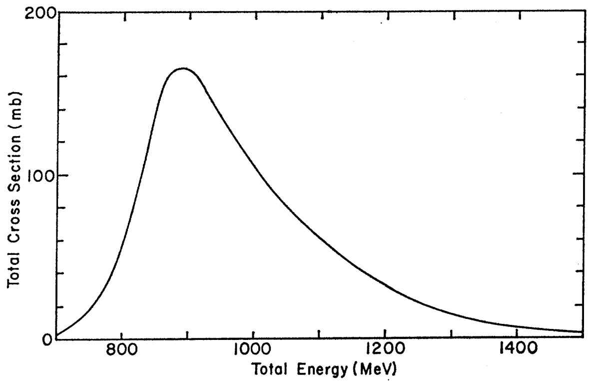

FIG. 3. Total cross section for $I=\frac{1}{2}$ p-wave $K\pi$ scattering versus center-of-mass energy.

factor is calculated explicitly. The amplitude, which shows more rapid variation, is obtained by taking the appropriate quotient.

IV. RESULTS

A graph of the total cross section as a function of energy is presented in Fig. 3. A cutoff of $3.87m_\pi$ gives a peak at 890 MeV. The general shape of the curve is similar to previous results. The cutoff is noticeably lower, but the width is still on the same order as previous calculations using N/D. We derive a value of about 210 MeV which is comparable to the results of Fulco, Shaw, and Wong and about four times the experimental value. One is led to doubt whether the inclusion of further channels will significantly improve the situation and that the defect is inherent in N/D.

The large change in the cutoff from previous results conclusively demonstrates the strong influence of the $K^*\pi$ channel on the $K\pi$ amplitude, and future multichannel models should include it. This conclusion and the simple method for calculating the coupling constants for inclusion of such channels seem to be the major results of this work.

ACKNOWLEDGMENTS

The author would like to express his gratitude to Professor Ngee-Pong Chang, Professor Michael Arons, and Professor Gordon Kane for helpful discussions and encouragement.

PHYSICAL REVIEW D VOLUME 2, NUMBER 1 1 JULY 1970

Forward Proton Compton Scattering and Continuous Dispersion Sum Rules*

YU-CHIEN LIU AND IAN J. MCGEE
Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada
(Received 6 March 1970)

The new invariant amplitudes of Bardeen and Tung for nucleon Compton scattering, which are free of both kinematic singularities and zeros, are examined. The forward scattering amplitude, and the continuous-dispersion sum rules derived therefrom, are obtained. Using the data of a recent calculation by Damashek and Gilman, tests of these sum rules are shown to be quite satisfactory, indicating the validity of the dispersion relation, the good parametrization of the forward proton Compton scattering amplitude, and the presence of a $J=0$ fixed pole within the accuracy of present experiment.

I. INTRODUCTION

HISTORICALLY, dispersion relations as applied to particle physics were first derived and analyzed by Gell-Mann, Goldberger, and Thirring[1] in 1954. Owing to kinematical complexity and experimental unfeasibility, a full-scale analysis[2] of nucleon Compton scattering was not available until a recent effort in the accurate measurement of the unpolarized total photoabsorption cross section.[3] This permitted a calculation of the real part[3] of the spin-averaged forward amplitude from threshold to 20 GeV, although the comparison of such a calculation with experiment has yet to be done.[4]

The form of the invariant amplitudes for (nucleon) Compton scattering was first investigated over ten years ago. In 1958, Prange[5] wrote down six invariant amplitudes based on the principles of Lorentz, gauge, parity, and charge-conjugation invariance. They were

* Work supported in part by the National Research Council of Canada.

[1] M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. **95**, 1612 (1954).

[2] See the references quoted in R. Köberle, Phys. Rev. **166**, 1558 (1968). Also P. S. Baranov, L. V. Fil'kov, and G. A. Sokol, Fortschr. Physik **16**, 595 (1968); G. C. Fox and D. Z. Freedman, Phys. Rev. **182**, 1628 (1969).

[3] M. Damashek and F. J. Gilman, Phys. Rev. D **1**, 1319 (1970).

[4] S. J. Brodsky, A. C. Hearn, and R. G. Parsons, Phys. Rev. **187**, 1899 (1969).

[5] R. E. Prange, Phys. Rev. **110**, 240 (1958). Actually he dealt with electron Compton scattering.

later modified by Lapidus and Chou,[6] and Hearn and Leader,[7] to eliminate the kinematic singularities, but, unfortunately, the kinematic zeros (constraints) remained. A satisfactory set of amplitudes possessing the necessary analytic properties shared by the invariant amplitudes of other processes (e.g., $\pi N \to \pi N$, $\gamma N \to \pi N$) was presented after a recent attempt of Bardeen and Tung[8] acting primarily toward that purpose.

In this new set of invariant amplitudes,[8] the Mandelstam representations can be written down without *ad hoc* subtractions.[9] The low-energy theorems also appear quite naturally. In particular, the forward-dispersion relation can be treated directly in terms of the invariant amplitudes without any constraints.

The extension of the forward dispersion relation to the continuous dispersion sum rules (CDSR) and the continuous-moment sum rules (CMSR) is straightforward. At the moment, the real part[3] of the amplitude must be obtained via the dispersion relation because an experimental value is not available. Numerical analysis using these data show that the extended sum rules are well satisfied. This indicates that the parametrization of Damashek and Gilman[3] on the forward proton Compton scattering amplitude is a good one, and confirms the existence of a fixed pole[10] proposed earlier.

In Sec. II we discuss the new amplitudes of Bardeen and Tung,[8] together with a comparison of the work of Hearn and Leader.[7] In Sec. III the ordinary forward dispersion relation is extended. A distinction is made between the CDSR and the CMSR. Numerical calculations are also carried out and discussed with conclusions presented in Sec. IV.

II. NEW INVARIANT AMPLITUDES

For the process $N(p_1)+\gamma(k_1)\to N(p_2)+\gamma(k_2)$, the scattering amplitude $\bar{u}(p_2)\epsilon_\mu^*(k_2)M_{\mu\nu}\epsilon_\nu(k_1)u(p_1)$ can be expressed in the following form according to Bardeen and Tung[8,11]:

$$M_{\mu\nu}=\sum_{i=1}^{6}\mathfrak{L}_iA_i,$$

$$\mathfrak{L}_1=K^2\delta_{\mu\nu}-2K_\mu K_\nu,$$

$$\mathfrak{L}_2=-\tfrac{1}{2}K^2(\gamma_\mu i\gamma K\gamma_\nu-\gamma_\nu i\gamma K\gamma_\mu)+P\cdot K(K_\mu i\gamma_\nu+i\gamma_\mu K_\nu)-i\gamma K(K_\mu P_\nu+P_\mu K_\nu),$$

$$\mathfrak{L}_3=mi\gamma K\delta_{\mu\nu}-P\cdot K\delta_{\mu\nu}-\tfrac{1}{2}K^2(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)-K_\mu\tfrac{1}{2}(i\gamma Ki\gamma_\nu-i\gamma_\nu i\gamma K)-\tfrac{1}{2}(i\gamma_\mu i\gamma K-i\gamma Ki\gamma_\mu)K_\nu-m(K_\mu i\gamma_\nu+i\gamma_\mu K_\nu)+(K_\mu P_\nu+P_\mu K_\nu),$$

$$\mathfrak{L}_4=-K^2(P_\mu i\gamma_\nu+i\gamma_\mu P_\nu)+P\cdot K(K_\mu i\gamma_\nu+i\gamma_\mu K_\nu)+i\gamma K(K_\mu P_\nu+P_\mu K_\nu)-P\cdot Ki\gamma K\delta_{\mu\nu}-mK^2\delta_{\mu\nu}+2mK_\mu K_\nu,$$

$$\mathfrak{L}_5=-K^2P_\mu P_\nu+P\cdot K(K_\mu P_\nu+P_\mu K_\nu)+\tfrac{1}{2}[P^2K^2-(P\cdot K)^2]\delta_{\mu\nu}-P^2K_\mu K_\nu,$$

$$\mathfrak{L}_6=-i\gamma KP_\mu P_\nu+\tfrac{1}{2}P\cdot K(P_\mu i\gamma_\nu+i\gamma_\mu P_\nu)-\tfrac{1}{4}P\cdot K(\gamma_\mu i\gamma K\gamma_\nu-\gamma_\nu i\gamma K\gamma_\mu)+\tfrac{1}{4}mK^2(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)+\tfrac{1}{2}mP\cdot K\delta_{\mu\nu}+\tfrac{1}{2}P^2i\gamma K\delta_{\mu\nu}-i\gamma KK_\mu K_\nu+\tfrac{1}{2}m^2(K_\mu i\gamma_\nu+i\gamma_\mu K_\nu)-\tfrac{1}{2}m(K_\mu P_\nu+P_\mu K_\nu)+\tfrac{1}{4}mK_\mu(i\gamma Ki\gamma_\nu-i\gamma_\nu i\gamma K)+\tfrac{1}{4}m(i\gamma_\mu i\gamma K-i\gamma Ki\gamma_\mu)K_\nu,\quad(1)$$

where

$$K=\tfrac{1}{2}(k_1+k_2),\quad P=\tfrac{1}{2}(p_1+p_2),\quad K^2=\tfrac{1}{4}t,\quad P\cdot K=-\tfrac{1}{4}(s-u),\quad P^2=\tfrac{1}{4}t-m^2,$$

and

$$P^2K^2-(P\cdot K)^2=\tfrac{1}{4}(su-m^4).$$

s and t are the standard Mandelstam variables.

The proof of gauge invariance for $\mathfrak{L}_i$ is immediate. Using the obvious fact that in the c.m. system

$$(P+K)\cdot\epsilon^*(k_2)=(P+K)\cdot\epsilon(k_1)=0$$

[in the gauges $\epsilon_4(k_2)=\epsilon_4(k_1)=0$], one may further eliminate, if desired, the four-vector P_μ (P_ν) in terms of K_μ (K_ν).

That the A_i's are free of both kinematic singularities and kinematic zeros[12] can be seen from an explicit form of the s-channel helicity amplitudes[13]:

$$f_{\frac{1}{2}1;\frac{1}{2}1}{}^s=-\cos(\tfrac{1}{2}\theta_s)(1/8m)\{2[(s-m^2)^2+m^2t]A_4+m(m^4-su)A_5-[(s-m^2)^2-m^2t]A_6\},$$
$$f_{-\frac{1}{2}1;\frac{1}{2}1}{}^s=\cos^2(\tfrac{1}{2}\theta_s)\sin^2(\tfrac{1}{2}\theta_s)[(s-m^2)^2/8m\sqrt{s}\,]\times[2mA_4+\tfrac{1}{2}(s+m^2)A_5+mA_6],$$
$$f_{-\frac{1}{2}1;-\frac{1}{2}1}{}^s=-\cos^3(\tfrac{1}{2}\theta_s)[(s-m^2)^2/8m](2A_4+mA_5+A_6),$$
$$f_{-\frac{1}{2}-1;\frac{1}{2}1}{}^s=-\sin(\tfrac{1}{2}\theta_s)[1/8m\sqrt{s}]\times\{(s+m^2)tA_1-m(s-m^2)tA_2+2[(s-m^2)^2+(m^4-su)]A_3\},\quad(2)$$
$$f_{\frac{1}{2}-1;\frac{1}{2}1}{}^s=-\cos(\tfrac{1}{2}\theta_s)\sin^2(\tfrac{1}{2}\theta_s)[(s-m^2)^2/8s]\times(2A_1+2A_3),$$
$$f_{\frac{1}{2}-1;-\frac{1}{2}1}{}^s=-\sin^3(\tfrac{1}{2}\theta_s)[(s-m^2)^2/8ms\sqrt{s}]\times[(s+m^2)A_1+m(s-m^2)A_2+2m^2A_3],$$

[6] L. I. Lapidus and C. Kuang-Chao, Zh. Eksperim. i Teor. Fiz. **37**, 1714 (1959) [Soviet Phys. JETP **10**, 1213 (1960)].
[7] A. C. Hearn and E. Leader, Phys. Rev. **126**, 789 (1962).
[8] W. A. Bardeen and Wu-Ki Tung, Phys. Rev. **173**, 1423 (1968).
[9] D. Holliday, Ann. Phys. (N. Y.) **24**, 289 (1963).
[10] M. J. Creutz, S. D. Drell, and E. A. Paschos, Phys. Rev. **178**, 2300 (1969).
[11] We use the Pauli metric.

[12] P. Ader, M. Capdeville, and H. Navelet, Nuovo Cimento **56A**, 315 (1968).
[13] We follow the convention of H. D. I. Abarbanel and M. L. Goldberger [Phys. Rev. **165**, 1594 (1968)] for the helicity amplitudes. The f's are normalized to

$$d\sigma/d\Omega=[(4\pi)^{-1}(m/\sqrt{s})]^2\sum_{[\lambda]}|f_{[\lambda]}|^2$$

in the c.m. system. Notice that there are a few misprints in the expression of $f_{\frac{1}{2}1;\frac{1}{2}1}{}^s$ and $f_{-\frac{1}{2}-1;\frac{1}{2}1}{}^s$ in Ref. 8.

as well as the form of the regularized, parity-conserving, t-channel $[\gamma(-k_2)+\gamma(k_1)\to N(p_2)+\bar{N}(-p_1)]$ helicity amplitudes:

$$\begin{aligned}
f_{\frac{1}{2}\frac{1}{2};11}{}^t+f_{-\frac{1}{2}-\frac{1}{2};11}{}^t&=[t/4m(t-4m^2)^{1/2}]\\
&\quad\times[(t-4m^2)A_1+(s-u)A_3],\\
f_{\frac{1}{2}\frac{1}{2};11}{}^t-f_{-\frac{1}{2}-\frac{1}{2};11}{}^t&=[(\sqrt{t})/4m][mtA_2-(s-u)A_3],\\
f_{\frac{1}{2}-\frac{1}{2};11}{}^t/\sin\theta_t&=\tfrac{1}{4}tA_3,\\
f_{\frac{1}{2}\frac{1}{2};1-1}{}^t/\sin^2(\tfrac{1}{2}\theta_t)\cos^2(\tfrac{1}{2}\theta_t)&=-[t(t-4m^2)^{1/2}/2m]\\
&\quad\times[mA_4-\tfrac{1}{8}(t-4m^2)A_5],\\
f_{\frac{1}{2}-\frac{1}{2};1-1}{}^t/\sin(\tfrac{1}{2}\theta_t)\cos^3(\tfrac{1}{2}\theta_t)+f_{-\frac{1}{2}\frac{1}{2};1-1}{}^t/\sin^3(\tfrac{1}{2}\theta_t)\cos(\tfrac{1}{2}\theta_t)&=[t^{3/2}(t-4m^2)^{1/2}/2m]A_4,\\
f_{\frac{1}{2}-\frac{1}{2};1-1}{}^t/\sin(\tfrac{1}{2}\theta_t)\cos^3(\tfrac{1}{2}\theta_t)-f_{-\frac{1}{2}\frac{1}{2};1-1}{}^t/\sin^3(\tfrac{1}{2}\theta_t)\cos(\tfrac{1}{2}\theta_t)&=[t(t-4m^2)/4m]A_6,
\end{aligned}\tag{3}$$

where

$$\cos\theta_s=1+2st/(s-m^2)^2$$

and

$$\cos\theta_t=(s-u)/[(t-4m^2)t]^{1/2}.$$

(The u channel is again $\gamma N\to\gamma N$, the same as the s channel.)

The connection of this new set of A_i's with the familiar Hearn-Leader[7] amplitudes can be found easily by comparing the t-channel helicity amplitudes in the two representations. This is due to simple EE (equal-equal) kinematics in the t channel. Denoting the Hearn-Leader invariant amplitudes by A_i' $(i=1,6)$, we first have

$$\begin{aligned}
f_{\frac{1}{2}\frac{1}{2};11}{}^t&=[p_t(A_1'+A_2')-2E_tA_3'\\
&\quad+mk_t\cos\theta_t(A_4'+A_5')]/2m,\\
f_{-\frac{1}{2}-\frac{1}{2};11}{}^t&=[p_t(A_1'+A_2')+2E_tA_3'\\
&\quad+mk_t\cos\theta_t(A_4'+A_5')]/2m,\\
f_{\frac{1}{2}-\frac{1}{2};11}{}^t&=\sin\theta_t\,E_tk_t(A_4'+A_5')/2m,\\
f_{\frac{1}{2}\frac{1}{2};1-1}{}^t&=-[p_t(A_1'-A_2')+mk_t\cos\theta_t(A_4'-A_5')]/2m,\\
f_{\frac{1}{2}-\frac{1}{2};1-1}{}^t&=\sin\theta_t[-E_tk_t(A_4'-A_5')-2p_tk_tA_6']/2m,\\
f_{-\frac{1}{2}\frac{1}{2};1-1}{}^t&=\sin\theta_t[E_tk_t(A_4'-A_5')-2p_tk_tA_6']/2m,
\end{aligned}\tag{4}$$

where $p_t=\frac{1}{2}(t-4m^2)^{1/2}$ and $k_t=\frac{1}{2}(t)^{1/2}=E_t$. A comparison of (3) and (4) leads to[8]

$$\begin{aligned}
A_1&=(2/t)[(A_1'+A_2')-((s-u)/4m)(A_4'+A_5')],\\
A_2&=-(4/mt)[A_3'-((s-u)/8m)(A_4'+A_5')],\\
A_3&=(1/2m)[A_4'+A_5'],\\
A_4&=(su-m^4)^{-1}[\tfrac{1}{2}(s-u)(A_4'-A_5')-(t-4m^2)A_6'],\\
A_5&=-[8/(su-m^4)][\tfrac{1}{2}(A_1'-A_2')+mA_6'],\\
A_6&=-[2/(su-m^4)][\tfrac{1}{2}t(A_4'-A_5')-(s-u)A_6'].
\end{aligned}\tag{5}$$

Another feature of the A_i's emerges from imposing gauge invariance (charge conservation) in $\mathcal{L}_i$ directly. It makes the structure of the nucleon Born terms[14] completely different from the pure hadron case. By considering the special cases of forward scattering (for which only $\mathcal{L}_4$, $\mathcal{L}_5$, and $\mathcal{L}_6$ survive) as well as backward scattering (for which only $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ are left), a disentanglement of the nucleon Born terms into the contribution to respective A_i's can be made with relative ease. The result is the same as inserting into Eq. (5) the old Born terms.[9] Both agree with Table II of Ref. 8. From that table one sees that the Born terms contribute not only to the single spectral function $[(m^2-s)^{-1}+(m^2-u)^{-1}$ for crossing-even, $(m^2-s)^{-1}-(m^2-u)^{-1}$ for crossing-odd amplitudes$]$, but also to the double spectral function $[(m^2-s)^{-1}(m^2-u)^{-1}$, crossing-even amplitudes only$]$ of the Mandelstam representations. This removes the previous problem adding *ad hoc* subtraction[9] constants to the Mandelstam representation for the Hearn-Leader amplitudes (A_i''s). The low-energy theorems[8] for A_i to second order in the photon energy (which arise purely from a study of the kinematic structure of the amplitudes) can be derived by inspection, since the A_i's do not have any kinematic singularities or zeros.

III. FORWARD-DISPERSION SUM RULES

In the forward direction, $\mathcal{L}_1=\mathcal{L}_2=\mathcal{L}_3=0$,

$$\mathcal{L}_4=-P\cdot Ki\gamma K\delta_{\mu\nu},\quad \mathcal{L}_5=-\tfrac{1}{2}(P\cdot K)^2\delta_{\mu\nu},$$
$$\mathcal{L}_6=-\tfrac{1}{4}P\cdot K(\gamma_\mu i\gamma K\gamma_\nu-\gamma_\nu i\gamma K\gamma_\mu).$$

Therefore the forward scattering amplitude reduces to

$$M=\chi_2^\dagger[(s-m^2)^2/8m][-(2A_4+mA_5)\boldsymbol{\varepsilon}_2^*\cdot\boldsymbol{\varepsilon}_1+A_6i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}_2^*\times\boldsymbol{\varepsilon}_1]\chi_1.\tag{6}$$

By comparing with the f_1 and f_2 of Gell-Mann *et al.*[1] in the laboratory system, we obtain[15]

$$f_1(\nu)=-[(s-m^2)^2/32\pi m](2A_4+mA_5)$$

and

$$f_2(\nu)=[(s-m^2)^2/32\pi m]A_6,\tag{7}$$

where $\nu=(s-m^2)/2m$ is the laboratory energy of the photon. The crossing symmetry of f_1 (even) and f_2 (odd) in ν follows from the crossing symmetry of A_4 and A_5 (even) and A_6 (odd) under the transformation $s\rightleftarrows u$.

Because of the zero mass of the photon, the real parts of f_1 and f_2 do not have singularities at the pole position ($s=u=m^2$, or $\nu=0$). In fact, from (7) and the Born

[14] That is,

$$[ie\gamma_\mu+\mu\gamma_\mu\gamma k_2]\cdot[-i\gamma(P+K)+m]\cdot[ie\gamma_\nu+\mu\gamma k_1\gamma_\nu]/(m^2-s)$$
$$+[ie\gamma_\nu-\mu\gamma_\nu\gamma k_1]\cdot[-i\gamma(P-K)+m]\cdot[ie\gamma_\mu-\mu\gamma k_2\gamma_\mu]/(m^2-u).$$

μ is in units of $e/2m$.

[15] In terms of the Hearn-Leader amplitudes, one would have $f_1(\nu)=\frac{1}{2}(-A_1'+A_2')+\frac{1}{2}\nu(A_4'-A_5')=A_2'-\nu A_5'=-A_1'+\nu A_4'$; the last two equalities follow from one of the constraints at $t=0$: $(A_1'+A_2')-\nu(A_4'+A_5')=0$.

terms for A_4, A_5, and A_6, we have

$$f_1(0) = -\alpha/m \quad \text{and} \quad f_2(0) = -(\alpha\kappa^2/2m^2)\nu \tag{8}$$

($\alpha = e^2/4\pi$, $\mu = \kappa e/2m$). It is only under such circumstances that one can derive the CMSR,[16] in addition to the more physical CDSR[17] (see below).

In the following discussion we confine our attention to the amplitude $f_1(\nu)$, because its imaginary part is related to the unpolarized total absorption cross section and a numerical calculation is possible.

The ordinary dispersion relation is well known[1]:

$$\mathrm{Re} f_1(\nu) = f_1(0) + \frac{2\nu^2}{\pi} P \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'} \frac{\mathrm{Im} f_1(\nu')}{\nu'^2 - \nu^2}. \tag{9}$$

Equation (9) summarizes the analytic properties of $f_1(\nu)$ in ν; i.e., there is a branch cut starting from $\nu_0 = \mu + \mu^2/2m$ to ∞; a subtraction is necessary and is performed at a convenient point $\nu = 0$, where [from (8)] $f_1(0) = -\alpha/m$.

Remembering these properties and invoking the Regge-pole model[18] for $f_1(\nu)$,[3,10] viz.,

$$f_1(\nu) \underset{\nu \geqslant N}{\longrightarrow} \frac{c_P}{4\pi} i\nu - \frac{c_{P'}}{4\pi} \frac{e^{-i\frac{1}{2}\pi\alpha_{P'}}}{\sin\frac{1}{2}\pi\alpha_{P'}} \nu^{\alpha_{P'}} + C, \tag{10}$$

one can derive the following CMSR:

$$\int_0^N \frac{d\nu}{\nu} \nu^{2\beta} [\cos\pi\beta \, \mathrm{Im} f_1(\nu) - \sin\pi\beta \, \mathrm{Re} f_1(\nu)] = \frac{c_P}{4\pi} N^{1+2\beta} \frac{\sin\frac{1}{2}\pi(1+2\beta)}{1+2\beta} + \frac{c_{P'}}{4\pi} \frac{N^{\alpha_{P'}+2\beta}}{\sin\frac{1}{2}\pi\alpha_{P'}} \times \frac{\sin\frac{1}{2}\pi(\alpha_{P'}+2\beta)}{\alpha_{P'}+2\beta} - C N^{2\beta} \frac{\sin\pi\beta}{2\beta}, \tag{11}$$

where $\beta > 0$. If $\mathrm{Re} f_1(\nu)$ were available from experiment, Eq. (11) would allow the fixed-pole residue C to be probed more effectively, for in the ordinary version of finite-energy sum rules (FESR), where $\beta = 0, 1, 2, 3, \ldots$, this term is present only in the lowest moment $\beta = 0$ and vanishes otherwise.

Furthermore, if $f_1(\infty)$ is known, a subtracted CMSR can also be written:

$$\int_0^N \frac{d\nu}{\nu} \nu^{2\beta} \left[\cos\pi\beta \left(\mathrm{Im} f_1(\nu) - \frac{c_P}{4\pi}\nu\right) - \sin\pi\beta \, \mathrm{Re} f_1(\nu)\right] = \frac{c_{P'}}{4\pi} \frac{N^{\alpha_{P'}+2\beta}}{\sin\frac{1}{2}\pi\alpha_{P'}} \frac{\sin\frac{1}{2}\pi(\alpha_{P'}+2\beta)}{\alpha_{P'}+2\beta} - C N^{2\beta} \frac{\sin\pi\beta}{2\beta} \quad (\beta > 0). \tag{12}$$

When $\alpha_{P'} + 2\beta = 2, 4, \ldots$, the first term on the right-hand side of (12) vanishes.[19] In such cases one is left with a sum rule for C, allowing a determination of this constant more easily than from Eq. (11).

As usual, however, the region between $\nu = 0$ and $\nu = \nu_0$ (branch point of the unitarity cut) is unphysical. In particular, the Born term (if present) is singular within this region. Such a type of CMSR is void of physical interest if a numerical calculation is intended. The well-known method[20] of overcoming this difficulty is to write, instead of (11),

$$\int_{\nu_0}^N \frac{d\nu}{\nu} \frac{\cos\pi\beta \, \mathrm{Im} f_1(\nu) + \sin\pi\beta \, \mathrm{Re} f_1(\nu)}{(\nu^2 - \nu_0^2)^\beta} - \tfrac{1}{2}\pi \frac{f_1(0)}{\nu_0^{2\beta}} = \frac{c_P}{4\pi} N^{1-2\beta} \frac{\sin\frac{1}{2}\pi(1-2\beta)}{1-2\beta} + \frac{c_{P'}}{4\pi} \frac{N^{\alpha_{P'}-2\beta}}{\sin\frac{1}{2}\pi\alpha_{P'}} \frac{\sin\frac{1}{2}\pi(\alpha_{P'}-2\beta)}{\alpha_{P'}-2\beta} - C N^{-2\beta} \frac{\sin\pi\beta}{2\beta} \quad (\beta < 1), \tag{13}$$

where $\mathrm{Re} f_1(\nu)$ is made to appear above ν_0 by means of the factor $(\nu^2 - \nu_0^2)^\beta$. We simply call the sum rules of this type CDSR,[21] rather than CMSR. That is, even if we made a change of variable (e.g., $k^2 = \nu^2 - \nu_0^2$) the integrand would not correspond to a "moment" in the ordinary sense; furthermore, the amplitude would lose its simple crossing symmetry in the new variable (e.g., k).

For completeness, we record another CDSR:

$$\int_{\nu_0}^N d\nu \frac{\nu}{(\nu^2 - \nu_0^2)^\beta} [\cos\pi\beta \, \mathrm{Im} f_1(\nu) + \sin\pi\beta \, \mathrm{Re} f_1(\nu)] = -\frac{c_P}{4\pi} N^{3-2\beta} \frac{\sin\frac{1}{2}\pi(3-2\beta)}{3-2\beta} - \frac{c_{P'}}{4\pi} \frac{N^{2+\alpha_{P'}-2\beta}}{\sin\frac{1}{2}\pi\alpha_{P'}} \times \frac{\sin\frac{1}{2}\pi(2+\alpha_{P'}-2\beta)}{2+\alpha_{P'}-2\beta} + C N^{2-2\beta} \frac{\sin\frac{1}{2}\pi(2-2\beta)}{2-2\beta} \quad (\beta < 1). \tag{14}$$

---

[16] This name refers to the sum rules of the type

$$\int_0^N d\nu \, \nu^\beta f(\nu),$$

with β varying continuously. See, for example, E. Ferrari and G. Violini, Phys. Letters **28B**, 684 (1969).

[17] Y. C. Liu and S. Okubo, Phys. Rev. Letters **19**, 190 (1967); Y. C. Liu and I. J. McGee, Phys. Rev. D **1**, 3123 (1970).

[18] $f_1(\nu)$ receives contributions from the Regge poles of quantum numbers: $P = (-1)^J$, $C = (-1)^J = \text{even}$, $G = (-1)^{J+I}$, namely P, P', and A_2, etc.

[19] Y.-C. Liu and S. Okubo, Phys. Rev. **168**, 1712 (1968); M. G. Olsson, Phys. Letters **26B**, 310 (1968).

[20] W. Gilbert, Phys. Rev. **108**, 1078 (1957); Y. C. Liu *et al.*, Ref. 17.

[21] This name is not intended as perfect. We simply want to emphasize that it is a dispersion sum rule, while noting that a name like CMSR could be misleading.

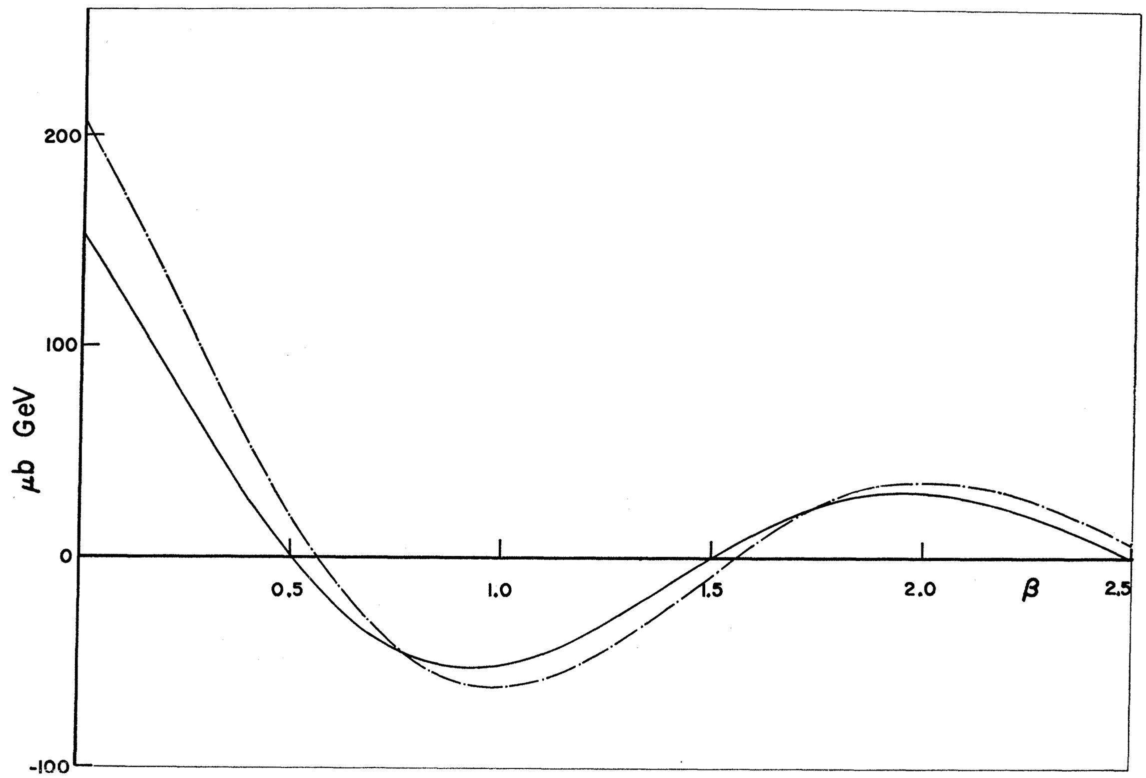

FIG. 1. Plot (broken curve) of the right-hand side of the CMSR (11) divided by a factor $N^{2\beta}$. The left-hand side coincides with this curve for all allowed values of β. The solid line is the contribution from the Pomeranchuk trajectory, which dominates because we have taken $N=20$ GeV.

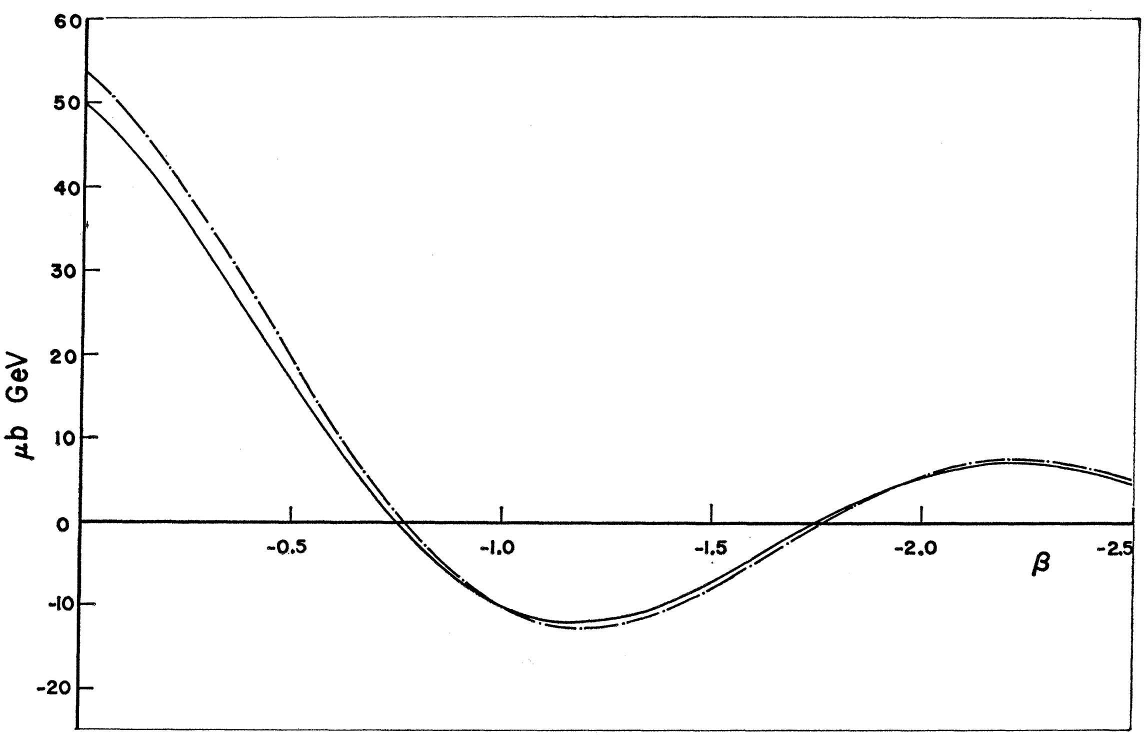

FIG. 2. Plot (broken curve) of the right-hand side of the CMSR (12) divided by a factor $N^{2\beta}$. The left-hand side coincides with this curve for all values of β shown in the diagram. The solid line is the contribution from the effective secondary Regge poles (P' and A_2), which vanishes at $\beta=0.75$, 1.75, . . . , for an effective intercept of 0.5. The fixed-pole term is nonvanishing there, but very small.

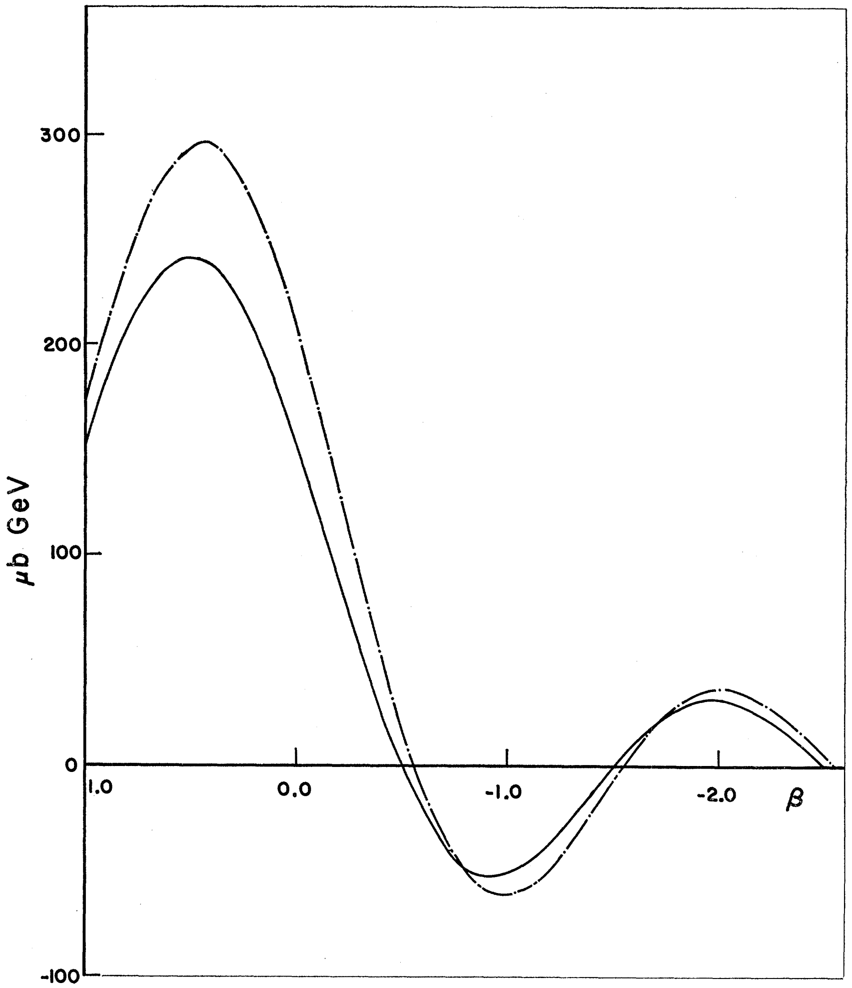

FIG. 3. Plot (broken curve) of the right-hand side of the CDSR (13) multiplied by a factor $N^{2\beta}$. For $\beta<0.4$ the left-hand side coincides with this curve. For $1.0>\beta>0.4$ both sides of (13) (not multiplied by the factor $N^{2\beta}$) agree with each other to within experimental accuracy. Numerically $\nu_0=0.150$ GeV, $f_1(0)=-3.0\,\mu$b GeV. The solid line is the contribution from the Pomeranchuk trajectory. The peak occurs at $\beta=0.5$, characteristic of the CDSR alone.

Numerical tests of the CMSR (11), (12) and of the CDSR (13), (14) are simple, using the full data set of Damashek and Gilman.[3] On the left-hand side, we have $\mathrm{Im}f_1(\nu)=(\nu/4\pi)\sigma_T(\nu)$, and $\mathrm{Re}f_1(\nu)$ is calculated from the ordinary dispersion relation (9). On the right-hand side a best fit to the high-energy data yielded $c_p=96.6$ μb, $c_{p'}=70.2$ μb, $\alpha_{p'}=0.5$, and $C=-2.5$ μb. To our surprise, all four sum rules give perfect equality on both sides for all allowed values of β, as shown in Figs. 1–4, with $N=20$ GeV. (In general they hold quite well for N lying between 2 and 20 GeV.) This indicates (1) an excellent parametrization and calculation of $\mathrm{Im}f_1(\nu)$ and $\mathrm{Re}f_1(\nu)$, respectively, performed in Ref. 3; (2) a reliable feedback of the $\mathrm{Re}f_1(\nu)$ calculated from ordinary dispersion relation into the CMSR or CDSR, eliminating doubts[22] that have been raised about such a procedure; (3) sizable evidence for the presence of the $J=0$ fixed pole ($C\neq0$).

Whether this fixed pole is really present or not can only be settled as soon as $\mathrm{Im}f_1(\nu)$ is more accurately analyzed in the higher energy region and $\mathrm{Re}f_1(\nu)$ is measured over a wider energy region. Of course, measurement of $\mathrm{Re}f_1(\nu)$ also tests the basic principles used to derive the dispersion relation, the CMSR and the CDSR.

[22] E. Ferrari and G. Violini, Ref. 16.

IV. CONCLUSION

A coherent study of the proton Compton scattering is made possible by combining a recently proposed set of truly invariant amplitudes and a dispersion calculation of the real part of the forward amplitude. A distinction is made between the oft-quoted CMSR and the more physical CDSR. Within the present experimental accuracy, all forward-dispersion sum rules revealed the presence of a $J=0$ fixed pole in the proton Compton scattering amplitude. It is hoped that measurements of the unpolarized total absorption cross section at higher energies, measurement of the real part in the physical region, as well as a partial-wave analysis of the (non-

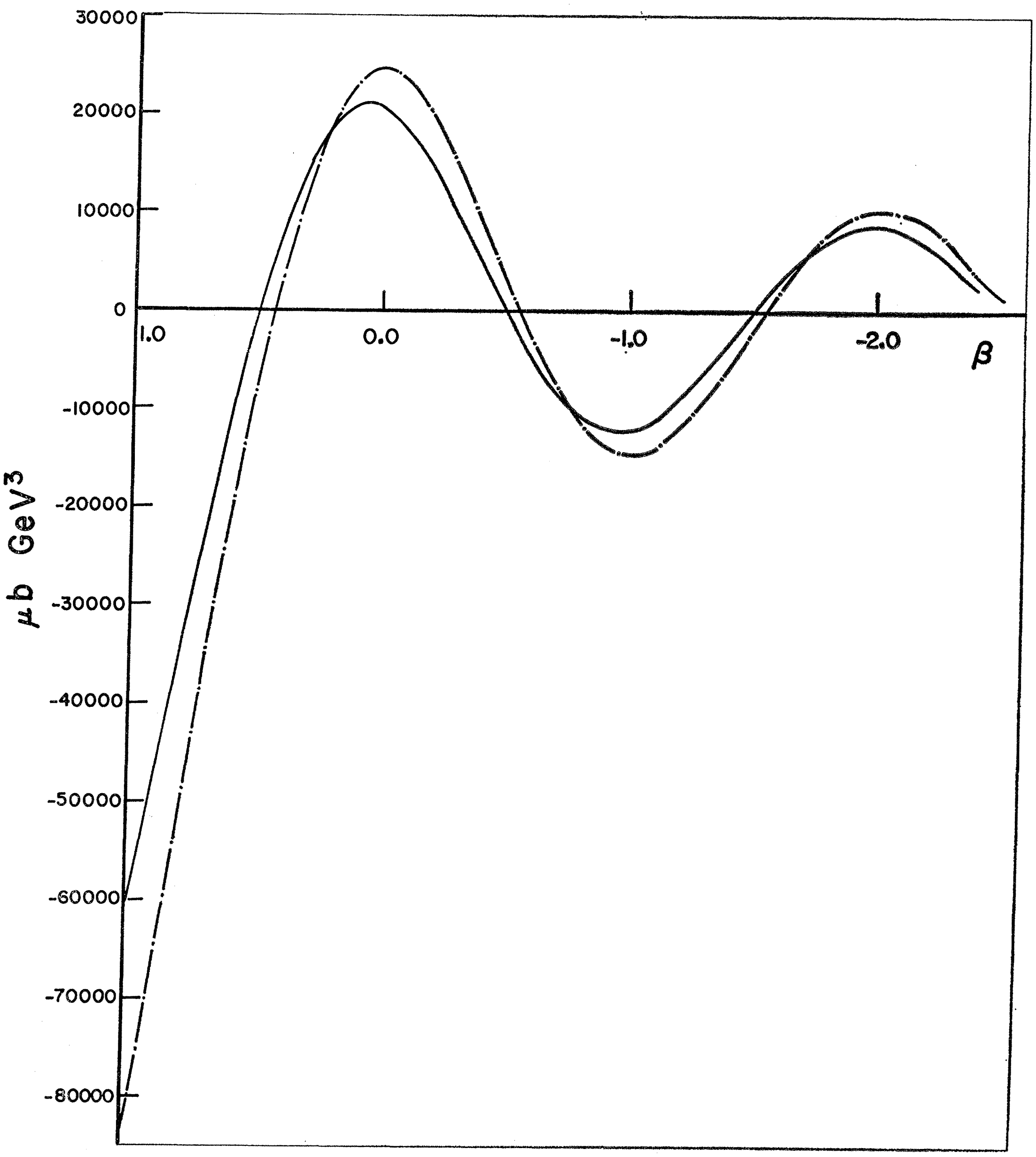

FIG. 4. Plot (broken curve) of the right-hand side of the CDSR (14) multiplied by a factor $N^{2\beta}$. The left-hand side coincides with this curve for all values of β shown in the diagram. The solid curve is the contribution from the dominating Pomeranchuk trajectory alone.

forward) amplitude, will all be possible in the near future to confirm or to disprove the conclusions reached here.

ACKNOWLEDGMENTS

We thank Wu-Ki Tung and S. D. Drell for useful communications.