Interpretation of Small-Angle $\pi^- p$ Elastic Scattering from 1.71 to 3.01 GeV/c*

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(Received 1 June 1970)

Elastic scattering at small angles in the reaction $\pi^- p \to \pi^- p$ has been investigated for Regge recurrences in the region 1.71–3.01 GeV/c. From examination of differential-cross-section data at low t values (see preceding paper), we find additional evidence for the existence of both the established $N_\gamma(2210)$ and the predicted $N_\alpha(2220)$. The resonance parameters obtained for the $N_\alpha(2220)$ are mass 2245 MeV, width 330 MeV, elasticity 0.15.

I. INTRODUCTION

In recent years pion-nucleon elastic scattering data in the low-energy region ($\lesssim 3$ GeV/c) have been appearing in increasing amounts and with increasing precision. In particular, very accurate total cross-section and differential cross-section data are now available, and some polarization data are also in the literature.[1] Perhaps the most traditional method of analyzing these low-energy data has been the phase-shift analysis in which the partial-wave composition of the data is determined.[2,3] Such analyses, in general, depend little upon specific models of the scattering mechanism. In the last several years, however, a number of attempts have been made to fit pion-nucleon elastic scattering data using a sum of direct-channel resonance amplitudes plus a background amplitude which is usually taken to have a Regge form.[4-6] Whereas the motivation of phase-shift analyses is the determination of the partial-wave amplitudes with its consequent "discovery" of resonances, the motivation of the resonance-plus-Regge-background models (interference models) is more complex. In the latter models the background amplitude and most of the resonance amplitudes are taken as determined or nearly determined (e.g., the elasticities of the resonances may be allowed some variation) and the aim is then (1) to fit the data and examine the validity of the model, and (2) to determine the parameters of the one or two suspected, but unknown, resonance amplitudes. The analysis of backward $\pi^- p$ elastic scattering by Barger and Cline[5] is an early example of the resonance-plus-Regge technique, while the more recent analysis of Coulter, Ma, and Shaw[6] incorporates the notion of duality to avoid the "double-counting" criticisms of the older interference models. In the present paper, new $\pi^- p$ elastic scattering data are analyzed using a simple resonance-plus-background parametrization; the spirit of the investigation is close to that of the phase-shift analysis in that little use is made of specific theoretical models, but the technique is similar to that of the interference model.

Impetus for the present analysis comes from a recent high-precision measurement of the $\pi^- p$ elastic scattering differential cross section at 13 beam momenta from 1.71 to 3.01 GeV/c and values of $-t \lesssim 0.7$ (GeV/c)^2 by an Iowa State University–Minnesota collaboration.[7] Examination of the data reveals a considerable departure from an e^{bt} form for the differential cross section; in particular, for fixed beam momentum one interpretation of the differential cross section is a slow t variation superposed on an e^{bt} background: This t variation is then found to vary with beam momentum. These facts suggest the presence of one or more resonance amplitudes. In the center of the ISU-Minn. data region ($\sim$2300 MeV) the most prominent resonances are the well-established $G_{17}(2190)$ (N_γ) and the unconfirmed $H_{19}(2220)$ (N_α). The G_{17} is established by phase-shift and other analyses,[2,3,8] while the strongest evidence at the present time for the H_{19} is the London Legendre-polynomial fits.[9] In the analysis presented here the existence of the $G_{17}(2190)$ is assumed, and an attempt is made to determine the presence or absence of an H_{19} amplitude resonating at 2200±70 MeV, the position predicted for the third recurrence on the N_α trajectory by extrapolation from the two lower known recurrences. The technique used is to lump the lower partial waves into a simply parametrized background amplitude to which is added a combination of resonance

* Work performed in part in the Ames Laboratory of the U. S. Atomic Energy Commission. Contribution No. 2760.

[1] G. Giacomelli, P. Pini, and S. Stagni, CERN Report No. CERN/HERA 69-1 (unpublished).

[2] C. Lovelace, in *Proceedings of the International Conference on Elementary Particles, Heidelberg, 1967*, edited by H. Filthuth (North-Holland, Amsterdam, 1968); A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[3] A. Donnachie, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968); R. J. Plano, in *Proceedings of the Lund International Conference on Elementary Particles, Lund, Sweden, 1969*, edited by G. von Dardel (Institute of Physics, Lund, Sweden, 1969).

[4] See, e.g., A. Donnachie and R. G. Kirsopp, Nucl. Phys. **B10**, 433 (1969); F. N. Dikmen, Phys. Rev. Letters **22**, 622 (1969); V. Barger and D. Cline, Phys. Rev. **155**, 1792 (1967).

[5] V. Barger and D. Cline, Phys. Rev. Letters **16**, 913 (1966).

[6] P. W. Coulter, E. S. Ma, and G. L. Shaw, Phys. Rev. Letters **23**, 106 (1969).

[7] M. Fellinger, E. Gutman, R. C. Lamb, F. C. Peterson, L. S. Schroeder, R. C. Chase, E. Coleman, and T. G. Rhodes, preceding paper, Phys. Rev. D **1**, 1777 (1970).

[8] A. Yokosawa, S. Suwa, R. E. Hill, R. J. Esterling, and N. E. Booth, Phys. Rev. Letters **16**, 714 (1966).

[9] W. Busza, D. G. Davis, B. G. Duff, F. F. Heymann, C. C. Nimmon, D. T. Walton, E. H. Bellamy, T. F. Buckley, P. V. March, A. Stefanini, J. A. Strong, and R. N. F. Walker, Nuovo Cimento **52A**, 331 (1967).

amplitudes. Fits to the data determine the existence of the resonance amplitudes.[10] The conclusion is that the $H_{19}(2220)$ amplitude is present, although other higher partial waves are not ruled out.

In Sec. II the theoretical model is presented and briefly justified, in Sec. III the fits to the data are given, and their validity discussed, and in Sec. IV the main conclusions are summarized.

II. THEORETICAL MODEL

We use a simple theoretical model for $\pi^- p$ elastic scattering in the 2.2-GeV/c region which lumps all resonances except the G_{17} ($l=4$, $I=\frac{1}{2}$, $J=\frac{7}{2}$) and H_{19} into a non-spin-flip diffractive background term of the form[11]

$$f_B=(\alpha+i)(k\sigma/4\pi)e^{\frac{1}{2}bt},$$

where α, σ, and b are constants, k is the magnitude of the three-momentum in the c.m. system, and t is the square of the four-momentum transfer from initial to final pion. We add to this term the resonance amplitudes f_{G17}, g_{G17}, f_{H19}, and g_{H19}, where f and g represent, respectively, the non-spin-flip and spin-flip amplitudes. For example, the G_{17} amplitudes are given by the forms

$$f_{G17}=\tfrac{2}{3}k^{-1}4a_{4-}P_4(\cos\theta),$$

$$g_{G17}=\tfrac{2}{3}k^{-1}(-1)a_{4-}\sin\theta\frac{dP_4(\cos\theta)}{d(\cos\theta)},$$

where P_4 is a Legendre polynomial and

$$a_{4-}=x/[(2/\Gamma)(E_0-E)+i].$$

Here x is the resonance elasticity, Γ is the resonance total width, E_0 is the resonance mass, and E is the total c.m. energy. The differential cross section is therefore

$$d\sigma/d\Omega=|f_B+f_{G17}+f_{H19}|^2+|g_{G17}+g_{H19}|^2.$$

Since there is no spin-flip background term, and the resonance amplitudes are much smaller than the background amplitude at small t, most of the t structure in the differential cross section will be determined by the interference between the non-spin-flip resonance terms and the large non-spin-flip background term. For example, at $-t\simeq 0.3$ (GeV/c)^2 the ratio of the background to the resonance contribution as determined from the fits (see Sec. III) varies from 6.3/0.05 at a laboratory beam momentum of 1.71 GeV/c to 6.3/0.18 at a laboratory beam momentum of 2.16 GeV/c.

Other resonances will not contribute significantly in the region investigated. Examination of known resonances with low masses ($E_0<2000$ MeV) shows that they all have relatively small amplitudes in the data region considered. There are two resonances, the $H_{3,11}(2420)$ and the $D_{13}(2030)$, which have masses in the data region. Lovelace[2] lists the elasticity of the $H_{3,11}$ as approximately half the elasticity of either the G_{17} or the H_{19}. Taking into account this and its smaller isospin coefficient ($\frac{1}{3}$ as opposed to $\frac{2}{3}$ for the G_{17} and H_{19}), the amplitude of the $H_{3,11}$ should be about one-eighth the amplitude of the G_{17} plus the H_{19}. Therefore, the effect of the $H_{3,11}$ resonance should be overshadowed by the G_{17} and H_{19}, and is ignored in the model. The D_{13} amplitude does not add much to the t structure, because $P_2(\cos\theta)$ is positive and decreases almost linearly in $-t$, behaving very much like the background as a function of t. Since the effects of the D_{13} are indistinguishable from those of the background, the D_{13} is also ignored in the analysis of the differential cross-section data. Further, any possible effects of the D_{13} or $H_{3,11}$ can be eliminated by considering only the data that lie well away from these two resonances and also lie close to the G_{17} and H_{19}. This is discussed below.

This crude model will only be valid in a restricted t region. Empirically, the model does not explain differential cross-section data for $-t\gtrsim 0.7$ (GeV/c)^2, i.e., it cannot reproduce the well-known second bump structure. The difference between the data and the predicted differential cross section becomes several times the measured error for the larger $-t$ values. Also, other effects have previously been put forward to explain the elastic scattering at large angles.[12]

Further, the model will not be valid at angles very near $\theta=0$. Previously, it was noted, in explaining the differential cross-section data, that the effect of the $D_{13}(2030)$ was similar to and indistinguishable from the background term and hence was ignored. According to Lovelace,[2] the elasticity of the D_{13} is approximately the same as that of the G_{17}, so that the D_{13} cannot be neglected (or lumped into the background) when addressing the total cross-section data. As expected, the D_{13} does appear as a significant effect in the total cross-section data. In summary, low angular momentum resonances such as the $D_{13}(2030)$ can be neglected for purposes of examining the detailed t dependence of the differential cross-section data in a restricted t range, but they cannot be neglected in fits to the total cross-section data if they have appreciable elasticities.

In the following analysis the model is applied primarily to the region $0.15<-t<0.6$ (GeV/c)^2. There is, in addition, a discussion of the total cross-section data.

[10] The method used in this work is similar, but not identical, to the phase-band analysis: see M. J. Moravcsik, Phys. Rev. **177**, 2587 (1969).

[11] The use of a non-Reggeized background is justified because (a) an attempt is made to keep the analysis as free of theoretical prejudices as possible, and (b) the practical effects of a Regge background are small because the variation of s, the c.m. energy squared over the data region, is only several hundred MeV.

[12] S. Suwa, A. Yokosawa, N. E. Booth, R. J. Esterling, and R. E. Hill, Phys. Rev. Letters **15**, 560 (1965), and references therein.

TABLE I. Fits of differential cross-section data for zero, one, and two resonance terms plus background.

No. of Resonances	Background σ (mb)	α	b $(\text{GeV}/c)^{-2}$	G_{17} resonance M (MeV)	Γ (MeV)	x	H_{19} resonance M (MeV)	Γ (MeV)	x	χ^2	$P(\chi^2)$ (%)
					Data group I[a]						
0	40.2	−0.05	8.71							775	$\sim 10^{-30}$
1	27.7	−0.71	7.46	2230	219	0.23				257	$\sim 10^{-4}$
2	33.1	−0.35	7.28	2260	239	0.09	2245	329	0.15	161	26
					Data group II[b]						
1	27.6	−0.72	7.45	2226	219	0.27				179	2×10^{-2}
2	33.4	−0.31	7.30	2313	240	0.12	2226	267	0.15	130	19
					Data group III[c]						
1	25.1	−1.05	8.06	2225	157	0.14				178	$\sim 10^{-4}$
2	33.5	−0.32	7.30	2224	186	0.08	2277	371	0.17	107	17

[a] Data range: $1.71 \leq P_L \leq 3.01$ GeV/c, $0.15 < -t < 0.6$ $(\text{GeV}/c)^2$; 159 data points.
[b] Data range: $1.71 \leq P_L \leq 3.01$ GeV/c, $0.15 < -t < 0.5$ $(\text{GeV}/c)^2$; 126 data points.
[c] Data range: $1.91 \leq P_L \leq 2.41$ GeV/c, $0.15 < -t < 0.6$ $(\text{GeV}/c)^2$; 103 data points.

III. ANALYSIS OF DATA

Three successive models were fitted to the data. The first model consists of a background term only, with α, σ, and b as parameters. The second model contains the background amplitude plus a G_{17} resonance amplitude with the additional G_{17} resonance parameters x, E_0, and Γ (six parameters total). The third model consists of the H_{19} resonance plus the G_{17} and the background. This adds the three H_{19} resonance parameters (nine parameters total). For each model the parameters were varied using the program MINFUN[13] until the least χ^2 was obtained. The ISU-Minn. data[7] at beam momenta 1.71, 1.81, 1.91, 2.01, 2.09, 2.16, 2.23, 2.31, 2.41, 2.51, 2.62, 2.76, and 3.01 GeV/c were fitted in the t range $0.15 < -t < 0.6$ $(\text{GeV}/c)^2$. The results of these fits are shown in Table I. In the table are shown the parameters for the background and for each resonance, the χ^2, and the χ^2 probability obtained from the fits of the three models to three different sections of the data. The zero-resonance fit refers to the model consisting of the background amplitude only, the one-resonance fit to the model consisting of background and G_{17} resonance terms, and the two-resonance fit to the model consisting of background, G_{17}, and H_{19} terms. Table I is divided into three groups of fits, each group containing fits over different data regions. The first group of three fits utilized the entire region of data considered, with beam momenta P_L of 1.71–3.01 GeV/c, and a $-t$ range 0.15–0.6 $(\text{GeV}/c)^2$. The second group contains the models fitted to data with beam momenta of 1.71–3.01 GeV/c and a more restricted $-t$ range of 0.15–0.5 $(\text{GeV}/c)^2$. The third group contains the models fitted to data with beam momenta in the restricted range 1.91–2.41 GeV/c, and a $-t$ range 0.15–0.6 $(\text{GeV}/c)^2$. The second and third groups of fits, for the reduced data regions, are discussed later in this section.

[13] R. J. Pankhurst, CERN Report No. D502, 1964 (unpublished).

A comparison of χ^2's and χ^2 probabilities for the first three fits of Table I indicates that the two-resonance model is superior to both the background model and to the background-plus-G_{17} model in fitting the data over the entire region of data considered. The differences can be seen visually in Fig. 1, which contains graphs of the data and the theoretical $d\sigma/dt$ for the three models at beam momenta 1.71 and 2.16 GeV/c. The background-only model is the worst fit. The relatively poor fit of the background-plus-G_{17} model indicates that some other effect is needed to explain the data. The good fit of the background-plus-G_{17}-plus-H_{19} model indicates that this model is a possible explanation of the t dependence of the differential cross section.

The large differences between the χ^2 probabilities for the zero-, one-, and two-resonance fits given at the top of Table I raise a number of questions about the analysis. For example, do the values of the parameters found by fitting agree with the values found by other methods? Does the data region analyzed sufficiently exclude effects at large values of $-t$? Is the assumption that the effects of the $D_{13}(2030)$ and $H_{3,11}(2420)$ resonances are minimal justified? How do the predictions of the two-resonance model compare with total cross-section and polarization data? Could a different background parametrization equally well fit the differential cross-section data? And, finally, could a resonance other than the H_{19} combine with the G_{19} to give equal or better results? A discussion of these questions is given below.

The error of the G_{17} parameters, calculated by MINFUN,[13,14] is 50 MeV for the mass and 40 MeV for the width. Lovelace[2] gives the G_{17} a mass of 2265 MeV and a width of 298 MeV. A comparison with Table I shows that the fitted mass of the G_{17} agrees with the

[14] The error for the central masses was also determined from the background-plus-G_{17}-plus-H_{19} fit by varying only the two resonance masses until the χ^2 probability dropped to 1%. The error in the masses for both the G_{17} and H_{19} was approximately 30 MeV. This error is roughly equivalent to the error given by MINFUN.

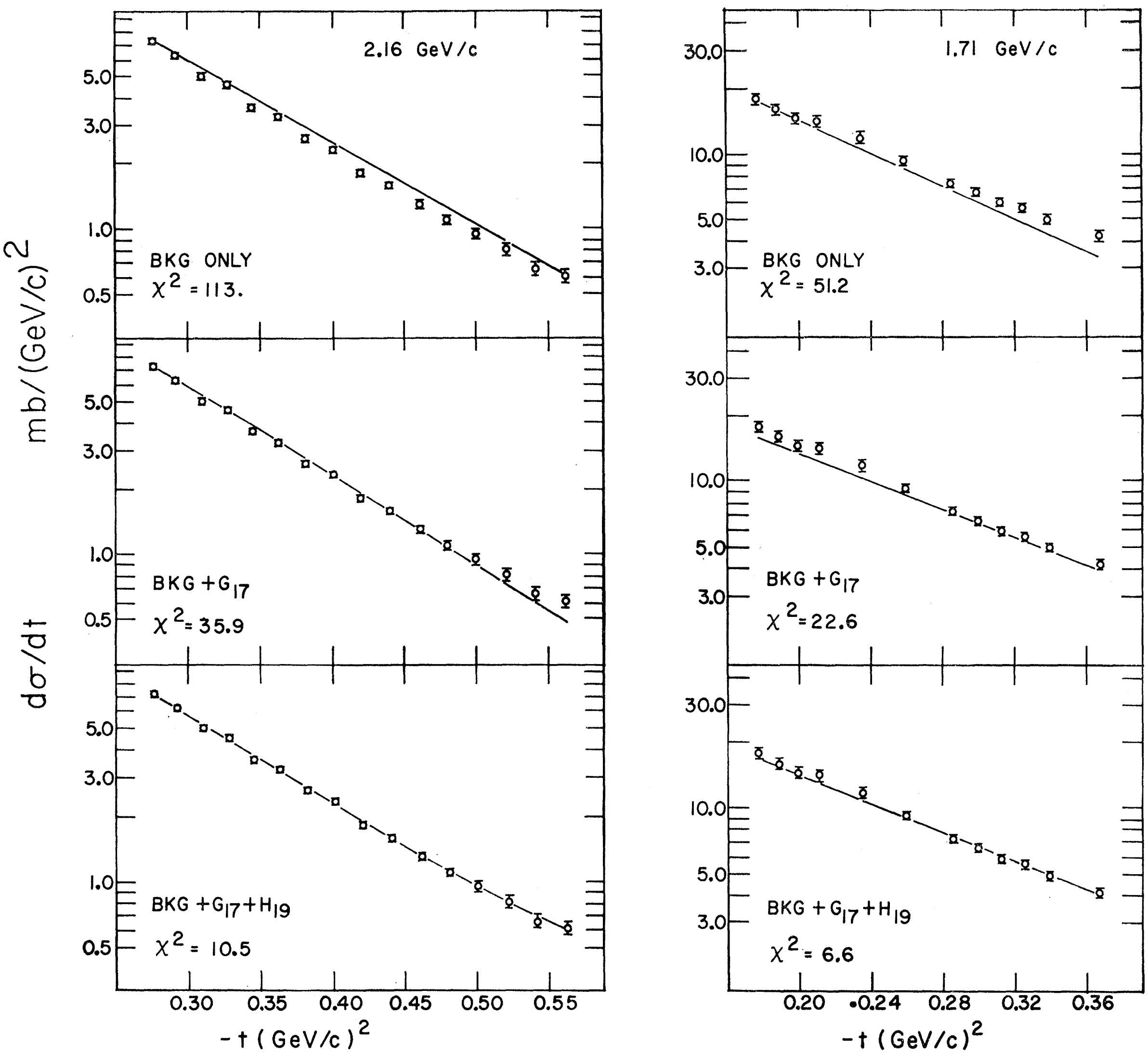

FIG. 1. Comparison of differential cross-section data from the ISU-Minn. group (Ref. 7) with the theoretical differential cross sections calculated from the three different models: background, background plus G_{17}, and background plus G_{17} plus H_{19}, at beam momenta 2.16 and 1.71 GeV/c.

mass as given by Lovelace. (The two width determinations are in rough agreement.) Further, the fitted G_{17} mass and width values agree with those given in the Particle Data Group tables.[15] The fitted H_{19} mass value of 2245 MeV agrees with the mass value of the third recurrence on the N_α Regge trajectory (2200±70 MeV) as calculated by extrapolating from the first two recurrences (938 and 1688 MeV)[15] using a linear Regge trajectory. Also the fitted value of α (-0.35), the real part of the background term, agrees with the value obtained from other sources[16] (-0.1 to -0.3).

To ensure that the large difference in χ^2 probability between the one- and two-resonance model fits, as seen in Table I, was not due to variations of t near -0.6 (GeV/c)^2, the same method was used to fit the data for the restricted t range $0.15<-t<0.5$ (GeV/c)^2. The results are shown in Table I. Although the fit for the one-resonance model over the reduced t range is somewhat better than the one-resonance fit over the unreduced t range, the two-resonance fit is still clearly superior, and it is concluded that results obtained over the entire t range from 0.15 to 0.6 (GeV/c)^2 are not strongly dependent upon the exact value of the maximum $-t$ in the data sample.

Varying the lower limit of the fitted t range would not alter the results of the analysis because of the scarcity of data there. For example, raising the lower $-t$ limit

[15] A. Barbaro-Galtieri, S. E. Derenzo, L. R. Price, A. Rittenberg, A. H. Rosenfeld, N. Barash-Schmidt, C. Bricman, Matts Roos, Paul Söding, and C. G. Wohl, Rev. Mod. Phys. **42**, 87 (1970).

[16] M. N. Focacci and G. Giacomelli, CERN Report No. 66-18 (unpublished).

from 0.15 to 0.25 $(\text{GeV}/c)^2$ eliminates only 12 out of 159 data. Further, the effect of eliminating these low-$(-t)$ points from the fit has been partially investigated during the fits to the restricted beam-momentum range, since the beam momentum cut also eliminates the low-$(-t)$ points (see preceding paper and following paragraph). The conclusion is that the discrimination between the one- and two-resonance fits is not affected by varying the lower $-t$ limit of the fits.

To better eliminate any effects the $H_{3,11}$ or D_{13} might have, two fits were made for beam momenta in the restricted range $1.91\leq P_L\leq 2.41$ GeV/c over the t range $0.15\leq -t\leq 0.6$ $(\text{GeV}/c)^2$. This momentum range, corresponding to an energy range 2120–2330 MeV, is between the masses of the $D_{13}(2030)$ and $H_{3,11}(2420)$. This energy range is far enough from the central masses of the D_{13} and $H_{3,11}$ that the dominant resonance effects are due to the G_{17} and H_{19}. The results of these fits are shown in Table I. For these two fits, the difference in χ^2 probability between the one- and two-resonance models is the same as the difference in probability for the fits over the entire beam-momentum range. Hence our original assumption that the effects of the D_{13} and $H_{3,11}$ in the differential cross-section data can be lumped in the background appears reasonable. In other words, the χ^2 probabilities are not strongly dependent upon the limits of the beam-momentum range fitted.

When total cross-section data from $1.719\leq P_L\leq 2.665$ GeV/c taken from Carter *et al.*[1] are compared with the theoretical total cross section predicted from the two-resonance model, a large discrepancy is noted. For 23 data points, a comparison between the predicted total cross-section values and the Carter *et al.* data resulted in a χ^2 of 157. Also the peak in the data is at beam momentum 2.05 GeV/c (2170 MeV), while the peak in the predicted values is at beam momentum 2.18 GeV/c (2240 MeV). The difference in the values of the masses for these two peaks is almost twice the error calculated for the central masses of the two resonances. It is clear that the two-resonance model, as it stands, cannot explain the total cross-section data. It is noted, as explained in Sec. II, that the total cross section is proportional to the imaginary part of the scattering amplitude at $t=0$, where the $D_{13}(2030)$ can no longer be lumped into the background. A two-parameter fit to the Carter *et al.* total cross-section data was made with the two-resonance model plus an additional D_{13} term. The values for the central mass (2060 MeV) and width (293 MeV) of the D_{13} were taken from Lovelace,[2] and σ and the D_{13} elasticity were varied. The other eight parameters from the two-resonance model were taken from the nine-parameter fit to the differential cross-section data (values in Table I). σ was varied in increments of 1.0 mb from 24.1 to 33.1 mb. The D_{13} elasticity was varied in increments of 0.06 from 0.06 to 0.36. The best results were for $\sigma=31.1$ mb and $x_{D13}=0.24$ with $\chi^2=34.2$, which is to be compared to $\chi^2=157$ given above for the two-resonance model without the D_{13}. The fit with the D_{13} had a probability of about 4%, which undoubtedly could have been improved if σ and x_{D13} has been varied continuously. This indicates that the two-resonance model can interpret the total cross-section data if the D_{13} dependence is no longer lumped into the background. In other words, the D_{13} is important in the total cross-section data, but does not contribute significantly to the t dependence of the differential cross section in our data region: $0.15<-t<0.6$ $(\text{GeV}/c)^2$.

TABLE II. Fits of differential cross-section data for other resonance combinations.

Terms included	χ^2	$P(\%)$
Bkg$+H_{19}$	200	0.34
Bkg$+G_{17}+H_{1,11}$	160	28
Bkg$+G_{17}+I_{1,11}$	158	31
Bkg$+G_{17}+I_{1,13}$	162	24

To determine if the model of the G_{17} plus H_{19} plus a background is unique in explaining the differential cross-section data, fits were also made for several other models involving different resonance combinations over the entire data region $1.71\leq P_L\leq 3.01$ GeV/c, $0.15<-t<0.6$ $(\text{GeV}/c)^2$. Six additional models were fitted: a background-plus-H_{19} resonance model, and five models with a background plus a G_{17} resonance in combination with the resonances $H_{1,11}$, $I_{1,11}$, $I_{1,13}$, F_{15}, and F_{17}, respectively. As before, the background parameters were σ, α, and b, and the resonance parameters were E_0, Γ, and x. The χ^2 and χ^2 probability for these fits are shown in Table II.

The background-plus-H_{19} model fits the data better than the background-plus-G_{17} model (see Table I); however, there is still a significant difference between the H_{19} alone and the G_{17}-plus-H_{19} model (see Table I). One concludes that the G_{17} is still important in interpreting the data. The better fit of the background-plus-H_{19} model compared with the background-plus-G_{17} model and the larger elasticity of the H_{19} compared with the G_{17} in the background-plus-G_{17}-plus-H_{19} model (see Table I) does suggest that more of the structure in the data is due to the H_{19} than to the G_{17}.

In Table II the three two-resonance models (background plus G_{17} plus $H_{1,11}$, background plus G_{17} plus $I_{1,11}$, and background plus G_{17} plus $I_{1,13}$) all give fits to the differential cross-section data that are as good as the background-plus-G_{17}-plus-H_{19} model. The model with the $H_{1,11}$ is expected to fit well, since the $H_{1,11}$ amplitude differs from the H_{19} amplitude only by a constant in the non-spin-flip term and a sign change in the spin-flip term. The constant in the non-spin-flip term has little influence, since the two terms can be made equivalent by varying the elasticity. The spin-flip term is small and does not contribute much to the differential cross-section structure. The indistinguish-

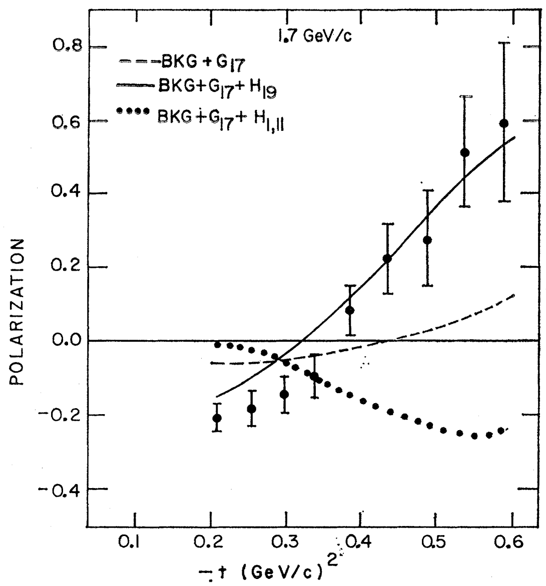

FIG. 2. Comparison of polarization data from Esterling *et al.* (Ref. 1) with theoretical polarization calculated from the three different models: background plus G_{17}, background plus G_{17} plus H_{19}, and background plus G_{17} plus $H_{1,11}$, at beam momentum 1.70 GeV/c.

ability between the I and the H resonances in the present data is seen by studying the Legendre polynomial in the non-spin-flip term, which determines most of the differential cross-section structure. For the angle region of the data used ($0.66<\cos\theta<0.85$) the two Legendre polynomials P_5 and P_6 do not differ much. The difference is even less when P_4+P_5 is compared with P_4+P_6. Since the Legendre polynomials are so similar for the H and I resonances, it is not expected that the difference between them is readily separated, and the analysis of the data bears this out.

Further, an analysis of total cross-section data would not be able to separate the H and I resonances either. The central masses for all resonances in the fits for these three two-resonance models are between 2210 and 2260 MeV. The widths of the resonances are between 250 and 360 MeV. These parameter values are close enough to the equivalent parameter values in the background-plus-G_{17}-plus-H_{19} model that a partial fit to total cross-section data (as before, utilizing an added D_{13} resonance) is expected to give equivalent results. Hence an analysis of total cross-section data will not eliminate any of these three models as possible interpretations.

TABLE III. Analysis of polarization data for various resonance combinations.

Terms included	χ^2
Bkg+G_{17}	213
Bkg+G_{17}+H_{19}	110
Bkg+G_{17}+$H_{1,11}$	397
Bkg+G_{17}+$I_{1,11}$	125
Bkg+G_{17}+$I_{1,13}$	366
Bkg+H_{19}	147

Polarization data, consisting of 33 points collected by Esterling *et al.*[1] at lab momenta 1.7, 1.88, 2.07, 2.27, and 2.50 GeV/c, were examined to try to eliminate some of the possible models, and to verify the above analysis of the differential cross-section data. The parameters obtained from fitting the differential cross section are used to calculate a theoretical polarization $P(\theta)$, which is given by

$$P(\theta)=2\,\mathrm{Im}(fg^*),$$

where $f=f_B+f_{R1}+f_{R2}$ and $g=g_{R1}+g_{R2}$, and where R_1 denotes the G_{17} and R_2 the other resonance. The theoretical polarization was then compared with the polarization data for the t range $0.15<-t<0.6$ (GeV/c)^2. The results are shown in Table III, and Fig. 2, a graph of one comparison of $P(\theta)$ with the data for the models: background plus G_{17} alone, background plus G_{17} plus H_{19}, and background plus G_{17} plus $H_{1,11}$, all at $P_L=1.7$ GeV/c. The analysis of the polarization data agrees with the analysis of the differential cross-section data: The background-plus-G_{17} model does not adequately explain the data, while the background-plus-G_{17}-plus-H_{19} model gives much better results (a χ^2 of 110 compared with 213). The model of the background plus H_{19} only and the model of the background plus G_{17} plus $I_{1,11}$ have a χ^2 comparable with the favored background-plus-G_{17}-plus-H_{19} model, while the models with the $H_{1,11}$ and $I_{1,13}$ both have very high χ^2's (see Table III). The large χ^2's in the polarization analysis are expected since the parameters were not fitted to these data.

The analysis of the polarization data appears to eliminate the $H_{1,11}$ and $I_{1,13}$, both of which have spins $l+\frac{1}{2}$. The reason for the large difference in polarization between resonances with spin $l+\frac{1}{2}$ and those with spin $l-\frac{1}{2}$ (e.g., the H_{19}) is due to the sign change in the spin-flip amplitude. The resonances with spin $l+\frac{1}{2}$ add oppositely to the polarization of the background plus G_{17}, compared to resonances with spin $l-\frac{1}{2}$. The polarization analysis indicates the dominant resonance must have spin $l-\frac{1}{2}$. In conclusion, comparison of the theoretical polarizations with the polarization data of Esterling *et al.* eliminates all models except the background plus G_{17} plus $I_{1,11}$, and the favored background plus G_{17} plus H_{19}.

Fits to the differential cross-section data were also made with the following models: background plus G_{17} plus F_{15} and background plus G_{17} plus F_{17}. In both cases the lowest χ^2 was obtained when the elasticity of the F resonance was negative, a physical impossibility. The elasticities of the resonances were unconstrained during the fits. It is assumed that if the elasticities had been constrained to be positive, the best fit would have been obtained when the elasticity of the F resonance was zero. This is equivalent to the background-plus-G_{17} model. The poor fit for the F_{15} and F_{17} is due to their Legendre polynomials. In the data region P_3 increases as $\cos\theta$ increases, while P_4 and P_5 decrease as $\cos\theta$ increases. Also P_3 has the opposite sign of P_4 and P_5 in the data region. Hence, whenever the G_{17} or H_{19} amplitude interferes destructively with the background, the F_{15} or F_{17} amplitude interferes constructively. The best fits were obtained with negative F elasticities. This has the same effect as switching the sign of the Legendre polynomial. This further supports the view that the angle dependence in the differential cross-section data must be close to the G_{17}, H_{19} combination.

Since the differential cross-section data points rise slightly from a pure exponential decay at $-t$ near 0.6 $(\mathrm{GeV}/c)^2$, an investigation was made to determine if a "better" background would influence the above conclusions. Fits using a four-parameter background were made with the G_{17} plus background and with the G_{17} plus H_{19} plus background. A t^2 term was included in the exponential of the background so that the background amplitude became

$$f_B = (\alpha+i)(k\sigma/4\pi)\exp(\tfrac{1}{2}bt+ct^2),$$

where all parameters are the same as before and c is a constant. Fits were made over the entire data range $1.7 \leq P_L \leq 3.01$ GeV/c and $0.15 < -t < 0.6$ $(\mathrm{GeV}/c)^2$. The fit for the background plus G_{17} resulted in a χ^2 of 191 and a probability of 1.6%. The fit for the background plus G_{17} plus H_{19} resulted in a χ^2 of 155 and a probability of 31.6%.

The addition of a ct^2 term to the background improves the one-resonance fit but does not significantly alter the two-resonance fit. The reason for this lies in the slight upward curve in the data at larger angles. The G_{17} cannot contribute to this, since P_4 goes to zero in this range. P_5 is near its maximum in this area and does contribute. The ct^2 will have the same effect as P_5 in the H_{19} term. It is noted, however, that the ct^2 term in the background cannot replace the H_{19} amplitude, since the two-resonance fits are still preferred over the one-resonance fits by a factor of approximately 20.

IV. CONCLUSION

A one-resonance model does not explain either differential cross-section or polarization data, while a two-resonance model fits differential cross-section, total cross-section, and polarization data well in all data regions studied, although the additional presence of the D_{13} resonance is needed to explain total cross-section data. Polarization data excludes resonances with spin $l+\frac{1}{2}$, such as the $H_{1,11}$ and $I_{1,13}$, as the second resonance to be added to the G_{17}. It is concluded that $\pi^- p$ elastic scattering for $-t \lesssim 0.6$ $(\mathrm{GeV}/c)^2$ in the momentum region 1.7–3.0 GeV/c can be adequately interpreted by the presence of the two resonances, the G_{17}, and H_{19}, that is, the Regge recurrences $N_\gamma(2210)$ and $N_\alpha(2220)$, interfering with a simple background. The H_{19} is not conclusively established, however, since the data can be equally explained by the presence of the two resonances, the G_{17} and $I_{1,11}$. However, there is no known Regge recurrence for the $I_{1,11}$ at the mass value derived from the fit (2230 MeV).[17]

ACKNOWLEDGMENTS

The authors would like to express their gratitude to Professor H. B. Crawley, Professor L. S. Schroeder, Professor F. C. Peterson, and Professor R. C. Lamb for valuable discussions and criticisms, to Dr. R. A. Jespersen for advice on fitting routines, and especially to Professor Lamb, Professor Peterson, and Professor Schroeder for making their data available in advance of publication.

[17] Another argument for favoring the H_{19} over the $I_{1,11}$ interpretation may be made using the L-excitation three-quark model of the baryons. The H_{19} in this model is an $L=4$, $S=\frac{1}{2}$, $J=\frac{9}{2}^+$ second excited state of the $SU(6)$ 56^+-plet ground state, while the $I_{1,11}$ is an $L=5$, $S=\frac{1}{2}$ or $\frac{3}{2}$, $J=\frac{11}{2}^-$ second excited state of the 70^--plet. The model predicts the $I_{1,11}$ mass to be several hundred MeV higher than the H_{19} mass, and both masses to be somewhere above 2200 MeV. The observed resonance mass being 2245 MeV, the natural identification is with the H_{19} state since no lower H_{19} states have been observed. For details, see B. T. Feld, *Models of Elementary Particles* (Blaisdell, Waltham, Mass., 1969), especially Chap. 16.