Possible Origin for Parity Conservation in Meson-Nucleon Scattering

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(Received 16 April 1970)

We consider parity-nonconserving meson-nucleon scattering and show that self-consistency conditions imposed by a bootstrap-type calculation require that parity be conserved when the meson is pseudoscalar. For a scalar meson, parity is not conserved.

I. INTRODUCTION

SELF-CONSISTENCY is one of the most fundamental of physical principles. It has been suggested that the necessity for a self-maintaining system to achieve self-consistency is responsible for a large number of physical facts, for example, the transformation properties of the $SU(3)$-breaking interaction.[1] Certainly, its role in the bootstrap philosophy is well known. Several years ago, Zachariasen and Zemach[2] suggested that parity conservation in strong interactions also be regarded as a result of a bootstrap-type self-consistency rather than an imposed initial condition. They considered πN scattering in an N/D formalism with only nucleon exchange and achieved encouraging results.

We reconsider this problem, using finite-energy sum rules (FESR) at fixed u, and include the $N^*(1236)$ in the exchange. We find that, with just nucleon exchange, parity is nonconserved for a scalar-meson interaction, and conserved for a pseudoscalar-meson interaction. When the N^* is included, the results do not change significantly (10–15% in the worst case). We find that the cutoff inherent in the FESR affects only the percent accuracy of the result and not the result itself.

In Sec. II we review the formalism for parity-nonconserving meson-nucleon scattering[2,3] and we discuss the sum rule that will be used. The calculation is described in Sec. III, and in Sec. IV our results are discussed and compared with those of Ref. 2.

II. FORMALISM

Consider the parity-nonconserving scattering of a nucleon (p,p') and a meson (q,q'). Four amplitudes are needed to describe the scattering:

$$T=\bar{U}(p')[A+\gamma_5 C+\gamma\cdot\tfrac{1}{2}(q+q')B+\gamma_5\gamma\cdot\tfrac{1}{2}(q+q')D]U(p) \quad (1)$$

or

$$T=\bar{U}(p')[f_1+\sigma\cdot q'\sigma\cdot q f_2+\sigma\cdot q f_3+\sigma\cdot q' f_4]U(p), \quad (2)$$

with

$$\begin{aligned} f_1&=[(E+M)/2W][A+(W-M)B],\\ f_2&=[(E-M)/2W][-A+(W+M)B],\\ f_3&=(q/2W)(C+DW),\\ f_4&=(q/2W)(-C+DW). \end{aligned} \quad (3)$$

If we decompose the scattering into helicity amplitudes, we find

$$\begin{aligned} T_{++}&=(1+\cos\theta)^{1/2}(f_1+f_2+f_3+f_4),\\ T_{+-}&=\frac{\sin\theta}{(1+\cos\theta)^{1/2}}(f_1-f_2-f_3+f_4),\\ T_{--}&=(1+\cos\theta)^{1/2}(f_1+f_2-f_3-f_4),\\ T_{-+}&=\frac{\sin\theta}{(1+\cos\theta)^{1/2}}(-f_1+f_2-f_3+f_4). \end{aligned} \quad (4)$$

If one ignores the exterior kinematic factors (which disappear when the kinematic singularities are removed), one finds

$$\begin{aligned} f_1+f_2&=\tfrac{1}{2}(T_{++}+T_{--}),\\ f_1-f_2&=\tfrac{1}{2}(T_{+-}-T_{-+}),\\ f_3+f_4&=\tfrac{1}{2}(T_{++}-T_{--}),\\ f_3-f_4&=\tfrac{1}{2}(T_{+-}+T_{-+}). \end{aligned} \quad (5)$$

Using (3), one obtains

$$\begin{aligned} \tfrac{1}{2}(T_{++}-T_{--})&=qD,\\ \tfrac{1}{2}(T_{+-}+T_{-+})&=qC/W,\\ \tfrac{1}{2}(T_{++}+T_{--})&=AM/W+(B/W)(EW-M^2),\\ \tfrac{1}{2}(T_{+-}-T_{-+})&=(E/W)A+(B/W)(MW-ME). \end{aligned} \quad (6)$$

For $W=M$ and $\mu/M<1$, the last two equations become

$$\begin{aligned} \tfrac{1}{2}(T_{++}+T_{--})&\cong A-MB,\\ \tfrac{1}{2}(T_{+-}-T_{-+})&\cong MB. \end{aligned} \quad (7)$$

These equations will be used in discussing the amplitudes derived from a specific Lagrangian.

We wish to consider both nucleon and N^* exchange. For the nucleon, assume an interaction of the form[4]

$$L=g\bar{\psi}\gamma_5(a+b\gamma_5)\psi\varphi, \quad (8)$$

[1] R. Dashen, Y. Dothan, S. Frautschi, and D. Sharp, Phys. Rev. **143**, 1185 (1966).
[2] F. Zachariasen and C. Zemach, Phys. Rev. **138**, B441 (1965).
[3] V. Singh, Phys. Rev. **129**, 1889 (1963).
[4] γ_5 is taken to be anti-Hermitian.

with a pseudoscalar meson.[5] In the direct channel, one obtains the isospin-$\frac{1}{2}$ amplitudes,

$$A^{1/2}(s,u)=-\binom{0}{3}\frac{2b^2Mg^2}{s-M^2},\quad B^{1/2}(s,u)=-\binom{0}{3}\frac{g^2(a^2+b^2)}{s-M^2},$$

$$C^{1/2}(s,u)=\binom{0}{3}\frac{2abMg^2}{s-M^2},\quad D^{1/2}(s,u)=0. \tag{9}$$

The crossed amplitudes due to nucleon exchange are easily obtained from these:

$$A^N(s,u)=-\binom{2}{-1}\frac{2b^2Mg^2}{u-M^2},$$

$$B^N(s,u)=+\binom{2}{-1}\frac{g^2(a^2+b^2)}{u-M^2}, \tag{10}$$

$$C^N(s,u)=\binom{2}{-1}\frac{2abMg^2}{u-M^2},\quad D^N(s,u)=0.$$

When the exchanged state is the N^*, the invariant amplitudes can be found from the interaction

$$L=\frac{M^*\Gamma}{3q_*{}^3(E^*+M)}\partial^u\bar{\psi}(a+b\gamma_5)\psi_u\varphi. \tag{11}$$

We use the propagator[6]

$$\bar{\Delta}^{\mu\nu}(k)=3g_{\mu\nu}-\gamma_\mu\gamma_\nu-\frac{4k_\mu k_\nu}{M^{*2}}+\frac{\gamma_\mu(\gamma\cdot k)k_\nu+k_\mu(\gamma\cdot k)\gamma_\nu}{M^{*2}}. \tag{12}$$

This has the calculational advantage of commuting with γ_5.

One obtains for the contribution due to N^* exchange

$$A^{N*}(s,u)=-\gamma(M^*)\binom{1}{4}\{-3q^{*2}Z_u{}^*[a^2(M+M^*)+b^2(M-M^*)]+a^2(M-M^*)(E^*+M)^2+b^2(M+M^*)(E^*-M^2\}/(u-M^{*2})=\binom{1}{4}A^*,$$

$$B^{N*}(s,u)=-\gamma(M^*)\binom{1}{4}\times\frac{3q^{*2}Z_u{}^*(a^2+b^2)-a^2(E^*+M)^2-b^2(E^*-M)^2}{u-M^{*2}}=\binom{1}{4}B^*, \tag{13}$$

$$C^{N*}(s,u)=-\gamma(M^*)\binom{1}{4}\frac{2abM^*(-3q^{*2}Z_u{}^*-E^{*2}+M^2)}{u-M^{*2}}=\binom{1}{4}C^*,$$

$$\gamma(M^*)=M^*\Gamma/3q^{*3}(E^*+M),$$

$$Z_u{}^*=1-(s+M^{*2}-2M^2-2\mu^2)/2q^{*2},$$

$$q^{*2}=[(M+M^*)^2-\mu^2][(M-M^*)^2-\mu^2]/4M^{*2}.$$

Note that $D=0$ for both N and N^* intermediate states. Returning to Eq. (6), we see that this implies that $T_{++}=T_{--}$. We know from Jacob and Wick[7] that parity conservation requires $T_{++}=T_{--}$ and $T_{+-}=T_{-+}$, the latter also being required by time-reversal invariance. This would imply that some portion of the total amplitude is still parity conserving without imposing any exterior conditions.

We assume the usual Regge form for the amplitudes

$$R(s,u)=-\frac{\beta(u)s^\alpha}{\sin\pi\alpha(M^2)}(1+e^{-\pi\alpha i}), \tag{14}$$

and near the pole we rewrite it as

$$R(s,u)=\frac{-\beta(u)s^\alpha(1+e^{-i\pi\alpha})}{\sin\pi\alpha(M^2)+\pi\alpha'(M^2)\cos\pi\alpha(M^2)(u/M^2-1)+\cdots},$$

so that

$$\text{Residue } R(s,u)|_{u=M^2}=-2\beta(M^2)/\pi\alpha'(M^2).$$

Identifying this with the direct-channel nucleon amplitudes [Eq. (9)], one obtains the residues

$$M^2\beta_A(M^2)=\binom{0}{3}\pi b^2Mg^2\alpha'(M^2),$$

$$M^2\beta_B(M^2)=\binom{0}{3}\pi(\tfrac{1}{2}g^2)(a^2+b^2)\alpha'(M^2), \tag{15}$$

$$M^2\beta_C(M^2)=-\binom{0}{3}\pi Mg^2ab\alpha'(M^2).$$

[5] The scalar case may be obtained from the pseudoscalar one by the substitution $b\to -a$.

[6] E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963).

[7] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

The sum rule is of the usual fixed-u type. One considers an amplitude $F^u(s,u)$ such that

$$\lim_{s\to\text{large}}\int_{-\infty}^{\infty}\text{Im}[F^u(s,u)-R(s,u)]ds=0.$$

Then one obtains

$$\int_0^{N_r}\text{Im}F^u(s,u)ds=\frac{\beta(M^2)N_r^{\alpha+1}}{\alpha+1}\xrightarrow[u\to M^2]{}N_r\beta(M^2),\quad(16)$$

and in the first moment

$$\int_0^{N_r}s\,\text{Im}F^u(s,u)ds=\frac{\beta(u)N_r^{\alpha+2}}{\alpha+2}\xrightarrow[u\to M^2]{}\beta(M^2)\frac{N_r^2}{2}.\quad(17)$$

III. CALCULATION

We wish to consider u physical. Therefore, in Eqs. (10) and (12), replace s by u. Using (15), (16), (10), and (12), one obtains the following sum rules for the isovector and isoscalar meson, respectively:

$$\begin{aligned}
&\tfrac{2}{3}Mg^2b^2+\tfrac{4}{3}A^*=2b^2f,\\
&-\tfrac{1}{3}(M^2/\sigma)(a^2+b^2)g^2+\tfrac{4}{3}(M^{*2}/\sigma)B^*=(a^2+b^2)f,\\
&-\tfrac{2}{3}abMg^2+\tfrac{4}{3}C^*=-2abf,\\
&-2Mg^2b^2+A^*=2b^2f,\\
&(M^2/\sigma)(a^2+b^2)g^2+M^{*2}B^*/\sigma=(a^2+b^2)f,\\
&abMg^2+C^*=-2abf,\\
&f=\tfrac{1}{2}Mg^2NR\alpha'(M^2).
\end{aligned}\quad(18)$$

In the equations for the B amplitude, we have replaced N_r^2 by $2M\sigma N_r$, in order to have the same factor appear on the right-hand side of all equations. We want to calculate the ratio b/a. To eliminate the cancellations when only N exchange is considered in the above equations, we consider the linear combinations

$$\begin{aligned}
&\int_0^{N_r}\text{Im}[A^u(s,u)-R_A(s,u)]ds\\
&\qquad\pm\int_0^{N_r}s\,\text{Im}[B^u(s,u)-R_B(s,u)]ds=0,\\
&\int_0^{N_r}\text{Im}[C^u(s,u)-R_C(s,u)]ds\\
&\qquad\pm\int_0^{N_r}s\,\text{Im}[B^u(s,u)-R_B(s,u)]ds=0.
\end{aligned}\quad(19)$$

TABLE I. Tabulation of results of Zachariasen and Zemach (Ref. 2).

	Isoscalar pion	Isovector pion
$b=0$, $a\neq0$	No solution because of repulsive $P_{1/2}$ force	Solution for restricted values of (input coupling)/(output coupling)
$b\neq0$, $a=0$	Solution if $1<\frac{\text{input coupling}}{\text{output coupling}}<\infty$	No solution because of repulsive $S_{1/2}$ force
$ab\neq0$	No interaction since $b^2/a^2<0$	Parity-violating solution for restricted values of coupling ratio

For the isovector meson we find

$$\begin{aligned}
(3b^2+a^2)f_1&=\tfrac{1}{3}Mg^2\left(2b^2-\frac{a^2M}{\sigma}-\frac{b^2M}{\sigma}\right)+\frac{4}{3}\left(A^*+\frac{M^{*2}}{\sigma}B^*\right),\\
(b^2-a^2)f_2&=\tfrac{1}{3}Mg^2\left(2b^2+\frac{a^2M}{\sigma}+\frac{b^2M}{\sigma}\right)+\frac{4}{3}\left(A^*-\frac{M^{*2}}{\sigma}B^*\right),\\
(a-b)^2f_3&=\tfrac{1}{3}Mg^2\left(-2ab-\frac{a^2M}{\sigma}-\frac{b^2M}{\sigma}\right)+\frac{4}{3}\left(C^*+\frac{M^{*2}}{\sigma}B^*\right),\\
-(a+b)^2f_4&=\tfrac{1}{3}Mg^2\left(-2ab+\frac{a^2M}{\sigma}+\frac{b^2M}{\sigma}\right)+\frac{4}{3}\left(C^*-\frac{M^{*2}}{\sigma}B^*\right),
\end{aligned}\quad(20)$$

where the subscript on the f merely distinguishes which linear combination of amplitudes it appears in. The isoscalar case is derived from the above by removing the $\frac{4}{3}$ from the N^* contribution and multiplying the first term by -3. For the N^* contribution zero, we plot the four equations for f and ask for what b/a they are equal. We then add the N^* contribution and repeat the process. We vary σ to check on the cutoff dependence. The results are shown in Figs. 1–8.[8]

[8] Figures 1–4 are valid for both isovectors and isoscalar mesons. Figures 5–8 are valid for isovector mesons only. The isoscalar case is almost identical, with the same results.

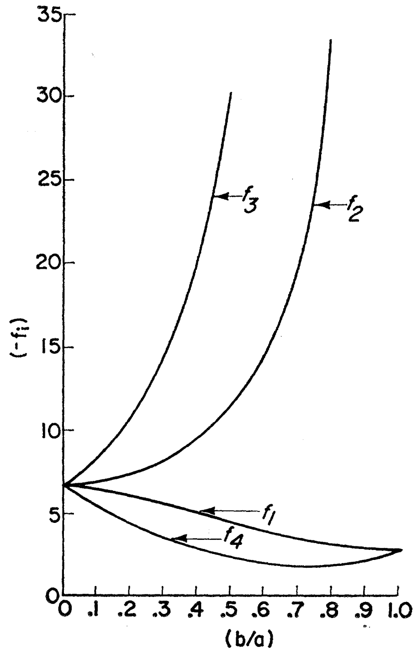

FIG. 1. Nucleon exchange only, $N_r=0.29$ BeV².

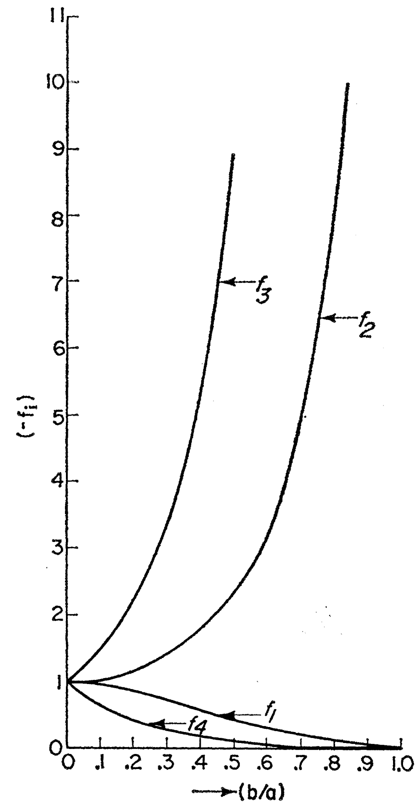

FIG. 3. Nucleon exchange only, $N_r=1.8$ BeV².

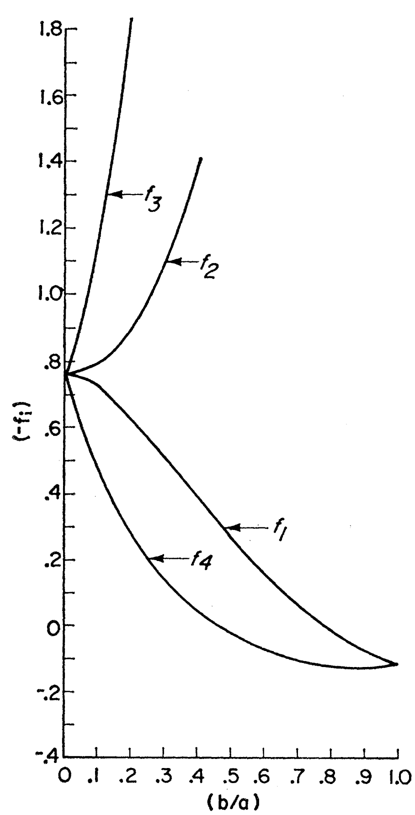

FIG. 2. Nucleon exchange only, $N_r=1$ BeV².

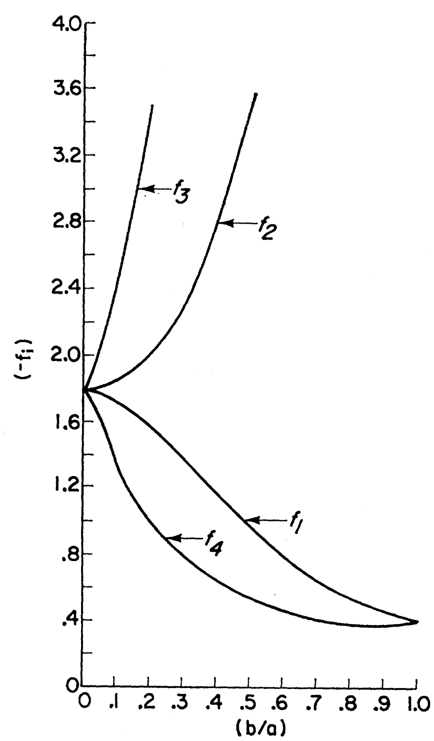

FIG. 4. Nucleon exchange only, $N_r=2.36$ BeV².

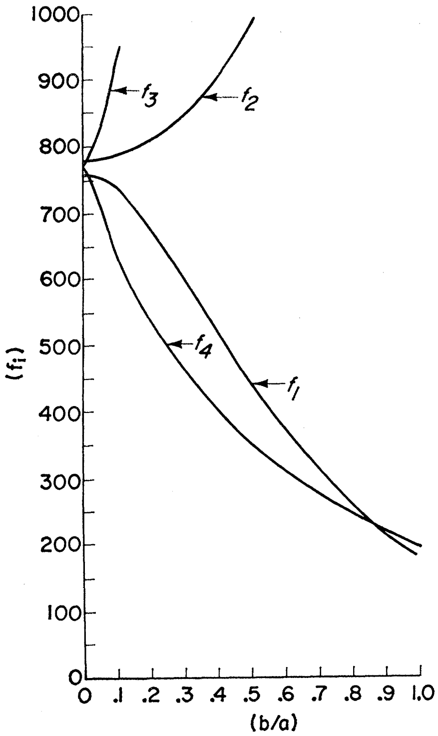

FIG. 5. Nucleon and $N^*(1236)$ exchange, isovector meson, $N_r = 0.29$ BeV^2.

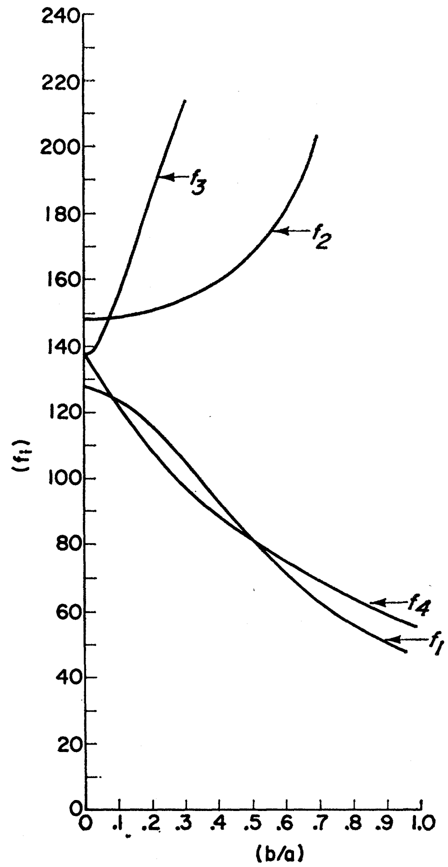

FIG. 6. Nucleon and $N^*(1236)$ exchange, isovector meson, $N_r = 1$ BeV^2.

IV. DISCUSSION

From the graphs, we see that $b/a=0$ is the only solution consistent with all four equations for only nucleon exchange. When N^* exchange is included, there is no b/a giving exact consistency, but the point of closest approach of the four equations is $b/a=0$ and the separation of the equations at this point is at worst 15% of the total value of the equations. For the lower cutoffs, the percent separation is less than 5%.

The results of Zachariasen and Zemach (ZZ) are shown in Table I. A comparison of our result with this collection of answers might seem misleading at first glance; however, one should note that when ZZ say a solution does or does not exist, they are referring to the existence of an acceptable mass value. We can obtain the same results as in the first two cases above by simply calculating the exchanges as they do and observing the signs of the forces. ZZ do not seem to discuss the self-consistency of the coupling strengths in the parity-conserving case, and, although they indicate that acceptable mass values for $b\neq 0$ and $a=0$ do exist, we find that self-consistency conditions on the couplings cause this solution to vanish. For the parity-nonconserving interaction, we agree for the isoscalar meson, but not for the isovector one, as there we find no parity-nonconserving interactions at all.

There are at least two possible viewpoints about the origin of parity in self-consistency. The first is that if one sets up a bootstrap problem in the ordinary way, one would get as an output a self-consistent mass and a self-consistent parity-conserving coupling. If the

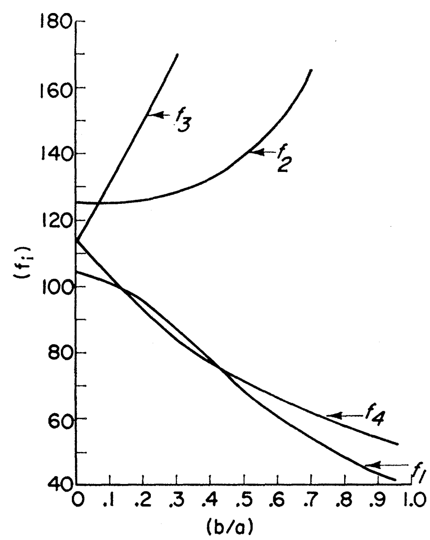

FIG. 7. Nucleon and $N^*(1236)$ exchange, isovector meson, $N_r = 1.8$ BeV^2.

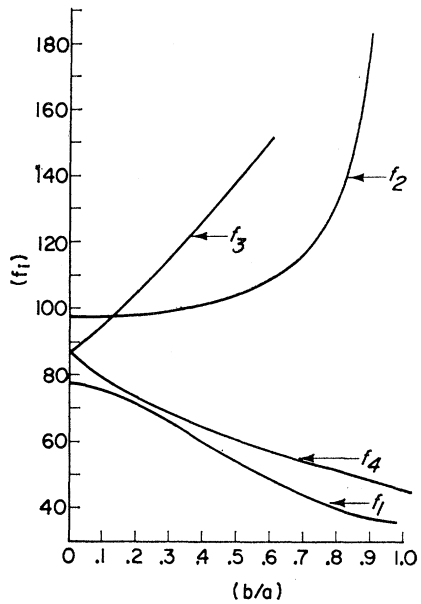

FIG. 8. Nucleon and $N^*(1236)$ exchange, isovector meson, $N_r = 2.36$ BeV^2.

bootstrap failed, there would be no mass and no coupling strength, i.e., the existence of parity conservation is linked to the existence of an exchange force strong enough to bind a particle.

A second possible idea is that there are two levels of self-consistency in a bootstrap problem. The first level is required because of the existence of the bootstrap dynamics. It is a relation between the spatial structure in which a body moves and the body itself. It is, however, exterior to the body. Out of this would come spatial conservation laws. The second level is imposed upon the dynamics itself; it is interior to the body and from it would come the self-consistent mass and coupling constant. Here one would expect parity conservation to emerge from a bootstrap calculation but to be independent of the strength of the dynamics.[9]

Our results would seem to indicate that the second viewpoint is correct. Explicitly, if one considers the isovector nucleon exchange part of Eq. (17), one gets

$$f(3b^2+a^2) = \tfrac{1}{3}Mg^2(b^2-a^2),$$
$$f(b^2-a^2) = \tfrac{1}{3}Mg^2(3b^2+a^2),$$
$$f(a-b)^2 = -\tfrac{1}{3}Mg^2(a+b)^2,$$
$$-f(a+b)^2 = \tfrac{1}{3}Mg^2(a-b)^2,$$

with $\sigma = M$. Absorb $\frac{1}{3}Mg^2$ into f:

$$f = \frac{b^2-a^2}{3b^2+a^2} = \frac{3b^2+a^2}{b^2-a^2},$$

$$-f = \frac{(a+b)^2}{(a-b)^2} = \frac{(a-b)^2}{(a+b)^2},$$

or

$$(-a^2+b^2) = \pm(3b^2+a^2) \rightarrow b^2=0, \quad b^2=-a^2$$
$$(a+b)^2 = \pm(a-b)^2 \rightarrow ab=0, \quad b^2=-a^2.$$

We thus obtain two solutions, one of which we can discard as physically unrealizable. The graphs can be regarded as another form of this same calculation, where the value of f at which $b/a=0$ is not important. An identical calculation can be performed, with the N^* included, with similar results.

In summary, we have found that self-consistency requires that parity be conserved for a pseudoscalar (isoscalar and isovector) meson and violated for a scalar meson. This conservation (nonconservation) appears to be independent of the existence of a self-consistent mass.

ACKNOWLEDGMENT

I would like to thank Gordon Kane for suggesting this topic and for several helpful discussions.

[9] This idea was originally suggested by R. F. Dashen and S. Frautschi [Phys. Rev. **143**, B1172 (1966)] in connection with the existence of hadronic weak and electromagnetic properties, independent of the existence of the elementary particle.