Comments and Addenda

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in Physical Review Letters *and are not appropriate for regular articles. It includes only the following types of communications: (1) comments on papers previously published in* The Physical Review *or* Physical Review Letters; *(2) addenda to papers previously published in* The Physical Review *or* Physical Review Letters, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section may be accompanied by a brief abstract for information retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.*

Reggeization of the Spinor in Yang-Mills Theory*

ERNEST ABERS†
Department of Physics, University of California, Los Angeles 90024

AND

ROBERT A. KELLER AND VIGDOR L. TEPLITZ
Laboratory of Nuclear Science, Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 22 May 1970)

We extend the second-order part of the calculations of Gell-Mann *et al.* (on Reggeization of a spinor interacting with a massive, neutral vector meson) to the case of a massive, isovector vector meson interacting with the conserved vector current. We find that, in second order, the $I=\frac{1}{2}$ spinor remains a Regge pole, and that no isotopic spin-$\frac{3}{2}$ trajectory appears at the nonsense point. We also discuss the application of the Mandelstam counting procedure.

SEVERAL years ago Gell-Mann, Goldberger, Low, Marx, Singh, and Zachariasen[1–5] studied the conditions under which an elementary particle in conventional field theory can lie on a Regge trajectory for all values of the coupling constant. They showed, in particular, that in the theory of a neutral massive vector meson and a spinor, interacting via a conserved current, the spinor lies on a Regge trajectory in second and fourth order. A condition-counting argument for this result, independent of perturbation theory, has been given by Mandelstam.[6]

The purpose of the present note is to extend in lowest order the work of Gell-Mann *et al.* to the case of an isotopic triplet of massive vector mesons interacting with an isotopic doublet of spinors. This is the theory first considered by Yang and Mills.[7] We deal here only with second-order diagrams, but it should be emphasized that Reggeization of an elementary particle in field theory requires the satisfaction of nontrivial con-

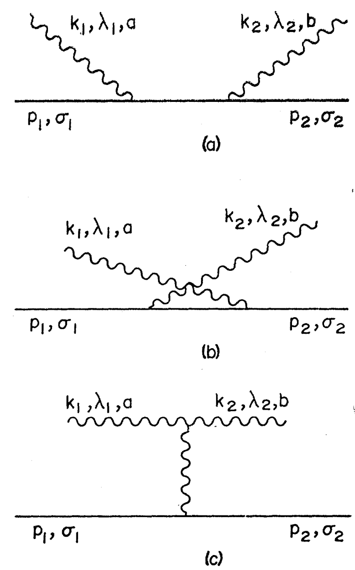

FIG. 1. Second-order diagrams. For neutral vector-meson scattering, only *a* and *b* are present. In the Yang-Mills theory, *c* must be included.

* This work forms part of a dissertation by the second author to be submitted to the Department of Physics, MIT, in partial fulfillment of the requirements for the Ph.D. degree. This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT (30–1) 2098.

† Alfred P. Sloan Foundation Fellow.

[1] M. Gell-Mann and M. L. Goldberger, Phys. Rev. Letters **9**, 275 (1962); **10**, 39 (1963).

[2] M. Gell-Mann, M. L. Goldberger, F. E. Low, and F. Zachariasen, Phys. Letters **4**, 265 (1963).

[3] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[4] M. Gell-Mann, M. L. Goldberger, V. Singh, and F. Zachariasen, Phys. Rev. **133**, B161 (1964).

[5] See also M. Gell-Mann, M. L. Goldberger, and F. E. Low, Rev. Mod. Phys. **36**, 640 (1964); H. Cheng and T. T. Wu, Phys. Rev. **140**, B465 (1965).

[6] S. Mandelstam, Phys. Rev. **137**, B949 (1965): see also E. Abers and V. Teplitz, *ibid.* **158**, 1365 (1967); **165**, 1934(E) (1968).

[7] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).

ditions in second order.[8] In scalar-vector scattering, for example, the scalar fails to Reggeize in second order.[4,9]

Gell-Mann *et al.*[3] in the neutral theory consider the diagrams of Figs. 1(a) and 1(b). The amplitude is given by

$$\begin{aligned}M_G=&\epsilon_{\lambda_2}{}^{\nu*}(k_2)\bar{u}_{\sigma_2}(p_2)\gamma_\nu[(p_1+k_1)\cdot\gamma-m]^{-1}\gamma_\mu u_{\sigma_1}(p_1)\\&\times\epsilon_{\lambda_1}{}^{\mu}(k_1)+\epsilon_{\lambda_2}{}^{\nu*}(k_2)\bar{u}_{\sigma_2}(p_2)\gamma_\mu[(p_1-k_2)\cdot\gamma-m]^{-1}\\&\times\gamma_\nu u_{\sigma_1}(p_1)\epsilon_{\lambda_1}{}^{\mu}(k_1).\end{aligned}\tag{1}$$

They compute the leading contribution to the asymptotic (in $z=\cos\theta_s$) behavior of the even-parity-conserving helicity amplitudes. They find

$$\begin{aligned}f_{11}{}^+&=-\frac{\gamma^2}{\sqrt{2}}\frac{E+m}{8\pi Wk^2}\frac{(E-m-\omega)^2}{W-m}+O(1/z)\,,\\f_{00}{}^+&=-\frac{\gamma^2}{\sqrt{2}}\frac{E+m}{4\pi Wk^2}\frac{\lambda^2}{W-m}+O(1/z)\,,\\f_{01}{}^+&=\frac{\gamma^2}{\sqrt{2}}\frac{E+M}{4\pi\sqrt{2}Wk^2}\frac{\lambda(E-M-\omega)}{W-m}+O(1/z)\,,\\f_{-1-1}{}^+&=-\frac{\gamma^2}{\sqrt{2}}\frac{E+m}{8\pi Wk^2}(W-m)\frac{1}{z}+O(1/z^2)\,,\\f_{-10}{}^+&=-\frac{\gamma^2}{\sqrt{2}}\frac{E+m}{4\pi\sqrt{2}Wk^2}\lambda\frac{1}{z}+O(1/z^2)\,,\\f_{-11}{}^+&=\frac{\gamma^2}{\sqrt{2}}\frac{E+m}{8\pi Wk^2}(E-\omega-m)\frac{1}{z}+O(1/z^2)\,,\end{aligned}\tag{2}$$

where the subscripts refer to the meson helicities. The nontrivial condition for Reggeization is the factorization of the coefficients of z^0 in the sense amplitudes $f_{\lambda_1\lambda_2}$ ($\lambda_1\lambda_2=0,+1$) and z^{-1} in the nonsense amplitudes.

In the Yang-Mills theory, (1) is modified by the presence of isospin coefficients $\tau_b\tau_a$ for the s-channel pole term and $\tau_a\tau_b$ for the u-channel pole term and by the inclusion of the meson-exchange diagram required by current conservation. The question is then: Does the presence of the extra diagram exactly compensate in the second-order leading z behavior for the u-channel isospin factor and restore the Gell-Mann *et al.* result of (2) with an over-all factor of $\tau_b\tau_a$? The answer to the question is affirmative. The amplitude in the Yang-Mills theory is

$$\begin{aligned}M_Y=&\tau_b\tau_a\epsilon_{\lambda_2}{}^{\nu*}(k_2)\bar{u}_{\sigma_2}(p_2)\gamma_\nu[(p_1+k_1)\cdot\gamma-m]^{-1}\\&\times\gamma_\mu u_{\sigma_1}(p_1)\epsilon_{\lambda_1}{}^{\mu}(k_1)+\tau_a\tau_b\epsilon_{\lambda_2}{}^{\nu*}(k_2)\bar{u}_{\sigma_2}(p_2)\\&\times\gamma_\mu[(p_1-k_2)\cdot\gamma-m]^{-1}\gamma_\nu u_{\sigma_1}(p_1)\epsilon_{\lambda_1}{}^{\mu}(k_1)\\&+[(\tau_a\tau_b-\tau_b\tau_a)/(t-m^2)]\epsilon_{\lambda_2}{}^{\nu*}(k_2)\bar{u}_{\sigma_2}(p_2)\\&\times[g_{\mu\nu}\gamma\cdot(k_1+k_2)-(2k_2-k_1)_\mu\gamma_\nu-(2k_1-k_2)_\nu\gamma_\mu]\\&\times u_{\sigma_1}(p_1)\epsilon_{\lambda_1}{}^{\mu}(k_1).\end{aligned}\tag{3}$$

Note that current conservation is expressed by the fact that $M_Y\to 0$ when $\epsilon_{\lambda_2}{}^*(k_2)$ [or $\epsilon_{\lambda_1}(k_1)$] is replaced by k_2 (or k_1). Rewriting (3) as

$$M_Y=\frac{G^s}{s-m^2}\tau_b\tau_a+\frac{G^u}{u-m^2}\tau_a\tau_b+\frac{G^t}{t-m^2}(\tau_a\tau_b-\tau_b\tau_a)\tag{4}$$

and recalling

$$t\to 2k_s{}^2z\quad\text{and}\quad u\to -2k_s{}^2z\quad\text{as}\quad z\to\infty\,,\tag{5}$$

we see that the Yang-Mills spinor Reggeizes provided the leading contribution of $(G^u-G^t)/z$ to the sense amplitude is z^{-1} and to the nonsense amplitude is z^{-2}. To see that is so, we write G^u as

$$\begin{aligned}G^u=\bar{u}_{\sigma_2}(p_2)(&2\gamma\cdot\epsilon_{\lambda_1}p_1\cdot\epsilon_{\lambda_2}{}^*-2\gamma\cdot\epsilon_{\lambda_2}{}^*k_2\cdot\epsilon_{\lambda_1}\\&+2\gamma\cdot k_2\epsilon_{\lambda_1}\cdot\epsilon_{\lambda_2}{}^*-\gamma\cdot k_2\gamma\cdot\epsilon_{\lambda_2}{}^*\gamma\cdot\epsilon_{\lambda_1})u_{\sigma_1}(p_1).\end{aligned}\tag{6}$$

G^t is given by

$$\begin{aligned}G^t=\bar{u}_{\sigma_2}(p_2)(&-2\gamma\cdot\epsilon_{\lambda_1}k_1\cdot\epsilon_{\lambda_2}{}^*-2\gamma\cdot\epsilon_{\lambda_2}{}^*k_2\cdot\epsilon_{\lambda_1}\\&+2\gamma\cdot k_2\epsilon_{\lambda_1}\cdot\epsilon_{\lambda_2}{}^*)u_{\sigma_1}(p_1).\end{aligned}\tag{7}$$

The reduction by one power of z from (2) of the asymptotic contribution to $f_{\lambda_2\lambda_1}$ of $(G^u-G^t)/z$ can be verified by direct computation of

$$\bar{u}_{\sigma_2}(p_2)\gamma\cdot\epsilon_{\lambda_1}(p_1+k_1)\cdot\epsilon_{\lambda_2}{}^*u_{\sigma_1}(p_1)\tag{8}$$

and

$$\bar{u}_{\sigma_2}(p_2)\gamma\cdot k_2\gamma\cdot\epsilon_{\lambda_2}{}^*\gamma\cdot\epsilon_{\lambda_1}u_{\sigma_1}(p_1)\,.\tag{9}$$

The essential point, however, is as follows: Choosing

$$\begin{aligned}&\epsilon_{\lambda_1}=-\lambda_1(\hat{x}+i\lambda_1\hat{y})/\sqrt{2}\,,\quad\lambda_1=\pm1\\&\epsilon_{\lambda_1}=m^{-1}(k,E_k\hat{z})\,,\quad\lambda_1=0\\&\epsilon_{\lambda_2}=-\lambda_2(\hat{x}\cos\theta-\hat{z}\sin\theta-i\lambda_2\hat{y})/\sqrt{2}\,,\quad\lambda_2=\pm1\\&\epsilon_{\lambda_2}=m^{-1}(k,E_k\hat{k}_2)\,,\quad\lambda_2=0\end{aligned}\tag{10}$$

we see that, since $\epsilon_{\lambda_2}{}^*\cdot p_1\sim z^1$ and $\epsilon_{\lambda_2}{}^*\cdot k_1\sim z^1$ while $\epsilon_{\lambda_2}\cdot(k_1+p_1)\sim z^0$, the contribution of (8) is lowered one power of z from the contribution of the first terms in (6) and (7). The second and third terms in (6) and (7) have an effective asymptotic behavior of z^1 from the factors $k_2\cdot\epsilon_{\lambda_1}$ and $\epsilon_{\lambda_1}\cdot\epsilon_{\lambda_2}{}^*$; the matrices $\gamma\cdot\epsilon_{\lambda_2}{}^*$ and $\gamma\cdot k_2$, although z dependent, merely juggle the $\cos\frac{1}{2}\theta$ and $\sin\frac{1}{2}\theta$ factors in the helicity-eigenstate spinors. Similarly, the expression (9), which is the fourth term in (6), has an effective z^0 relative to the z^1 of the other terms in (6) and (7). Hence the Yang-Mills spinor Reggeizes.

It should be noted that in addition to the Reggeization of the elementary spinor, the vanishing of $(G^u-G^t)/z$ shows that there is no nonsense-choosing $I=\frac{3}{2}$ trajectory at $J=\frac{1}{2}$. The question of whether the Reggeization persists in higher orders must await further progress on the problem of renormalizing massive Yang-Mills fields.

The vanishing of $(G^u-G^t)/z$ further shows that the above results generalize from $SU(2)$ isospinor-isovector scattering to a theory with any representation of any internal symmetry group as long as only the three diagrams of Fig. 1 enter in second order and gauge invariance is required.

[8] E. Abers, M. Cassandro, I. Muzinich, and V. Teplitz, Phys. Rev. **170**, 1331 (1968)

[9] D. Shapero, Phys. Rev. **186**, 1697 (1969).

The above result raises two further second-order questions. It has been shown by Abers *et al.*[8] that in the model of Gell-Mann *et al.* the daughter trajectories do not factorize, i.e., there are three degenerate trajectories in second order near each negative integer in the $l=J-\frac{1}{2}$ plane. It is conceivable, although perhaps unlikely, that the degeneracy is absent in the Yang-Mills theory. It is similarly possible to reinvestigate scalar-vector scattering with isotopic spin in second order.

The answers to these questions will depend on the representation of the internal symmetry group.

Finally, Srivastava has asked what the Mandelstam counting procedure[10] predicts in problems with isospin. The answer is that it generalizes straightforwardly: The numbers of conditions and parameters are multiplied by the number of internal symmetry channels. It is thus easy to see from Mandelstam's procedure that spinor-vector scattering with isospin must Reggeize provided it is calculated in a theory with s-channel unitarity. If, however, the theory does not have s-channel unitarity, the spinor need not Reggeize. An example of this latter situation is the theory with isospin but without the meson-exchange diagram of Fig. 1(c) for which it is easy to check that the sense amplitudes in second order have $e^{i\delta}\sin\delta$ violating unitarity by increasing proportionally to s. There is thus a curious and intimate connection among three distinct properties of spinor-vector scattering: gauge invariance, asymptotic behavior of s-channel, partial-wave amplitudes, and s-channel Regge-pole residues.

We are grateful to Professor Y. Srivastava for raising the Yang-Mills question, and to J. Bronzan, D. Dicus. H. Goldberg, and K. Johnson for helpful conversations,

[10] For spinor-vector scattering without isospin, Mandelstam's procedure lies in noting the $J^P=\frac{1}{2}^+$ sense amplitudes must satisfy s-channel unitarity and analyticity but contain six arbitrary constants (three subtraction constants and three Castillejo-Dalitz-Dyson parameters) which are constrained by three $s=0$ conditions and six threshold conditions. The excess of conditions requires that any two unitary theories agree. In particular, the $J^P=\frac{1}{2}^+$ partial waves computed continuation in J must agree with the $J^P=\frac{1}{2}^+$ partial waves computed directly from the field theory.

PHYSICAL REVIEW D VOLUME 2, NUMBER 8 15 OCTOBER 1970

Paraparticles of Infinite Order

JAMES B. HARTLE*
Department of Physics, University of California, Santa Barbara, California 93106

AND

ROBERT H. STOLT† AND JOHN R. TAYLOR†
Department of Physics and Astrophysics, University of Colorado, Boulder, Colorado 80302

(Received 25 May 1970)

We show that with one exception every possible statistical type of particle consistent with the cluster properties of quantum mechanics can be uniquely identified by a single pair of integers (p,q). A particle of type (p,q) has states corresponding to all Young diagrams whose first p columns (and only these) have arbitrary depth and whose first q rows (and only these) have arbitrary length. Parabosons and parafermions of order p are included in this scheme as types $(0,p)$ and $(p,0)$, respectively. If p and q are nonzero, then the particle is of infinite order. The only exception to this classification scheme is a particle with states corresponding to all Young diagrams.

It has recently been shown[1] that all first-quantized theories of identical particles consistent with the cluster laws of quantum mechanics fall into one of two categories: those of finite order and those of infinite order. The finite-order particles may be further classified as parafermions and parabosons of order $p=1, 2, 3, \ldots$ and correspond in a natural way to the second-quantized parafields with the same names.[2]

It is the purpose of this paper to show that infinite-order particles can be very simply classified. The reasons for doing this are two: First, it seems desirable to have a classification of all theoretically possible statistical types of particle; and second, we believe that it may be possible to construct a second-quantized theory of infinite-order particles once their properties are better understood (although we have not actually done so).

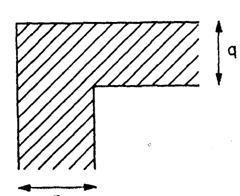

FIG. 1. The $[p,q]$ envelope in the space of Young diagrams contains p columns of arbitrary depth and q rows of arbitrary length.

* Supported in part by the National Science Foundation.
† Supported in part by the U. S. Air Force under Contract No. AFOSR-30-67.

[1] R. H. Stolt and J. R. Taylor, Phys. Rev. D 1, 2226 (1970).
[2] R. H. Stolt and J. R. Taylor, Nucl. Phys. D19, 1 (1970).