the polarization vector as $\epsilon=(\boldsymbol{\varepsilon},0;0)$ by making use of the freedom to choose a gauge, and assuming $\sigma_+=\sigma_-$, $B_+=B_-$ for simplicity, the scattering amplitude given by Eq. (5) becomes

$$\mathfrak{M}=4i\sigma e m_p E_\gamma e^{Bt/2}\eta(1-\eta)\times\left[\frac{\boldsymbol{\varepsilon}\cdot(\mathbf{r}-\frac{1}{2}\boldsymbol{\delta})}{(\mathbf{r}-\frac{1}{2}\boldsymbol{\delta})^2+m_\pi^2}+\frac{\boldsymbol{\varepsilon}\cdot(\mathbf{r}+\frac{1}{2}\boldsymbol{\delta})}{(\mathbf{r}+\frac{1}{2}\boldsymbol{\delta})^2+m_\pi^2}\right]$$
$$=4i\sigma e m_p E_\gamma e^{Bt/2}\times\left[\frac{\boldsymbol{\varepsilon}\cdot(\mathbf{r}-\frac{1}{2}\boldsymbol{\delta})}{m^2+\boldsymbol{\delta}^2-2\mathbf{r}\cdot\boldsymbol{\delta}/(1-\eta)}+\frac{\boldsymbol{\varepsilon}\cdot(\mathbf{r}+\frac{1}{2}\boldsymbol{\delta})}{m^2+\boldsymbol{\delta}^2+2\mathbf{r}\cdot\boldsymbol{\delta}/\eta}\right],\quad(13)$$

where the first term inside the brackets corresponds to the π^- scattering, the second to the π^+.

From the viewpoint of old-fashioned perturbation theory, the virtual pion is considered to be on the mass shell, with energy not being conserved at the dissociation vertex. For the diagram in which the π^- scatters, the $\pi\pi$ system in the intermediate state has an invariant mass squared

$$m_{\mathrm{int}}^2=[(\mathbf{r}-\tfrac{1}{2}\boldsymbol{\delta})^2+m_\pi^2]/\eta(1-\eta)=m^2+\boldsymbol{\delta}^2-2\mathbf{r}\cdot\boldsymbol{\delta}/(1-\eta)\quad(14)$$

in this view. The violation of energy conservation in dissociation is $\Delta E=m_{\mathrm{int}}^2/2E_\gamma$, and the energy denominator $1/\Delta E$ is clearly visible in Eq. (13). [If the photon were a particle with mass, the corresponding expression would be $\Delta E=(m_{\mathrm{int}}^2-m_\gamma^2)/2E_\gamma$.]

At zero momentum transfer, $\boldsymbol{\delta}=0$ and $m_{\mathrm{int}}=m$, i.e., the photon dissociates into a $\pi\pi$ system of the same mass as the final one. (In coordinate space, the dissociation takes place a long distance, $\approx E_\gamma/m^2$, away from the target particle, but the transverse separation of the pions remains $\lesssim 1/m_\pi$.) The energy denominator then gives $\mathfrak{M}$ a factor m^{-2}, analogous to the Ross-Stodolsky factor.[14] Away from the forward direction, m_{int} can be either smaller or larger than m. If $\mathbf{r}$ is small, m_{int} is larger than m, and this results in the diffraction peak being steeper for diffractive production of low-mass systems. Equation (13) does not contain the pion mass explicitly, so effects such as the Ross-Stodolsky factor and the steepening of the diffraction peak at low mass can probably be believed to be very general effects, not depending on the specific model involving dissociation into two pions. In determining the steepening of the diffraction peak, the terms $\boldsymbol{\delta}^2=-t$, $2\mathbf{r}\cdot\boldsymbol{\delta}/(1-\eta)$, and $2\mathbf{r}\cdot\boldsymbol{\delta}/\eta$ are to be compared with m^2. As a result, in $\gamma p\rightarrow K^+K^-p$ there should be very little steepening of the diffraction peak even near the K^+K^- threshold. On the other hand, in $pp \to pn\pi^+$ at threshold, the relevant difference in masses squared is $(m_p+m_\pi)^2-m_p^2=0.3$ BeV^2, so a good deal of steepening is possible.

V. CORRECTIONS TO MODEL

A. Form Factors

The theoretical basis for calculating simple diagrams such as those of Fig. 1 as an approximation to the true diffraction dissociation amplitude rests in the presumed domination of that amplitude, in the sense of dispersion theory, by singularities which lie close to the physical region. In the case of the Drell-Söding process, the nearby singularities are the pion poles, at the zeros of $(k-q_\pm)^2-m_\pi^2 \equiv 2\omega^2(1-t/4\omega^2)[1\pm(q/\omega)\cos\theta]$. These poles are closest for $\pi\pi$ systems produced with small momentum transfer from the target, and with decay direction in the Jackson frame near to forward or backward. In the lab frame, this corresponds to both pions having small transverse momentum, and the one which interacts with the target having the smaller fraction of the photon's longitudinal momentum. At the edge of the physical region, for $s\to\infty$, $t\to 0$, $\cos\theta\to\mp 1$, the quantity $(k-q_\pm)^2-m_\pi^2$ approaches as close as $-m_\pi^2$ to 0, if $\omega \gg m_\pi$.

When the pion poles are far from the physical point under consideration, the diagrams of Fig. 1 are no longer a good approximation to the amplitude. Thinking in terms of either field theory or dispersion theory, other diagrams become important. The pion propagators $1/[(k-q_\pm)^2-m_\pi^2]$ therefore need to be multiplied by "form factors" $F\,[(k-q_\pm)^2-m_\pi^2]$, where $F(0)=1$. The function $F(x)$ cannot be specifically associated with the dissociation vertex, so to call it a form factor is somewhat misleading.[15] However, it is expected to fall off as x becomes negative on a scale of roughly 0.5–1.0 BeV^2, like the proton's electromagnetic form factor, since it must correspond to masses of intermediate states like 3π or $\pi\rho$ in the dispersion relation; or since in coordinate space it must preserve the long-range part of the pion exchange amplitude, while removing the part corresponding to distances $\lesssim 0.8$ F, where other processes are important.

As a qualitative model of the form factor, I have used $F(x)=e^{Ax}$, where $A\approx 1.0-2.0$ BeV^{-2}. The uncertainty in $F(x)$ produces a fairly large uncertainty in the predicted cross section at high masses, but has little effect in the region of the ρ^0, as shown in Sec. VI. It would be interesting to extract $F(x)$ from the data, by studying the cross section as a function of $(k-q_+)^2-m_\pi^2$ or $(k-q_-)^2-m_\pi^2$, in a region of high $\pi\pi$ masses, where one of the two diagrams dominates because of the form factors, and ρ^0 production is negligible. An attempt has been made to calculate effective form factors like F in terms of angular momentum barrier penetration factors[16]; and data for the reaction $pp\to pn\pi^+$ have been fitted in this way.[17] An alternative approach to calculating F would be that of the absorption model: to remove low partial waves, suitably defined, from the simple pole diagram.

Another way to look at the form factor is as a cutoff on the transverse momenta of the pions. For example, $F(x)=e^{Ax}$ corresponds to a transverse momentum distribution falling off like $e^{-p_\perp^2/2p_0^2}$, where $p_0=(4A)^{-1/2}$, exclusive of a small effect due to the elastic scattering. The value $A=2.0$ BeV^{-2} corresponds to $p_0=350$ MeV, which is typical of what one observes in most reactions at high energy.

B. Final-State Scattering

One could include corrections to the diagrams of Fig. 1, in which the outgoing pions are elastically scattered from each other in the final state, as illustrated in Fig. 2. Other intermediate channels could also be included; e.g., the photon could dissociate into $\pi^0\omega^0$, which could scatter to form $\pi^+\pi^-$ after elastic scattering on the target particle. However, such inelastic channels should be of little importance for $\gamma p\to\pi^+\pi^-p$, the sometimes large number of them being nullified by the smallness of inelastic amplitudes, and by the effect of form factors. Corrections of the final-state rescattering type may be thought of as unitarity corrections, requiring in some sense that the total probability for the photon to dissociate into *something* is no larger than 1. They are distinct from the form-factor corrections discussed above, since they can be significant even in the case of the long-range (very nearby singularity) part of the pion exchange.

The effect of rescattering at low $\pi\pi$ masses is tied up with the question of resonance production, and is discussed in Sec. V C. At high masses, the $\pi\pi$ scattering amplitude is presumably mainly imaginary, so Fig. 2 corresponds to final-state absorption, which reduces the amplitude due to Fig. 1. Calculating Fig. 2 by requiring both of the pions which scatter in the final state to be on their mass shells, and assuming a pure imaginary $\pi\pi$ elastic amplitude with $d\sigma/dt\propto e^{\beta t}$, yields a correction to Fig. 1 of $-\sigma_{\pi\pi}/8\pi\beta\approx -20\%$ at low transverse momenta. The quantity $\sigma_{\pi\pi}/8\pi\beta$, which equals $2\sigma_{\rm el}/\sigma_{\rm total}$ for $\pi\pi$ scattering, corresponds to one-half of the absorption calculated by Gottfried and Jackson[18] for low partial

[15] In the case of $\gamma p\to\pi^+\pi^-p$, the function $F(x)$ results from the diffraction scattering vertex alone. Corrections to the $\pi\pi\gamma$ vertex and to the π propagator cancel each other according to a theorem of Francis Low, Phys. Rev. **110**, 974 (1958). The theorem arises because the dispersion relation for the $\pi\pi\gamma$ vertex contains a subtraction constant, given by the pion charge, plus an integral over the absorptive part, which vanishes because no real intermediate states can connect a pion (which has spin 0) with a $\pi\gamma$ system (which has helicity 1). This theorem does not negate the qualitative arguments given about the function $F(x)$, however.

[16] H. Dürr and H. Pilkuhn, Nuovo Cimento **40**, 899 (1965); G. Wolf, Phys. Rev. **182**, 1538 (1969).

[17] E. Colton, P. E. Schlein, E. Gellert, and G. A. Smith, UCLA Report No. UCLA-1027, 1968 (unpublished).

[18] K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).

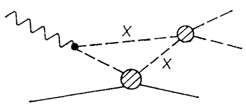


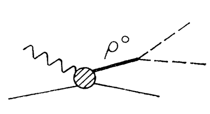
FIG. 2. Feynman diagram for final-state scattering correction. There are really two diagrams, since either π^+ or π^- can scatter on the proton. The lines marked by $\times$ are put on the mass shell in the calculation.

waves in $2\to 2$ amplitudes, since, in their case, absorption in the initial state has the same effect as absorption in the final state. (Initial state absorption might be important in our case also, e.g., due to absorption of the incident virtual ρ^0 in the vector-dominance model.) The final-state scattering has only a minor effect on the angular distribution of the pions, since elastic scattering is peaked so sharply forward. For example, at high masses and high transverse momenta, Fig. 1 gives a transverse momentum distribution $\propto \exp(-2Ap_\perp^2)$, while Fig. 2 gives $\exp[-2Ap_\perp^2/(1+2A/\beta)]$, where $2A/\beta\approx 0.4$.

C. Resonances

Reactions in which one can attempt to study the diffraction dissociation process are without exception complicated by the existence of resonances near threshold in the diffractively produced system: for $\gamma p\to\pi^+\pi^- p$, the ρ^0; for $\pi p\to \pi\rho p$, the A_1; for $Kp\to K^*(890)\pi p$, the Q; for $pp\to pn\pi^+$ or $pp\to pp\pi^+\pi^-$, or $pp\to p$ (missing mass), the nucleon resonances at 1.40 BeV ($J^P=\frac{1}{2}^+$) and 1.69 BeV ($J^P=\frac{5}{2}^+$). The presence of these resonances close to where the simple dissociation models predict peaks in the mass spectrum may or may not be understandable by extending the notion of duality to reactions involving Pomeranchukon exchange.[19] The original hope of explaining peaks such as the A_1 as purely due to "Deck effect," analogous to Fig. 1, has largely vanished in the face of better data[20] and the realization that the Deck effect cannot give peaks narrower than a few hundred MeV. A narrow A_1, which would contradict the Deck model, has not been observed in diffractive production on nuclei, however.[21] The reaction $\gamma p\to K^+K^-$ is especially simple from the standpoint of resonances, because the only known diffractively produced resonance is the ϕ, which is very narrow, so resonance effects

FIG. 3. Feynman diagram representing photoproduction and decay of ρ^0.

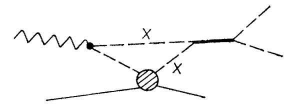


[19] G. Chew and A. Pignotti, Phys. Rev. Letters **20**, 1078 (1968). By itself, the duality concept has only qualitative content. An interesting attempt to implement the concept by means of the Veneziano formula, treating the Pomeranchukon like a particle, has been discussed by S. Pokorski and H. Satz, Nucl. Phys. **B19**, 113 (1970), and H. Satz and K. Schilling, Nuovo Cimento **67A**, 511 (1970).

[20] J. Ballam *et al.*, Phys. Rev. Letters **21**, 934 (1968).

[21] J. Allard *et al.*, Nuovo Cimento **46A**, 737 (1966); A. Cnops *et al.*, Phys. Rev. Letters **21**, 1609 (1968).

FIG. 4. Final-state-scattering correction dominated by a resonance.



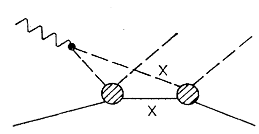
should be negligible over most of the K^+K^- mass spectrum.

Various attempts have been made to incorporate resonance production into the Drell-Söding or Deck type of model, using the theory of final-state interactions.[5] I prefer not to take this approach. It is complicated in the case of $pp\to pn\pi^+$ by the fact that an $N^*(1400)$ which decays into $n\pi^+$ could just as well be formed from one of the other open channels such as $p\pi^+\pi^-$ and $pp\pi^0$. In the case of $\gamma p\to\pi^+\pi^- p$, channels like $\gamma p\to\pi^0\omega^0 p$ and $\gamma p\to\bar{p}pp$ should still be able to contribute to $\gamma p\to\rho^0 p$, even though they are closed at the ρ mass, if one thinks of the ρ^0 along the lines of a bootstrap model.

I prefer, therefore, to consider the amplitude for resonance production, $\gamma p\to\rho^0 p$, as a separate dynamical object from the Drell-Söding "background" term. For the purpose of data fitting, one must simply parametrize its magnitude, phase, helicity dependence, mass dependence (e.g., with or without $1/m_{\pi\pi}^4$), and t dependence (e.g., with or without the steepening at small t characteristic of the Söding mechanism).

In adding a background term, such as the Drell-Söding process, to a resonance production amplitude, one stands in danger of "double counting." In order to avoid double counting, I recommend the following prescription: *Multiply the background term in a given partial wave by $e^{i\delta}\cos\delta$*, where δ is the phase shift in that partial wave. The resonance term is proportional to $e^{i\delta}\sin\delta$, with a magnitude which one can only parametrize as discussed above. In terms of a Breit-Wigner form, $e^{i\delta}\sin\delta=m_\rho\Gamma/(m_\rho^2-m^2-im_\rho\Gamma)$ and $e^{i\delta}\cos\delta=(m_\rho^2-m^2)/(m_\rho^2-m^2-im_\rho\Gamma)$, where the width Γ is a function of m^2, going to zero at threshold. The factor $e^i\cos\delta$ gives the background amplitude the phase $e^{i\delta}$ suggested by Watson's theorem,[23] and forces it to go through zero at the peak of the resonance, where $m=m_\rho$, $\delta=\frac{1}{2}\pi$. Far from the resonance, $e^{i\delta}\cos\delta$ goes to 1, so the background term is unmodified. Final-state interaction theory leads naturally to the $e^{i\delta}\cos\delta$ prescription—see, e.g., footnote 18 of Ref. 14. Further arguments in favor of it follow.

FIG. 5. Double-scattering correction. There are really two diagrams, since either π^+ or π^- can scatter first. The lines marked by $\times$ are put on the mass shell in the calculation.



[22] F. Zachariasen, Phys. Rev. Letters **7**, 112 (1961).

[23] I. Aitchison and C. Kacser, Phys. Rev. **173**, 1700 (1968); John Gillespie, *Final State Interactions* (Holden-Day, San Francisco, 1964).

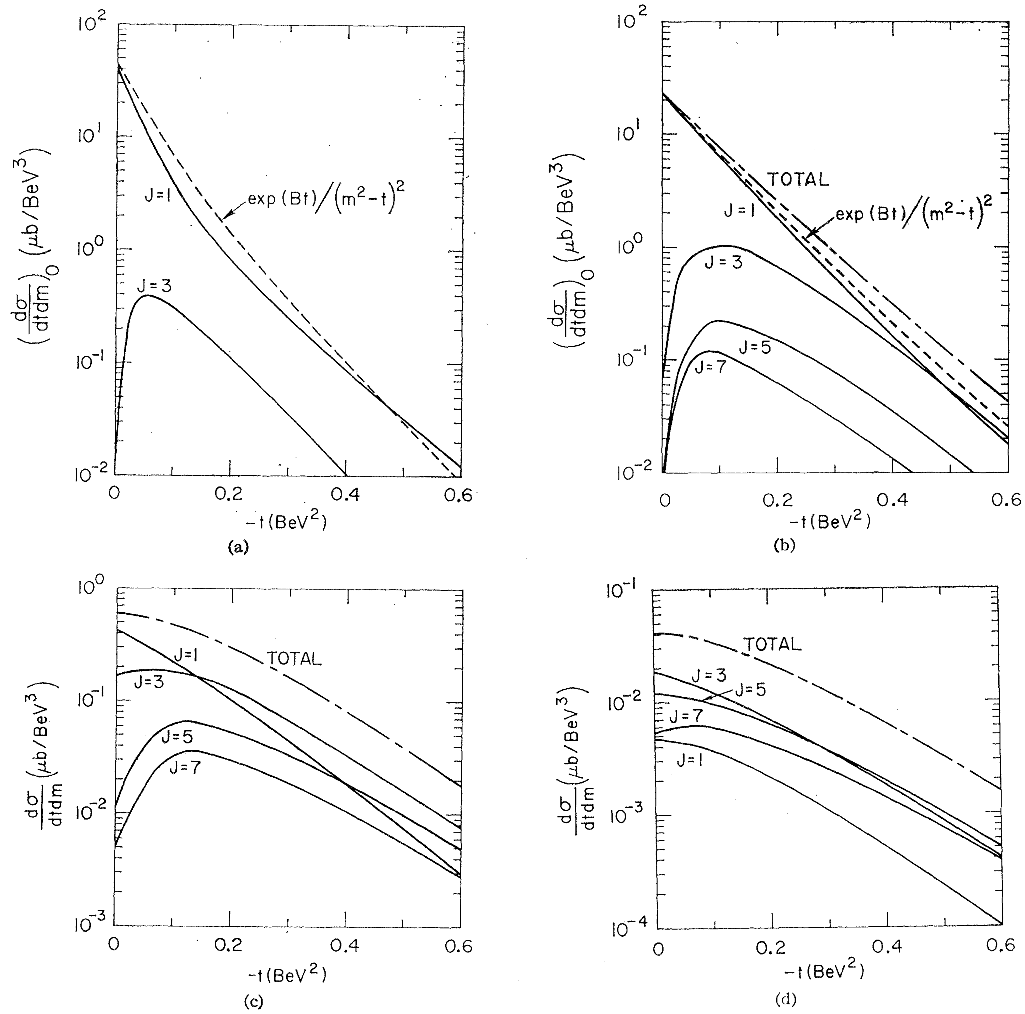


FIG. 6. Momentum transfer dependence of the Drell-Söding process for various angular momentum states. The parameters chosen were $E_\gamma = 10$ BeV, $\sigma_+ = \sigma_- = 30$ mb, $B_+ = B_- = 9$ BeV^{-2}, and $A = 2$ BeV^{-2}. Even partial waves are not present because $\pi^+ p$ and $\pi^- p$ scattering were assumed identical. The dashed curve in (a) and (b) shows the function $e^{Bt}/(m^2-t)^2$, which fits the p-wave cross section approximately. The $\pi\pi$ masses are $m = 0.4$ BeV in (a), 0.760 BeV in (b), 1.64 BeV in (c), and 2.50 BeV in (d). Note that ρ^0 production is neither included here, nor in Figs. 7 and 8.

In an infinite momentum frame for the dissociating system, such as the lab frame, the lifetime of the ρ is $E_\gamma / m_\rho \Gamma$, which is long compared to the time associated with dissociation and scattering, $2E_\gamma / m_\rho^2$. It is therefore natural to treat the ρ as a stable elementary object, in considering the dynamics of its production. Following the ideas of Sec. V B, one should therefore add to the graphs of Fig. 1 a resonance production term (Fig. 3) and a rescattering term (Fig. 4). It is easy to show that adding the *absorptive part* of this rescattering term (the part corresponding to the pions which form the ρ being on the mass shell) to Fig. 1 is equivalent to multiplying Fig. 1 by $e^{i\delta}\cos\delta$.

A final argument for $e^{i\delta}\cos\delta$ comes from considering a model in which a nonrelativistic bound state of two particles is diffractively excited into a continuum state, as a result of one of its constituents elastic scattering on a target particle. (One could think of the coherent breakup of a deuteron,[6,24] and then imagine that the np system possessed an $I = 0$ resonance.) The scattering amplitude, in single-scattering impulse approximation, is given by

$$T = T_{\mathrm{el}}(\mathbf{\Delta}) \int \psi_{\mathrm{final}}^*(\mathbf{r}) \psi_{\mathrm{init}}(\mathbf{r}) e^{i\mathbf{\Delta}\cdot\mathbf{r}/2} d\mathbf{r}\,, \qquad (15)$$

where $\psi(\mathbf{r})$'s are nonrelativistic wave functions for the two-body system at relative coordinate $\mathbf{r}$, $T_{\mathrm{el}}(\mathbf{\Delta})$ is the amplitude for elastic scattering of one of the constituents on the target, and $\mathbf{\Delta}$ is the three-momentum transfer. In the case of a repulsive δ-function shell potential $[\lambda\delta(r-a)]$ for the two-body system (the

[24] R. Glauber, Phys. Rev. 99, 1515 (1955).

lowest-lying resonance being considered metastable), it is easy to show that the scattering amplitude for producing an s-wave state in the continuum is

$$T=(e^{i\delta}\cos\delta+Ce^{i\delta}\sin\delta)T_{\text{plane wave}}, \qquad (16)$$

where $T_{\text{plane wave}}$ is obtained from (15) by replacing $\psi_{\text{final}}(\mathbf{r})$ with the plane wave corresponding to no final-state scattering. The phase shift δ depends on the energy of the continuum state, which is analogous to the mass of a diffractively produced system. The coefficient C of the resonance term depends on that energy and also on the parameters describing the potential.

D. Double Scattering

In the forward direction, one would expect the diffractive production amplitude for $\gamma p\rightarrow\pi^+\pi^-p$ to be proportional to the cross section for absorbing the $\pi^+\pi^-$ system on the target proton. According to the single-scattering diagrams (Fig. 1), that cross section is given by $\sigma_++\sigma_-$. This needs to be corrected by a shadow term, corresponding to the fact that if one of the pions has been absorbed, absorption of the other one does not add to the cross section.

One way to estimate the shadow effect is to consider a sum of absorptive Gaussian potentials for πp scattering, which leads to a total cross section σ, and differential cross section $d\sigma/dt\propto e^{Bt}$ in the eikonal approximation. Doubling the strength of this potential, i.e., reducing the mean-free path at every point in coordinate space by a factor of 2, should correspond to scattering of the $\pi\pi$ system. Keeping only linear and quadratic terms in σ, this leads to a total cross section $2\sigma-\sigma^2/8\pi B$, and a differential cross section $d\sigma/dt\propto[e^{Bt/2}-(\sigma/16\pi B)e^{Bt/4}]^2$. The correction to the total cross section (forward amplitude) amounts to about 15%. In going away from $t=0$, the shadow effect falls more slowly than the single scattering, since it comes more from small impact

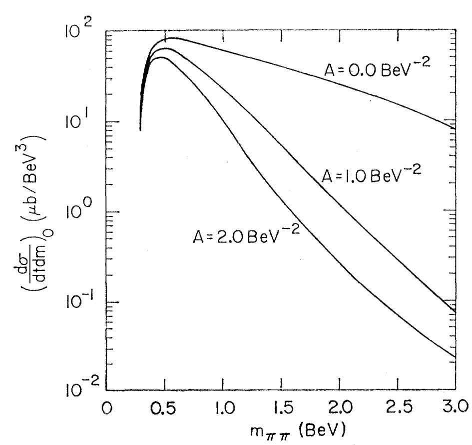


FIG. 7. Dependence of the Drell-Söding process in the forward direction on the $\pi\pi$ system, for values of the form factor parameter A. These parameters were $E_\gamma=20$ BeV^{-2}, $\sigma_+=\sigma_-=30$ mb, and $B_+=B_-=9$ BeV^{-2}.

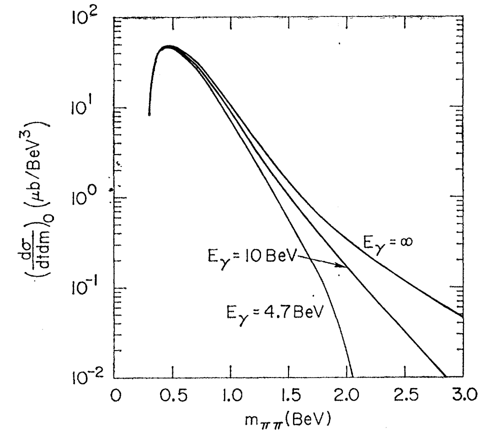


FIG. 8. Dependence of the Drell-Söding process in the forward direction on the mass of the $\pi\pi$ system, for various values of the photon energy. (Other parameters as in Fig. 6.)

parameters, and will produce a dip in the differential cross section around $t=-0.8$ BeV^2, where the two become equal in magnitude. This interference mechanism is known to produce dips in scattering on nuclei,[25] and has been proposed as the source of the dips observed in reactions like $\pi^-p\rightarrow\pi^0n$ as well.[26] It would be interesting to look for it in $\gamma p\rightarrow\pi^+\pi^-p$, where the $\pi\pi$ system may be intermediate between an elementary particle and a nucleus in the degree to which it acts as a composite object.

A somewhat more refined approximation to the shadow effect, which still assumes that the target proton interacts with the two pions individually, is offered by the double-scattering diagrams of Fig. 5. Computing these diagrams in the $s\rightarrow\infty$, $t\rightarrow 0$ limit, by requiring the intermediate particles between the elastic scatterings to be on the mass shell, ignoring the recoil energy of the nucleon via $m_p\rightarrow\infty$, and letting $m_\pi\rightarrow 0$, yields a correction of the form $\sigma_+\sigma_-\times 8\pi B$, as given by the optical model, multiplied by a factor $1-e^{-Bp_\perp^2}$ which reduces the shadow effect for pions with low transverse momenta. This factor results from the possibility, which was neglected in the optical model, of the two pions having different impact parameters on the proton.

VI. NUMERICAL RESULTS

The momentum transfer dependence of the Drell-Söding process is shown in Fig. 6, for various masses and partial waves of the $\pi\pi$ system. (Of the corrections discussed in Sec. V, only the form-factor effects were included in calculating this and the succeeding figures; in particular, ρ^0 production was omitted.) At low masses, the $\pi\pi$ system is mainly p wave. The diffraction peak is significantly steeper than the e^{Bt}, with $B=9$ BeV^{-2}, assumed for πp scattering, and is given ap-

[25] H. Hsiung *et al.*, Phys. Rev. Letters **21**, 187 (1968); M. Fellinger *et al.*, *ibid.* **22**, 1265 (1969).
[26] F. Henyey *et al.*, Phys. Rev. **182**, 1579 (1969).

proximately by $e^{Bt}/(m^2-t)^2$. (See Sec. IV.) At high masses, higher angular momentum states predominate because of the form factors, which limit transverse momenta. Also, the diffraction peak becomes flatter.

The mass dependence of the forward cross section is shown in Fig. 7, for various strengths of the form factor. The cross section is concentrated at low masses by the form factors, which are already significant at the mass of the ρ^0. The true form factors are not known, but must correspond roughly to $A \approx 1$–2 BeV^{-2}. The effect of this uncertainty on the cross section can be seen.

The energy dependence of the cross section, exclusive of that due to the energy dependence of πp scattering, is shown in Fig. 8. At 4.7 BeV, the high masses are cut off by the minimum momentum transfer; the cross sections have a factor e^{Bt}, where $B \approx 9$ BeV^{-2} and $t < t_{\min} \approx -(m^2/2E_\gamma)^2$. The cross section in the limit $E_\gamma \to \infty$ is not very different from that at $E_\gamma = 10$ BeV, as expected for a diffractive process, and this justifies the use of that limit for qualitative discussions.

The mass dependence of the forward cross section for $\gamma p \to K^+K^-p$ is shown in Fig. 9. This process is interesting because it could be studied at low K^+K^- masses without interference from resonance production—the ϕ, not included in the figure, being very narrow. The form-factor effects are strong even near threshold because of the relatively large kaon mass, so the cross section is rather small.

At the ρ mass, the amplitude for producing a p-wave $\pi^+\pi^-$ pair via the Drell-Söding process is on the order of 20% of the amplitude for producing it through the ρ in the vector-dominance model,[14] and the two amplitudes are 90° out of phase if πp and ρp scattering have the same phase. The result of adding a *constant* background term with this magnitude and phase to a Breit-Wigner resonance is shown in Fig. 10. A significant asymmetry is produced, and the maximum is shifted downward by ≈ 15 MeV. The asymmetry produced by the actual Söding term would be somewhat greater, because of its $(m_\rho/m_{\pi\pi})^2$ behavior at $t=0$. The result of correcting the background term for double counting by means of the factor $e^{i\delta}\cos\delta = (m^2-m_\rho^2)/(m^2-m_\rho^2+im_\rho\Gamma)$ is also shown. Its effect is rather small: It raises the apparent cross section for ρ production by $\approx 5\%$, but does not affect the position of the peak.

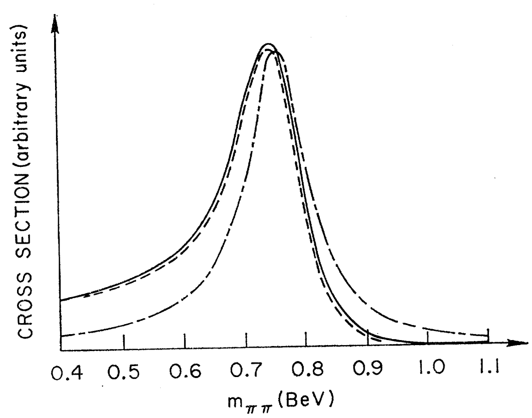


FIG. 10. Dash-dot curve: pure Breit-Wigner resonance, with constant width; solid curve: Breit-Wigner resonance+constant interfering background amplitude; dashed curve: like the solid curve, but background term multiplied by $e^{i\delta}\cos\delta$, where the Breit-Wigner term is $e^{i\delta}\sin\delta$.

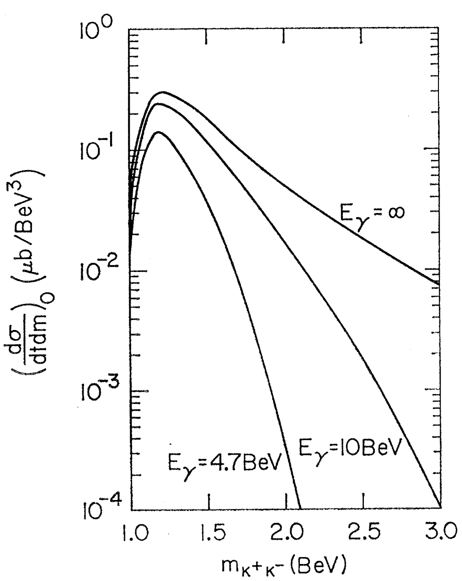


FIG. 9. Mass dependence of the forward Drell-Söding cross section for $\gamma p \to K^+K^-p$, assuming $E_\gamma = 10$ BeV, $\sigma_+ = \sigma_- = 20$ mb, and $A = 2$ BeV^{-2}.

VII. COMMENTS ON EXPERIMENT

The Drell-Söding process has been included as a background term in a number of fits to $\gamma p \to \pi^+\pi^-p$ in the region of the ρ.[2,27] It provides a natural explanation for the observed skewing of the resonance peak toward low masses. In a recent experiment,[2] that skewing was found to be consistent with the Söding model, and inconsistent with a simple $(m_\rho/m_{\pi\pi})^4$ Ross-Stodolsky factor (though perhaps consistent with a factor like $[m_\rho^2/(m_{\pi\pi}^2-t)]^2$). The same experiment finds evidence for the Drell-Söding process occurring outside of the p wave; the Y_0^4 moment of the $\pi\pi$ angular distribution agrees with the Söding model, in which it results mainly from interference of the 3^- partial wave of the background term with the 1^- due to the resonance.

At masses above the ρ^0 region, it is somewhat difficult to test the Drell-Söding model, because both the predicted and the observed cross sections are rather small.[28] One must also avoid the effects of other processes, such as nucleon resonance production, by requiring the π^+p and π^-p invariant masses to be reasonably large, and the momentum transfer to be small.

[27] H. Alvensleben *et al.*, Phys. Rev. Letters **23**, 1058 (1969).
[28] G. McClellan *et al.*, Phys. Rev. Letters **23**, 718 (1969).