The Veneziano fit, including the $\pi\Delta$ mechanism, is far better than that of Roberts and Wagner[48] to fewer data.

There is substantial Δ production at incident momenta as low as 456 MeV/c (1339-MeV c.m.) rather far below the threshold for production of a Δ of mass 1236 MeV.

With the results of the fit to "isobar" model predictions, we have calculated partial-wave inelasticities in the S_{11}, P_{11}, P_{13}, D_{13}, and D_{33} partial waves, as a function of energy.

These results are only qualitatively in agreement with those following from the CERN elastic phase-shift analysis.[8] We find inelasticity in the S_{11}, P_{13}, and D_{13} waves at lower energies than that anlysis. The increasing inelasticities obtained here are consistent with the interpretation that P_{11} and D_{13} waves resonate at energies higher than those available in the present experiment.

The results of our analysis and that of the Saclay group[46] from 530 to 760 MeV/c are compatible.

We have derived values of the P_{11} branching ratios into πN, $\pi\Delta$, and σN where σ is an $I=J=0$ interacting $\pi\pi$ pair. The πN branching ratio is between 43% at 1339 MeV and 60% at 1402 MeV. The branching ratio between $\pi\Delta$ and σN depends, not surprisingly, on the form of σ used. In the preferred form the inelasticity is almost entirely through σN: With the Veneziano-Lovelace form of δ_{00} the ratio of $\pi\Delta$ to σN couplings is about unity and increases with energy.

[48] R. G. Roberts and F. Wagner, Nuovo Cimento 64A, 206 (1969).

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Measurement of the K^- Cascade Time in Liquid Helium*

J. G. Fetkovich, J. McKenzie,† B. R. Riley, and I-T. Wang
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
AND
K. Bunnell‡
Argonne National Laboratory, Argonne, Illinois 60439 and Northwestern University, Evanston, Illinois 60201
AND
M. Derrick, T. Fields, L. G. Hyman, and G. Keyes§
Argonne National Laboratory, Argonne, Illinois 60439
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The average cascade time of K^- mesons has been measured in a helium bubble chamber. In a sample of about 36 000 stopped K^-, 44 τ decays at rest were found, corresponding to a cascade time of $(3.2\pm0.5)\times10^{-10}$ sec. This result agrees with a previous measurement.

INTRODUCTION

When a negative meson stops in a material medium, it forms a mesonic atom with a constituent nucleus by replacing an electron. The meson, initially in a highly excited level, cascades down through the lower levels accessible to it until it decays or undergoes a nuclear reaction. It is sometimes important to know the atomic state from which nuclear capture occurs. For this reason, studies of the cascade process are necessary.

The first detailed attempt to understand the cascade process in liquid helium was made in a theoretical study by Day.[1] One might expect naively that in helium both electrons are rapidly ejected by internal Auger transitions, and then the K-mesonic ion further deexcites by electromagnetic transitions. If so, the cascade would go through circular orbits, and nuclear interactions might be expected to proceed mainly from the $3d$ and $2p$ states. Day showed that collisional effects might be important enough to considerably modify that expectation. In particular, he concluded that because of collisional Stark-effect mixing of orbital angular momentum states, most π^- and K^- would undergo nuclear capture from relatively high s states.

It is possible also to study the cascade process experimentally. The easiest parameter to measure is the average cascade time. The two measurements[2,3] of the

* Work performed under the auspices of the U. S. Atomic Energy Commission.
† Now at University College, London, England.
‡ Now at Stanford Linear Accelerator Center, Stanford, Calif. 94305.
§ Now at Université Libre de Bruxelles, Bruxelles, Belgium.
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π^- cascade time give a mean result of $(3.16\pm0.22)\times10^{-10}$ sec. Fetkovich and Pewitt[2] showed that the unmodified theory of Day implied a cascade time orders of magnitude shorter than measured, but that a modification, in which Stark mixing was turned off, was in reasonable agreement with experiment. These authors also calculated cascade times for K^- in helium corresponding to the modified Day theory, and suggested that a measurement with K^- could be very useful in distinguishing between several alternatives. This measurement was subsequently made[4] by using the τ-decay mode and yielded a K^- cascade time of $(2.5\pm0.4)\times10^{-10}$ sec. This result, according to the analysis of Ref. 2, implied substantial p-state absorption of K^- in helium.

Recently, the yields, energies, and widths of some mesonic x-ray lines in helium have been measured. A model of the cascade process has been developed which is reasonably consistent with all the x-ray data.[5] In this model, atomic capture occurs when the meson replaces one of the two atomic electrons, ending in an orbit of about the same energy and radius. The next step is an Auger transition, ionizing the atom. Until the meson reaches the lower levels, deexcitation proceeds mainly through external Auger transitions in collisions with other helium atoms. The same collisions, by Stark-effect mixing, induce transitions between orbital angular momentum states of the same principal quantum number. This mixing causes some nuclear reactions to occur from s states of high excitation. Once the lower levels are reached, depopulation proceeds predominantly by electromagnetic dipole radiation and nuclear interactions. The only important difference between this picture and that considered by Day is in the strength of the Stark-mixing effect.

This model implies that in liquid helium, pions are absorbed in about equal numbers from s and p states, while kaon absorption proceeds mainly from p states. It also implies that the π^- and K^- cascade times are 1.2×10^{-11} and 3.0×10^{-11} sec, respectively. The discrepancies between these values and the measurements are perhaps due to the trapping of a small fraction of mesons in highly excited, metastable circular orbits.[6] A recent detailed calculation by Russell[7] supports this hypothesis for K^-, but not π^-.

Certain characteristics of some reactions of stopped mesons in helium are sensitive to the orbital angular momentum of the state of absorption. One experiment[8] involving pion interactions in helium has been done which suggests that the absorption of stopped π^- is predominantly from non-s-states. However, a later experiment[9] finds a contradictory result. Furthermore, other experiments,[10] in which $\Sigma^\pm$ and Λ were produced by stopped K^-, indirectly indicate that kaons interact predominantly from s states.

In summary, the cascade process in helium is currently not well understood. One of the critical experimental numbers, the K^- cascade time, has been measured only once. In this situation it seemed desirable to have an independent measurement of this parameter.

EXPERIMENTAL METHOD

The cascade time was measured in this experiment using part of an exposure of the ANL–CMU helium bubble chamber to the stopping K^- beam at the ZGS. The bubble chamber is a cylinder of 25-cm diameter and 35-cm depth, with a central magnetic field of 40.94 kG. The beam[11] and chamber[12] are described in detail elsewhere.

We may define[2] the average cascade time T_c to be

$$T_c=\sum_{i=1}^{r} n_iT_i\Big/\sum_{i=1}^{r} n_i. \tag{1}$$

Here T_i is the time that the average K^- spends in the ith channel from initial capture into a mesonic-atom orbit until decay or nuclear absorption. The number of mesons cascading through channel i is n_i. The total number of different channels available is r. Different channels are distinguished by the mesonic-atom states through which the kaons cascade. In this experiment, apparently all T_i satisfy

$$T_i\ll\tau, \tag{2}$$

if trapping is not important. Then we can write

$$T_c\simeq\tau N_d/N_s. \tag{3}$$

In this equation τ is the mean decay lifetime of the K^-; N_d is the number of K^- observed to decay at rest, out

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[9] Chun-Ming Leung, Nuovo Cimento Letters **2**, 389 (1969).

[10] J. G. Fetkovich, in Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, 1969, edited by A. R. Bodmer and L. G. Hyman, p. 451 (unpublished); K. Bunnell, M. Derrick, T. Fields, L. G. Hyman, and G. Keyes, Phys. Rev. D **2**, 98 (1970).

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[12] M. Derrick, T. Fields, L. Hyman, J. Loken, K. Martin, E. G. Pewitt, J. G. Fetkovich, and J. McKenzie, in *Proceedings of the International Conference on Instrumentation for High-Energy Physics, Stanford, California, 1966* (International Union of Pure and Applied Physics and U. S. Atomic Energy Commission, Washington, D. C., 1966), p. 264.

of a total sample of N_s stopping K^-. It should be noted that if the trapping hypothesis is the correct explanation of the large cascade time in helium, then Eq. (2) is not satisfied, and T_c measures just the fraction of K^- which are trapped, instead of the average cascade time.

In measuring N_d, we use only the τ-decay mode in order to avoid experimental ambiguities arising from the difficulty of distinguishing muons and pions in the more frequent two-body decays of K^-.

ANALYSIS OF DECAYS

The film was scanned for events having the τ-decay topology. The scanning region in each of the three views was geometrically limited so that difficult areas (i.e., near the walls, within reflective flares, etc.) were excluded. A template was applied on the scan table to exclude decays of obviously high-momentum kaons. This template tested that the sagitta of the last four centimeters of beam track was within acceptable limits. The resulting momentum cutoff is discussed in connection with Fig. 2 (below). Another requirement imposed on the scan table was that there be no scatter of the K^- before decay.

In a scan of 61 864 frames, 278 τ candidates were found satisfying all the above criteria. These events were measured and then subjected to additional geometric cuts, as listed in Table I. In this table, the track FRMS refers to the TVGP parameter which measures the goodness of fit of the fitted space curve to the measured track. The FRMS cuts tend to filter out tracks having scatters which were not detected by the scanners. The π length cut was to ensure high scanning efficiency and good momentum resolution. The K length cut had the same effect, but was applied mainly to ensure that enough length was available for unambiguous use of the stopping template.

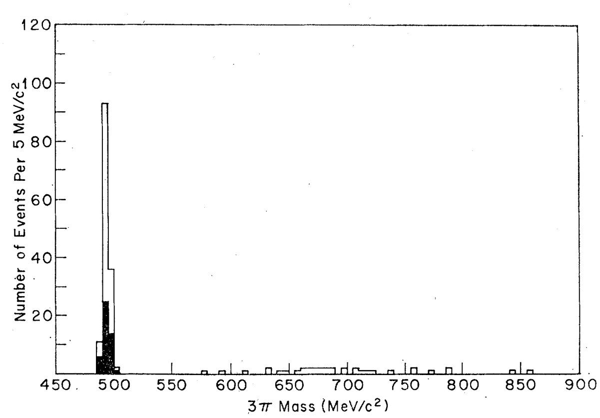

FIG. 1. Histogram of the 3π invariant masses, for all accepted τ^- candidates, calculated using unfitted parameters. The 46 shaded events in the K^- mass peak represent events for which a subsequent one-constraint τ^- fit yielded a K^- end momentum consistent with zero. The 96 unshaded events in the peak represent decays in flight. There are 37 background events including two with mass greater than 900 MeV which are not shown.

TABLE I. Geometric cuts applied to TVGP[a] output for each τ candidate.

Fiducial volume	10-cm radius×24-cm cylinder
K^- dip	$\lvert\lambda\rvert \leq 20°$
K^- length	$l_K \geq 6$ cm
K^- FRMS[b]	FRMS≤ 30 μm
$\pi^\pm$ length	$l_\pi \geq 2$ cm

[a] Three-view geometry program.
[b] Film root-mean-square deviation of measured points from the projected fitted space curve.

Of the 278 τ candidates measured, 182 passed the geometrical cuts of Table I. For these events, the mass and momentum of the decaying incident particle were computed assuming the incident track to be unmeasured (zero constraint). The resulting mass distribution is given in Fig. 1. It is clear that the kaon peak contains no significant background. Of the 37 events outside the peak, 33 were found to yield one-constraint fits to $\Lambda \rightarrow p\pi^-$. Thus, these events are presumed to be $\Lambda\pi^-$ production with the Λ decaying very close to the production vertex. The other four were not unambiguously identified. Of the 142 events in the K^- mass peak, 46 events, shaded in Fig. 1, had calculated momenta consistent with zero (see below). The remaining 96 represent decays in flight.

The events within the K^- peak of Fig. 1 were next subjected to one-constraint fits to the τ hypothesis. In these fits, the K^- track was assumed unmeasured. This was done in lieu of using all four constraints, because the kinematics program may have difficulty treating events in which the beam particle is at rest or nearly at rest. In any case, for decays at rest, measurement of the K^- track adds little information to the fit. The resulting K^- momentum spectrum at decay is shown in Fig. 2. The momentum resolution obtained with one-constraint fits is evidently more than adequate. The

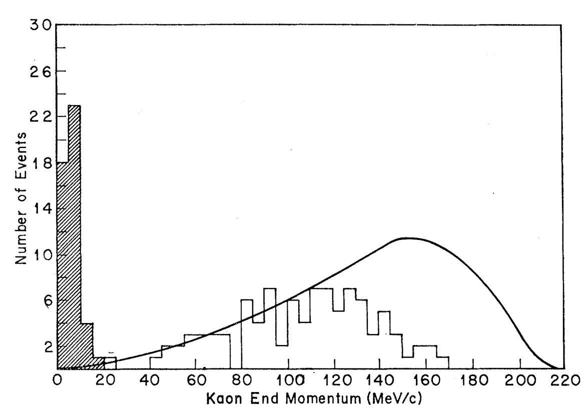

FIG. 2. Distribution of K^- end momentum for the events in the mass peak of Fig. 1. The plotted momenta are from a one-constraint fit in which the beam track was considered unmeasured. The 46 shaded events satisfy the at-rest criterion described in the text.

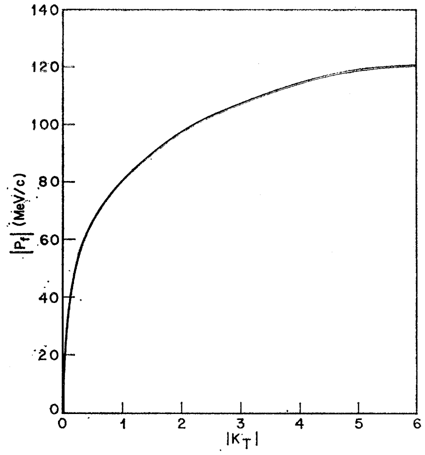

FIG. 3. Absolute value of final kaon momentum versus absolute value of average K_T. Negative end momenta (corresponding to overstopped tracks) yield $K_T<0$, while $P_f>0$ (apparent in-flight interaction) yields $K_T>0$.

curve in Fig. 2 is the expected momentum spectrum of K^- τ decays. It was calculated by using the Bethe-Bloch equation for stopping power and folding in the measured momentum spectrum of K^- incident on the chamber. Effects of interactions in flight are expected to be small, and were ignored. The curve is (absolutely) normalized to the measured number of K^- observed to stop in this experiment. Relative to this curve, the experimental spectrum is attenuated at high momenta because of the stopping template criterion applied at the scan table to the K^- track. One sees that the template eliminates few events below about 120 MeV/c, and accepts none above about 160 MeV/c.

The zero-momentum peak in Fig. 2 has a calculated width of $\sigma_p = 7.3$ MeV/c. We arbitrarily accept as decays at rest all events with fitted end momentum less than $3\sigma_p = 21.9$ MeV/c. The 46 events satisfying this cut are shown shaded in Figs. 1 and 2. The fraction of decays at rest lost by this cut is presumed negligible. There is a background of decays in flight in the stopping peak which is evaluated by integrating the curve in Fig. 2 from zero to $3\sigma_p$. The result, one event, is subtracted from the total, resulting in an estimated 45 decays at rest.

The scanning efficiency for accepted τ decays was measured by double-scanning 20% of the film containing 39 bona fide τ's. The single-scan efficiency was found to be $(87\pm5)\%$.

In order to calculate N_d, the number of K^- decays at rest, the following formula was used:

$$N_d = N_\tau / B\epsilon_\tau (1-F_\pi). \quad (4)$$

Here N_τ is the estimated number of τ decays at rest, B is the τ-decay branching ratio for K^-, ϵ_τ is the average scanning efficiency for finding τ decays, and F_π is the fraction of τ's eliminated by the pion length cut of 2 cm due to scattering, decay, or insufficient pion range. The result is

$$N_d = \frac{45}{(0.0554)(0.88)(1-0.104)} = (1.03\pm0.15)\times10^3. \quad (5)$$

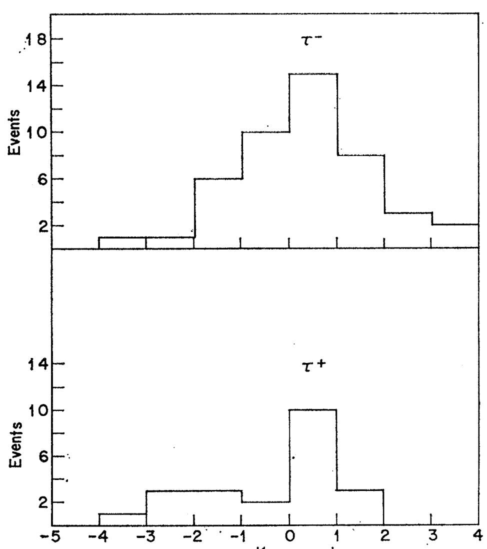

FIG. 4. Distribution of K_T for 46 τ^- decays at rest and 22 τ^+ decays at rest.

ANALYSIS OF STOPS

In order to determine the number of K^- stops in the same film [N_s of Eq. (3)], 905 frames were scanned for stopping K^- leading to any reaction. These 905 frames were evenly distributed, in groups of $\sim$75, throughout the frames scanned for decays. In all phases of scanning, measuring, and analysis, the kaon tracks were treated identically to those in the decay events, except for the addition of a K-test cut as discussed below. Since most reactions of the stopped mesons are kinematically underconstrained, one has only the directly measured K^- momentum available to determine whether the interaction was in flight or at rest. The resolution of this measurement is $\sim$80 MeV/c for

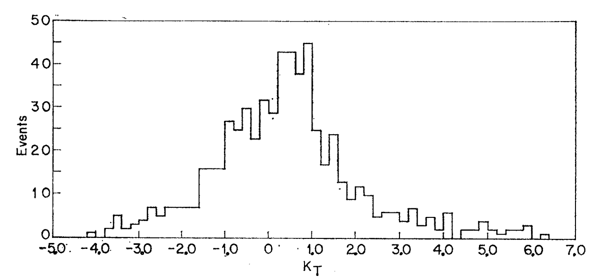

FIG. 5. Distribution of K_T for 613 accepted stopping K^- candidates. There are 10 more accepted candidates with $K_T>7$ which are not shown.

stops, and so the at-rest determination is more complicated than for the τ decays.

Due to the nonlinearity of the range-momentum relation, the distribution of measured momenta for stopped particles has a peculiar shape. It is convenient, therefore, to do the analysis in terms of a related parameter which is more nearly Gaussian distributed. This parameter is the TVGP "KTEST" defined at the track midpoint by

$$K_T = \frac{K(\text{range}) - K(\text{fit})}{\sigma_K(\text{fit})}. \tag{6}$$

Here, K(range) is the curvature that the track would have (in the absence of multiple Coulomb scattering) if the particle stopped at the measured end point. K(fit) is the curvature from the space-curve fit to the measured points. It has an estimated error of $\pm\sigma_K$(fit). Negative values of K_T correspond to "overstopped" tracks. Interactions in flight yield K_T distributed about positive values. Figure 3 shows the relationship between end momentum and average K_T. The K_T distribution for a sample of known stopped kaons (the 46 at-rest negative τ decays described above, and 22 K^+ τ decays) is shown in Fig. 4.

In the scan for stopped K^-, 1438 events were accepted for measurement, and 623 of these passed all the cuts. The attenuation was mainly due to restrictive geometrical cuts. The 905 frames were all double scanned with average over-all efficiency of $(99.9\pm0.1)\%$. The K_T distribution of these events is shown in Fig. 5. With a minimum beam track length of 6 cm and a field greater than 40 kG, the shape of a slow or stopping K^- orbit uniquely identifies it. Therefore, there is no π^- contamination of the beam. One sees, however, from the tail at high K_T, that there is a contamination of in-flight events which must be subtracted from the total. Correction for this contamination by comparison of the K_T distribution for known stops (Fig. 4) with that of Fig. 5 is in principle possible, but it would be difficult to eliminate the possibility, for example, that the interaction cross section rises sharply below, say, 50 MeV/c. Figure 3 shows that interactions occurring between zero and 50 MeV/c would have K_T distributions virtually indistinguishable from the stopped events. However, the high-momentum resolution of this chamber allows a reasonably small upper limit to be placed on the in-flight contamination by invoking unitarity.

The correction to the observed number of stopped K^- for interactions in flight was necessary only for inelastic interactions. (An "elastic" K^- interaction is defined, for the purposes of this paper, to be one in which the K^- reemerges without any other visible prongs. All others are called inelastic.) Events in which elastic scatters were observed were eliminated from both the numerator and denominator of Eq. (3) at the scan table. Events in which such scatters were undetected are present in the numerator and denominator in the same proportion. We calculate the number of inelastic interactions occurring with $K_T<3$ to be less than 4% of stops. This is the maximum allowed by unitarity including s- and p-wave contributions. In what follows, we take the correction for in-flight inelastic interactions to be $(2\pm2)\%$.

We find the total number of K^- stops in the film [N_s of Eq. (3)] to be

$$N_s = R(N_s'/\epsilon_s)F. \tag{7}$$

In this expression, R is the ratio of the number of frames scanned for τ^- decays to the number scanned for K^- stops. The average scanning efficiency for stops is ϵ_s. The term N_s' represents the number of acceptable K^- interactions and decays observed (Fig. 5) with $K_T\leq3$. The factor F is a correction factor for the following effects: decays in flight (-2.5%), interactions in flight (-2.7%), and loss of events due to the requirement $K_T\leq3$ $(+3\%)$.

We find

$$\begin{aligned} N_s' &= 592\pm24, \\ \epsilon_s &= (99.9\pm0.1)\times10^{-2}, \\ F &= 0.98\pm0.04, \\ R &= 68.4, \end{aligned} \tag{8}$$

and thus

$$N_s = (39.6\pm2.3)\times10^3.$$

RESULTS AND DISCUSSION

We calculate the average cascade time according to (3) using (5) and (8). The result is

$$T_c = (3.2\pm0.5)\times10^{-10}\ \text{sec},$$

where we have used $\tau=1.234\times10^{-8}$ sec. This result agrees with the previously measured value[4] of $(2.5\pm0.4)\times10^{-10}$ sec.

It should be pointed out that the measured quantity, which we have called T_c, is greater than the actual cascade time in that it includes the time for the K^- to moderate from an initial momentum P to atomic capture. The value of P is determined by the resolution obtained in measuring the momentum of the K^- in τ decays at rest (Fig. 2). In this experiment this is 7.3 MeV/c, corresponding to a K^- velocity $\beta=0.014$. The moderation time from this velocity to atomic capture is estimated[13] to be $\sim10^{-13}$ sec. This is, however, probably a serious underestimate for liquid helium.

[13] E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).